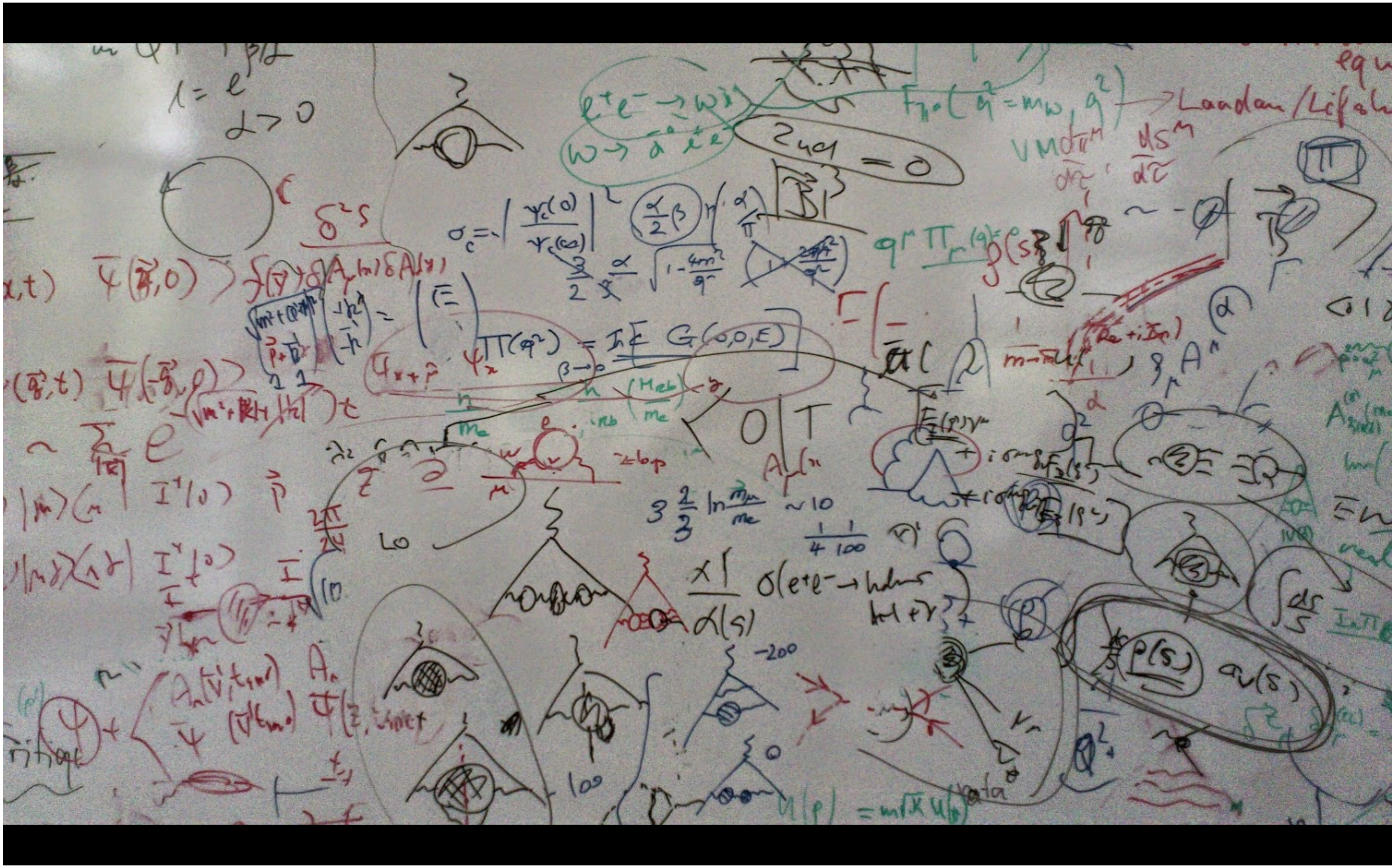

Lepton $g-2$ Moments: what they tell us

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Matter To The Deepest “Conference 2025” ,
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Abstract:
The magnetic moments of leptons (g_ℓ-factors) and the associated anomalies of the muon and electron play a decisive role in high-precision verification of the Standard Model. These quantities are not only among the most precisely measured in particle physics, but can also be predicted with high accuracy. At the same time, they offer promising insights into physics beyond the Standard Model. The key relationship for testing possible new heavy states is

$$\frac{\delta a_\ell}{a_\ell} \propto \frac{m_\ell^2}{M^2} \quad (M \gg m_\ell),$$

where m_ℓ is the mass of the lepton and M is the mass scale of new physics. This equation shows that the fractional change in a_ℓ is proportional to the square of the lepton mass over the square of the new physics scale, making the muon about $(m_\mu/m_e)^2 \sim 4 \times 10^4$ times more sensitive to Beyond Standard Model (BSM) physics than the electron. For the muon, theory and experiment now agree to 9 decimal places, so that $\delta a_\mu \sim \frac{\alpha}{\pi}\frac{m_\mu^2}{M_{\mathrm{NP}}^2}$ can effectively probe intermediate mass scales beyond M_t up to about 1 TeV (essentially ruled out directly by LHC).

Outline of Talk:

- ❖Introduction
- ❖Theory Status
- ❖Hadronic Stuff: the Limitation to Theory
- ❖Hadronic Vacuum Polarization (HVP) – Data & Status
- ❖Ab initio Method: HVP via Lattice QCD
- ❖What is the Problem?
- ❖Muon g-2 Status and the SM
- ❖Electron g-2 Status and its Future
- ❖Conclusion

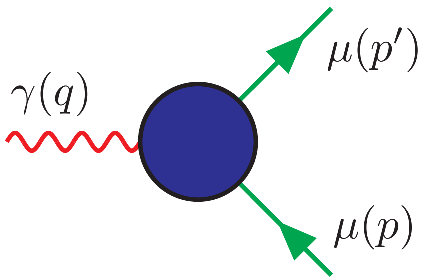
Introduction

Particle with spin $\vec{s} \Rightarrow$ magnetic moment $\vec{\mu}$ (internal current circulating)

$$\vec{\mu} = g_\mu \frac{e\hbar}{2m_\mu c}\,\vec{s} \;\; ; \quad g_\mu = 2\,(1 + a_\mu)$$

$$\boxed{\text{Dirac: } g_\mu = 2 \quad , \quad a_\mu = \frac{\alpha}{2\pi} + \cdots \text{ muon anomaly}}$$

Electromagnetic Lepton Vertex



$$= (-\mathrm{i}e)\,\bar{u}(p')\left[\gamma^\mu F_1(q^2) + \mathrm{i}\,\frac{\sigma^{\mu\nu} q_\nu}{2m_\mu} F_2(q^2)\right] u(p)$$

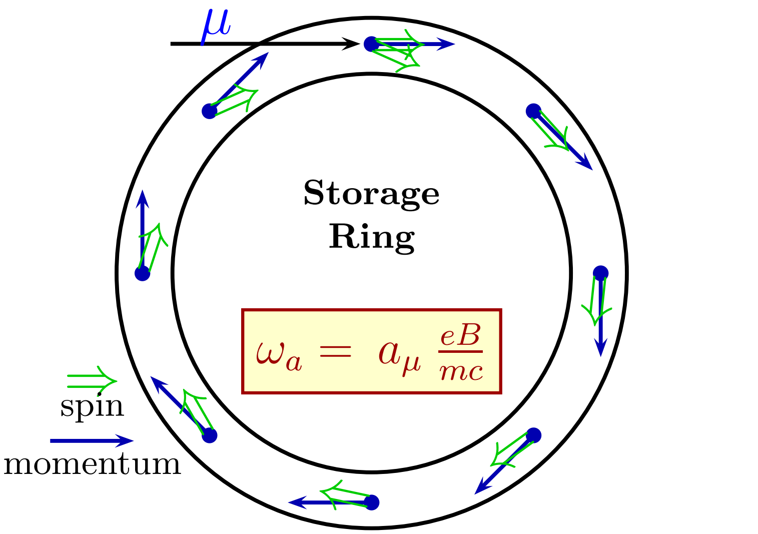
$$F_1(0) = 1 \;\; ; \quad F_2(0) = a_\mu$$

the simplest object
you can think of
in the static limit

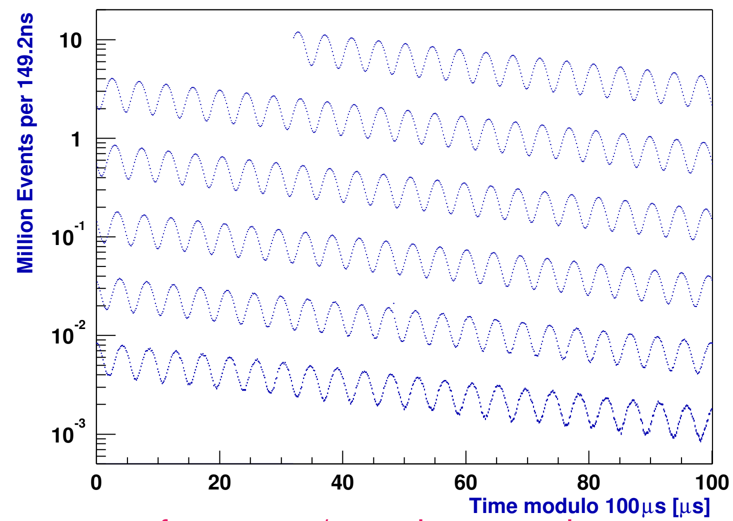
a_μ responsible for the Larmor (spin) precession $\Rightarrow$need polarized muons orbiting
Shoot protons on target producing pions which decay by P violating weak process

$$\boxed{\pi^+ \to \mu^+ \nu_\mu \;;\;\; \mu^+ \to e^+ \nu_e \bar{\nu}_\mu}$$

Larmor precession $\vec{\omega}$ of beam of spin particles in a homogeneous magnetic field $\vec{B}$



actual precession $\times$ 2 $\sim 12'$/circle



frequency/counting experiment

Magic Energy: $\vec{\omega}$ is directly proportional to $\vec{B}$ at magic energy $\sim$ 3.1 GeV

$$\vec{\omega}_a = \frac{e}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2-1}\right)\vec{\beta}\times\vec{E}\right]^{E\sim 3.1\,\mathrm{GeV}}_{\text{at "magic } \gamma\text{"}} \simeq \frac{e}{m}\left[a_\mu \vec{B}\right]$$

CERN, BNL and FNAL g-2 experiments

Stern, Gerlach 22: $g_e = 2$; Kusch, Foley 48: $g_e = 2\,(1.00119 \pm 0.00005)$

Crucial: 3.1 GeV muons life-time in lab frame $\gamma\tau_\mu$ 29 times longer!

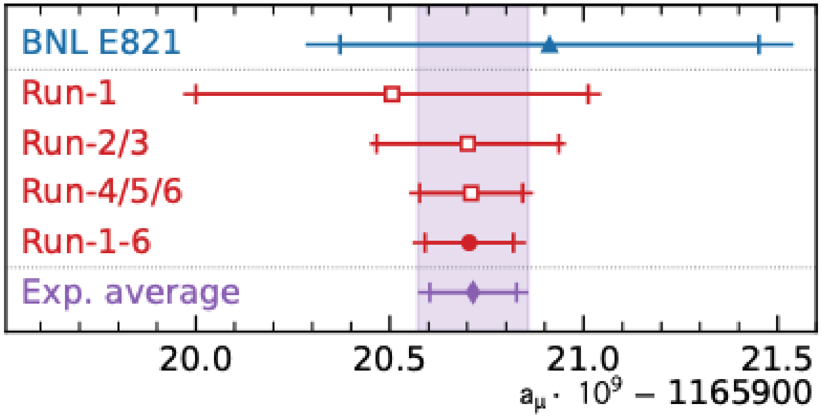
$$a_\mu^{\rm exp} = (11\,659\,207.15 \pm 0.80 \pm 0.95)[1.24]) \times 10^{-10} \text{ FNAL/BNL}$$

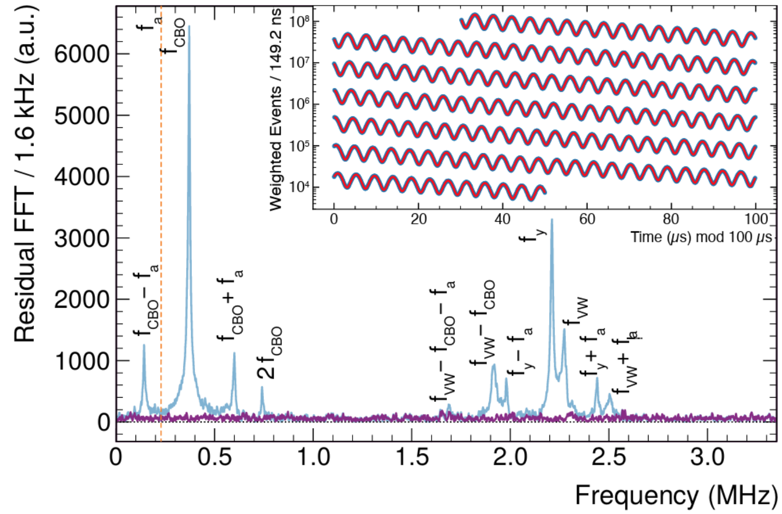
Precision: (124 ppb) today vs. (540 ppb) at BNL

The exciting new Fermilab Muon $g-2$ measurement applying sophisticated improved technology brilliantly confirmed the 2004 final BNL result and triggers a new serious attack on the SM.

$$a_\mu[\text{BNL}] = 11659209.1(5.4)(3.3)[6.3] \Longrightarrow a_\mu[\text{FNAL}] = 11659207.05(1.14)(0.95)[1.48]$$

in units 10^{-10}; FNAL error down by factor 4! now: comparable syst and stat errors





Left: BNL vs. FNAL21/23/25. Right: $N(t) = N_0\, e^{-t/\gamma\tau_\mu} \times \{1 + A\, \cos\left[\omega_a t - \varphi_0\right]\}$ modulo well understood corrections N_0 normalization, A P-violation asymmetry

To come – novel complementary experiment at J-PARC: $\vec{E} = 0$ cavity in place of magic γ determined by $\left(a_\mu - \frac{1}{\gamma^2 - 1}\right) = 0$ at CERN, BNL and Fermilab

- ultra relativistic (CERN, BNL, Fermilab)) vs. ultra cold (J-PARC) muons

  $\Rightarrow$very different systematics

a_μ measured via a ratio of frequencies (measurement of a_μ and B)

$$B = \frac{\hbar \omega_p}{2\mu_p}\,,\ \omega_a = \frac{e a_\mu}{m_\mu c}\,B\,,\ \mu_\mu = \left(1+a_\mu\right)\frac{e\hbar}{2m_\mu c} \Leftrightarrow \mu_\mu = \left(1+a_\mu\right)\frac{\hbar}{2}\,\frac{\omega_a}{a_\mu B} = \left(\frac{1}{a_\mu}+1\right)\frac{\omega_a}{\omega_p}\,\mu_p \Leftrightarrow$$

$$\boxed{a_\mu^{\rm exp} = (g_\mu/2-1) = \frac{\omega_a/\tilde{\omega}_p}{\mu_\mu/\mu_p - \omega_a/\tilde{\omega}_p} = \frac{\mathcal{R}}{\lambda-\mathcal{R}}}$$

in terms of 3 frequency measurables

- ❒ $\tilde{\omega}_p = (e/m_\mu)\langle B\rangle$ free proton NMR frequency
- ❒ $\mathcal{R} = \omega_a/\tilde{\omega}_p$ Larmor precession from BNL/FNAL
- ❒ $\lambda = \omega_L/\tilde{\omega}_p = \mu_\mu/\mu_p$ from hyperfine splitting of muonium

$\mu_p'/\mu_B = 1.5209931551(62) \times 10^{-3}; m_\mu/m_e = 206.7682827(46)$ (Codata 2024)
$\lambda = 3.183345137(85)$ (Codata 2008); $\mathcal{R}' = 0.00370730088(36)(29)$ (FNAL)
Experiment essentially limited at 120 ppb by μ_μ/μ_p from CODATA

Theory Status

all of SM counts

$$a_\mu^{\rm SM} = F_2(0) \quad = \quad a_\mu^{(\rm QED+EW+HVP_{LO}+HVP_{NLO}+HVP_{NNLO}+HLbL_{LO}+HLbL_{NLO})}$$

$$= \quad 0.00116591810 \pm 0.00000000043$$ White Paper 2020 (WP20)

My 2007 value: $a_\mu^{\rm SM} = 0.00116591793(68)$ in arXiv:hep-ph/0703125. Value up 81.0-79.3=1.7 in 10^{-10}, error down 30%
Davier et al. arXiv:0906.5443 incl. τ-decay spectra got $a_\mu^{\rm had}[ee+\tau] = 705.3 \pm 4.5$ vs $a_\mu^{\rm had}[\rm BMW] = 707.5 \pm 5.5$

News in Theory: $\rm HVP_{LO}$ and $\rm HLbL_{LO}$ calculations in **lattice QCD**, impact by BMW 2019 and later validations

Major shifts due to lattice QCD results:
$\rm HVP_{LO} = 693.1(4) \Rightarrow 713.2(6.1)$ and $\rm HLbL_{LO} = 90(17) \Rightarrow 112.6(9.6)$

New puzzle: data driven dispersion relation approach (DRA) versus ab initio lattice QCD approach (BMW,..., LQCD)

5 Numbers to establish the "g-2 Test" as $\delta \mathbf{a}_\mu(\mathbf{NewPhysics}) \equiv \mathbf{a}_\mu^{\rm exp} - \mathbf{a}_\mu^{\rm SM}$

$\omega_a,\ \omega_p,\ \lambda$ from experiment and $a_\mu^{\rm HVP_{LO}}$, $a_\mu^{\rm HLbL_{LO}}$ from theory, other SM contributions well under control!

Theory as now in units $\times 10^{-11}$:

	a_μ[SM]	precision	$\delta a_\mu^{\mathrm{SM-exp}}$	S.D.	comment
MB17	116591783[35]	(300 ppb)	-306 ± 72	4.3 σ	DRA vs BNL
WP20	116591810[43]	(369 ppb)	-279 ± 76	3.7 σ	DRA vs BNL
GM25	dito	dito	-262 ± 45	5.9 σ	DRA vs FNAL 25
GM21	116592019[38]	(325 ppb)	-40 ± 44	0.9 σ	BMW vs FNAL 21
WP25	116592033[62]	(530 ppb)	-38 ± 63	0.6 σ	LQCD&DRA vs WA

		QED	+LOHVP	+EW+HLbL+HOHVP	
a_μ^{the}	=	0.00 1165	92	033	(62)
a_μ^{exp}	=	0.00 1165	92	072	(12)

agrees to 9 decimal places! Does SM prediction match 10 digits precision?

$$\delta a_\mu = a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{SM}} = 38 \pm 63 \times 10^{-11} = \Delta a_\mu^{\mathrm{BSM}} ???$$

Note: EW contributes 12.9 σ; HLbL contributes 9.6 σ and HOHVP contribute -8.2 σ

For details I refer to my book 2017 and references therein. The QED prediction of a_μ is given by (see Aoyama et al. 12, Laporta 17 , Steinhauser et al. 17, Volkov 19/24)

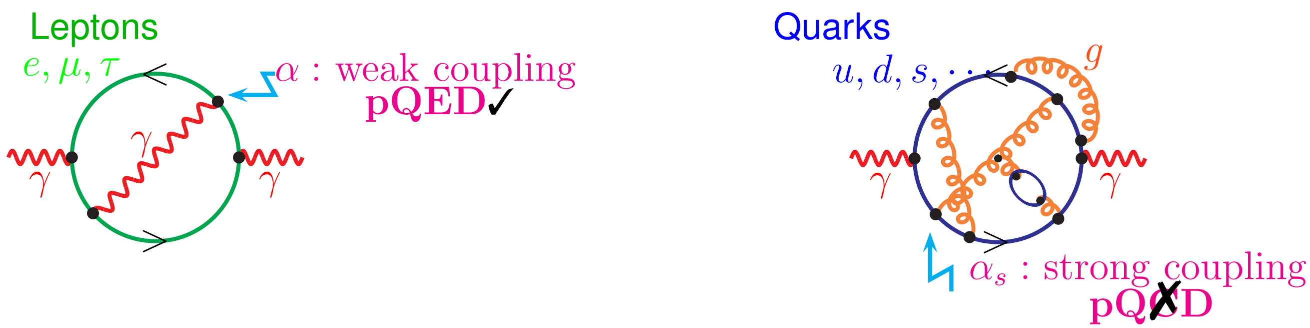
$$\begin{aligned} a_\mu^{\mathrm{QED}} &= \frac{\alpha}{2\pi} + 0.765\,857\,423(16)\left(\frac{\alpha}{\pi}\right)^2 + 24.050\,509\,82(28)\left(\frac{\alpha}{\pi}\right)^3 + 130.8734(60)\left(\frac{\alpha}{\pi}\right)^4 + 750.010(872)\left(\frac{\alpha}{\pi}\right)^5 \\ &= 116584718.8(2) \times 10^{-11} \end{aligned}$$

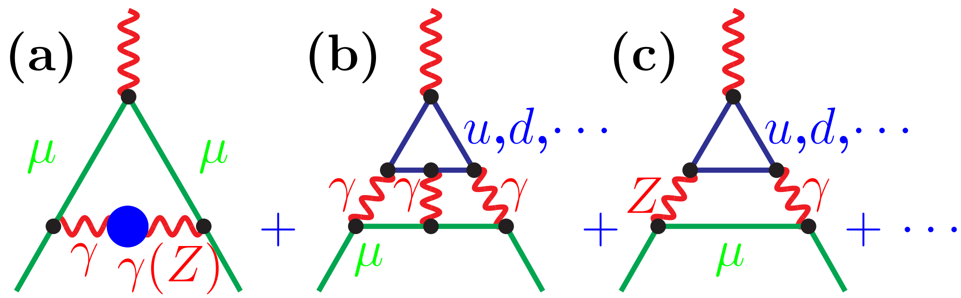
For the weak contributions we have (see Czarnecki, Marciano, Stöckinger et al. ...)

$$a_\mu^{\mathrm{EW}} = \underbrace{194.79(1) \times 10^{-11}}_{\text{LO}} - \underbrace{40.38(36) \times 10^{-11}}_{\text{NLO}} = 154.4(4) \times 10^{-11}$$

❐ Hadronic Stuff: the Limitation to Theory

General problem in electroweak precision physics:
contributions from hadrons (quark loops) at low energy scales





(a) Hadronic vacuum polarization $O(\alpha^2), O(\alpha^3)$ — Light quark loops

(b) Hadronic light-by-light scattering $O(\alpha^3)$ — ↓

(c) Hadronic effects in 2-loop EWRC $O(\alpha G_F m_\mu^2)$ — Hadronic "blobs"

Evaluation of non-perturbative effects:

- ❒ data in conjunction with Dispersion Relations (DR),
- ❒ low energy effective modeling, RLA, HLS, ENJL
- ❒ lattice QCD

(a) HVP via dispersion integral over $e^+e^- \to \mathrm{hadrons}$-data (1 independent amplitude to be determined by one specific data set), HLS global fit, lattice QCD

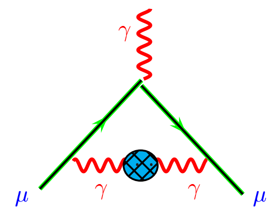
(b) HLbL via Resonance Lagrangian Approach (RLA) (CHPT extended by VDM in accord with chiral structure of QCD) [Knecht, Nyffeler], $\gamma\gamma \to \mathrm{hadrons}$-data driven dispersive approach [Colangelo et al.] (28/19 independent amplitudes to be determined by as many independent data sets), lattice QCD [Blum et al., Wittig, Meyer et al., ...]

(c) quark and lepton triangle diagrams: VVV = 0 by Furry ⇒only VVA (of $f\bar{f}Z$-vertex) contributes ⇒ABJ anomaly is perturbative and non-perturbative simultaneously i.e. leading effects calculable (anomaly cancellation) [de Rafael, Knecht, Perrottet, Melnikov, Vainshtein]

Hadronic Vacuum Polarization (HVP) – Data & Status

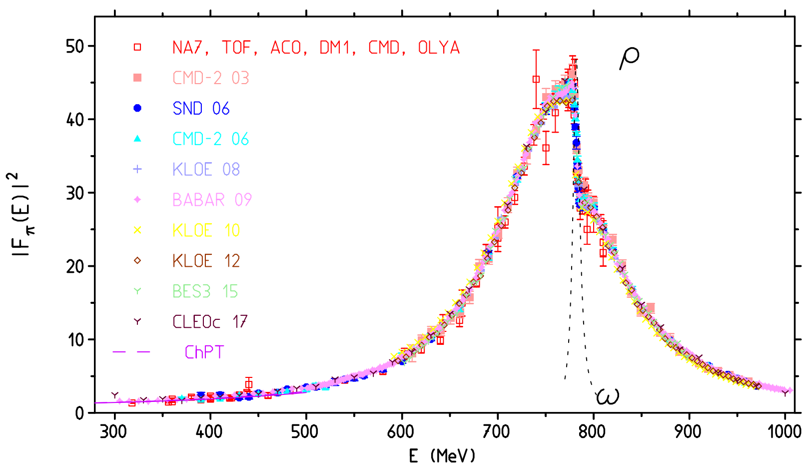
Leading non-perturbative hadronic contributions $a_\mu^{\rm had}$ can be obtained in terms of $R_\gamma(s) \equiv \sigma^{(0)}(e^+e^- \to \gamma^* \to \text{hadrons})/\frac{4\pi\alpha^2}{3s}$ data via Dispersion Relation (DR):

$$a_\mu^{\rm had} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \left(\int\limits_{4m_\pi^2}^{E^2_{\rm cut}} ds\, \frac{R_\gamma^{\rm data}(s)\; \hat{K}(s)}{s^2} + \int\limits_{E^2_{\rm cut}}^{\infty} ds\, \frac{R_\gamma^{\rm pQCD}(s)\; \hat{K}(s)}{s^2} \right)$$

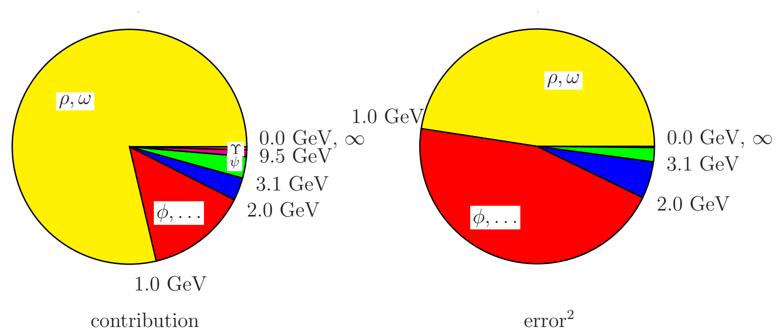


- Experimental error implies theoretical uncertainty!
- Low energy contributions enhanced: $\sim 75\%$ come from region $4m_\pi^2 < m_{\pi\pi}^2 < M_\phi^2$

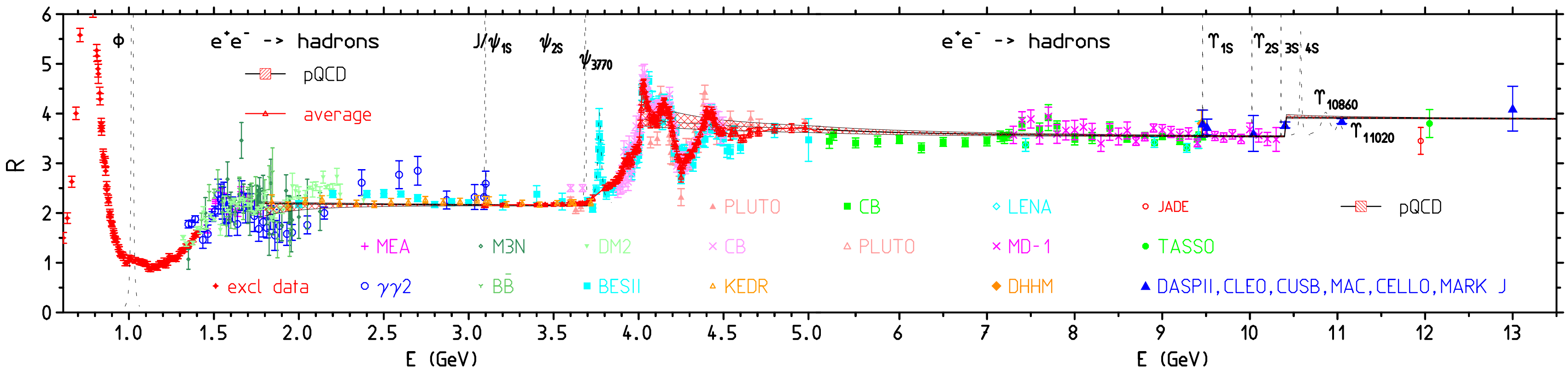
Data: NSK, KLOE, BaBar, BES3, CLEOc, [CMD-3?]



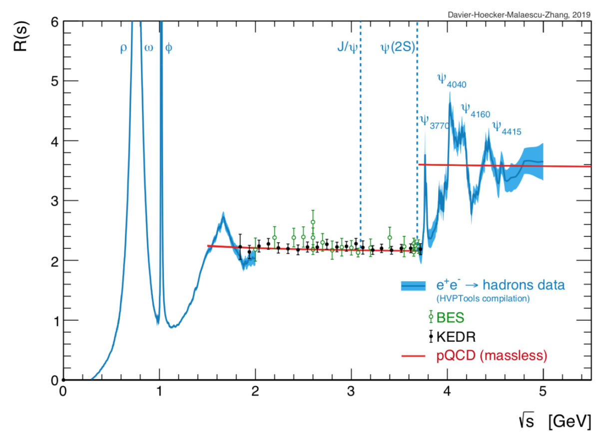
$$a_\mu^{\rm had(1)} = (697.17 \pm 4.18)\; 10^{-10}$$
e^+e^-–data based



Higher energies:

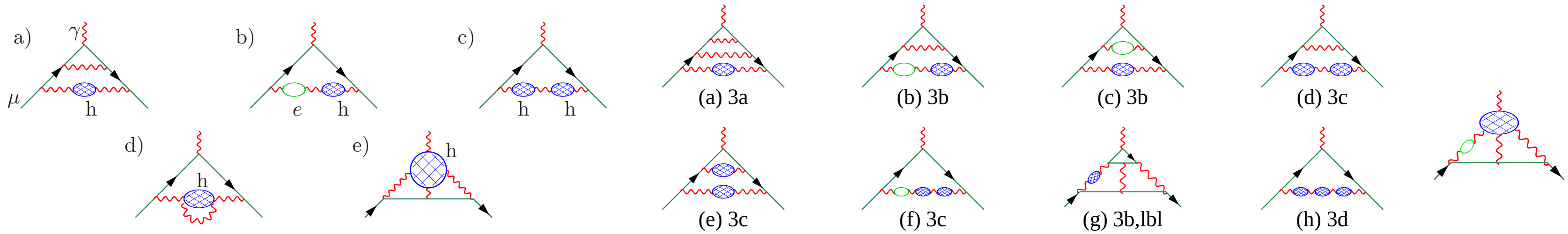


The compilation of $R(s)$–data utilized.



$R(s)$ compilation by Davier et al.

NLO and NNLO HVP Effects:



Hadronic higher order contributions: involving LO, NLO and NNLO vacuum polarization, and light-by-light scattering insertions.

Barbieri,Remiddi 74,Krause 97, Kurz et al 14, my 2023 update below

Summary of the hadronic and weak contributions (based on dispersive approach)

$a_\mu^{\mathrm{had}(1)}$	=	$(697.17 \pm 4.18)\ 10^{-10}$	(LO)
$a_\mu^{\mathrm{had}(2)}$	=	$(-10.89 \pm 0.067)\ 10^{-10}$	(NLO)
$a_\mu^{\mathrm{had}(3)}$	=	$(1.242 \pm 0.010)\ 10^{-10}$	(NNLO)
$a_\mu^{\mathrm{had,LbL}}$	=	$(10.34 \pm 2.88) \times 10^{-10}$	(HLbL)
a_μ^{weak}	=	$(15.4 \pm 0.4\ 10^{-10}$	(LO+NLO) .

LO HVP and LO HLbL to be reset by LQCD results!

- uncertainties of the subleading NLO and NNLO and EW below 1 σ (DRA save)

Ab initio Method: HVP via Lattice QCD

Primary object for HVP in LQCD: e.m. quark current correlator in Euclidean configuration space

$$\langle J_\mu(\vec{x},t)\, J_\nu(\vec{0},0)\rangle\,,\quad J_\mu = \tfrac{2}{3}\bar{u}\gamma_\mu u - \tfrac{1}{3}\bar{d}\gamma_\mu d - \tfrac{1}{3}\bar{s}\gamma_\mu s + \cdots$$

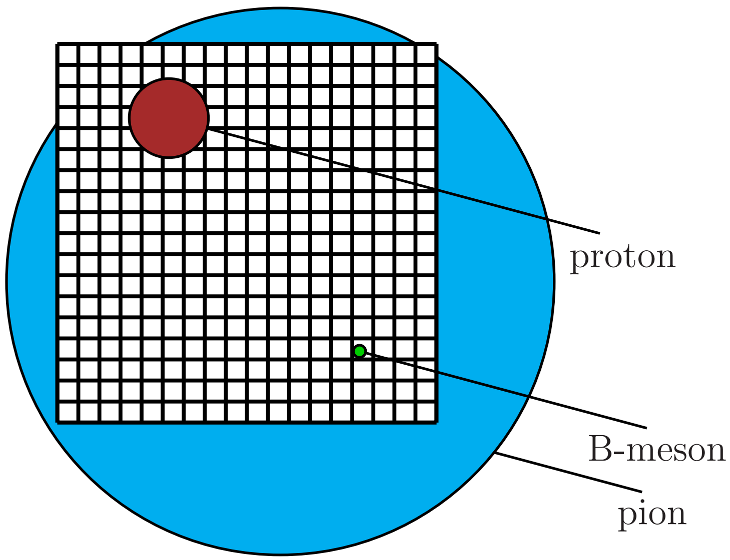
A Fourier transform yields the bare vacuum polarization function $\Pi(Q^2)$

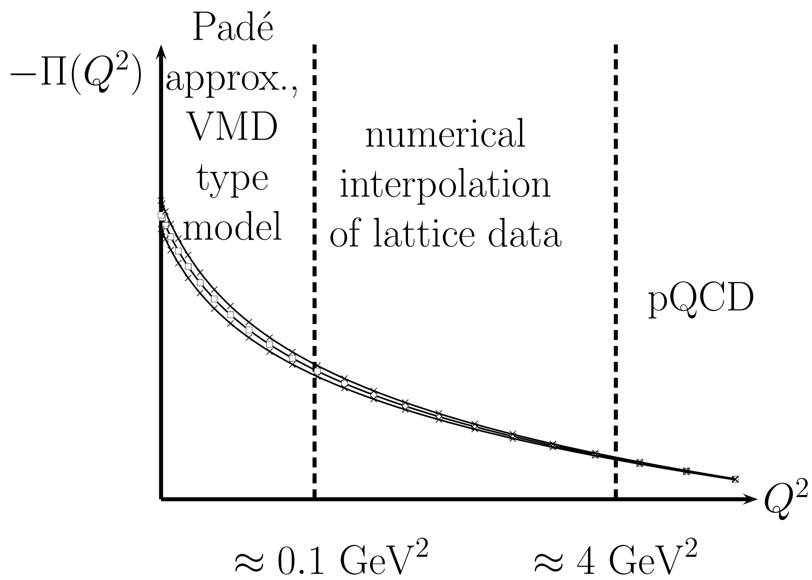
$$\Pi_{\mu\nu}(Q) = \int \mathrm{d}^4x \mathrm{e}^{\mathrm{i}\,Qx}\,\langle J_\mu(x)\, J_\nu(0)\rangle = \left(Q_\mu Q_\nu - \delta_{\mu\nu}\, Q^2\right)\,\Pi(Q^2)$$

needed to calculate

$$a_\mu^{\mathrm{HVP}} = 4\alpha^2 \int_0^\infty \mathrm{d}Q^2\, f(Q^2)\,\left\{\Pi(Q^2) - \Pi(0)\right\}$$ with $f(Q^2)$ a known kernel function

Limitations here: finite box $V = L^4$, lattice spacing a; extrapolations needed





Lattice QCD (LQCD) now a competitive method! From first principles straight forward approach

- LQCD lattice in finite box: momenta are quantized $Q_{\rm min} = 2\pi/L$ where L is the lattice box length. $Q_{\rm min} \to 0 \Leftrightarrow L \to \infty$ infinite volume limit
- $Q_{\rm min} = 2\pi/L$ with $m_\pi a L \gtrsim 4$ for $m_\pi \sim 200$ MeV, such that $Q_{\rm min} \sim 314$ MeV

  - lattice data: $Q^2 > (2\pi/L)^2$
  - extrapolate to $Q^2 = 0$ via Padé's or complete via HVP data
  - required accuracy: need LQCD data down to $Q^2_{\rm min} \approx 0.1$ GeV^2

- about 44% of the low x contribution to $a_\mu^{\rm had}$ is not covered by data yet

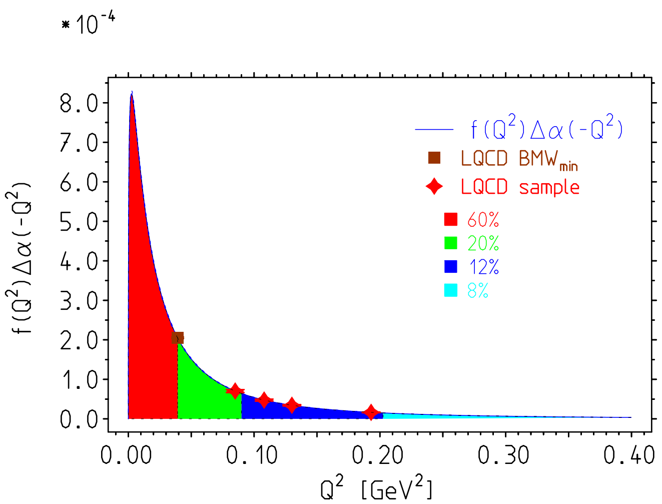
Data from BMW, Mainz/CLS-24, RBC/UKQCD combined and averaged by WP25 group yield:

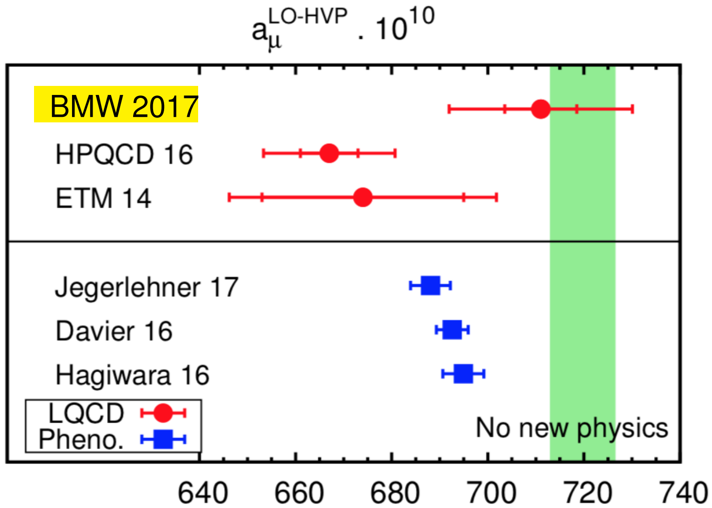
$$a_\mu^{\rm SD}({\rm iso}) = \underset{9.6\%}{69.06(22) \times 10^{-10}},\ a_\mu^{\rm W}({\rm iso}) = \underset{33.1\%}{236.16(42) \times 10^{-10}},\ a_\mu^{\rm LD}({\rm iso}) = \underset{57.2\%}{407.9(5.0) \times 10^{-10}}\ .$$

$a_\mu^{\rm HVP,\ LO}({\rm iso})\Big|^{\rm lat}_{\rm Avg.1} = 713.1(5.0) \times 10^{-10}$ including IB corrections $a_\mu^{\rm HVP,\ LO}({\rm cor})\Big|^{\rm lat}_{\rm Avg.1} = 713.2(6.1) \times 10^{-10}$

Individual flavor contributions from light (u,d) amount to about 90%, strange about 8% and charm about 2%.

As off 2017: LQCD results for the leading order $a_\mu^{\rm HVP}$, in units 10^{-10} [Brookhaven, Zeuthen, Mainz, Edinburgh, ... groups]: a first hint for a larger HVP value





Left: Percent contributions from ranges. Right: Plot from Budapest-Marseille-Wuppertal 17

Most recent WP25: a large HVP result now established

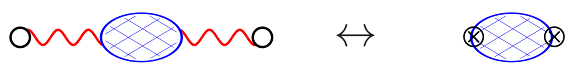
Collaboration	$a_\mu^{\rm HVP,\,LO}$	$a_\mu^{\rm HVP,\,LO}\vert_{\rm corr}$
BMW-20	707.5(5.5)	707.2(5.5)
Mainz/CLS-24	724.5(7.1)	724.5(7.1)
RBC/UKQCD-24	734.5(13.0)	729.4(13.0)

$$a_\mu^{\rm HVP,\,LO}\Big|^{\rm LQCD}_{\rm WP25} = 713.2(6.1)\times 10^{-10}$$

❐ What is the Problem?

Note: in spite of the fact of a heroic effort in calculating and modeling QED correction to hadronic processes (see Jadach, Ward, Placzek, Was, Czyz, German et al., Piccinini et al.,...) some puzzles remain:

- $e^+e^- \to \mathrm{hadrons}$ experiments measure the photon propagator, i.e., a mix of QCD and QED which is difficult to disentangle (requires removing external QED corrections, radiation from hadrons not well understood [sQED]). Lattice QCD “measures” the hadronic blob directly (external QED corrections switched off)



$\langle A(x)\, A(0) \rangle$ photon propagator $\leftrightarrow$ $\langle j(x)\, j(0) \rangle$ current correlator

Also:

- τ data in many respects are much simpler than e^+e^- ones, because the charged $\rho^\pm$ does not mix with other states in the dominant low energy range below about 1.05 GeV (up to above the ϕ, below ρ')!

In contrast understanding NC data requires modeling: CHPT + spin 1 resonances (VMD) $\Rightarrow$ Resonance Lagrangian Approach e.g. HLS (massive Yang-Mills) $\Rightarrow$ dynamical widths, dynamical mixing of $\gamma, \rho^0, \omega, \phi$

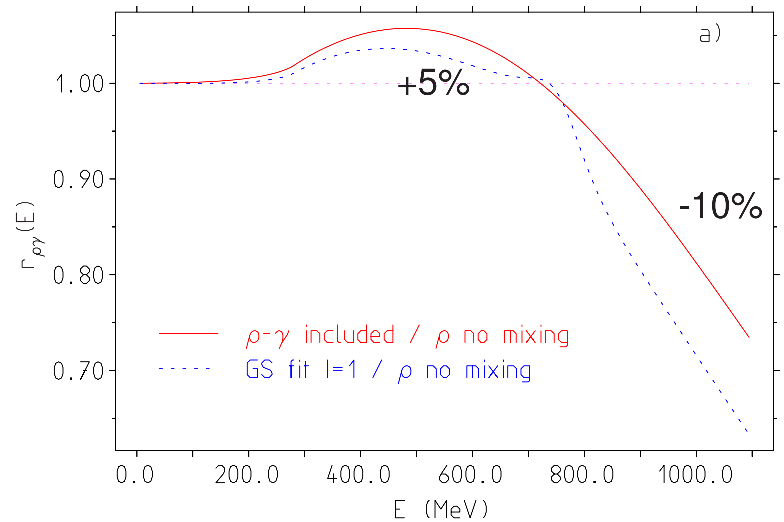
❐ Commonly forgotten: mixing of ρ^0, ω, ϕ with the photon [$\rho^0 - \gamma$ mixing], i.e., effect concerning relation photon propagator and one-particle irreducible blob, and is included in NC $e^+e^- \to \pi^+\pi^-$ data, absent in CC $\tau \to \nu_\tau \pi\pi$ data!
As a QED effect must be disentangled from e^+e^- data!

❐ Accounting the $\rho - \gamma$ interference resolves τ (charged channel) vs. e^+e^- (neutral channel) puzzle,
F.J.& R. Szafron, M. Benayoun et al. 2011

$\rho - \gamma$ interference
(absent in charged channel)
often mimicked by large shifts
in M_ρ and Γ_ρ

$-\mathrm{i}\,\Pi^{\mu\nu\,(\pi)}_{\gamma\rho}(q) =$  .

$$v_0(s) = r_{\rho\gamma}(s)\, R_{\mathrm{IB}}(s)\, v_-(s)$$



- not τ spectra require to be corrected for missing $\rho - \gamma$ mixing!
  rather this QED effect has to be eliminated from the e^+e^- data. Indeed absent in LQCD data!

- for getting had blob in e^+e^- the $\gamma - \rho^0$ mixing has to be removed!

- for the I=1 part of $a_\mu^{\mathrm{had}}[\pi\pi]$ results in (JS 2011)

$$\delta a_\mu^{\mathrm{had}}[\rho\gamma] \simeq (5.1 \pm 0.5) \times 10^{-10}\,,$$

as a correction applied for the range [0.63,0.96] GeV.

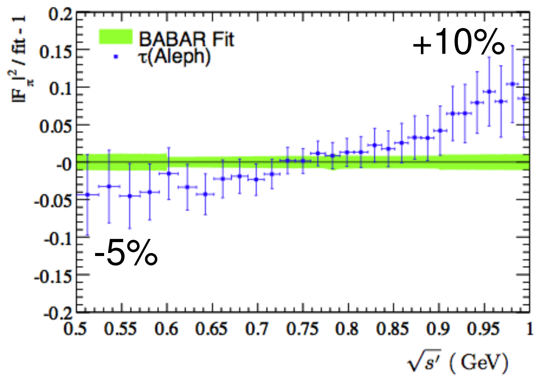
To be added to standard e^+e^- based $a_\mu^{\text{HVP, LO}}$: (in JS11 we subtracted it wrongly from the τ data)

$$a_\mu^{\text{HVP, LO}}|_{\text{cor}} = a_\mu^{\text{HVP, LO}} + \delta a_\mu^{\text{had}}[\rho\gamma] \simeq (702.3 \pm 4.2) \times 10^{-10}$$

Fits well with Davier et al. arXiv:0906.5443 incl. τ-decay spectra $a_\mu^{\text{had}}[ee+\tau] = (705.3 \pm 4.5) \times 10^{-10}$

At least a part of the missing shift with respect to LQCD data.

Best "proof" for our $\gamma - \rho^0$ mixing profile (Davier et al. 2009):



❐ The issue of photon radiation by hadrons like pions remains (sQED vs RLA ?).

Advantage lattice QCD!

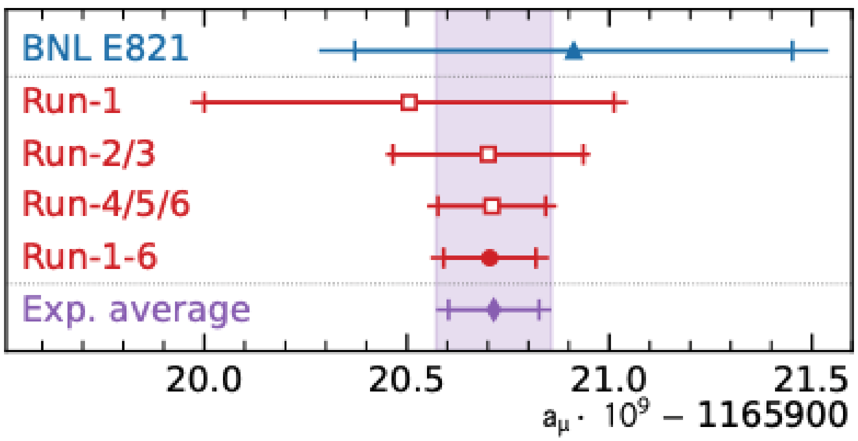
Muon g-2 Status and the SM

The exciting new Fermilab Muon $g-2$ measurement applying sophisticated improved technology brilliantly confirmed the 2004 final BNL result and triggers a new serious attack on the SM.

$a_\mu[\mathrm{BNL}] = 11659209.1(5.4)(3.3)[6.3] \Longrightarrow a_\mu[\mathrm{FNAL}] = 11659207.05(1.14)(0.95)[1.48]$

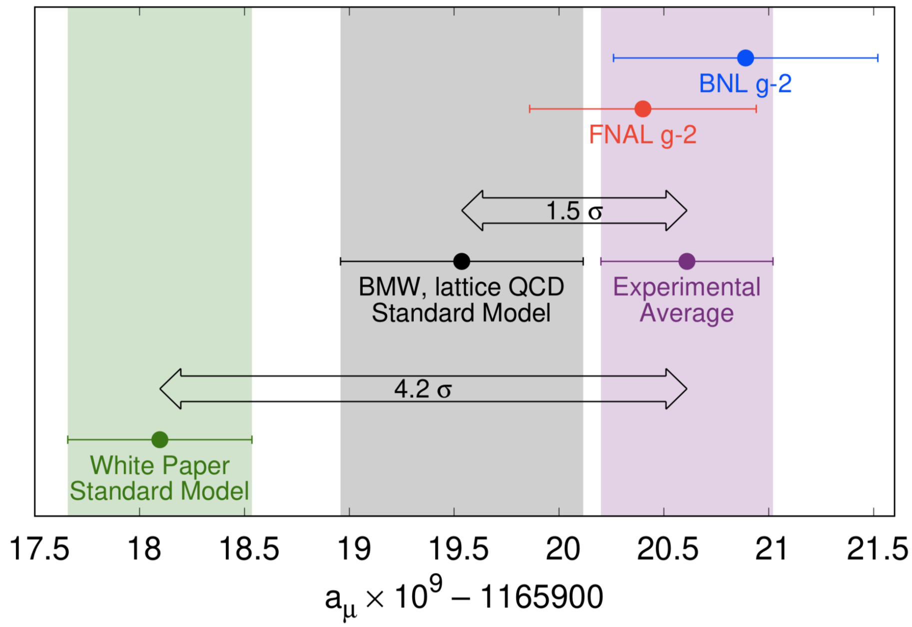
$a_\mu[\mathrm{FNAL/BNL}] = 11659207.15(0.80)(0.95)[1.24]$ (124 ppb) vs. (540 ppb) at BNL

in units 10^{-10}; FNAL error down by factor 4! now: comparable syst and stat errors



BNL vs. FNAL21/23/25

Compare 2021 status after FNAL Run-1: There's something in the air.



Muon g-2 plot incl. BMW lattice QCD after FNAL Run-1

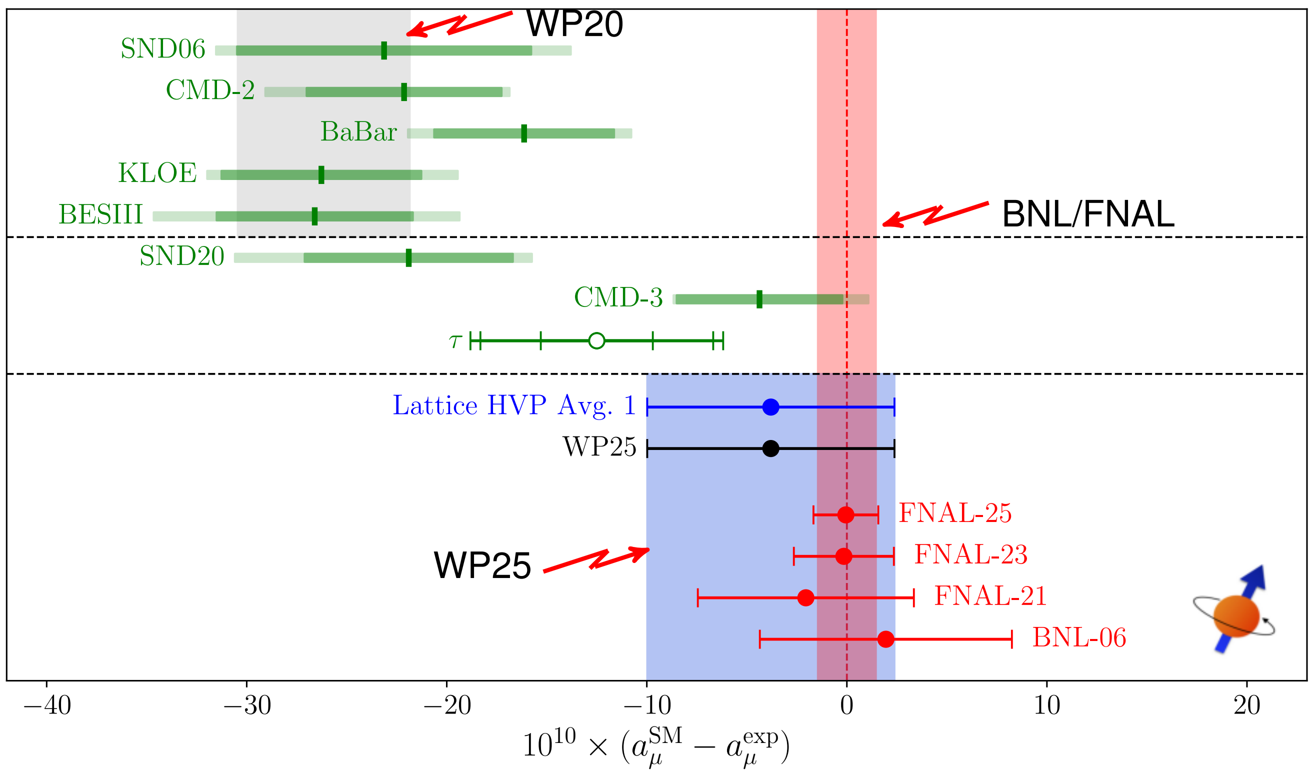
WP20 Gap $\Delta a_\mu^{\rm BSM}$ is 1.6 times EW contribution! Big effect missing?

Still: theory leading uncertainty: HVP (and HLbL); DR+Data vs lattice QCD

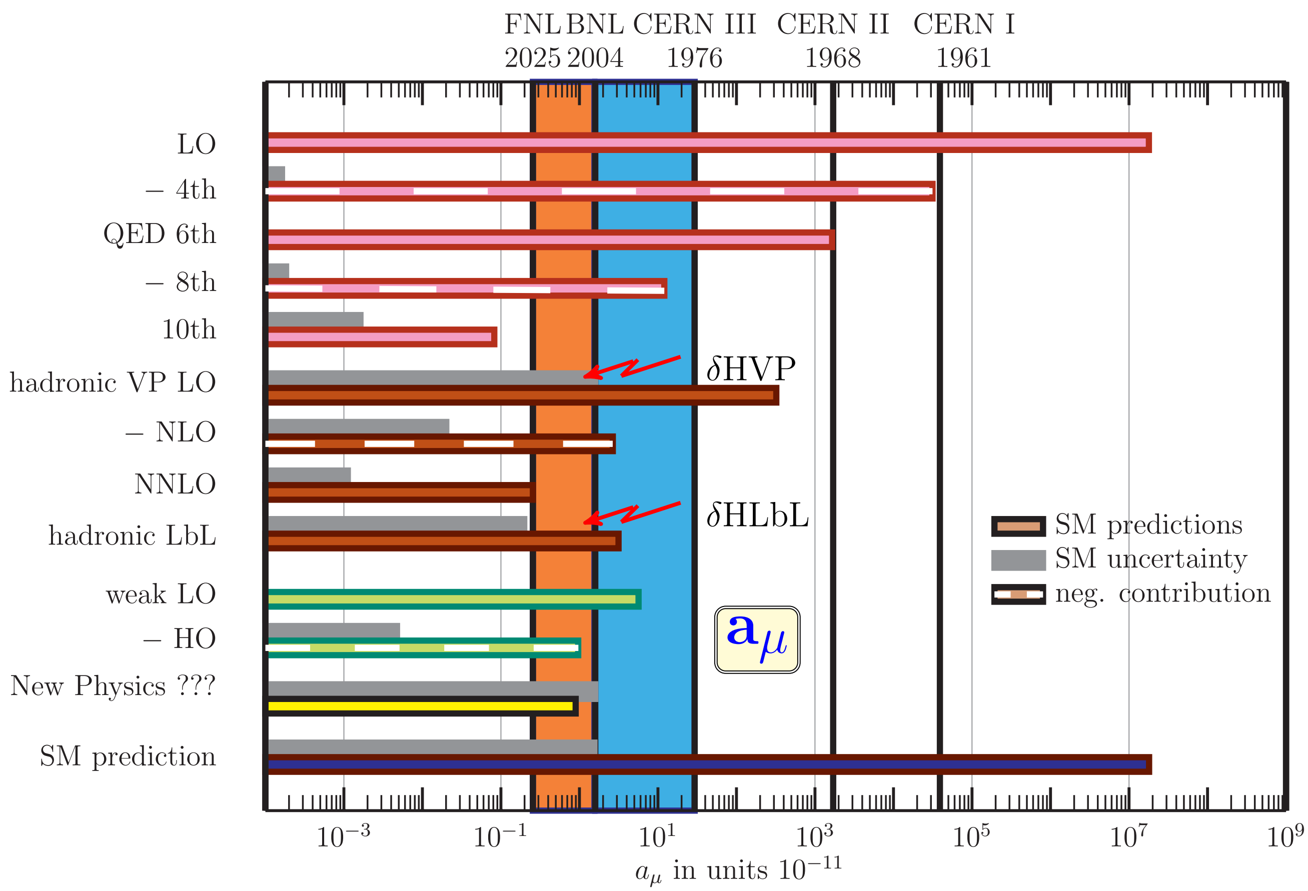
Note: $\delta a_\mu^{\rm HVP-LO}[\mathrm{LQCD-DR}] = 14.4(6.8) \times 10^{-10}$ vs $a_\mu^{\rm EW} = 15.44 \pm 0.04 \times 10^{-10}$

Now some might be shocked: $\boxed{\text{a 5.6 } \sigma \text{ missing BSM turned into a "validates the SM up to the 0.37 ppm level"}}$,

as highlighted in Fodor, Davier et al. 2024 [BMW/DMZ-24]. Status of a_μ in fall 2025:



While the consensus HVP(/HLbL) result from WP20 indicates a missing effect of $5.6\,\sigma$ between theory and experiment, the HVP(/HLbL) results from lattice QCD lead to an agreement with the SM at the level of $0.6\,\sigma$; graph from WP25. Puzzles regarding the true value of $a_\mu^{\rm HVP,\,LO}$ remain unsolved: KLOE vs. BaBar, CMD3 vs. SND20, NC e^+e^- data vs. CC τ data, the data-driven dispersive approach, and LQCD calculations each face their own challenges.



Present muon $g-2$ experiments testing various contributions. New Physics ? = deviation $(a_\mu^{\rm exp}-a_\mu^{\rm the})/a_\mu^{\rm exp}$. Limiting theory precision: hadronic vacuum polarization (HVP) and hadronic light-by-light (HLbL)

*** digging deeper and deeper ***

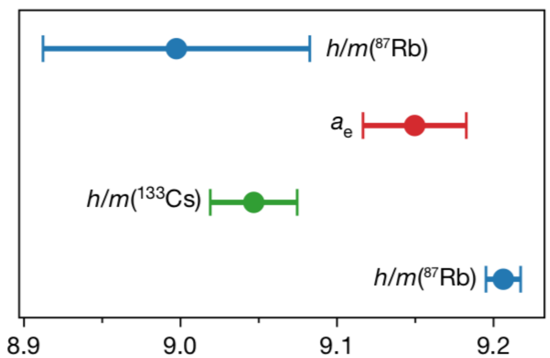
Electron g-2 Status and its Future

2018 $\alpha^{-1}(\mathrm{Cs18}) = 137.035999046(27) \Rightarrow a_e^{\mathrm{exp}} - a_e^{\mathrm{the}} = (-84 \pm 36) \times 10^{-14} \Rightarrow$ - 2.3 σ

Oops! δa_e sign differs from δa_μ! Challenge for BSM theory!

2020 $\alpha^{-1}(\mathrm{Rb20}) = 137.035999206(11) \Rightarrow a_e^{\mathrm{exp}} - a_e^{\mathrm{the}} = (51 \pm 30) \times 10^{-14} \Rightarrow$ + 1.7 σ

α Rb20 vs Cs18 differ by $\boxed{5.4\ \sigma}$! Note α is basic SM input parameter!



Electron anomaly $a_e - 0.00115965218050$:
from -2.3 σ in 2018 to +1.7 σ in 2020, and +2.3 σ 2024.

Largely a QED test, highly sensitive to $\alpha_{\mathrm{e.m.}}$.

Updates:

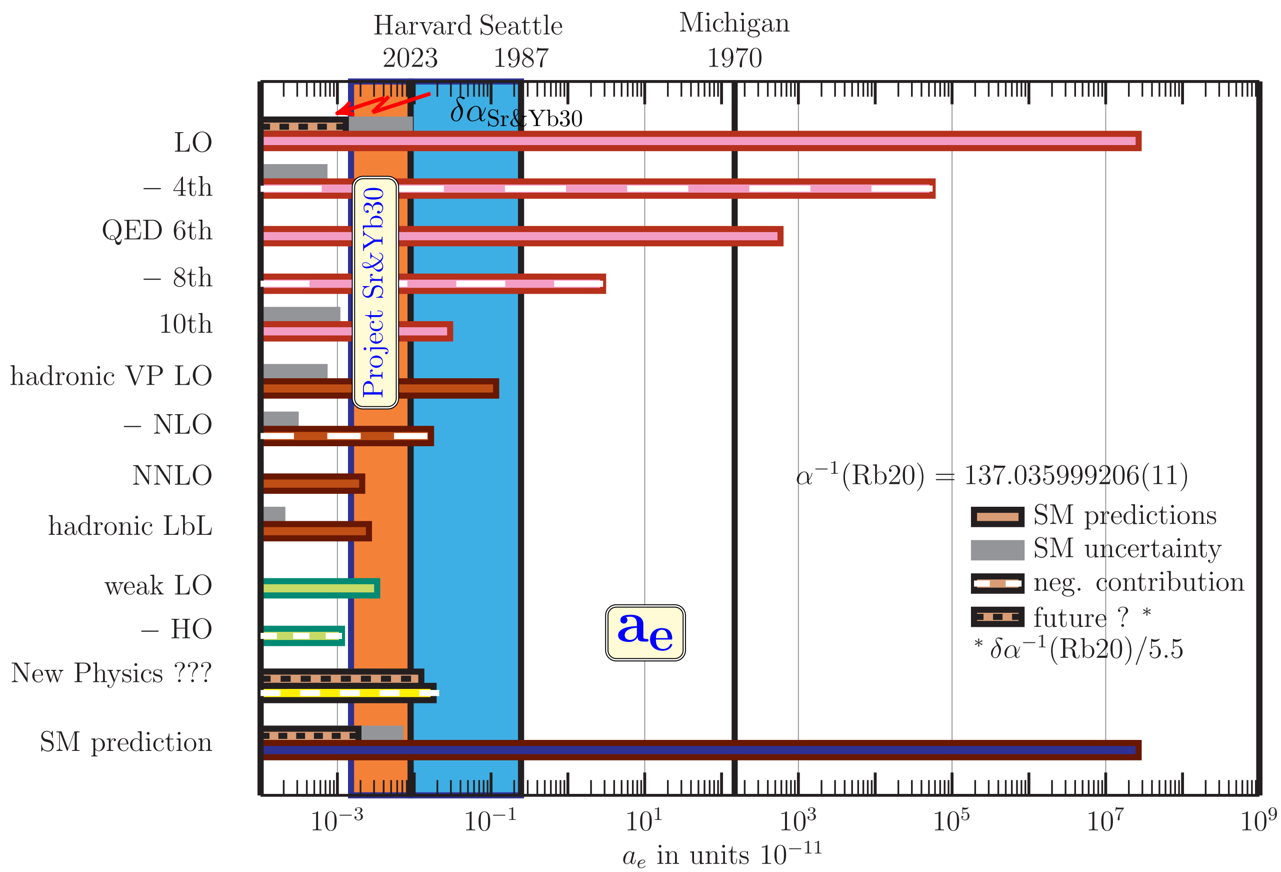
❖2022 update from $a_e^{\mathrm{exp}} = 0.00115965218073(28)$ to $a_e^{\mathrm{exp}} = 0.00115965218059(13)$ Gabrielse et al.

❖2024 $a_e^{\mathrm{LO-HVP}}$ rescaled by $a_\mu^{\mathrm{LO-HVP}}$ lattice vs. dispersive result, i.e.,

$a_e^{\mathrm{LO-HVP}} = 1.871(11) \times 10^{-12}$ to $a_e^{\mathrm{LO-HVP}} = 1.923(09) \times 10^{-12}$

❖2024 change in universal 5-loop coefficient to $A_1^{(10)} = 5.873(128)$ Volkov, Aoyama et al.

$a_e^{\mathrm{the}} = 0.001159.65218023(9) \Rightarrow a_e^{\mathrm{exp}} - a_e^{\mathrm{the}} = (36 \pm 16) \times 10^{-14} \Rightarrow$ + 2.3 σ

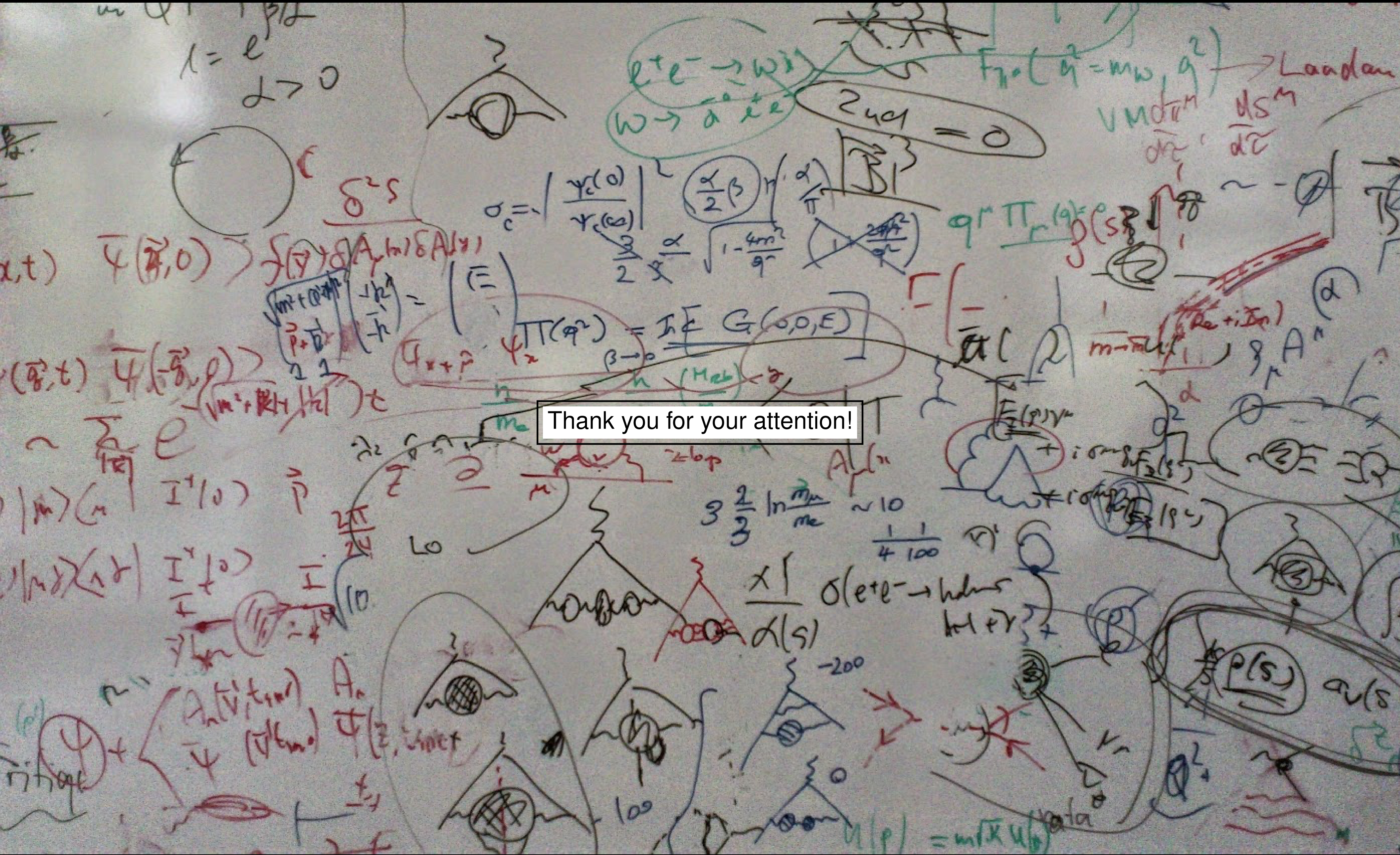


Status and sensitivity of the a_e experiments testing various contributions. The error is dominated by the uncertainty of α(Rb20) from atomic interferometry. "New Physics" ? = deviation $(a_e^{\mathrm{exp}} - a_e^{\mathrm{the}})/a_e^{\mathrm{exp}}$, i.e., essentially absent presently. The blue band illustrates the improvement by the Harvard/Northwestern U. experiment. The orange band shows the possible progress by the Sr&Yb30 atomic interferometry project AION.

In fair agreement with the SM!

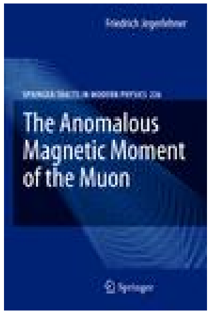
Conclusions

- One of the most prominent signs of BSM physics has gone: advantage SM, as by LHC searches!
- Does not mean hunting for BSM physics becomes less a main goal for research in particle physics
- What does the a_μ and a_e tell us?
  - High precision physics may be more challenging than expected, both $a_\mu^{\exp}$ and a_μ^{the}
  - Need alternative experiment on $a_\mu^{\exp}$: so far all experiment CERN, BNL, FNAL use magic γ. New to come: J-PARC ultra cold muons in $\vec{E} = 0$ field
  - MuonE (elastic μe-scattering) determination of HVP [Abbiendi, Venanzoni et al.] at CERN (measures directly what LQCD calculates)
  - New α determination as an input for a_e AION project (with Strontium, Yttrium ions)
  - Need understand inconsistencies in data required for DRA to HVP: KLOE vs. BaBar, SND20 vs CMD-3, $e^+e^- \to \pi^+\pi^-$ vs $\tau^- \to \pi^0\pi^-\nu_\tau$ data (NC vs CC); for HLbL need more a
  - Never trust consensus papers! High precision physics is extremely important but extremely challenging.
- Beyond BSM science fiction: Scrutinizing precise SM prediction and progress in determining SM parameters like M_t and M_H (requiring high precision Higgs/Top-quark pair factory) a big challenge, important also for better understanding early cosmology. But that's another story!



Thank you for your attention!

For further reading:



Jegerlehner F., The Anomalous Magnetic Moment of the Muon.
Springer Tracts in Modern Physics, Vol 274 (2017). Springer, Cham
(693 pages on one number to 8 digits)

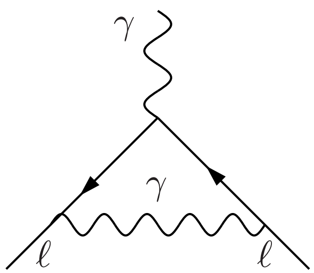
Book update comment: Lepton_Moments2024 on my WEB page https://people.physics.hu-berlin.de/~fjeger/

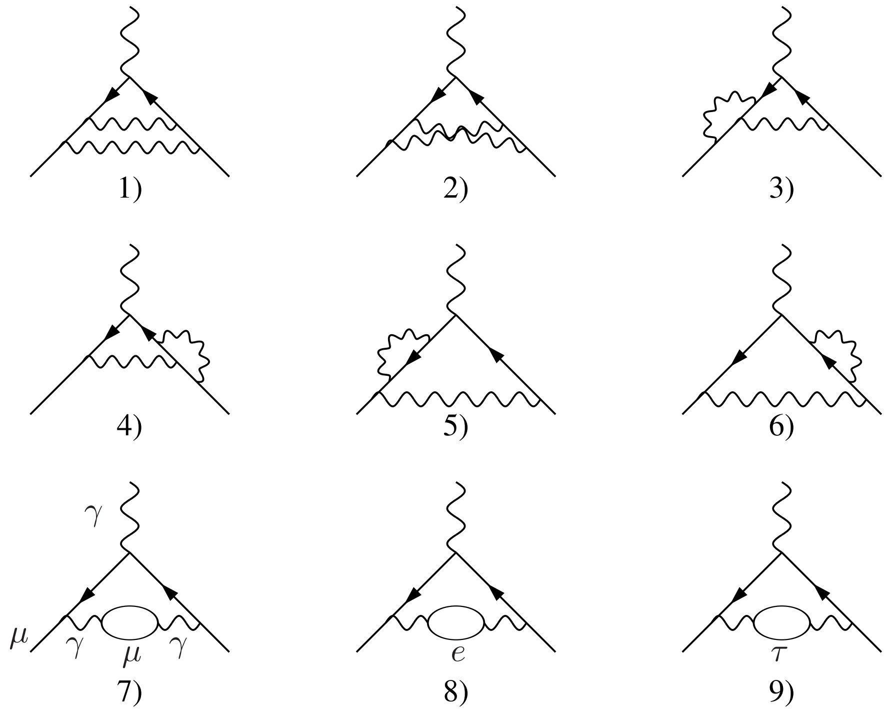
In the "To the memory of M.Veltman" Volume:
F. Jegerlehner, "The Standard Model of Particle Physics as a Conspiracy Theory and the Possible Role of the Higgs Boson in the Evolution of the Early Universe," Acta Phys. Polon. B **52**, no.6-7, 575-605 (2021) doi:10.5506/APhysPolB.52.575 [arXiv:2106.00862 [hep-ph]].

Backup Slides

On $a_\mu^{\rm QED}$

Universal contributions: a_μ internal muons loops only





$$a^{(2)}_{\ell\,\text{universal}} = \frac{1}{2}\left(\frac{\alpha}{\pi}\right)$$

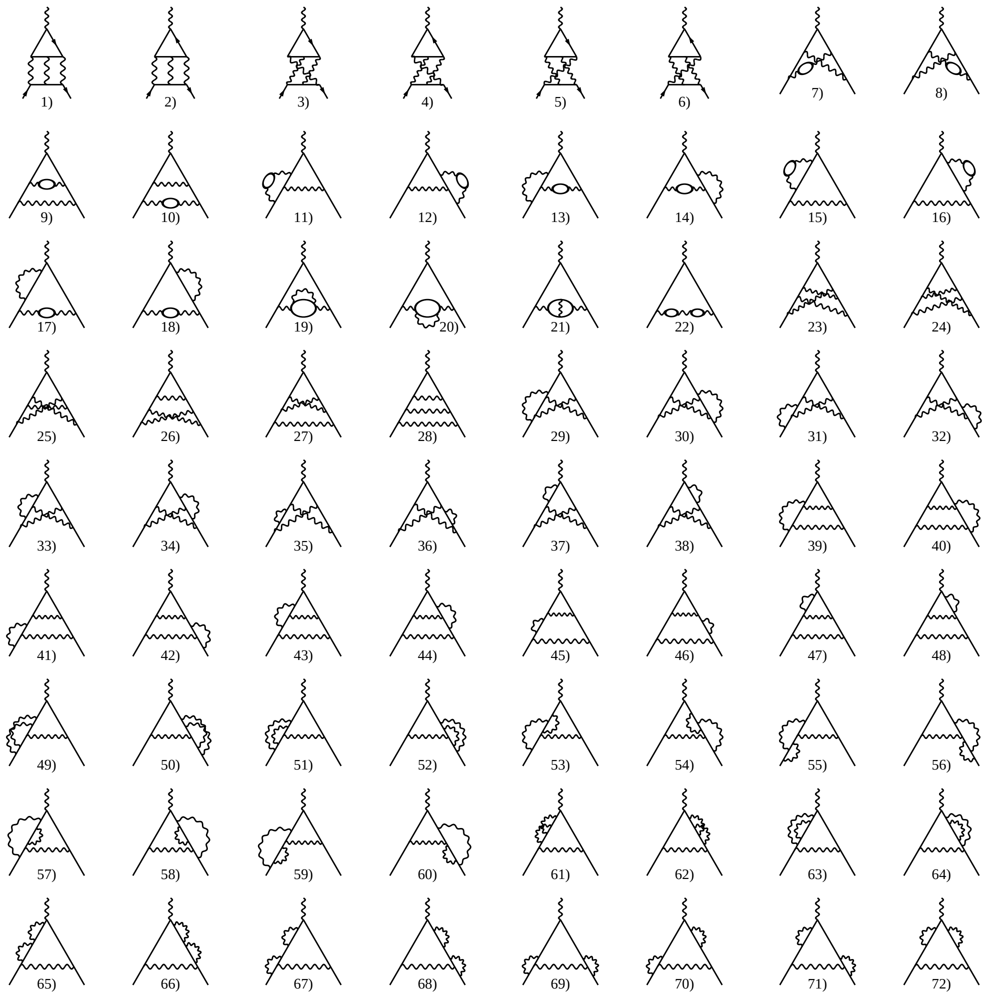
Schwinger 1948

$$a^{(4)}_{\ell\,\text{universal}}[1-7] = \left[\frac{197}{144} + \frac{\pi^2}{12} - \frac{\pi^2}{2}\ln 2 + \frac{3}{4}\zeta(3)\right]\left(\frac{\alpha}{\pi}\right)^2$$

Peterman 57, Sommerfield 57

Universal 3–loop contribution:

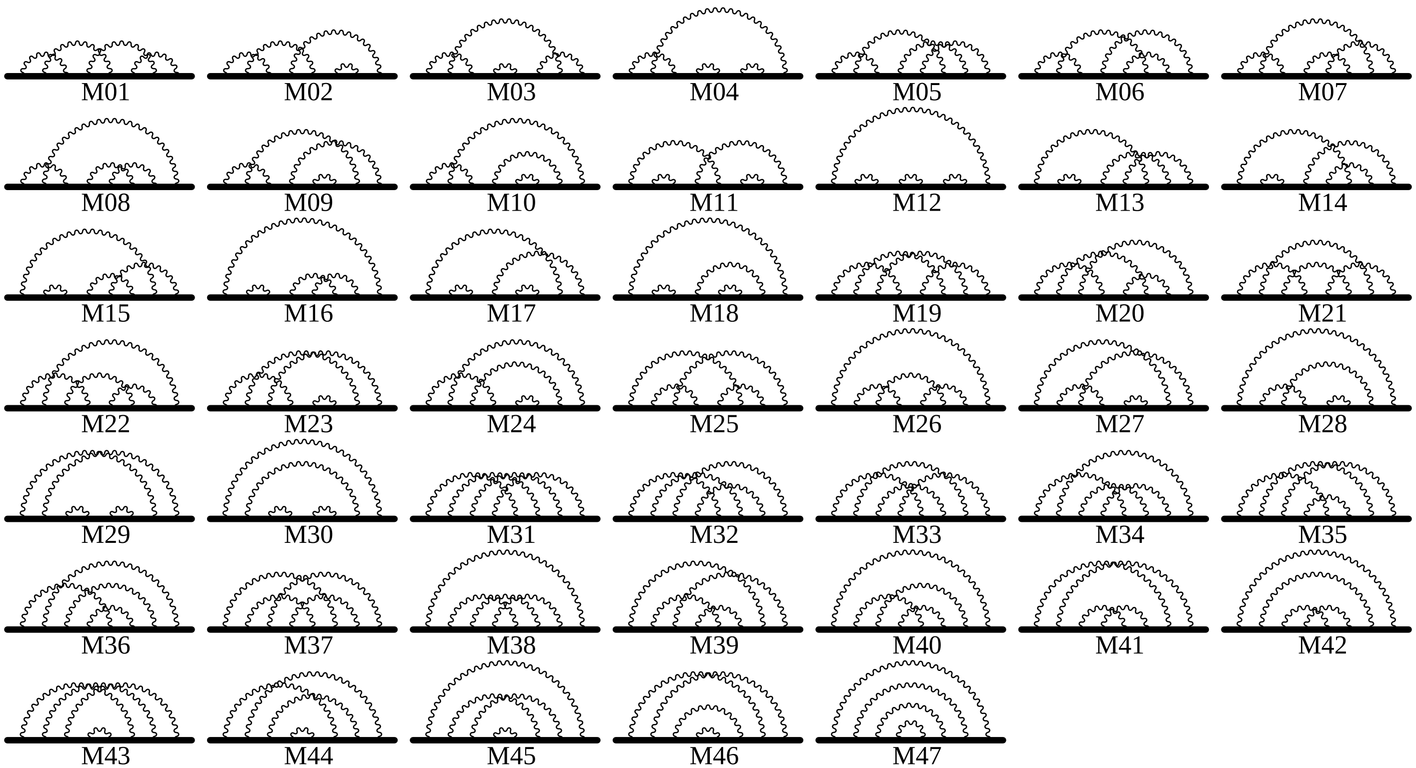
(Remiddi et al., Remiddi, Laporta 1996 [27 years for 72 diagrams])



$$
\begin{aligned}
a_{\ell\,\mathrm{uni}}^{(6)} \;=\; & \Big[\frac{28259}{5184}+\frac{17101}{810}\pi^2 \\
& -\frac{298}{9}\pi^2\ln 2+\frac{139}{18}\zeta(3) \\
& +\frac{100}{3}\left\{\mathrm{Li}_4(\frac{1}{2})\right. \\
& \left.+\frac{1}{24}\ln^4 2-\frac{1}{24}\pi^2\ln^2 2\right\} \\
& -\frac{239}{2160}\pi^4+\frac{83}{72}\pi^2\zeta(3) \\
& -\frac{215}{24}\zeta(5)\Big]\left(\frac{\alpha}{\pi}\right)^3
\end{aligned}
$$

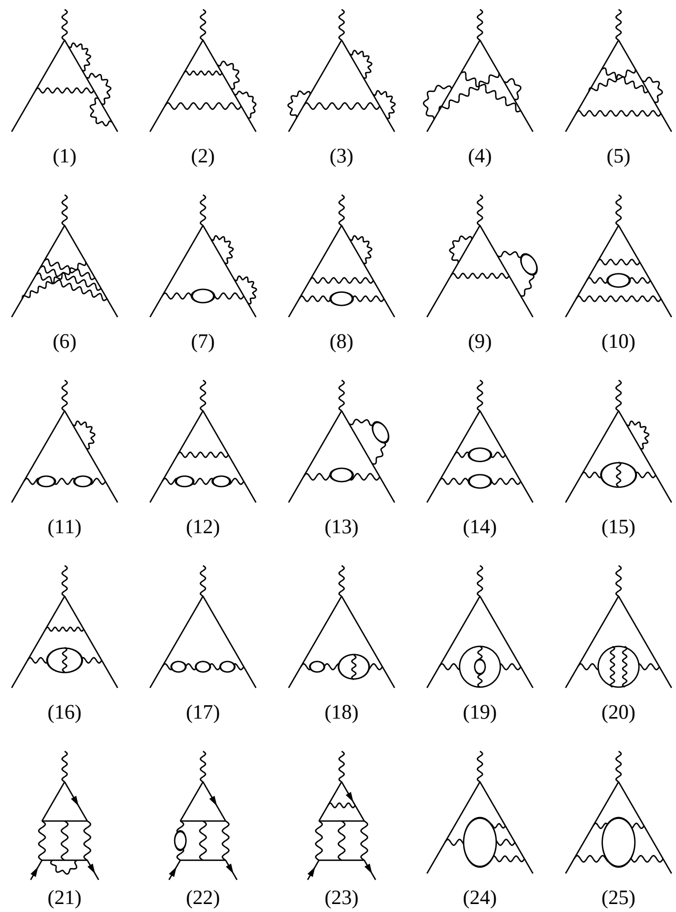
30 years of heroic efforts: demanding 4–loop contribution:

(Kinoshita et al., Aoyama, Hayakawa, Kinoshita and Nio 2007)



4-loop Group V diagrams. 47 self-energy-like diagrams of $M_{01} - M_{47}$ represent 518 vertex diagrams [by inserting the external photon vertex on the virtual muon lines in all possible ways].

Laporta's quasi exact universal 4–loop result



Sample diagrams of the 25 gauge-invariant subsets of the 891 universal $O(\alpha^4)$ contributions to a_ℓ.

Contribution to $A_1^{(8)}$ of the 25 gauge-invariant sets
amounts to know a_e^{QED} and $a_\mu^{\mathrm{QED\ universal}}$ to $O(\alpha^4)$ exactly
confirming Aoyama, Kinoshita & Nio numerical results

1	- 1.9710756168358189436456996553372644069880
2	- 0.1424873797998721572359452916848573709994
3	- 0.6219210635350725221040912234793176435540
4	1.0866983944758186876019614046906009723730
5	- 1.0405424100125820125394386209942499550940
6	0.5124620479679868704799540309091944655650
7	0.6904483475912615015281016003548025177320
8	- 0.0563360901705333159109594399102505959390
9	0.4092170284791885865905538336146384354250
10	0.3743579348118999490819538554149435787590
11	- 0.0913058400686967734264795669457888264810
12	0.0178536865498085781106917480565656491680
13	- 0.0341793760785627292101918809967262185800
14	0.0065041483818146409903657618974258022880
15	- 0.5724718621947819161527508499451810373110
16	0.1519895996858196396252805161065130420700
17	0.0008768658588899906979137489397137261650
18	0.0153252829020133808444974713451603186730
19	0.0111309139875173888309565009205701481230
20	0.0495132025595262351104722346512048517100
21	- 1.1388228764599745055631544311811117074240
22	0.5988420720314218204646495132017477278360
23	0.8222844858110343467198940487995984226060
24	- 0.8726573920771315179784019823814156103840
25	- 0.1179498687874207970627804934863463398290

HLbL Resonance Lagrangian vs Dispersive Approaches

Compare

e.g.	$a_\mu^{\rm HLbL}[{\rm RLA}] = 103(29) \times 10^{-11}$	(RLA, my book)
vs	$a_\mu^{\rm HLbL}[{\rm DRA}] = \;\; 92(19) \times 10^{-11}$	(DRA WP)
vs	$a_\mu^{\rm HLbL}[{\rm LQC}] = 107(14) \times 10^{-11}$	(LQCD Mainz)

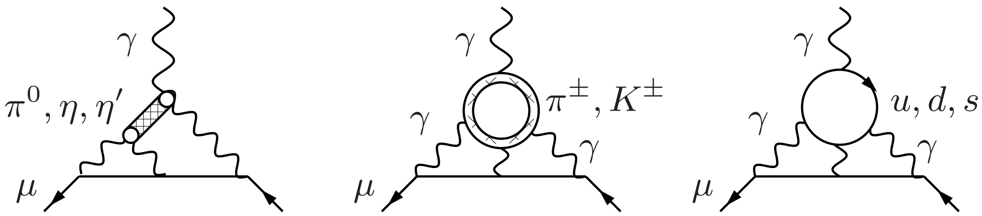
❒ besides lattice, the two approaches are not very different after all

❒ general data-driven DRA (Colangelo et al. 2015) reveals 19 essentially independent amplitudes (form-factors) which requires as many different channels to be measured (Bjinens et al. estimates 32)

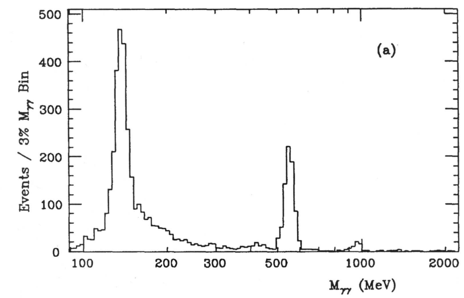
❒ only very few of the FFs are experimentally known, some single tag $\gamma\gamma^* \to \pi^0,\ \pi^+\pi^-,\ \pi^0\pi^0, \eta, f_1, \cdots$ All have been used by the RLA estimates as well.

❒ on experimental side $\gamma\gamma \to \mathrm{hadrons}$ is very difficult. Is subprocess of $e^+e^- \to e^+e^-\mathrm{hadrons}$

❒ fortunately dominated by single particle exchanges!

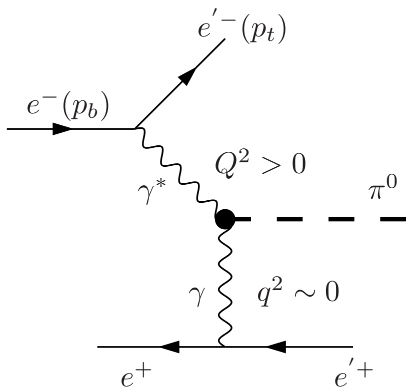


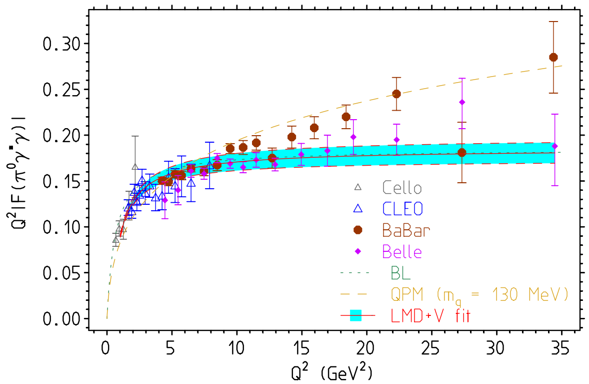
(a) [L.D.] (b) [L.D.] (c) [S.D.]

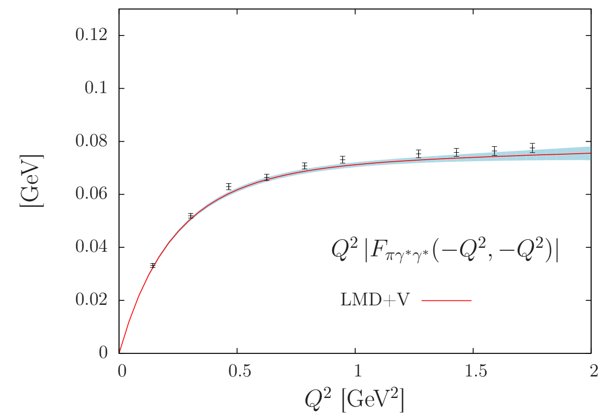


L: HLbL classes. R: HLbL facts, pronounced non-perturbative physics.

Gérardin, Meyer, Nyffeler 16





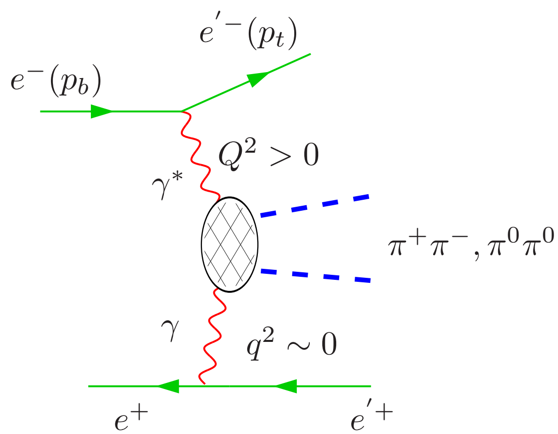


CELLO and CLEO measurement of the π^0 form factor $\mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2, -Q^2, 0)$ at high space–like Q^2. Outdated by BaBar? Belle conforms with theory expectations!. First measurements of $\mathcal{F}_{\pi^0*\gamma\gamma}(m_\pi^2; -Q^2, -Q^2)$ in lattice QCD

More recent LQCD results by RBC Collab.

Tree level: signal diagrams obscured by substantial background

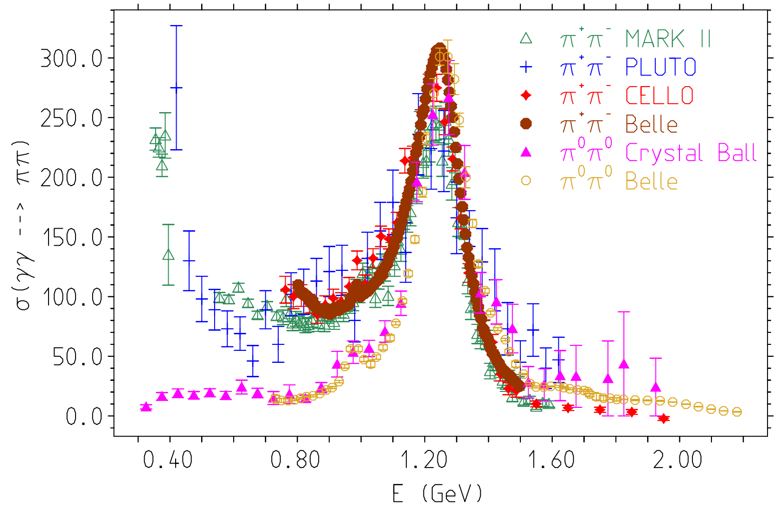
$e^+e^- \to e^+e^-\pi^0$	10 diagrams;	to extract $e^+e^- \to \gamma^*\gamma\pi^0 \to e^+e^-\pi^0$
$e^+e^- \to e^+e^-\pi^+\pi^-$	146 diagrams;	to extract $e^+e^- \to \gamma^*\gamma\pi^+\pi^- \to e^+e^-\pi^+\pi^-$
$e^+e^- \to e^+e^-\pi^0\pi^0$	60 diagrams;	to extract $e^+e^- \to \gamma^*\gamma\pi^0\pi^0 \to e^+e^-\pi^0\pi^0$



$$\frac{d\sigma_{\gamma^*\gamma}}{d|\cos\theta^*|} = \frac{d^3\sigma_{ee}}{dWd|\cos\theta^*|dQ^2}\frac{f}{2\frac{d^2L_{\gamma^*\gamma}}{dWdQ^2}(1+\delta)(\varepsilon/\varepsilon')\varepsilon'}.$$

The factors δ, ε, and f correspond to the radiative correction and ε' to the tagging efficiency.

Extracting $\gamma^*\gamma \to \pi\pi$ from $e^+e^- \to e^+e^-\gamma^*\gamma \to e^+e^-\pi\pi$ in a single tag experiment like Belle. Denoted by θ^* is the production angle of one of the π's

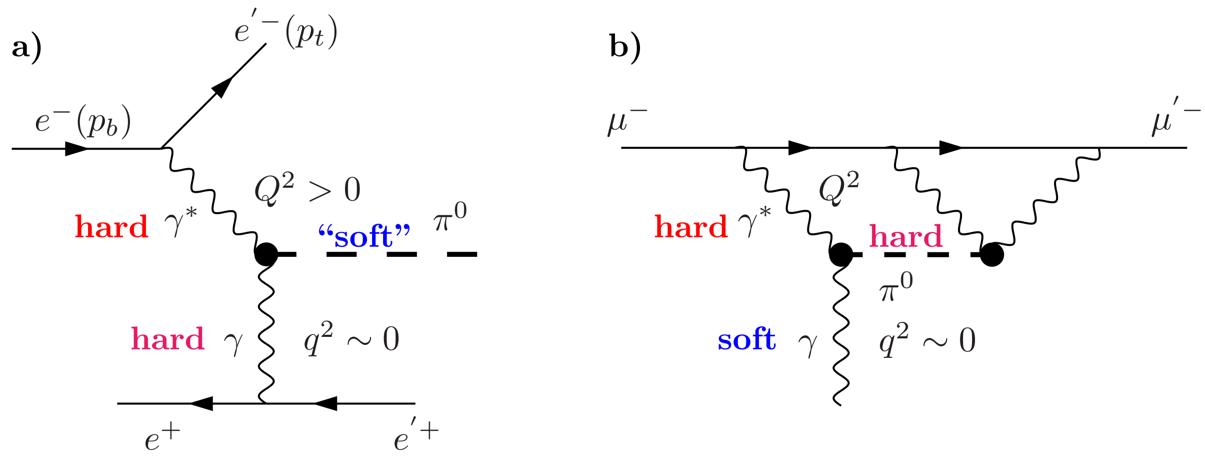


Di-pion production in $\gamma\gamma$ fusion. At low energy, we have direct $\pi^+\pi^-$ production and by strong rescattering $\pi^+\pi^- \to \pi^0\pi^0$, however, with a very much suppressed rate. Above about 1 GeV sQED is invalid, resolved $q\bar{q}$ couplings are seen. The $\pi^0\pi^0$ cross-section in this figure is enhanced by the isospin symmetry factor 2, by which it is reduced in reality.

❒ The DRA to HLbL is a great achievement providing a clear general setup to implement properly the available data. Unfortunately, the experimental database is not yet satisfactory and its improvement is a very challenging task for the future.

❒ In the end, LQCD must be the solution, but again HLbL is much more demanding than HVP, although a 10 % accuracy would be sufficient here.

A kinematics issue for HLbL:



a) Measurement of the π^0 form factor $\mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2,-Q^2,0)$ at high space–like Q^2,
b) needed at external vertex is $\mathcal{F}_{\pi^{0*}\gamma^*\gamma}(-Q^2,-Q^2,0)$ as $q_\gamma^\mu \approx 0$.

- in any case analytic continuation required, from real to virtual pion!

- actually $\pi^0\gamma^*\gamma^*$ (double tag) available from LQCD Mainz and RBC

A Comparison of WP25, WP20 and my Book:

Contribution	WP25	WP20	myBook17
HVP LO (lattice)	7132(61)	7116(184)	–
HVP LO (e^+e^-)	6947.9(41.8)	6931(40)	6894.6(32.5)
HVP LO ($e^+e^- + \tau$)	7045.0(62.0)	–	–
HVP NLO (e^+e^-)	−99.6(1.3)	−98.3(7)	−99.27(0.67)
HVP NNLO (e^+e^-)	12.4(1)	12.4(1)	12.24(0.10)
HLbL (pheno)	103.3(8.8)	92(19)	103.4(28.8)
HLbL NLO (pheno)	2.6(6)	2(1)	3(2)
HLbL (lattice)	122.5(9.0)	82(35)	–
HLbL (pheno + lattice)	112.6(9.6)	90(17)	–
QED	116 584 718.8(2)	116 584 718.931(104)	116 584 718.86(03)
EW	154.4(4)	153.6(1.0)	153.6(1.1)
HVP (LO + NLO + NNLO)	7045(61)	6845(40)	6807.6(31.9)
HLbL (pheno + lattice + NLO)	115.5(9.9)	92(18)	–
Total SM Value	116 592 033(62)	116 591 810(43)	116 591 783(35)

Comparison WP25, WP20 and my book (in units of 10^{-11}). “HLbL (lattice)” is WP20 + charm-loop. “HVP (LO + NLO + NNLO)” combines from HVP LO (lattice) [WP25] and HVP LO (e^+e^-) [WP20].

References

[1] G. W. Bennett et al. [Muon g-2 Collab.], Phys. Rev. D **73** (2006) 072003

[2] B. Abi et al. [Muon g-2], Phys. Rev. Lett. **126**, 141801 (2021)

[3] D. P. Aguillard et al. [Muon g-2], Phys. Rev. Lett. **131**, 161802 (2023); [arXiv:2506.03069 [hep-ex]] (2025).

[4] T. Aoyama, N. Asmussen, M. Benayoun, J. Bijnens, T. Blum, M. Bruno, I. Caprini, C. M. Carloni Calame, M. Cè and G. Colangelo, et al. Phys. Rept. **887**, 1-166 (2020) doi:10.1016/j.physrep.2020.07.006 [arXiv:2006.04822 [hep-ph]].

[5] R. Aliberti, T. Aoyama, E. Balzani, A. Bashir, G. Benton, J. Bijnens, V. Biloshytskyi, T. Blum, D. Boito and M. Bruno, et al. [arXiv:2505.21476 [hep-ph]].

[6] F. Jegerlehner, Springer Tracts Mod. Phys. **274**, pp.1-693 (2017) Springer, 2017, doi:10.1007/978-3-319-63577-4

[7] A. Boccaletti, S. Borsanyi, M. Davier, Z. Fodor, F. Frech, A. Gerardin, D. Giusti, A. Y. Kotov, L. Lellouch and T. Lippert, et al. [arXiv:2407.10913 [hep-lat]].

[8] L. Morel, Z. Yao, P. Cladé and S. Guellati-Khélifa, Nature **588**, no.7836, 61-65 (2020) doi:10.1038/s41586-020-2964-7

[9] X. Fan, T. G. Myers, B. A. D. Sukra and G. Gabrielse, Phys. Rev. Lett. **130**, no.7, 071801 (2023) doi:10.1103/PhysRevLett.130.071801 [arXiv:2209.13084 [physics.atom-ph]].

[10] S. Volkov, Phys. Rev. D **110**, no.3, 036001 (2024) doi:10.1103/PhysRevD.110.036001 [arXiv:2404.00649 [hep-ph]].

[11] T. Aoyama, M. Hayakawa, A. Hirayama and M. Nio, [arXiv:2412.06473 [hep-ph]].