Lepton flavor mixing and mass order are preserved in the 3HDM under flavor symmetry

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Introduction:

Standard Model is not flavor blind, however Yukawa matrices are unknown.
Lepton sector: $G_F = U(3) \times U(3)$
(or $G_F = U(3) \times U(3) \times U(3)$ if RH neutrinos exist).
No constraints on masses and neutrino mixing.
Fermion masses > 0 due to the Higgs doublet of the SM.

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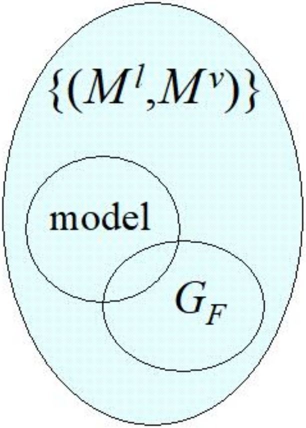
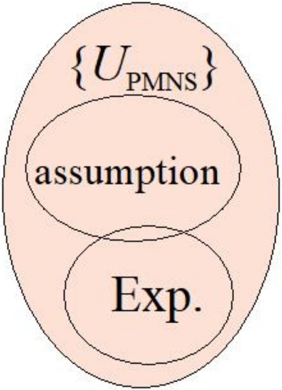
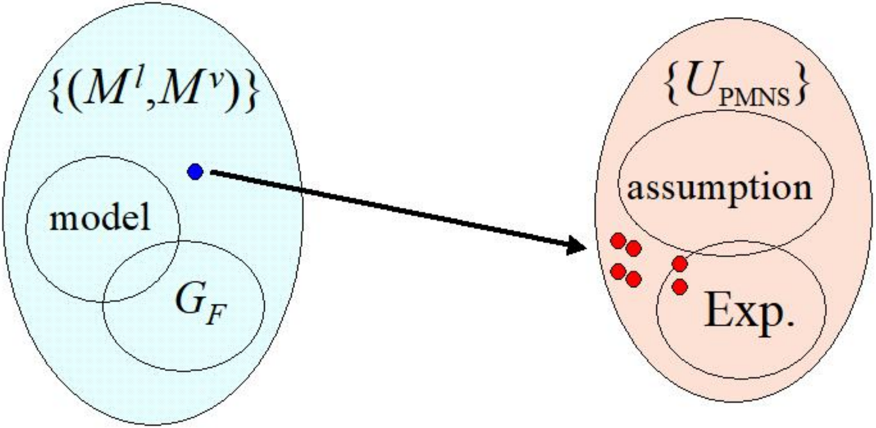
G_F cannot be $(U(3))^3$

Can we find any nontrivial $G_f \subset (U(3))^3$?
Or is it completely broken?

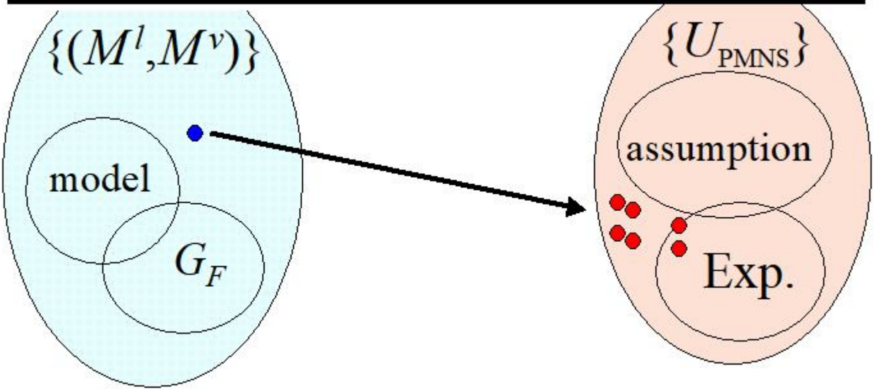

{(M^l,M^ν)}
model
G_F

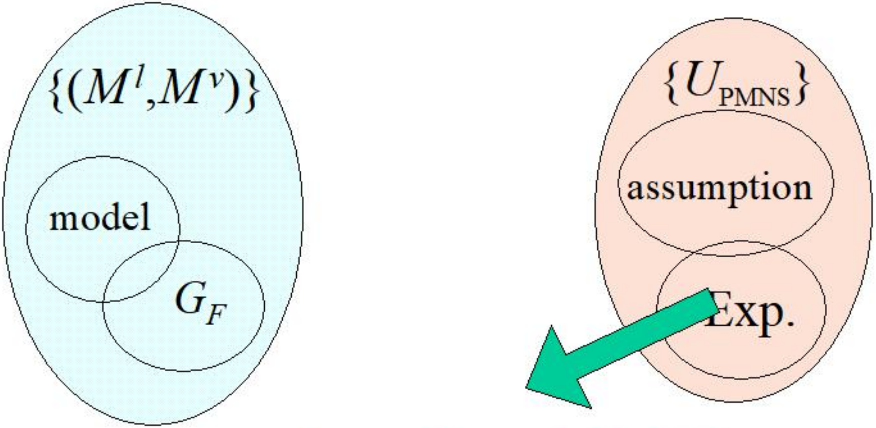

{U_PMNS}
assumption
Exp.


$\{(M^l, M^\nu)\}$
model
G_F
$\{U_{\mathrm{PMNS}}\}$
assumption
Exp.

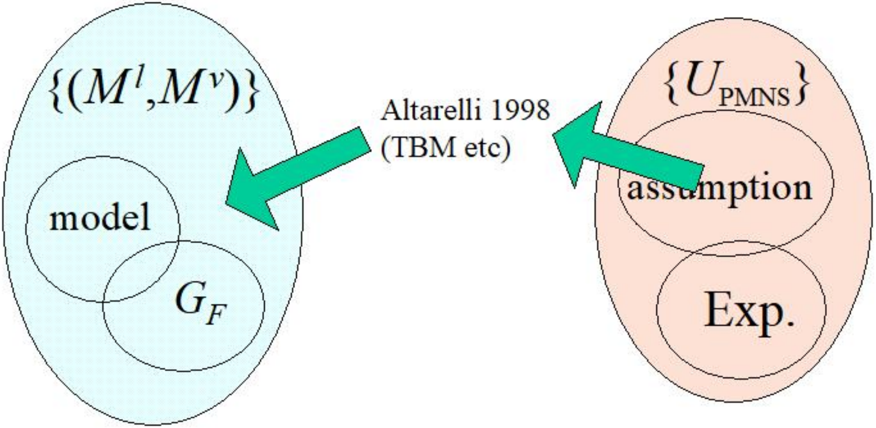
$$M^l = V^l M^l_{diag} U^{l\dagger} \longrightarrow U_{\mathbf{P}MNS} = U^{l\dagger} U^\nu$$

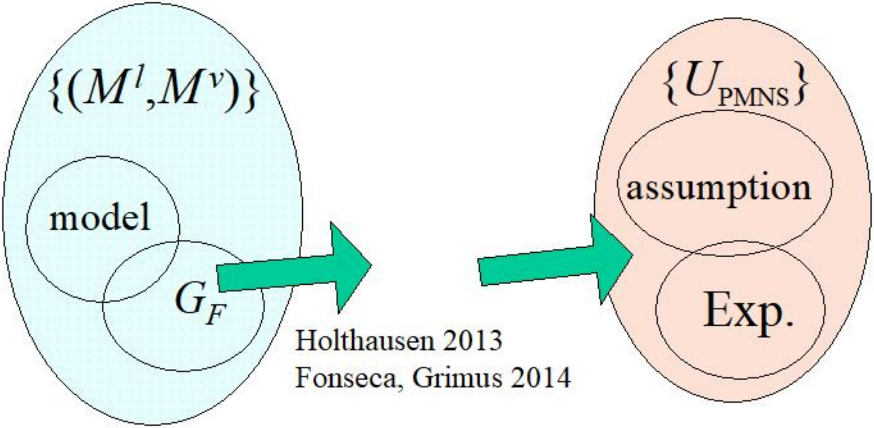
$$M^\nu = V^\nu M^\nu_{diag} U^{\nu\dagger}$$

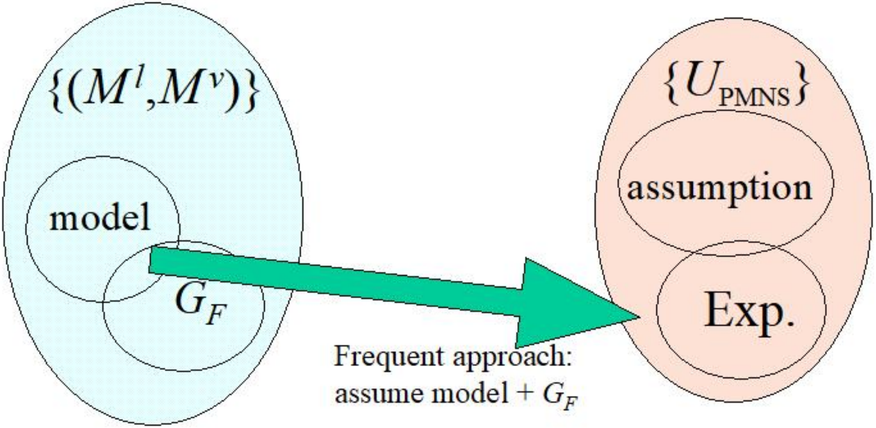




Kielanowski 2024, ($U_{\mathrm{PMNS}} = R_\nu\ D(\alpha,\beta,\gamma)\ R_l$ .....)
Karmakar 2024, (correlations among exp. parameters)


$\{(M^l, M^\nu)\}$
model
G_F
Altarelli 1998
(TBM etc)
$\{U_{\rm PMNS}\}$
assumption
Exp.


$\{(M^l, M^\nu)\}$
model
G_F
Holthausen 2013
Fonseca, Grimus 2014
$\{U_{\rm PMNS}\}$
assumption
Exp.


$\{(M^l, M^\nu)\}$
model
G_F
$\{U_{\text{PMNS}}\}$
assumption
Exp.
Frequent approach:
assume model + G_F

Methodology and assumptions:

Model is SM + ν_{Ri} + two extra Higgs doublets (3HDM)

ν_i have Dirac nature

$U_{\mathrm{P}MNS}$ must reflect lepton mass order

Flavor vectors are: L_L, l_R, ν_R and $(\Phi_1 \Phi_2 \Phi_3)^T$

Only source of flavor symmetry breaking are the Yukawa matrices

Does any G_F survive EWSB?

3HDM Yukawa term

$$\mathcal{L}^l = \quad -\overline{L}_{\alpha L}(h_i^l)_{\alpha\beta}\tilde{\Phi}_i l_{\beta R} + \text{H.c.},$$

$i = 1..3$
27 terms $+$ H.c.

Relevant Yukawa terms

$$\mathcal{L} \supset \mathcal{L}^{l} + \mathcal{L}^{\nu} \qquad (+\mathcal{L}^{\nu L} + \mathcal{L}^{\nu R} + \mathcal{L}^{\nu M})$$

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$\mathcal{L}^{\nu L}$ is not EW gauge invariant
$\mathcal{L}^{\nu R}$ has no effect on phenomenology
$\mathcal{L}^{\nu M}$ Majorana term not taken into account

Central question (3HDM):

Does any finite $G_F > U(1)$ exist such that $(U(3))^3 \to G_F$ is within phenomenological bounds after EWSB?

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At first: Can G_F accommodate the <u>mass ratios</u> of the charged leptons <u>and</u> the neutrinos?

Note: Some studies neglect masses and focus on PMNS only!

$$\mathcal{L}^l = -\overline{L}_{\alpha L}(h_i^l)_{\alpha\beta}\tilde{\Phi}_i l_{\beta R} + \text{ H.c.},$$

A C B

(irreducible representations of G_F)

$$\mathbf{3} \times \mathbf{3}' \times \mathbf{3}'' = n^l \mathbf{1} + \ldots$$

n^l is the number of possible contractions, that is the number of solutions for h^l of

$$(C^\dagger \otimes B^T \otimes A^\dagger) vec(h^l) = vec(h^l).$$

(similar for n^ν)

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h^l and h^ν define the mass matrices:

Solutions h^l, h^ν define the mass matrices

$$
\begin{aligned}
M^l &= -\frac{1}{\sqrt{2}} v_i^* h_i^l \\
M^\nu &= \frac{1}{\sqrt{2}} v_i h_i^\nu
\end{aligned}
$$

(v_i is VEV of ϕ_i)

M^l and M^ν have one of patterns P1 ... P7:

$$\mathrm{P1} = \begin{pmatrix} 0 & v_1 & 0 \\ 0 & 0 & v_2 \\ v_3 & 0 & 0 \end{pmatrix}$$

$$\mathrm{P2} = \begin{pmatrix} 0 & v_3 & v_2 \\ v_3 & 0 & v_1 \\ v_2 & v_1 & 0 \end{pmatrix}$$

$$\mathrm{P3} = \begin{pmatrix} v_2 & v_2 & v_2 \\ v_3 & v_3 & v_3 \\ v_1 & v_1 & v_1 \end{pmatrix}$$

$$P4 = \begin{pmatrix} v_2 & v_3 & v_1 \\ v_2 & v_3 & v_1 \\ v_2 & v_3 & v_1 \end{pmatrix}$$

$$P5 = \begin{pmatrix} v_1 & v_2 & v_3 \\ v_3 & v_1 & v_2 \\ v_2 & v_3 & v_1 \end{pmatrix}$$

$$P6 = \begin{pmatrix} 0 & v_1 + v_2 + v_3 & 0 \\ 0 & 0 & v_1 + v_2 + v_3 \\ v_1 + v_2 + v_3 & 0 & 0 \end{pmatrix}$$

$$P7 = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$$

Table: Occurrences of patterns P1 to P7 of $\{M^l, M^\nu\}$ of the 1-dimensional solutions, $n^l = 1, n^\nu = 1$, $|G| \leq 600$

$l \backslash \nu$	P1	P2	P3	P4	P5	P6	P7
P1	9559	0	238	0	0	0	0
P2	0	9	0	0	0	0	1
P3	237	0	108	0	0	0	0
P4	0	0	0	494	0	0	0
P5	0	0	0	0	7784	0	0
P6	0	0	0	0	0	498	0
P7	0	1	0	0	0	0	1515

$$\mathbf{3} \times \mathbf{3}' \times \mathbf{3}'' = n^l \mathbf{1} + \ldots$$
$$\mathbf{3} \times \mathbf{3}' \times \mathbf{3}'' = n^\nu \mathbf{1} + \ldots$$

Finite groups with $|G| \leq 600$ generate 28,807 inequivalent pairs $\{M^l, M^\nu\}$

$n^l \backslash n^\nu$	1	2	3
1	20,437	3816	0
2	3816	729	0
3	0	0	0

Group A_4 permits $n^l = n^\nu = 2$

$$M^l \sim \begin{pmatrix} 0 & \lambda_2 v_3 & \lambda_1 v_2 \\ \lambda_1 v_3 & 0 & \lambda_2 v_1 \\ \lambda_2 v_2 & \lambda_1 v_1 & 0 \end{pmatrix}$$

$$M^\nu \sim \begin{pmatrix} 0 & \mu_2 v_3 & \mu_1 v_2 \\ \mu_1 v_3 & 0 & \mu_2 v_1 \\ \mu_2 v_2 & \mu_1 v_1 & 0 \end{pmatrix},$$

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Eigenvalues of <u>sum of matrices</u>: notoriously difficult
$\rightarrow$ resort to numerical analysis.

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$$M^{\nu D} \sim \begin{pmatrix} 0 & \mu_2 v_3 & \mu_1 v_2 \\ \mu_1 v_3 & 0 & \mu_2 v_1 \\ \mu_2 v_2 & \mu_1 v_1 & 0 \end{pmatrix},$$

This structure is unique among $|G| < 600$
(besides T_7 for $n^l = 1, n^\nu = 2$)

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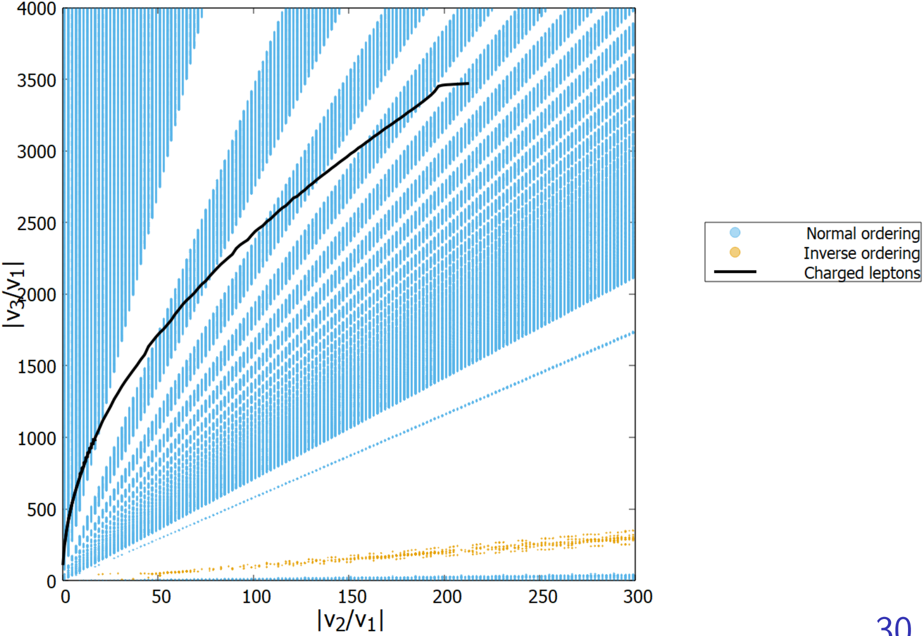
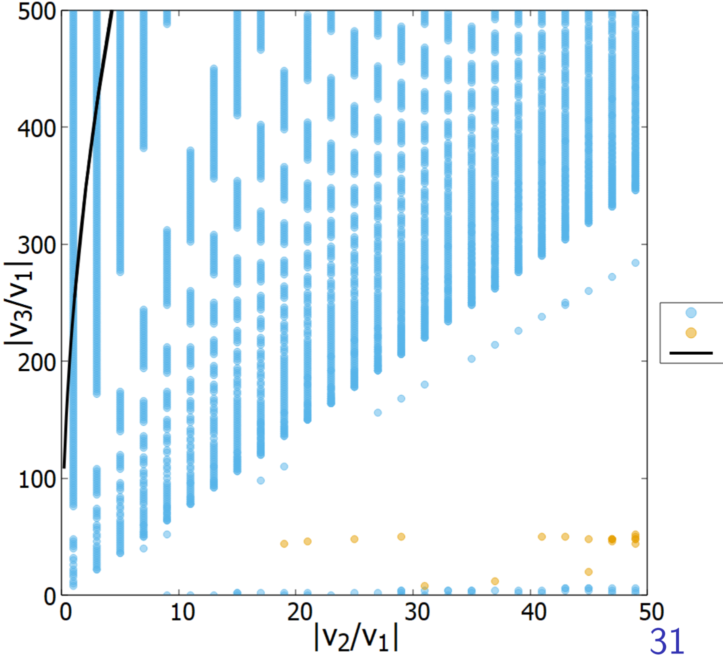
Free parameters are: $v_2, v_3, \lambda_2, \mu_2$ (8 real parameters).

$\chi^2(R_{NO}) < 1.0, \chi^2(R_{IO}) < 1.0$




500
400
300
200
100
0
0
10
20
30
40
50
|v3/v1|
|v2/v1|

Case: $n^l = n^\nu = 2$, $G_F = A_4$

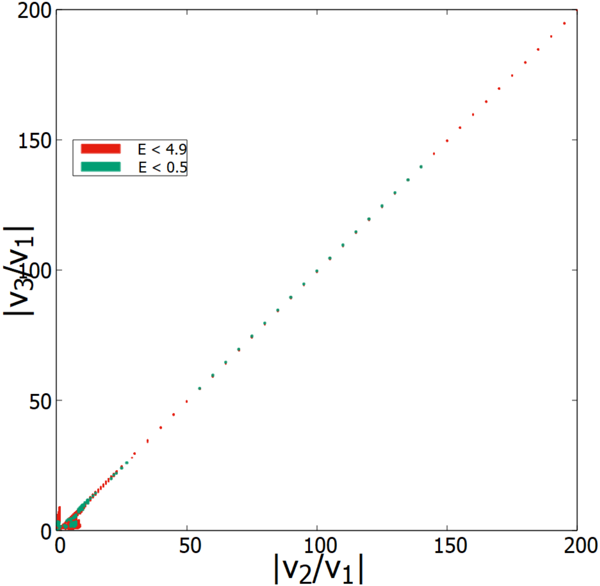
Masses of charged and neutral leptons (NO) can be accommodated in the 3HDM.

Neutrino masses:

$$
\begin{aligned}
m_1 &\approx 5.0 \times 10^{-6}\,\text{eV} \\
m_2 &\approx 0.86 \times 10^{-2}\,\text{eV} \\
m_3 &\approx 5.03 \times 10^{-2}\,\text{eV}.
\end{aligned}
$$

These values are within experimental bounds.

$$\chi^2(sin^2\theta_{12}) + \chi^2(sin^2\theta_{23}) + \chi^2(sin^2\theta_{13}) < 0.5$$



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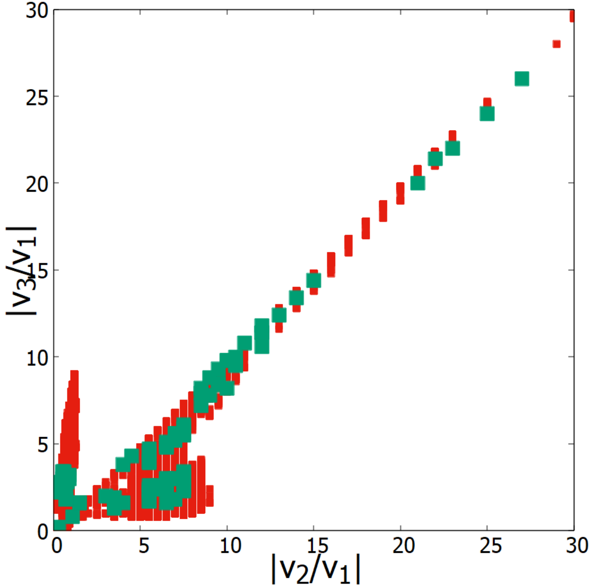
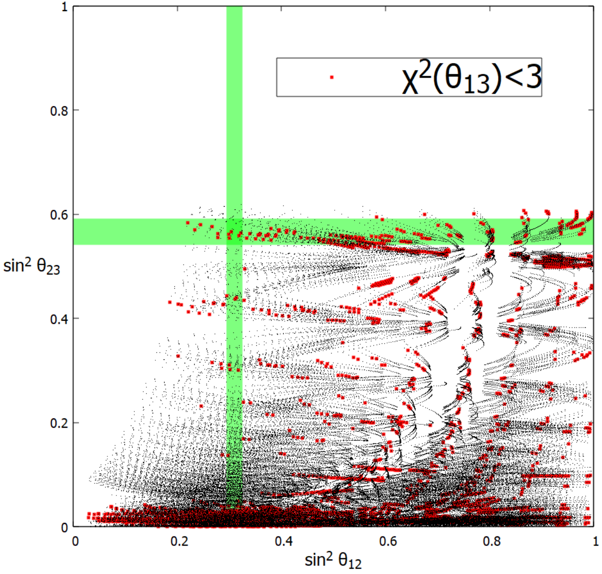
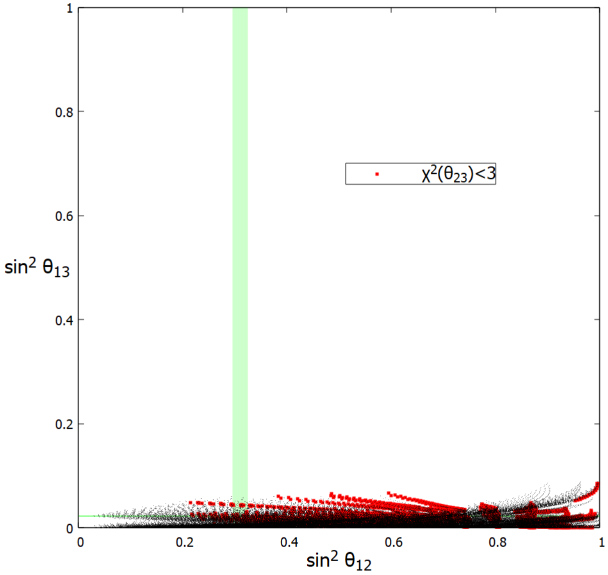
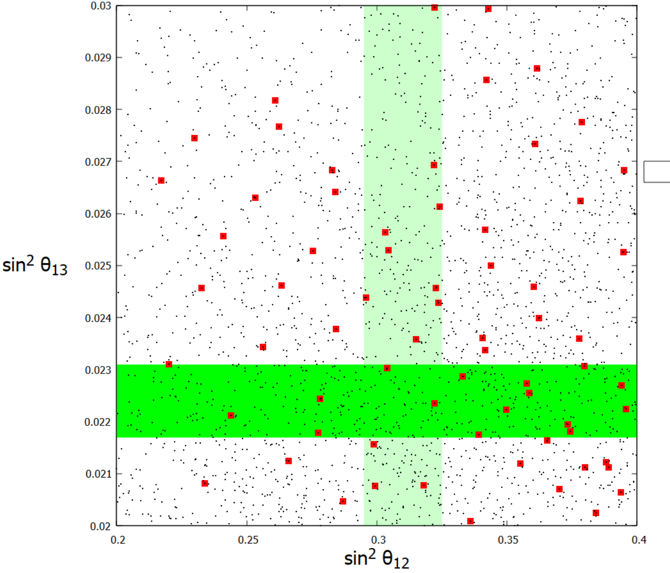


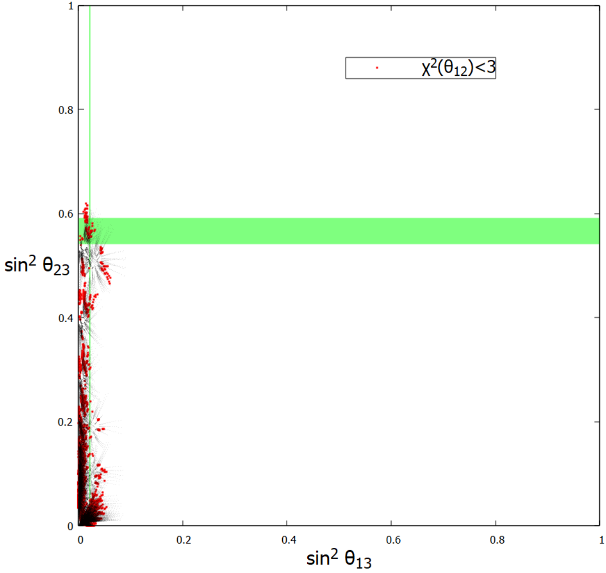


1
0.8
0.6
0.4
0.2
0
$\sin^2\theta_{23}$
0
0.2
0.4
0.6
0.8
1
$\sin^2\theta_{12}$
$\chi^2(\theta_{13})<3$

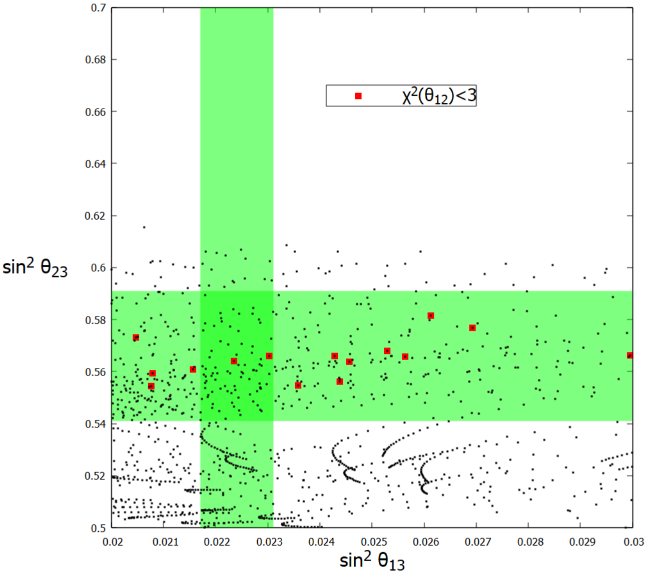

1
0.8
0.6
0.4
0.2
0
sin² θ13
χ²(θ23)<3
0
0.2
0.4
0.6
0.8
1
sin² θ12



0.03
0.029
0.028
0.027
0.026
0.025
0.024
0.023
0.022
0.021
0.02
$\sin^2\theta_{13}$
0.2
0.25
0.3
0.35
0.4
$\sin^2\theta_{12}$



1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1
sin² θ₂₃
sin² θ₁₃
χ²(θ₁₂)<3

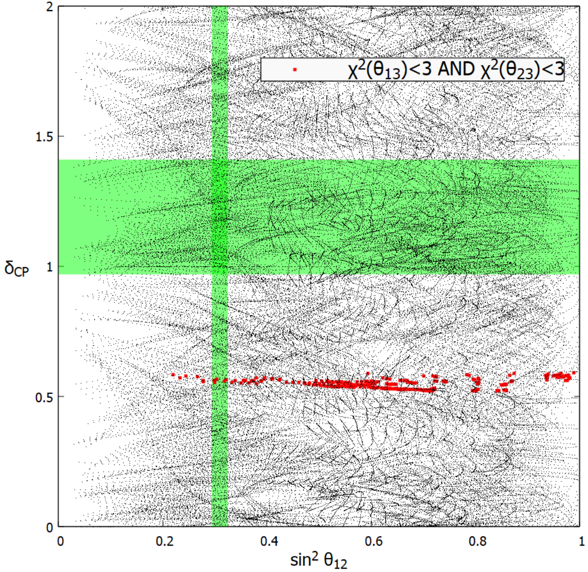


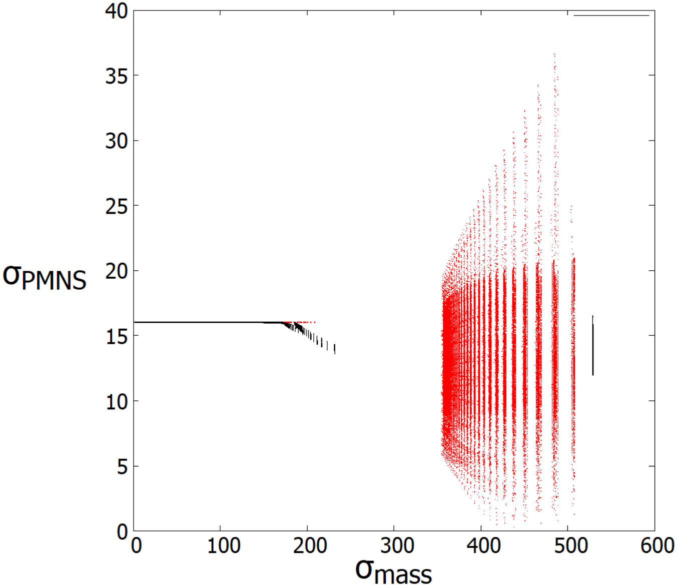
0.7
0.68
0.66
0.64
0.62
0.6
0.58
0.56
0.54
0.52
0.5
$\sin^2\theta_{23}$
0.02
0.021
0.022
0.023
0.024
0.025
0.026
0.027
0.028
0.029
0.03
$\sin^2\theta_{13}$
$\chi^2(\theta_{12})<3$

Case: $n^l = n^{\nu D} = 2$

PMNS Mixing angles can be accommodated
in the 3HDM, while respecting the lepton mass order

Calculated $\delta_{CP} \approx 0.6$, is 4σ away from 1.4.




40
35
30
25
20
15
10
5
0
σPMNS
0
100
200
300
400
500
600
σmass

Case: $n^l = n^{\nu D} = 2$, $G_F = A_4$

Observed in 8-dimensional parameter space:

calculated U_{PMNS} AND mass order are within exp. bounds

calculated U_{PMNS} OR mass ratios are within exp. bounds

calculated U_{PMNS} AND mass ratios are NOT within exp. bounds

calculated δ_{CP} is off by $\approx 4\sigma$

Viability of G_F

	mass order	mass ratios	mass order +PMNS	mass ratios +PMNS	G_F
1HDM (1,1) ($\sim$ SM)	✗	✗	✗	✗	
2HDM (1,1)	✓	✗	✗	✗	
3HDM (1,1)	✓	✗	✗	✗	
3HDM (1,(1,1))	✓	✓	✗	✗	T_7
3HDM (3,(1,1))	✓	✗	✗	✗	
3HDM ((1,1),1)	✓	✗	✗	✗	
3HDM ((1,1),3)	✓	✗	✗	✗	
3HDM ((1,1),(1,1))	✓	✓	✓	✗	A_4

Thank you!