YFS MC Approach to Precision Theory for Collider Physics: Origin, Development, and Outlook

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In collaboration with S. Jadach[1], W. Placzek, M. Skrzypek, Z.A. Was, S.A. Yost, and A. Siodmok

[1] Deceased.

In Memory of Prof. Stanislaw Jadach

- Sadly, Prof. Stanislaw Jadach passed away suddenly on Feb. 26, 2023
  CERN COURIER: May - June Issue, 2023:

Stanislaw Jadach 1947–2023

A leading light in radiative corrections

Stanislaw Jadach, an outstanding theoretical physicist, died on 26 February at the age of 75. His foundational contributions to the physics programmes at LEP and the LHC, and for the proposed Future Circular Collider at CERN, have significantly helped to advance the field of elementary particle physics and its future aspirations.

Born in Czerteż, Poland, Jadach graduated in 1970 with a master in physics from Jagiellonian University. There, he also defended his doctorate, received his habilitation degree and worked until 1992. During this period, whilst partly under martial law in Poland, Jadach took trips to Leiden, Paris, London, Stanford and Knoxville, and formed collaborations on precision theory calculations based on Monte Carlo event-generator methods. In 1992 he moved to the Institute of Nuclear Physics Polish Academy of Sciences (IFJ PAN) where, receiving the title of professor in 1994, he worked until his death.

Prior to LEP, all calculations of radiative corrections were based on first- and, later, partially second-order results. This limited the theoretical precision to the 1% level, which was unacceptable for experiments. In 1987 Jadach solved that problem in a single-author report, inspired by the classic work of Yennie, Frautschi and Suura, featuring a new calculational method for any number of photons. It was widely believed that soft-photon approximations were restricted to many photons with very low energies and that it was impossible to relate, consistently, the distributions of one or two energetic photons to those of any number of soft photons. Jadach and his colleagues solved this problem in their papers in 1989 for differential cross sections, and later in 1999 at the level of spin amplitudes. A long series of publications and computer programmes for Λ-summed perturbative Standard Model calculations ensued.

Most of the analysis of LEP data was based

Stanislaw Jadach made major contributions to the physics programmes at LEP and the LHC.

exclusively on the novel calculations provided by Jadach and his colleagues. The most important concerned the LEP luminosity measurement via Bhabha scattering, the production of lepton and quark pairs, and the production and decay of W and Z boson pairs. For the W-pair results at LEP2, Jadach and co-workers intelligently combined separate first-order calculations for the production and decay processes to achieve the necessary 0.5% theoretical accuracy, bypassing the need for full first-order calculations for the four-fermion process, which were unfeasible at the time. Contrary to what was deemed possible, Jadach and his colleagues achieved calculations that simultaneously take into account QED radiative corrections and the complete spin–spin correlation effects in the production and decay of two tau leptons. He also had success in the 1990s in novel simulations of strong interaction processes.

After LEP, Jadach turned to LHC physics. Among other novel results, he and his collaborators developed a new constrained Markovian algorithm for parton cascades, with no need to use backward evolution and predefined parton distributions, and proposed a new method, using a "physical" factorisation scheme, for combining a hard process at next-to leading order with a parton cascade, much simpler and more efficient than alternative methods.

Jadach was already updating his LEP-era calculations and software towards the increased precision of FCC-ee, and is the co-editor and co-author of a major paper delineating the need for new theoretical calculations to meet the proposed collider's physics needs. He co-organised and participated in many physics workshops at CERN and in the preparation of comprehensive reports, starting with the famous 1989 LEP Yellow Reports.

Jadach, a member of the Polish Academy of Arts and Sciences (PAAS), received the most prestigious awards in physics in Poland: the Marie Skłodowska-Curie Prize (PAS), the Marian Mięsowicz Prize (PAAS), and the prize of the Minister of Science and Higher Education for lifetime scientific achievements. He was also a co-initiator and permanent member of the international advisory board of the RADCOR conference.

Stanislaw (Staszek) was a wonderful man and mentor. Modest, gentle and sensitive, he did not judge or impose. He never refused requests and always had time for others. His professional knowledge was impressive. He knew almost everything about QED, and there were few other topics in which he was not at least knowledgeable. His erudition beyond physics was equally extensive. He is already profoundly and dearly missed.

Wiesław Płaczek Jagiellonian University, **Maciej Skrzypek** and **Zbigniew Wąs** Institute of Nuclear Physics and **Bennie Ward** Baylor University.

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OUTLINE

- Introduction
- Recapitulation of YFS Exact Amplitude-Based Resummation
- Approach to Precision Collider Physics: LHC, FCC, CPEC, CPPC, ILC, CLIC
- Improving the Collinear Limit in YFS Theory
- Outlook

Introduction

- How did we get started on our YFS journey?
  - 1986 MarkII Radiatvie Corrections Meeting Organized by G. Feldman at SLAC:
    - Preparation for 'Precision Z Physics' at SLC: Did not turn-on until 1989, MKII Observed $\sim$ 750 Z's
    - Staszek and I met in this Meeting.
- There was a No-Go Belief: Jackson-Scharre Naive Exponentiation-Based Methods – Nothing Better
- We started discussing whether the approach of Yennie, Frautschi, and Suura could do better –
  - It worked at the level of the amplitudes:
    - Could a MC realize all that?
    - Was renormalization group improvement alive?

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Introduction

- Discussion aided by my participation in the 27th Cracow School of Theoretical Physics
  - Long walks in the mountains
    - Staszek had already written MPI-PAE-PTH-87-6: "Yennie-Frautschi-Suura soft photons in Monte Carlo event generators"
    - We presented RG Improvement at the School.
- Proof of Principle:
  - "Exponentiation of Soft Photons in the Monte Carlo: The Case of Bonneau and Martin," University of Tennessee preprint UTHEP-88-0101, and SLAC-PUB-4543, Phys. Rev. D**38**, 2897 (1988)
  - "Multiphoton Monte Carlo for Bhabha Scattering at Low Angles," University of Tennessee preprint UTHEP-88-11-01, 1988, Phys. Rev. D40, 3582 (1989)"

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Introduction

- This was followed by "YFS2-The Monte Carlo for Fermion Pair Production at LEP/SLC with the Initial State Radiation of Two Hard and Multiple Soft Photons", CPC 56 (1990) 351
- $\Rightarrow$ KORALZ 3.8, BHLUMI 2.01
- "Final State Multiple Photon Effects in Fermion Pair Production at SLC/LEP," UTHEP-91-0903, Phys. Lett. B**274** (1992) 470.
- $\Rightarrow$ KORALZ 4.0, BHLUMI 4.04, ...

Introduction

- More applications followed:
  - BHWIDE, BHLUMI 2.30, YFSWW3, KORALW, KORALW&YFSWW3
- CEEX: Proof of Principle
  - "Coherent Exclusive Exponentiation CEEX: The Case of the Resonant e+e- Collision," CERN-TH-98-253, UTHEP-98-0801; Phys. Lett. B**449**, 97 (1999)
- $\Rightarrow$ KKMC, KKMC 4.22, KKMC-ee, KKMC-hh, KKMC-ee (C++), ...
- Applications: SLC, LEP1 and LEP2, BaBar, BELLE, BES, Φ-Factory, LHC
- Applications: TESLA, ILC, CLIC, FCC, SSC-RESTART, CEPC, CPPC, ...

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Introduction

- The Future of Precision Theory: Dictated by Future Accelerators – FCC, CLIC, ILC, CEPC, CPPC, $\cdots$
- Using FCC as an example, factors of improvement from $\sim$ 5 to $\sim$ 100 are needed from Theory
- Resummation is a key to such improvements in many cases:
  Today, we discuss amplitude-based resummation following the YFS MC methodology made possible by Staszek's seminal contributions.
- YFS $\rightarrow$ 'no limit to precision'
- See 1989 CERN Yellow Book article by Berends *et al.*

Introduction

- The Future of Precision Theory: Dictated by Future Accelerators – FCC, CLIC, ILC, CEPC, CPPC, $\cdots$

  Gianotti: 1/10/23 -- von der Leyen: 7/16/25, EU MFF Moonshots --FCC, clean aviation, ...

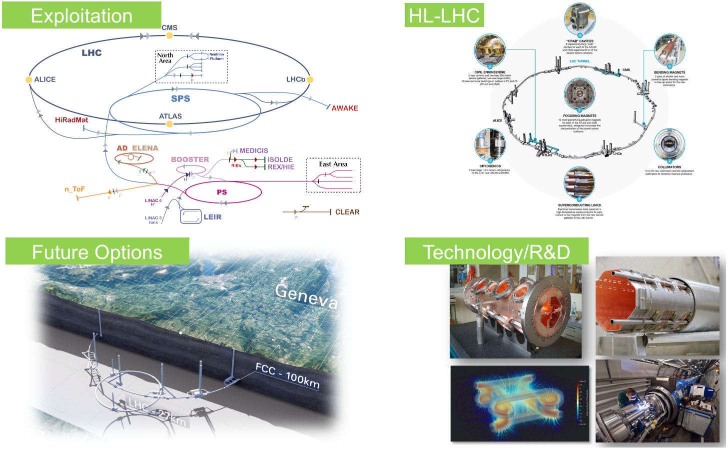


Figure: Future of CERN.

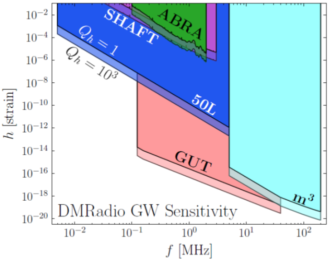
Introduction

- The Future of Precision Theory: Dictated by Future Accelerators – FCC, CLIC, ILC, CEPC, CPPC, $\cdots$
  Gianotti: 1/10/23

Theory

Some physics highlights:

- Higher-order calculations of background processes for LHC, HL-LHC and future colliders
- Axion physics and, in particular, studies for using axion haloscopes to detect high-frequency gravitational waves through oscillating electromagnetic signals sourced by spacetime distortions (arXiv: 2202.00695)
- String Theory: Exploring the swampland and how its conjectures can reveal information on the energy scales of nature (arXiv: 2205.12293)
- Bounds on the energy growth of gravitational amplitudes (arXiv: 2202.08280)



Other activities:

- Full restart of scientific activities and visitor programmes after Covid.
- TH served as a focal point for the physics community to discuss eco-friendly practices for organising scientific events and business travel. These issues were discussed in a dedicated Theory Institute, named “Sustainable HEP”

45

Figure: Future of CERN.

Introduction

- The Future of Precision Theory: Dictated by Future Accelerators – FCC, CLIC, ILC, CEPC, CPPC, $\cdots$
Grojean: 2/13/24

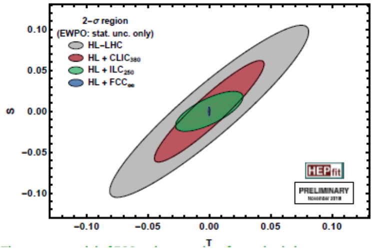
Tera-Z EW precision measurements.

▸ The target is to reduce syst. uncertainties to the level of stat. uncertainties.
(exploit the large samples and innovative control analyses)

▸ Exquisite $\sqrt{s}$ precision (100keV@Z, 300keV@WW) reduces beam uncertainties (EPOL)

➡ ~50 times better precision than LEP/LSD on EW precision observables

w/ stat. and param. only



Indirect sensitivity to 70TeV-scale sector connected to EW/Higgs

CG - 20 / 30 (For the impact of the theory uncertainties on the EW fit, see bonus slides) Feb. 13, 2024

Figure: Future of CERN.

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Introduction

- YFS methods are exact in the infrared but treat the collinear logs perturbatively in the $\bar{\beta}_n$ residuals
- DGLAP-based collinear factorization treats the collinear logs to all orders but has a non-exact IR limit
- In this talk, we present some new results for precision collider physics based on the usual YFS methods.
- We then investigate improving the collinear limit of YFS theory.
- A Key Point: Exact Amplitude-Based Resummation Realized on Evt-by-Evt Basis – Enhanced Precision for a Given Level of Exactness: LO, NLO, NNLO, NNNLO, ..., essential for future precision physics as exemplified by CERN <=> Computer Algebraic Methods essential!

Recapitulation of Exact Amplitude-Based Resummation Theory

$$d\bar{\sigma}_{\rm res} = e^{\rm SUM_{IR}(QCED)} \sum_{n,m=0}^{\infty} \frac{1}{n!m!} \int \prod_{j_1=1}^{n} \frac{d^3 k_{j_1}}{k_{j_1}}
\prod_{j_2=1}^{m} \frac{d^3 {k'}_{j_2}}{{k'}_{j_2}} \int \frac{d^4 y}{(2\pi)^4} e^{iy\cdot(p_1+q_1-p_2-q_2-\sum k_{j_1} - \sum {k'}_{j_2}) + D_{\rm QCED}}
\tilde{\bar{\beta}}_{n,m}(k_1, \ldots, k_n; k'_1, \ldots, k'_m) \frac{d^3 p_2}{p_2^0} \frac{d^3 q_2}{q_2^0}, \qquad (1)$$

where *new* (YFS-style) *non-Abelian* residuals $\tilde{\bar{\beta}}_{n,m}(k_1, \ldots, k_n; k'_1, \ldots, k'_m)$ have n hard gluons and m hard photons.

$\tilde{\bar{\beta}}_{n,m}: FCCee - need\ exact\ O\left(\frac{\alpha}{\pi}, \frac{\alpha}{\pi}L, \left(\frac{\alpha}{\pi}\right)^2, \left(\frac{\alpha}{\pi}\right)^2 L, \left(\frac{\alpha}{\pi}\right)^2 L^2, \left(\frac{\alpha}{\pi}\right)^3 L^2, \left(\frac{\alpha}{\pi}\right)^3 L^3, \left(\frac{\alpha}{\pi}\right)^4 L^4\right)$

$\Rightarrow$ *Computer Algebraic Methods* $\Leftrightarrow$ *Evaluation of Feynman Diagrams*

Here,

$$\begin{aligned} \mathrm{SUM_{IR}(QCED)} &= 2\alpha_s \Re B^{nls}_{QCED} + 2\alpha_s \tilde{B}^{nls}_{QCED} \\ D_{\mathrm{QCED}} &= \int \frac{d^3k}{k^0}\left(e^{-iky} - \theta(K_{max} - k^0)\right)\tilde{S}^{nls}_{QCED} \end{aligned} \tag{2}$$

where K_{max} is "dummy" and

$$\begin{aligned} B^{nls}_{QCED} &\equiv B^{nls}_{QCD} + \frac{\alpha}{\alpha_s} B^{nls}_{QED}, \\ \tilde{B}^{nls}_{QCED} &\equiv \tilde{B}^{nls}_{QCD} + \frac{\alpha}{\alpha_s} \tilde{B}^{nls}_{QED}, \\ \tilde{S}^{nls}_{QCED} &\equiv \tilde{S}^{nls}_{QCD} + \tilde{S}^{nls}_{QED}. \end{aligned} \tag{3}$$

"nls"$\equiv$ DGLAP-CS synthesization.

Shower/ME Matching: $\tilde{\bar{\beta}}_{n,m} \rightarrow \hat{\tilde{\bar{\beta}}}_{n,m}$
See Ann. of Phys. **323** (2008) 2147 and references therein for more details.

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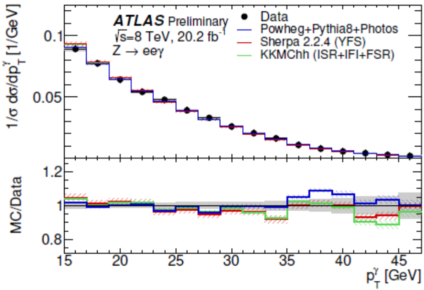
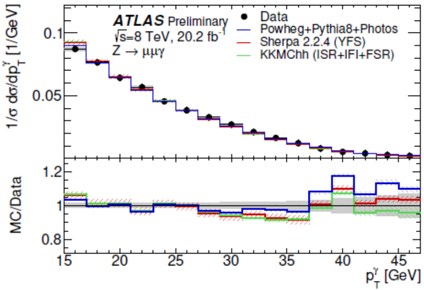
Approach to Precision Collider Physics: LHC, FCC, CPEC, CPPC, ILC, CLIC

- (HL-)LHC:
  $\mathcal{KK}$MChh: Exact $\mathcal{O}(\alpha^2 L)$ CEEX EW corrections matched to a Herwig parton shower (built-in) or to any other shower via Les Houches files

  (see also Liu *et al.*, to appear).$\Rightarrow$
  Recent ATLAS results on $Z\gamma$ production (Eur.Phys.J C **84** (2024) 195)





HL-LHC => Factor of ~ 10 smaller statistical errors => Test?

Approach to Precision Collider Physics: LHC, FCC, CPEC, CPPC, ILC, CLIC

- (HL-)LHC:
  $\mathcal{KK}$MChh: NISR shows effect of QED contamination in non-QED PDFs is below the errors on the PDFs:
- NISR –

$$\sigma(s) = \frac{3}{4}\pi\sigma_0(s) \sum_{q=u,d,s,c,b} \int d\hat{x}\, dzdr \int dx_q dx_{\bar{q}}\, \delta(\hat{x} - x_q x_{\bar{q}} z)$$
$$\times f_q^{h_1}(s\hat{x}, x_q) f_{\bar{q}}^{h_2}(s\hat{x}, x_{\bar{q}})\, \rho_I^{(0)}\left(\gamma_{Iq}(s\hat{x}/m_q^2), z\right)\, \rho_I^{(2)}\left(-\gamma_{Iq}(Q_0^2/m_q^2), r\right)$$
$$\times \sigma_{q\bar{q}}^{Born}(s\hat{x}z)\, \langle W_{MC} \rangle, \tag{4}$$

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- (HL-)LHC:
  $\mathcal{KK}$MChh: NISR shows effect of QED contamination in non-QED PDFs is below the errors on the PDFs:

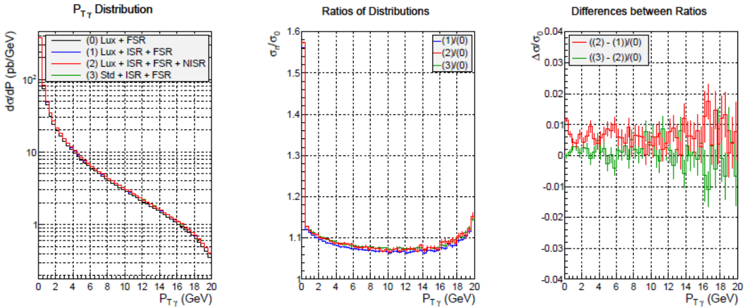

Figure 3(arXiv:2211.17177): The distribution for $P_{T\gamma}$ of the photon for which it is greatest for events with at least one photon and each lepton having $p_{Tl} > 25$ GeV, $\eta_l < 2.5$ calculated with (0) FSR only (black), (1) FSR + ISR (blue), and (2) FSR + ISR with NISR (red) for NNPDF3.1-LuxQED NLO PDFs. For comparison, (3) shows FSR + ISR with ordinary NNPDF3.1 NLO PDFs (green). The center graph shows ISR on/off ratios (1)/(0) (blue),(2)/(0) (red) and (3)/(0) (green). The right-hand graph shows the fractional differences $((1)-(2))/(0)$ in red and $((2)-(3))/(0)$ in green.

Approach to Precision Collider Physics: LHC, FCC, CPEC, CPPC, ILC, CLIC

- FCC-ee:
  BHLUMI and the Luminosity Theory Error – Current Purview
  (M. Skrzypek *et al.*, 2023 FCC Workshop, Krakow & MTTD2023; W. Placzek, here)

Bhabha cross sect. depends on detector acceptance angles

$$\sigma_{Bh} \simeq 4\pi\alpha^2\left(\frac{1}{t_{\rm min}} - \frac{1}{t_{\rm max}}\right) = 4\pi\alpha^2\left(\frac{t_{\rm max} - t_{\rm min}}{\bar{t}^2}\right), \quad \bar{t} = \sqrt{t_{\rm min} t_{\rm max}}$$

$\bar{t}$ is the characteristic scale of the process
$\bar{t}/s$ is the suppression factor between s- and t-channel contributions

Machine	$\theta_{\rm min} \div \theta_{\rm max}$ [mrad]	$\sqrt{s}$ [GeV]	$\bar{t}/s$	$\sqrt{\bar{t}}$ [GeV]
LEP	28÷50	M_Z	3.5×10^{-4}	1.70
FCCee	64÷86	M_Z	13.7×10^{-4}	3.37
FCCee	64÷86	240	13.7×10^{-4}	8.9
FCCee	64÷86	350	13.7×10^{-4}	13.0
ILC	31÷77	500	6.0×10^{-4}	12.2
ILC	31÷77	1000	6.0×10^{-4}	24.4
CLIC	39÷134	3000	13.0×10^{-4}	108

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- FCC-ee:
  BHLUMI and the Luminosity Theory Error – Current Purview

Lumi at FCCee$_{M_Z}$ – Forecast study

Forecast study for FCCee$_{M_Z}$			
Type of correction / Error	Published [1]	Strict	Redone
(a) Photonic $\mathcal{O}(L_e^2\alpha^3)$	0.10×10^{-4}	0.10×10^{-4}	0.10×10^{-4}
(b) Photonic $\mathcal{O}(L_e^4\alpha^4)$	0.06×10^{-4}	0.06×10^{-4}	0.06×10^{-4}
(b') Photonic $\mathcal{O}(\alpha^2 L^0)$		0.17×10^{-4}	0.17×10^{-4}
(c) Vacuum polariz.	0.6×10^{-4}	0.6×10^{-4}	0.6×10^{-4}
(d) Light pairs	0.5×10^{-4}	0.4×10^{-4}	0.27×10^{-4}
(e) Z and s-channel γ exch.	0.1×10^{-4}	0.1×10^{-4}	0.1×10^{-4}
(f) Up-down interference	0.1×10^{-4}	0.08×10^{-4}	0.08×10^{-4}
Total	1.0×10^{-4}	0.76×10^{-4}	0.70×10^{-4}

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- FCC-ee:
  BHLUMI and the Luminosity Theory Error – Current Purview

Lumi forecast at ILC and CLIC GeV

Forecast			
Type of correction / Error	ILC_{500}	ILC_{1000}	CLIC_{3000}
(a) Photonic $\mathcal{O}(L_e^2\alpha^3)$	0.13×10^{-4}	0.15×10^{-4}	0.20×10^{-4}
(b) Photonic $\mathcal{O}(L_e^4\alpha^4)$	0.27×10^{-4}	0.37×10^{-4}	0.63×10^{-4}
(c) Vacuum polariz.	1.1×10^{-4}	1.1×10^{-4}	1.2×10^{-4}
(d) Light pairs	0.4×10^{-4}	0.5×10^{-4}	0.7×10^{-4}
(e) Z and s-channel γ exch.	$1.0 \times 10^{-4(*)}$	2.4×10^{-4}	16×10^{-4}
(f) Up-down interference	$< 0.1 \times 10^{-4}$	$< 0.1 \times 10^{-4}$	0.1×10^{-4}
Total	1.6×10^{-4}	2.7×10^{-4}	16×10^{-4}

Note: Lattice methods with Jegerlehner's results allow, in principle, (c) -> (c)/6

$$\Delta\alpha_{had}(t) = \Delta\alpha_{had}(-Q_0^2)|_{lat} + [\Delta\alpha_{had}(t) - \Delta\alpha_{had}(-Q_0^2)]|_{pQCDAdler}$$

Approach to Quantum Gravity

- Cosmological Constant Result Still Obtains:
  (Phys. Dark Univ. **2** (2013) 97)

$$\rho_\Lambda(t_0) \cong \frac{-M_{Pl}^4(1+c_{2,eff}k_{tr}^2/(360\pi M_{Pl}^2))^2}{64}\sum_j \frac{(-1)^F n_j}{\rho_j^2} \times \frac{t_{tr}^2}{t_{eq}^2} \times \left(\frac{t_{eq}^{2/3}}{t_0^{2/3}}\right)^3$$

$$\cong \frac{-M_{Pl}^2(1.0362)^2(-9.194\times 10^{-3})}{64}\frac{(25)^2}{t_0^2} \cong (2.4\times 10^{-3}eV)^4.$$

$t_0 \cong 13.7\times 10^9$ yrs

$c_{2,eff} \cong 2.56\times 10^4$, cosmological index of the ST $\Rightarrow$

Constraints: BHs, etc., in progress.

Improving the Collinear Limit in YFS Theory

- Basic Formula for CEEX/EEX realization of the YFS resummation of

  $e^+e^- \to f\bar{f} + n\gamma,\ f = \ell, q,\ \ell = e, \mu, \tau, \nu_e, \nu_\mu, \nu_\tau,\ q = u, d, s, c, b, t$:

$$\sigma = \frac{1}{\text{flux}} \sum_{n=0}^{\infty} \int d\text{LIPS}_{n+2}\, \rho_A^{(n)}(\{p\},\{k\}), \tag{5}$$

- $$\rho_{\text{CEEX}}^{(n)}(\{p\},\{k\}) = \frac{1}{n!} e^{Y(\Omega;\{p\})} \bar{\Theta}(\Omega) \frac{1}{4} \sum_{\text{helicities } \{\lambda\},\{\mu\}} \left| \mathcal{M}\begin{pmatrix} \{p\}\,\{k\} \\ \{\lambda\}\,\{\mu\} \end{pmatrix} \right|^2. \tag{6}$$

  By definition, $\Theta(\Omega, k) = 1$ for $k \in \Omega$ and $\Theta(\Omega, k) = 0$ for $k \notin \Omega$, with

  $\bar{\Theta}(\Omega; k) = 1 - \Theta(\Omega, k)$ and

$$\bar{\Theta}(\Omega) = \prod_{i=1}^{n} \bar{\Theta}(\Omega, k_i).$$

- For Ω defined with the condition $k^0 < E_{\rm min}$, the YFS infrared exponent reads

$$
\begin{aligned}
Y(\Omega; p_a, ..., p_d) = {} & Q_e^2 Y_\Omega(p_a, p_b) + Q_f^2 Y_\Omega(p_c, p_d) \\
& + Q_e Q_f Y_\Omega(p_a, p_c) + Q_e Q_f Y_\Omega(p_b, p_d) \\
& - Q_e Q_f Y_\Omega(p_a, p_d) - Q_e Q_f Y_\Omega(p_b, p_c).
\end{aligned}
\tag{7}
$$

Improving the Collinear Limit in YFS Theory

- Here

$$
\begin{aligned}
Y_\Omega(p,q) \equiv & 2\alpha\tilde{B}(\Omega,p,q) + 2\alpha\Re B(p,q) \\
\equiv & -2\alpha\,\frac{1}{8\pi^2}\int\frac{d^3k}{k^0}\Theta(\Omega;k)\left(\frac{p}{kp}-\frac{q}{kq}\right)^2 \\
& + 2\alpha\Re\int\frac{d^4k}{k^2}\frac{i}{(2\pi)^3}\left(\frac{2p-k}{2kp-k^2}-\frac{2q+k}{2kq+k^2}\right)^2 .
\end{aligned} \tag{8}
$$

- Fundamental Idea of YFS: isolate and resum to all orders in α the infrared singularities so that these singularities are canceled to all such orders between real and virtual corrections.
  What collinear singularities are also resummed in the YFS resummation algebra?

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Improving the Collinear Limit in YFS Theory

- Focusing on the s-channel and s'-channel contributions, we have

$$
\begin{aligned}
Y_e(\Omega_I; p_1, p_2) &= \gamma_e \ln\frac{2E_{min}}{\sqrt{2p_1p_2}} + \frac{1}{4}\gamma_e + Q_e^2\frac{\alpha}{\pi}\left(-\frac{1}{2}+\frac{\pi^2}{3}\right),\\
Y_f(\Omega_F; q_1, q_2) &= \gamma_f \ln\frac{2E_{min}}{\sqrt{2q_1q_2}} + \frac{1}{4}\gamma_f + Q_f^2\frac{\alpha}{\pi}\left(-\frac{1}{2}+\frac{\pi^2}{3}\right),
\end{aligned}
\tag{9}
$$

  where

$$
\gamma_e = 2Q_e^2\frac{\alpha}{\pi}\left(\ln\frac{2p_1p_2}{m_e^2}-1\right), \quad \gamma_f = 2Q_f^2\frac{\alpha}{\pi}\left(\ln\frac{2q_1q_2}{m_f^2}-1\right), \tag{10}
$$

  $\Rightarrow$ The YFS exponent resums the collinear big log term $\frac{1}{2}Q^2\frac{\alpha}{\pi}L$ to the infinite order in both the ISR and FSR contributions.

- Can this be improved to the result of Gribov and Lipatov to exponentiate $\frac{3}{2}\frac{\alpha}{\pi}L$ via the QED form-factor?

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Improving the Collinear Limit in YFS Theory

- The YFS form factor derivation illustrated in Fig. 4

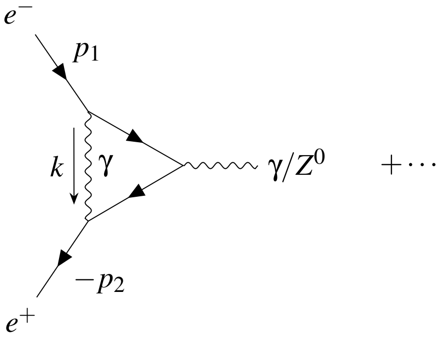

Figure: Virtual corrections which generate the YFS infrared function *B*. Self-energy contributions are not shown.

Improving the Collinear Limit in YFS Theory

- $\Rightarrow$ the amplitude factor

$$\mathcal{M}_\mu = \frac{\int d^4k}{(2\pi)^4}\frac{-i}{k^2+i\varepsilon}\bar{v}(p_2)(-iQ_e e)\gamma^\alpha \frac{i}{-\not{p}_2-\not{k}-m+i\varepsilon}(-ie)\gamma_\mu(v_A-a_A\gamma_5) \\ \frac{i}{\not{p}_1-\not{k}-m+i\varepsilon}(-iQ_e e)\gamma_\alpha u(p_1) \tag{11}$$

where $A=\gamma$ or Z.

Improving the Collinear Limit in YFS Theory

- Scalarising the fermion propagator denominators $\Rightarrow$

$$\mathcal{M}_\mu = -ie\frac{\int d^4k(-iQ_e{}^2e^2)}{(2\pi)^4}\frac{1}{k^2+i\varepsilon}\bar{v}(p_2)\gamma^\alpha\frac{-\not{p}_2-\not{k}+m}{k^2+2kp_2+i\varepsilon}\gamma_\mu(v_A-a_A\gamma_5)$$
$$\frac{\not{p}_1-\not{k}-m}{k^2-2kp_1+i\varepsilon}\gamma_\alpha u(p_1). \tag{12}$$

- Using the equations of motion

$$(\not{p}_1-\not{k}-m)\gamma_\alpha u(p_1) = \{(2p_1-k)_\alpha - \frac{1}{2}[\not{k},\gamma_\alpha]\}u(p_1), \qquad (a)$$
$$\bar{v}(p_2)\gamma^\alpha(-\not{p}_2-\not{k}+m) = \bar{v}(p_2)\{-(2p_2+k)^\alpha + \frac{1}{2}[\not{k},\gamma^\alpha]\}, \qquad (b). \tag{13}$$

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- $\Rightarrow$ Contribution to $2Q_e^2\alpha B(p_1,p_2)$ corresponding to the cross-term in the virtual IR function on the RHS of eq.(8):

$$2Q_e^2\alpha B(p_1,p_2)|_{\text{cross-term}} = \int d^4k \frac{(iQ_e{}^2e^2)}{8\pi^4}\frac{1}{k^2+i\varepsilon} \frac{(2p_1-k)(2p_2+k)}{(k^2-2kp_1+i\varepsilon)(k^2+2kp_2+i\varepsilon)}. \quad (14)$$

This term, together with the two squared terms in $2\alpha Q_e^2 B(p_1,p_2)$, leads to the exponentiation of $\frac{1}{2}Q_e^2\frac{\alpha}{\pi}L$.

Improving the Collinear Limit in YFS Theory

- The two commutator terms on the RHS of eq.(13), usually dropped, can be analyzed further: possible IR finite collinearly enhanced improvement of the YFS virtual IR function B.
- Isolate the collinear parts of k via the change of variables

$$k = c_1 p_1 + c_2 p_2 + k_\perp \tag{15}$$

where $p_1 k_\perp = 0 = p_2 k_\perp$, $\Rightarrow$ we have the relations

$$\begin{aligned} c_1 &= \frac{p_1 p_2}{(p_1 p_2)^2 - m^4} p_2 k - \frac{m^2}{(p_1 p_2)^2 - m^4} p_1 k \xrightarrow[CL]{} \frac{p_2 k}{p_1 p_2} \\ c_2 &= \frac{p_1 p_2}{(p_1 p_2)^2 - m^4} p_1 k - \frac{m^2}{(p_1 p_2)^2 - m^4} p_2 k \xrightarrow[CL]{} \frac{p_1 k}{p_1 p_2}, \end{aligned} \tag{16}$$

CL denotes the collinear limit $\equiv \mathcal{O}(m^2/s)$ dropped.

Improving the Collinear Limit in YFS Theory

- $\Rightarrow$ $(2p_1-k)^\alpha$ in eq.(13(a)) combines with the commutator term in eq.(13(b)) to produce

$$\begin{aligned}&\bar{v}(p_2)\{(2p_1-k)_\alpha\tfrac{1}{2}[\not k,\gamma^\alpha]\}\gamma_\mu(v_A-a_A\gamma_5)u(p_1)\\ &=\bar{v}(p_2)[\not k,\not p_1]\gamma_\mu(v_A-a_A\gamma_5)u(p_1)\\ &\underset{CL}{\longrightarrow}\bar{v}(p_2)[c_2\not p_2,\not p_1]\gamma_\mu(v_A-a_A\gamma_5)u(p_1)\\ &\underset{CL}{\longrightarrow}\bar{v}(p_2)(-2c_2p_1p_2)\gamma_\mu(v_A-a_A\gamma_5)u(p_1)\\ &\underset{CL}{\longrightarrow}\bar{v}(p_2)(-2p_1k)\gamma_\mu(v_A-a_A\gamma_5)u(p_1).\end{aligned} \tag{17}$$

- Similarly, $-(2p_2+k)^\alpha$ in eq.(13 (b)) combines with the commutator term in eq.(13(a)) to produce

$$\begin{aligned}&\bar{v}(p_2)\gamma_\mu(v_A-a_A\gamma_5)\{-(2p_2+k)^\alpha(-\tfrac{1}{2}[\not k,\gamma_\alpha])\}u(p_1)\\ &=\bar{v}(p_2)\gamma_\mu(v_A-a_A\gamma_5)[\not k,\not p_2]u(p_1)\\ &\underset{CL}{\longrightarrow}\bar{v}(p_2)\gamma_\mu(v_A-a_A\gamma_5)[c_1\not p_1,\not p_2]u(p_1)\\ &\underset{CL}{\longrightarrow}\bar{v}(p_2)\gamma_\mu(v_A-a_A\gamma_5)(2c_1p_1p_2)u(p_1)\\ &\underset{CL}{\longrightarrow}\bar{v}(p_2)\gamma_\mu(v_A-a_A\gamma_5)(2p_2k)u(p_1).\end{aligned} \tag{18}$$

- $\Rightarrow$ Shift of the factor $(2p_1 - k)(2p_2 + k)$ on the RHS of eq.(14) as

$$(2p_1 - k)(2p_2 + k) \underset{CL}{\longrightarrow} (2p_1 - k)(2p_2 + k) + 2p_1 k - 2p_2 k. \quad (19)$$

Improving the Collinear Limit in YFS Theory

- What does the term quadratic in the commutator (C^2) contribute?
- Superficial UV divergence $\Rightarrow$ Cannot naively drop $k_\perp$
- Proceed directly: we need

$$2Q_e^2\alpha B(p_1,p_2)\big|_{C^2}\mathcal{M}_{B\mu} \equiv \frac{\int d^4k(iQ_e{}^2e^2)}{8\pi^4}\frac{1}{k^2+i\varepsilon}\left.\frac{\frac{1}{4}\bar{v}(p_2)[\not k,\gamma^\alpha]\gamma_\mu[\not k,\gamma_\alpha](-ie)(v_A-a_A\gamma_5)u(p_1)}{(k^2-2kp_1+i\varepsilon)(k^2+2kp_2+i\varepsilon)}\right|_{CL}, \tag{20}$$

where we define

$$\mathcal{M}_{B\mu} = -ie\bar{v}(p_2)\gamma_\mu(v_A-a_A\gamma_5)u(p_1). \tag{21}$$

- *CL* now further restricted to contributions singular as $m^2/s \to 0$.

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Improving the Collinear Limit in YFS Theory

- Four terms in the numerator of eq.(20) from the respective sum of gamma matrix products $\{\not{k}\gamma^{\alpha}\gamma_{\mu}\not{k}\gamma_{\alpha} - \not{k}\gamma^{\alpha}\gamma_{\mu}\gamma_{\alpha}\not{k} - \gamma^{\alpha}\not{k}\gamma_{\mu}\not{k}\gamma_{\alpha} + \gamma^{\alpha}\not{k}\gamma_{\mu}\gamma_{\alpha}\not{k}\} =$ $\{\gamma^{\lambda}\gamma^{\alpha}\gamma_{\mu}\gamma^{\lambda'}\gamma_{\alpha} - \gamma^{\lambda}\gamma^{\alpha}\gamma_{\mu}\gamma_{\alpha}\gamma^{\lambda'} - \gamma^{\alpha}\gamma^{\lambda}\gamma_{\mu}\gamma^{\lambda'}\gamma_{\alpha} + \gamma^{\alpha}\gamma^{\lambda}\gamma_{\mu}\gamma_{\alpha}\gamma^{\lambda'}\}k_{\lambda}k_{\lambda'} \equiv$ $N_{\mu}^{\lambda\lambda'}k_{\lambda}k_{\lambda'}$
- This defines $N_{\mu}^{\lambda\lambda'}$.

Improving the Collinear Limit in YFS Theory

- Using standard methods, we need

$$I_\mu = 2\int_0^1 d\alpha_1 \int_0^{1-\alpha_1} d\alpha_2 \frac{\int d^n k' (i{Q_e}^2 e^2)}{8\pi^4} \left. \frac{\frac{1}{4}\bar{v}(p_2) N_\mu^{\lambda\lambda'} [\frac{k'^2}{n} g_{\lambda\lambda'} + \Delta_\lambda \Delta_{\lambda'}](-ie)(v_A - a_A\gamma_5)u(p_1)}{[k'^2 - \Delta^2 + i\varepsilon]^3} \right|_{CL}, \tag{22}$$

  where $\Delta = \alpha_1 p_1 - \alpha_2 p_2$.
- Equations of motion $\Rightarrow$ term involving Δ is not collinearly enhanced.

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Improving the Collinear Limit in YFS Theory

- The term contracted with $g_{\lambda\lambda'}$ gives us

$$I_\mu = \left\{ \frac{-3Q_e^2\alpha}{4\pi} \mathcal{M}_{B\mu} \right\}\Bigg|_{CL} \equiv 0 \qquad (23)$$

- $\Rightarrow$ No collinearly enhanced contribution from I_μ.
- Eq.(19) gives the complete collinear enhancement of B.
- Change in B does not affect its IR behavior – shift terms are IR finite $\Rightarrow$ Entire YFS IR resummation is unaffected.
- Shifted terms can be seen to extend the YFS IR exponentiation to obtain the entire exponentiated $\frac{3}{2}Q_e^2\alpha L$.

Improving the Collinear Limit in YFS Theory

- We have

$$2\alpha Q_e^2 \Delta B(p_1,p_2) = \frac{\int d^4k(iQ_e^2e^2)}{8\pi^4}\frac{1}{k^2+i\varepsilon}\frac{2p_1k-2p_2k}{(k^2-2kp_1+i\varepsilon)(k^2+2kp_2+i\varepsilon)}$$
$$= 2\int_{x_i\geq 0, i=1,2,3} d^3x\delta(1-x_1-x_2-x_3)\frac{\int d^4k'(iQ_e^2e^2)}{8\pi^4} \qquad (24)$$
$$\frac{2(p_1-p_2)p_x}{(k'^2-d+i\varepsilon)^3}$$

where $d = p_x^2$ with $p_x = x_1p_1 - x_2p_2$.

- $\Rightarrow$ We get

$$2Q_e^2\alpha\Re\Delta B(p_1,p_2) = Q_e^2\frac{\alpha}{\pi}L. \qquad (25)$$

- We see that indeed the entire term $\frac{3}{2}Q_e^2\frac{\alpha}{\pi}L$ is now exponentiated by our collinearly improved YFS virtual IR function B_{CL}

$$B_{CL} = B + \Delta B$$
$$= \int \frac{d^4k}{k^2}\frac{i}{(2\pi)^3}\left[\left(\frac{2p-k}{2kp-k^2}-\frac{2q+k}{2kq+k^2}\right)^2 - \frac{4pk-4qk}{(2pk-k^2)(2qk+k^2)}\right]. \qquad (26)$$

See S. Jadach, Durham talk, 2002, for integrated form of B_{CL}.

Improving the Collinear Limit in YFS Theory

- What about the real YFS IR algebra? Collinear enhancement desired in some applications
- $\Rightarrow$ Recall the original YFS EEX formulation of the respective algebra $\Rightarrow$ the formula for the YFS IR function $\tilde{B}$ given above in eq.(8).
- See Fig. 5.

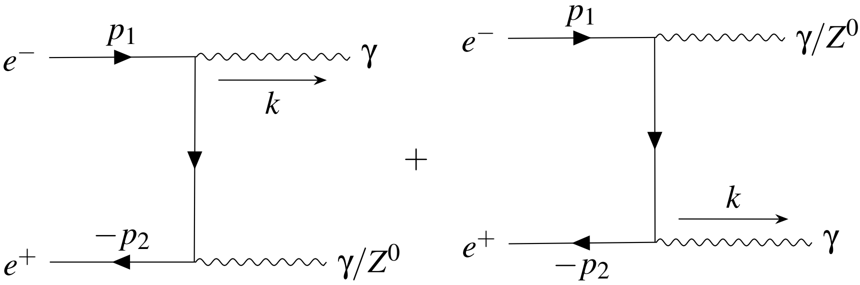

Figure: Real corrections which generate the YFS infrared function $\tilde{B}$.

Figure 2: Real corrections which generate the YFS infrared functi

Improving the Collinear Limit in YFS Theory

- Following the steps in the usual YFS algebra for real emission $\Rightarrow$

$$
\begin{aligned}
2\alpha Q_e^2 \tilde{B} \mathcal{M}_{B\mu}^{\dagger} \mathcal{M}_{B\mu'} = \frac{\int d^3k(-1)e^2 Q_e^2}{2k_0(2\pi)^3} & \left[\frac{\bar{u}(p_1)(2p_1^{\lambda} - k^{\lambda} + \frac{1}{2}[\not{k}, \gamma^{\lambda}])\gamma_{\mu}(v_A - a_A\gamma_5)v(p_2)}{k^2 - 2kp_1} \right. \\
& \left. + \frac{\bar{u}(p_1)\gamma_{\mu}(v_A - a_A\gamma_5)(-2p_2^{\lambda} + k^{\lambda} + \frac{1}{2}[\not{k}, \gamma^{\lambda}])v(p_2)}{k^2 - 2kp_2} \right] \\
& \left[\frac{\bar{v}(p_2)\gamma_{\mu'}(v_A - a_A\gamma_5)(2p_{1\lambda} - k_{\lambda} - \frac{1}{2}[\not{k}, \gamma_{\lambda}])u(p_1)}{k^2 - 2kp_1} \right. \\
& \left. \left. + \frac{\bar{v}(p_2)(-2p_{2\lambda} + k_{\lambda} - \frac{1}{2}[\not{k}, \gamma_{\lambda}])\gamma_{\mu'}(v_A - a_A\gamma_5)u(p_1)}{k^2 - 2kp_2} \right] \right|_{k^2=0} + K_{\mu\mu'}
\end{aligned}
\tag{27}
$$

where $K_{\mu\mu'}$ is infrared finite,

- $$\mathcal{M}_{B\mu} = \bar{v}(p_2)\gamma_{\mu}(v_A - a_A\gamma_5)u(p_1) \tag{28}$$

Improving the Collinear Limit in YFS Theory

- If we drop the commutator terms on the RHS of eq.(27) we recover the usual YFS formula for $2\alpha Q_e^2 \tilde{B}$.
- We again isolate collinearly enhanced contributions by using the representation in eq.(yfsalg4) for k, respecting the condition $k^2 = 0$. $\Rightarrow$ Maintain $0 = (c_1^2 + c_2^2)m^2 + 2c_1 c_2 p_1 p_2 - |k_\perp|^2$.
- $\Rightarrow$ Collinear enhancement of $\tilde{B}$:

$$2\alpha Q_e^2 \tilde{B}_{CL} = \frac{-\alpha Q_e^2}{4\pi^2} \int \frac{d^3k}{k_0} \left\{ \left(\frac{p_1}{kp_1} - \frac{p_2}{kp_2}\right)^2 + \frac{1}{kp_1}\left(2 - \frac{kp_2}{p_1 p_2}\right) + \frac{1}{kp_2}\left(2 - \frac{kp_1}{p_1 p_2}\right) \right\}. \tag{29}$$

- Agreement with Berends *et al.*

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Improving the Collinear Limit in YFS Theory

- What about CEEX?
- In Fig. 5, use of amplitude-level isolation of real IR divergences, K-S photon polarization vectors $\Rightarrow$

$$\mathcal{M}_\mu = \mathcal{M}_{B\mu}\mathfrak{s}_{CL,\sigma}(k), \tag{30}$$

with

$$\mathfrak{s}_{CL,\sigma}(k) = \sqrt{2}Q_e e\Bigg[-\sqrt{\frac{p_1\zeta}{k\zeta}}\frac{< k\sigma|\hat{p}_1 - \sigma >}{2p_1k} + \delta_{\lambda\,-\sigma}\sqrt{\frac{k\zeta}{p_1\zeta}}\frac{< k\sigma|\hat{p}_1\lambda >}{2p_1k} + \sqrt{\frac{p_2\zeta}{k\zeta}}\frac{< k\sigma|\hat{p}_2 - \sigma >}{2p_2k} + \delta_{\lambda\sigma}\sqrt{\frac{k\zeta}{p_2\zeta}}\frac{< \hat{p}_2\lambda|k - \sigma >}{2p_2k}\Bigg]. \tag{31}$$

Here, $\zeta \equiv (1,1,0,0)$ and $\hat{p} = p - \zeta m^2/(2\zeta p)$.

- Upon taking the modulus squared of $\mathfrak{s}_{CL,\sigma}(k)$ we see that the extra non-IR divergent contributions reproduce the known collinear big log contribution which is missed by the usual YFS algebra.

Outlook

- Amplitude-based resummation (Staszek's contributions thereto were essential) allows improved control of IR and Collinear limits
- MC realizations are needed for current and future precision collider physics, using residuals made amenable by computer algebraic methods.
- New, collinearly enhanced soft functions $\Leftrightarrow$ Higher level of accuracy for a given level of exactness in the IR-finite YFS hard photon residuals.
- Enhanced toolbox available to extend the (CEEX) YFS MC method to the other important processes at present and future colliders.
- Some New Physics may hang in the balance at both LHC, FCC, and other future colliders.
- We wish Staszek were still here to share the possible excitement with us, we really do miss him very much.