

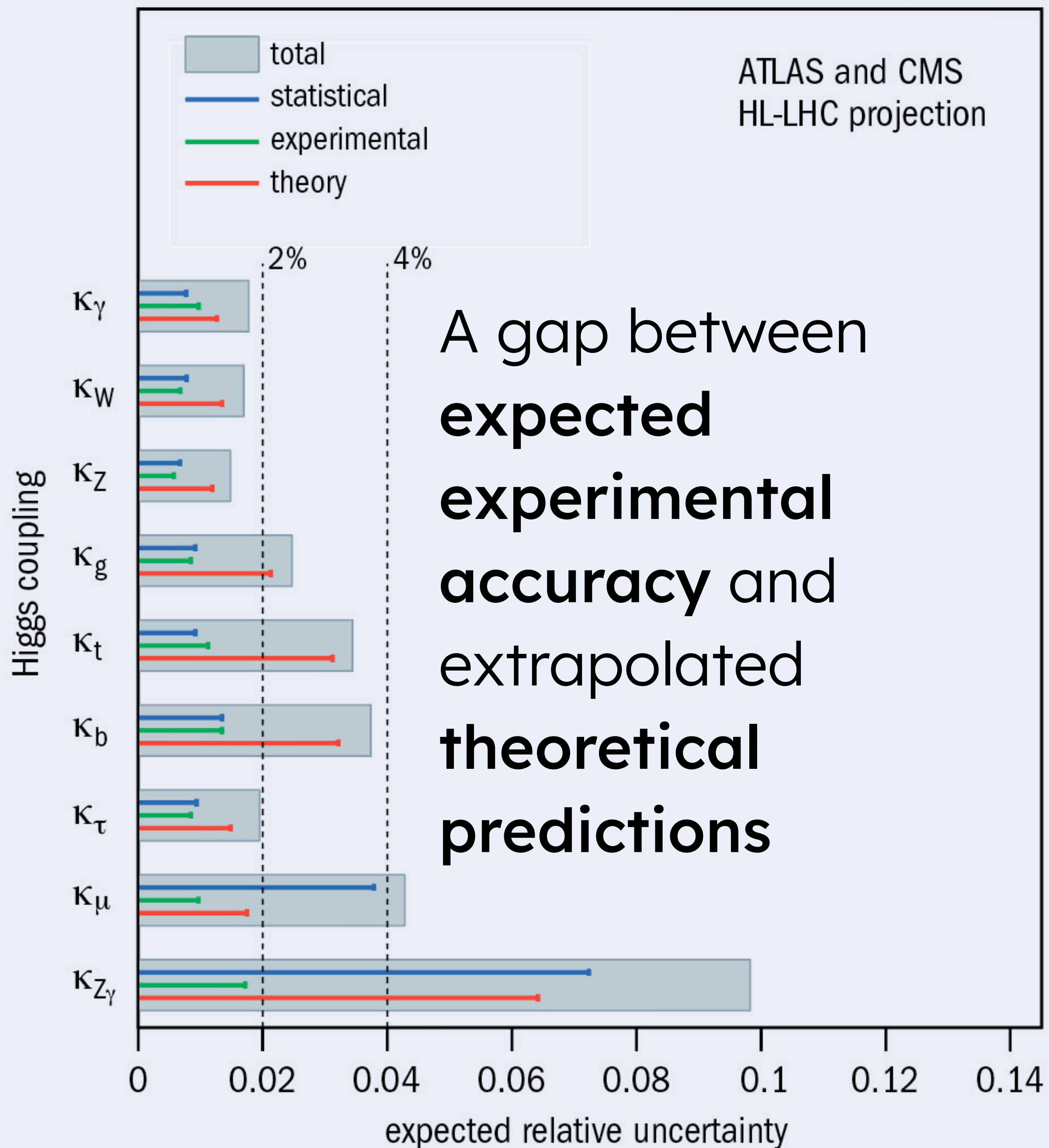
Qubits and Vacuum Amplitudes

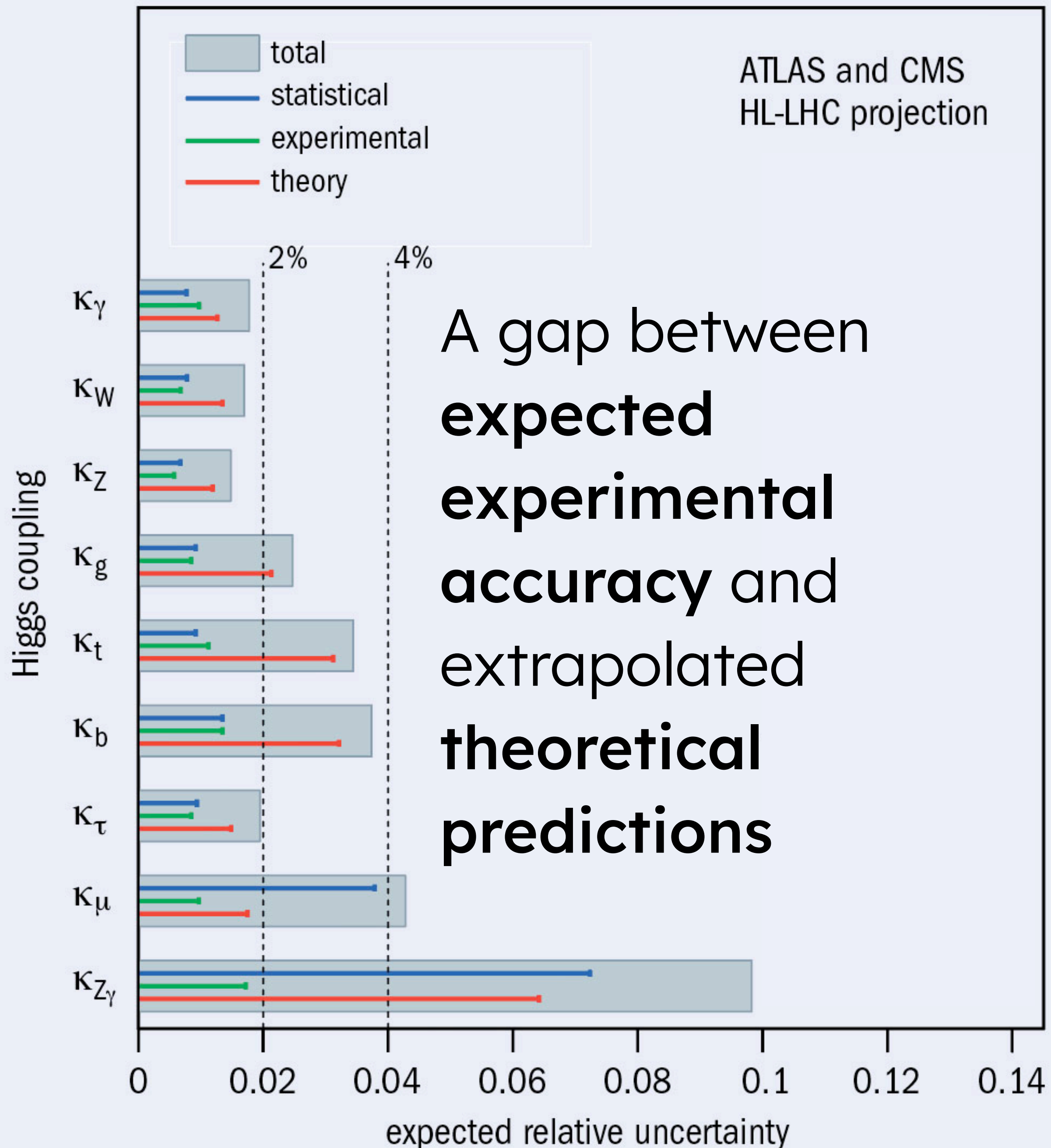
Germán RODRIGO

Matter To The Deepest

Katowice, Sep 15–19, 2025

Motivation





An imperative need to advance **theoretical predictions** for high-energy colliders beyond the current state of the art, to match precision measurements and unlock future discoveries

High-energy colliders
are quantum machines

$$\text{QM} \subset \text{QFT}$$

CMS

LHC

[Large Hadron Collider]

ALICE

ATLAS

High-energy colliders
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$$\text{QM} \subset \text{QFT}$$

The LHC is the
Largest Quantum Machine

CMS

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[Large Hadron Collider]

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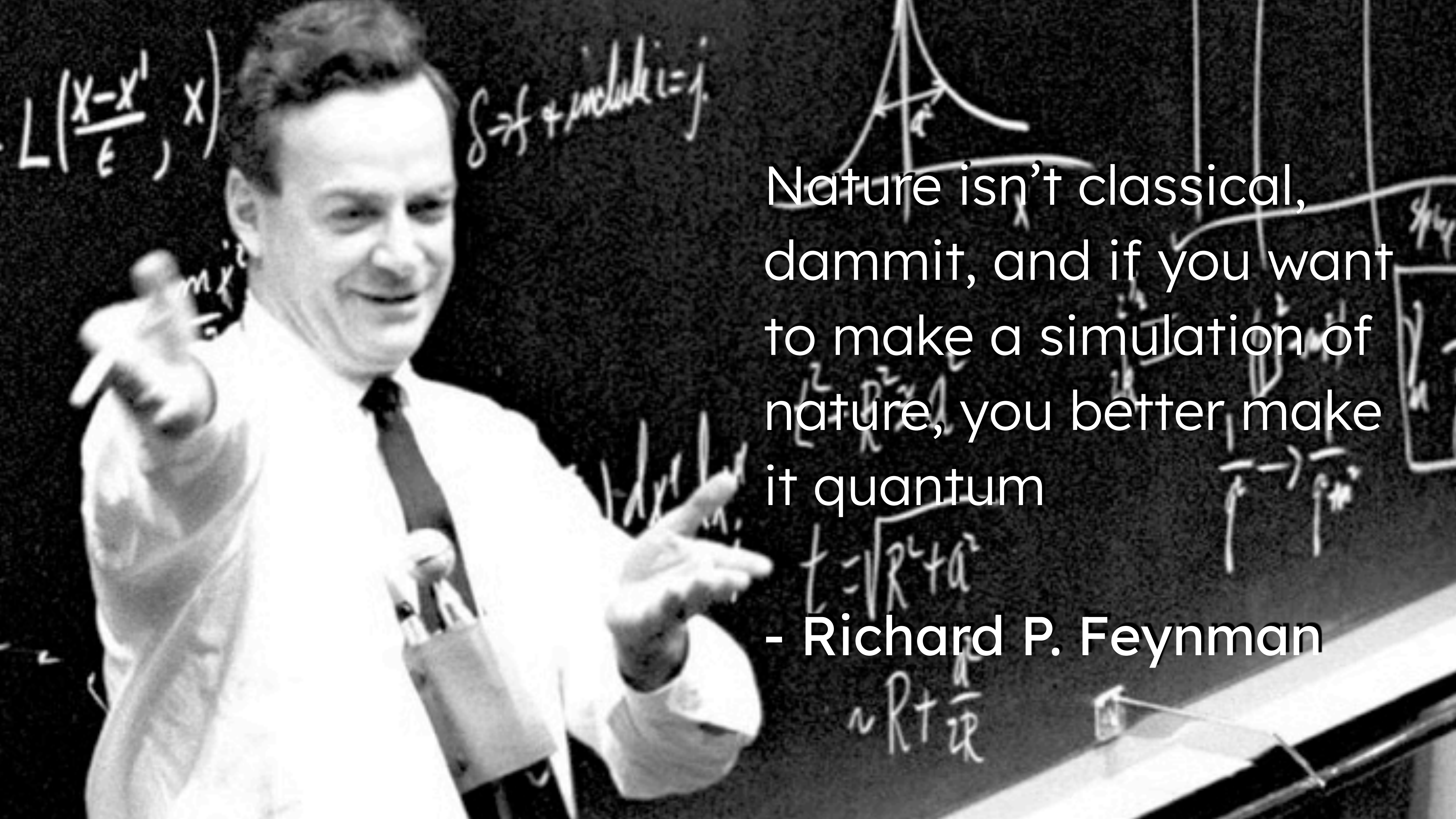
CERN is where the Bell
inequalities were born

CMS

LHC

[Large Hadron Collider]

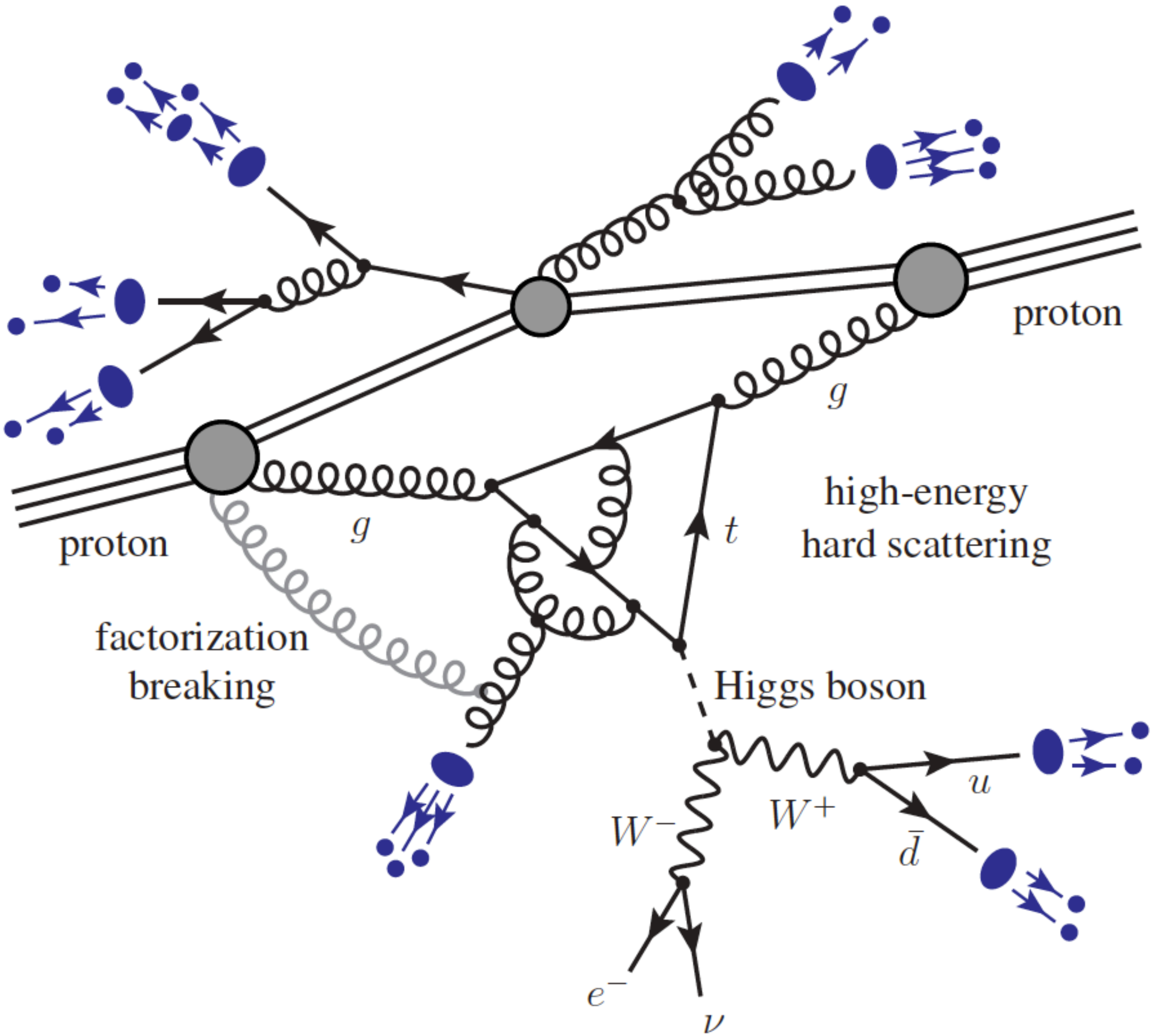
ATLAS



Nature isn't classical,
dammit, and if you want
to make a simulation of
nature, you better make
it quantum

- Richard P. Feynman

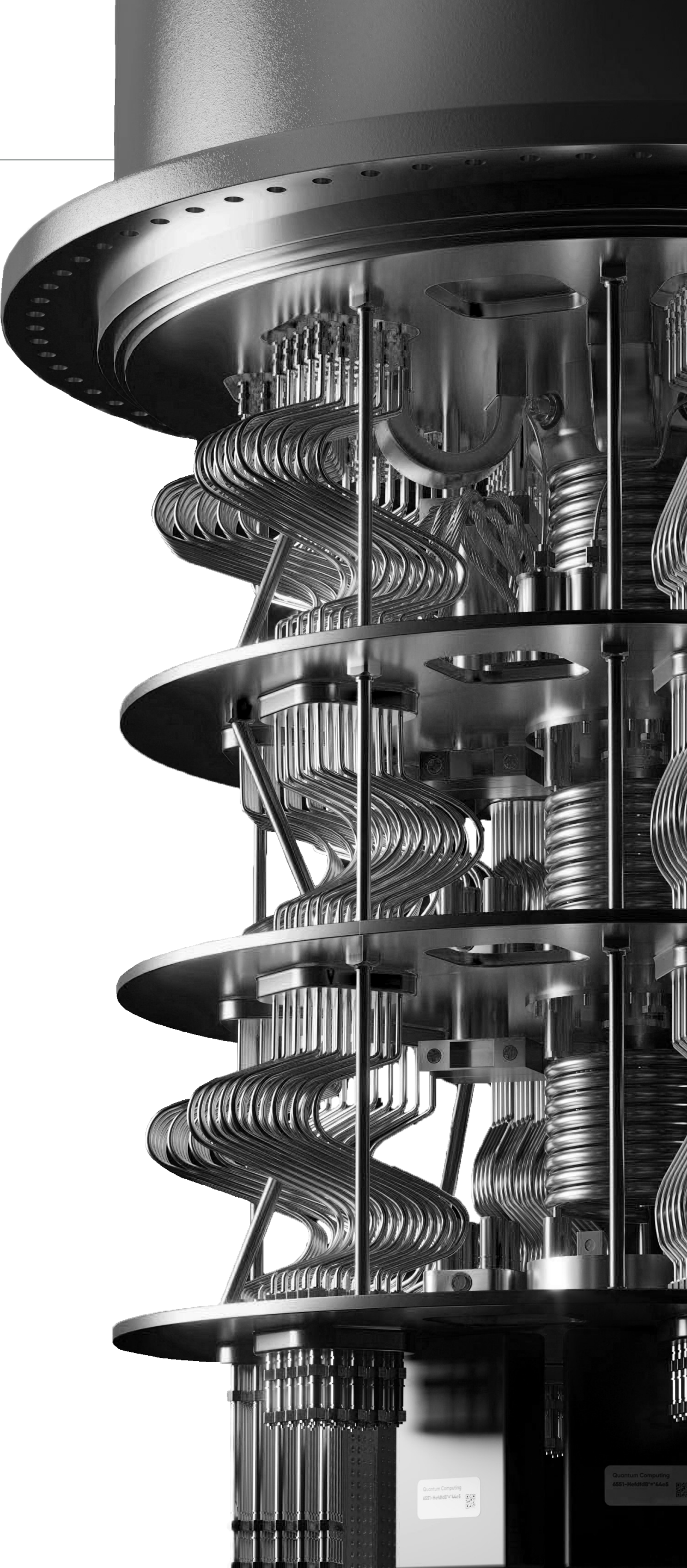
A QUANTUM EVENT GENERATOR AT HIGH PERTURBATIVE ORDERS?



HEP4QT



QT4HEP



Quantum at colliders

○ track reconstruction:

Mangano et al., [PRD 105, 076012 \(2022\)](#)
 Duckett, Facini, Jastrzebski, Malik, Scanlon, Rettie, [PRD 109, 052002 \(2024\)](#)
 Schwägerl, Issever, Jansen, Khoo, Kühn, Tüysüz, Weber, [2303.13249](#)

○ parton densities:

Pérez-Salinas, Cruz-Martínez, Alhajri, Carrazza, [PRD 103, 034027 \(2021\)](#)

○ fragmentation functions:

Martínez de Lejarza, Wu, Kyriienko, GR, Grossi, [2503.16073](#)

○ parton showers:

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○ quantum machine learning:

Guan, Perdue, Pesah, Schuld, Terashi, Vallecorsa, Vlimant, [MLST 2, 011003 \(2021\)](#)
 Wu et al., [JPG 48, 125003 \(2021\)](#)
 Felser, Trenti, Sestini, Gianelle, Zuliani, Lucchesi, Montangero, [npjQI 7, 111 \(2021\)](#)

○ Quantum integration:

Herbert, [Q6, 823 \(2022\)](#)
 Agliardi, Grossi, Pellen, Prati, [PLB 832, 137228 \(2022\)](#)
 Cruz-Martínez, Robbiati, Carrazza, [QST 9 \(2024\) 035053](#)
 Martínez de Lejarza, Cieri, Grossi, Vallecorsa, GR, [PRD 110 \(2024\) 074031](#), [QST 10 \(2025\) 025026](#)
 Williams, Pellen, [QST 10 \(2025\) 045017](#)
 Pyretzidis, Martínez de Lejarza, GR, [2506.19965](#)

○ tree-level helicity amplitudes:

Bepari, Malik, Spannowsky, Williams, [PRD 103, 076020 \(2021\)](#)

○ multiloop scattering amplitudes:

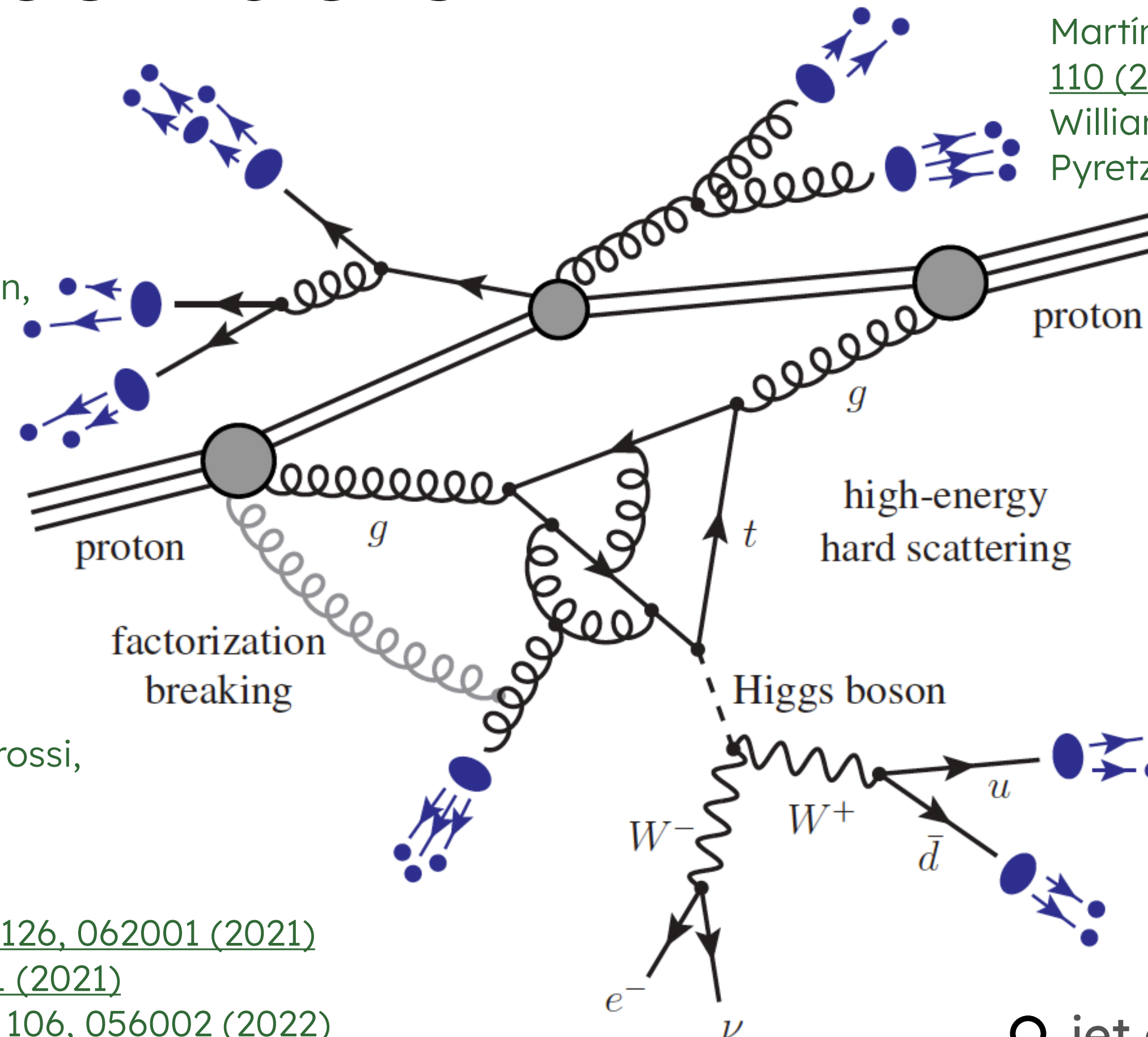
Ramírez, Rentería, GR, Sborlini, Vale Silva, [JHEP 2205, 100 \(2022\)](#)
 Clemente, Crippa, Jansen, Ramírez, Rentería, GR, Sborlini, Vale Silva, [PRD 108, 096035 \(2023\)](#)
 Ochoa, Uribe, Ramírez, GR, [2508.04019](#)

○ jets in a medium:

Barata, Du, Li, Qian, Salgado, [PRD 106, 074013 \(2022\)](#)
 Barata, Salgado, [EPJC 81, 862 \(2021\)](#)

○ jet clustering:

Wei, Naik, Harrow, Thaler, [PRD 101, 094015 \(2020\)](#)
 Pires, Bargassa, Seixas, Omar, [2101.05618](#)
 Pires, Omar, Seixas, [2012.14514](#)
 Martinez de Lejarza, Cieri, GR, [PRD 106, 036021 \(2022\)](#)



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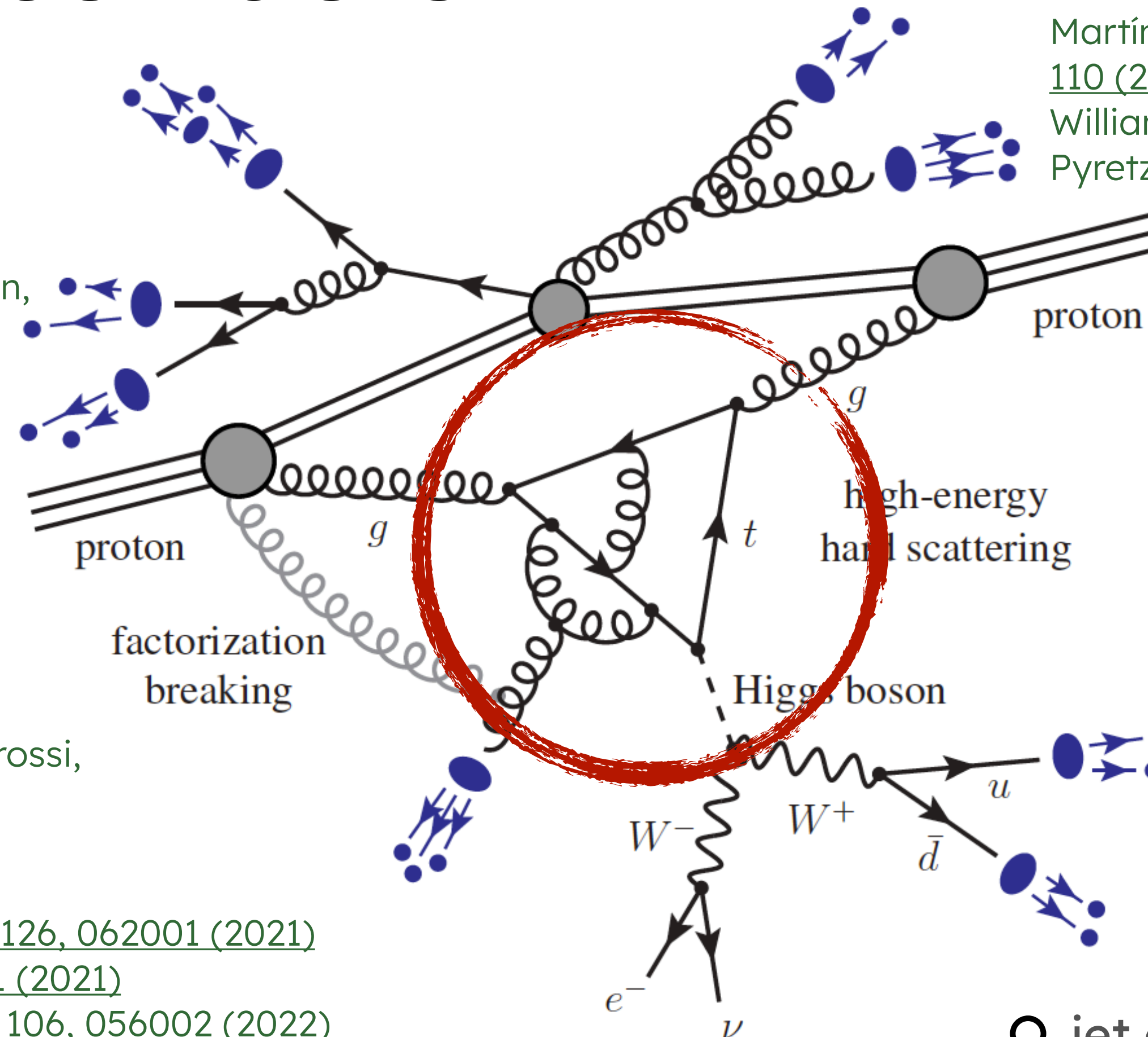
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A Feynman propagator is a sort of qubit

- A Feynman propagator describes a **quantum superposition** of propagation in both directions

$$G_F(q_i) = \frac{1}{q_i^2 - m_i^2 + i0} \equiv \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle)$$

- A Feynman diagram is a superposition of 2^n states



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- A Feynman diagram is a superposition of 2^n states
- If a particle returns to the point of emission: it travels back in time and thus **breaks causality**
 $\equiv$ cyclic configurations are nonphysical
- Causal configurations of Feynman diagrams are **Directed Acyclic Graphs (DAG)** in graph theory

Grover's based quantum algorithms

- **The $|e\rangle$ register** encodes the states of the edges/internal propagators: the qubit e_i is in the state $|1\rangle$ if the momentum flow of the corresponding edge is oriented in the direction of the original assignment, and $|0\rangle$ if it is in the opposite direction, initialised as a uniform superposition

$$|e\rangle = \frac{1}{\sqrt{N}} \sum_{x=0}^{N-1} |x\rangle, \quad N = 2^n$$

- **The $|a\rangle$ register** encodes acyclicity conditions, e.g. each subloop cycle tested with a **multi-controlled Toffoli gate** (+ NOT (Pauli-X) gates)

- **The Grover's marker** initialized to the Bell state $|out\rangle = |-\rangle = \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle)$

- **The oracle operator**

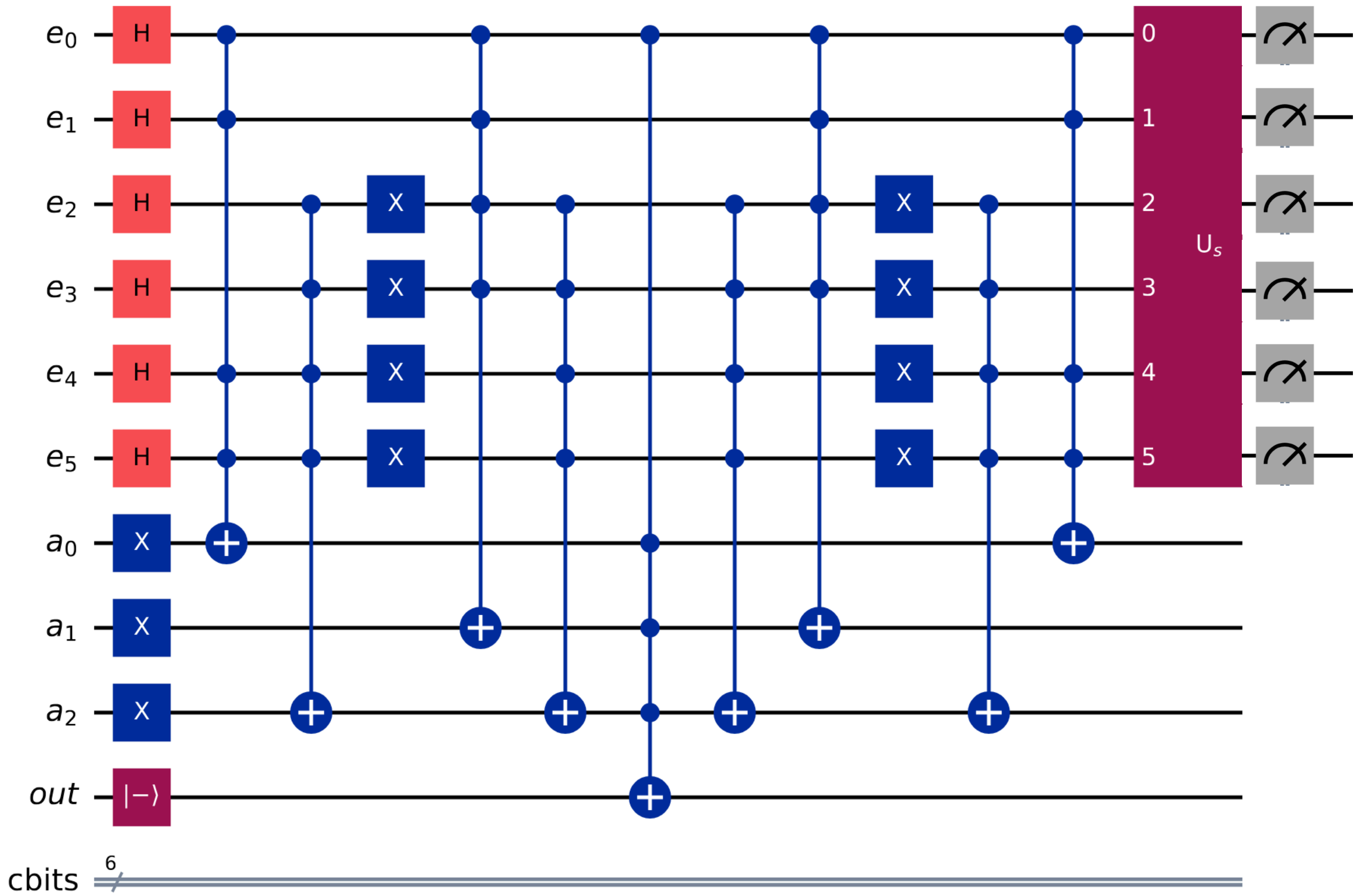
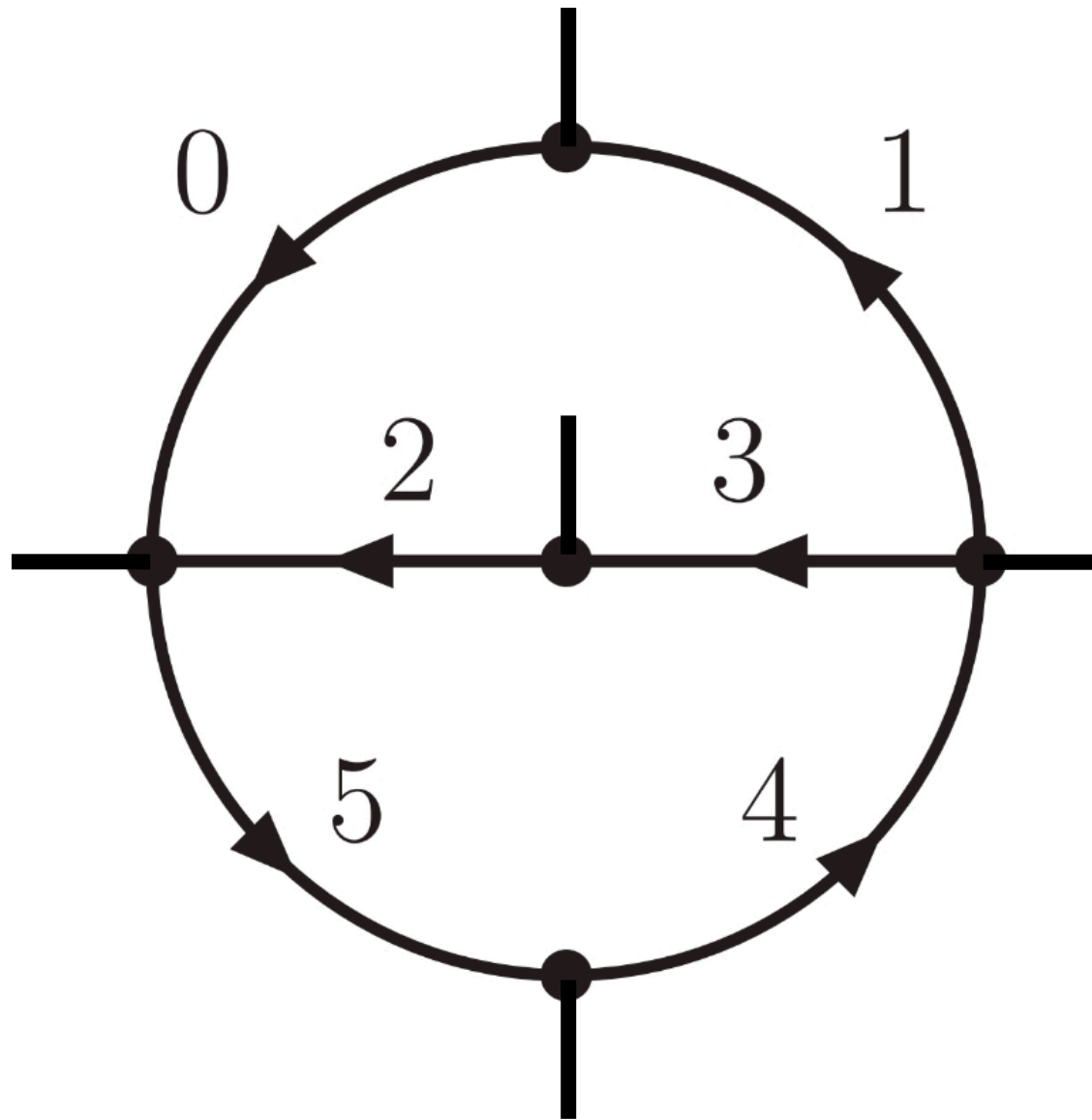
$$U_w |e\rangle |a\rangle |out\rangle = |e\rangle |a\rangle |out \otimes f(a, e_0)\rangle$$

$$|out \otimes 0\rangle = |out\rangle$$

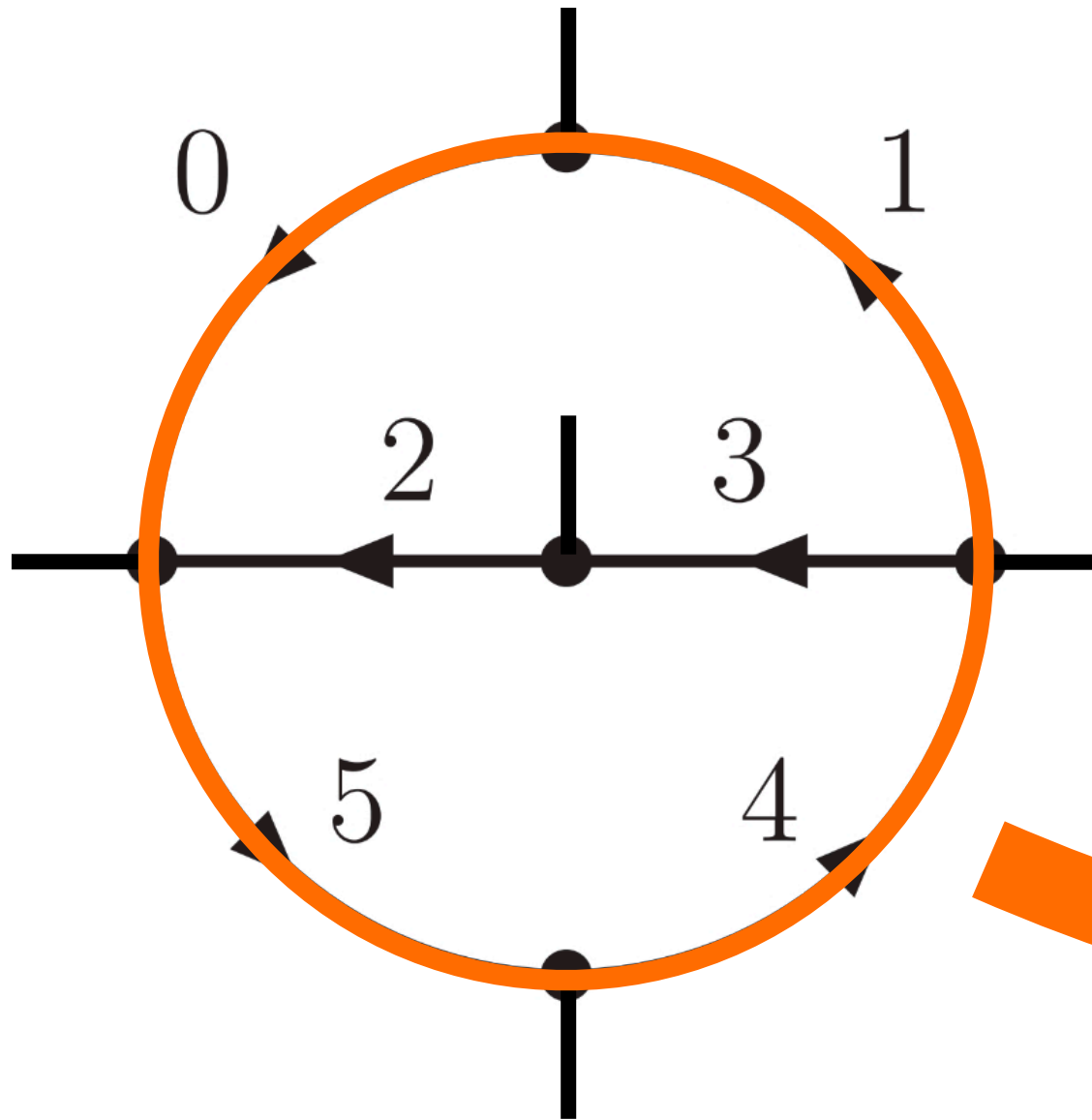
$$|out \otimes 1\rangle = -|out\rangle$$

- **The diffuser operator U_s** : amplifies probabilities of marked states through a rotation in the Hilbert space

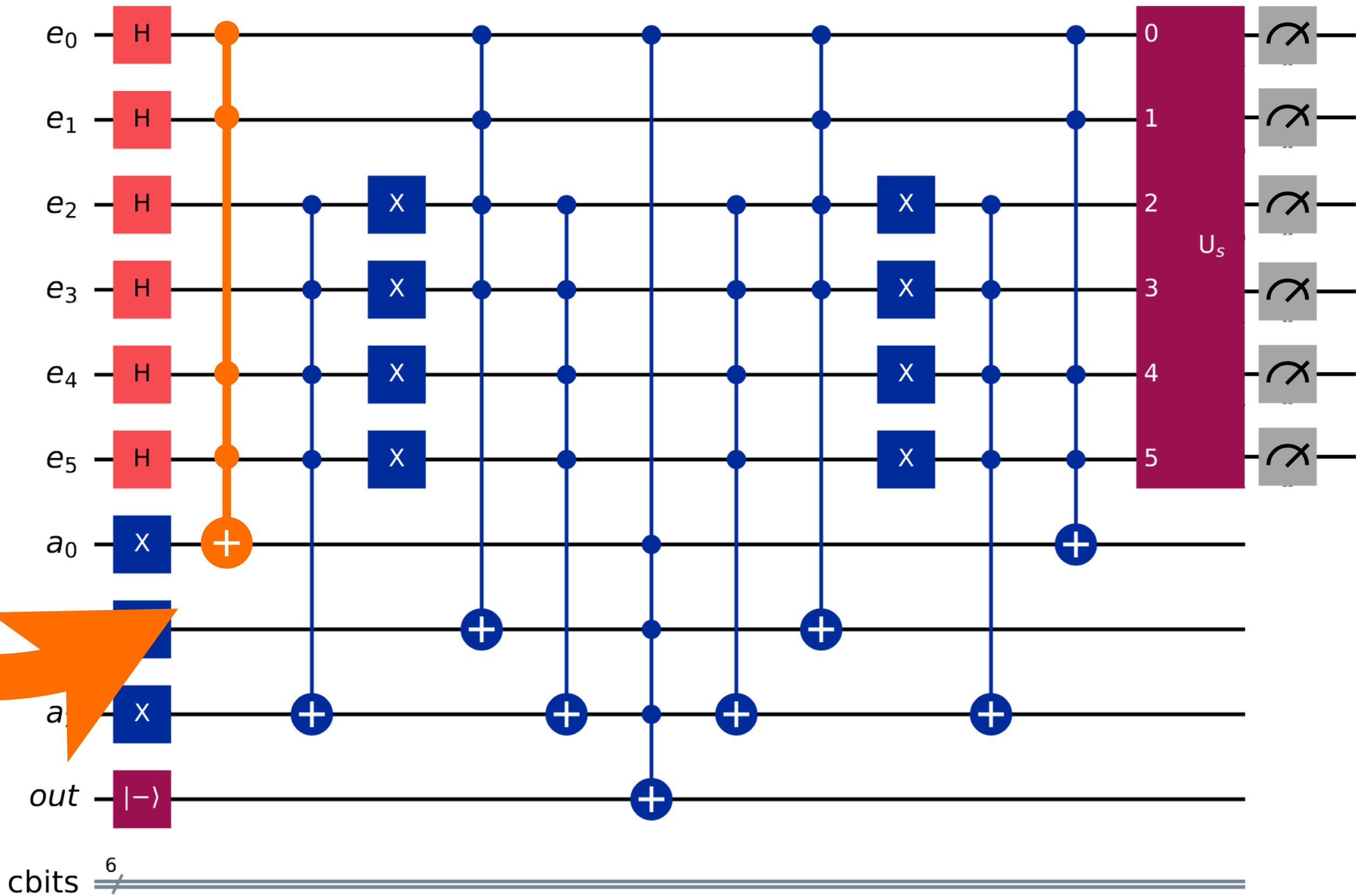
Oracle design: a two-loop topology with 6 edges



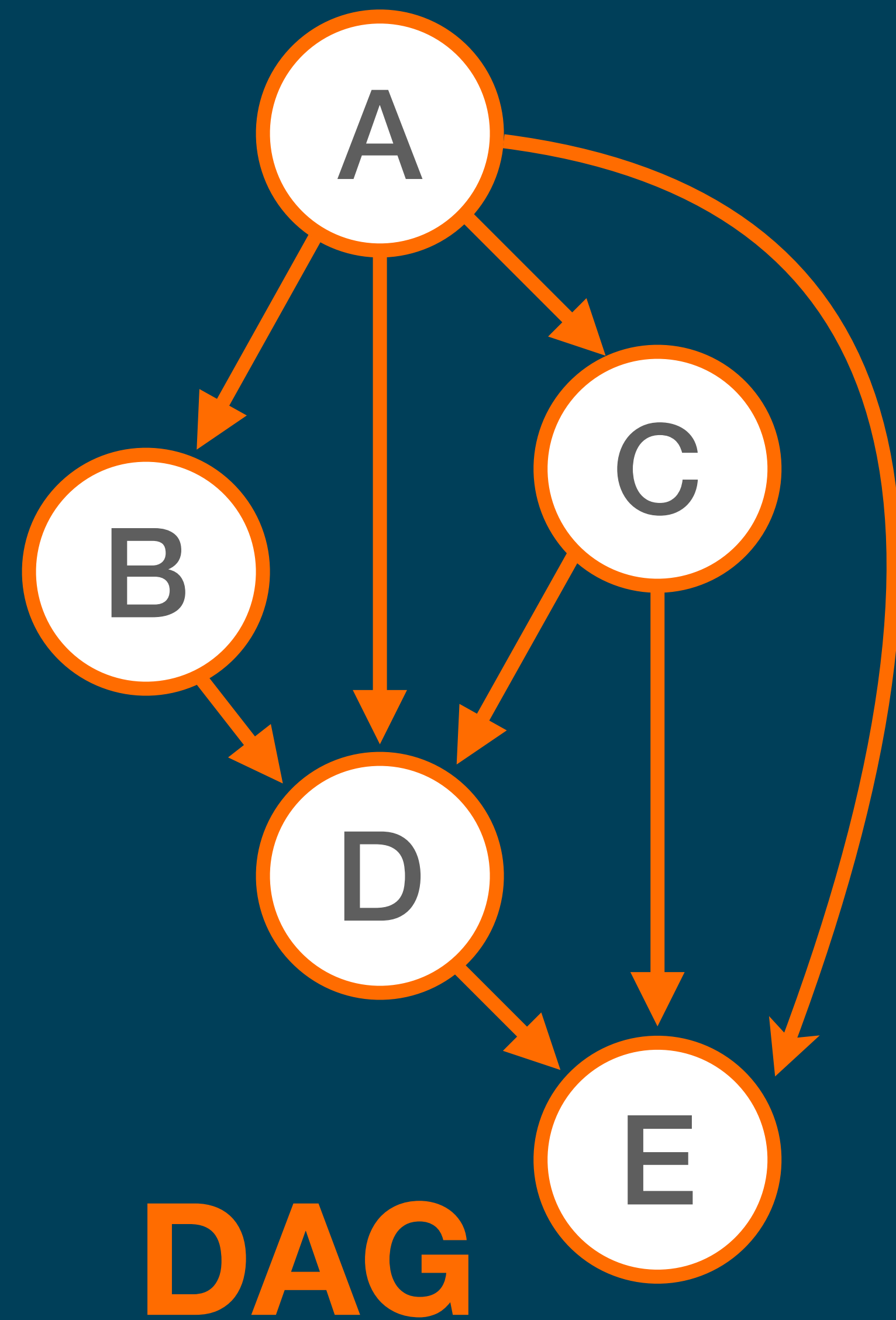
Oracle design: a two-loop topology with 6 edges



- Each loop cycle is mapped to a multi-controlled Toffoli gate



[Ramírez, Rentería, GR, [2404.03544](#)]



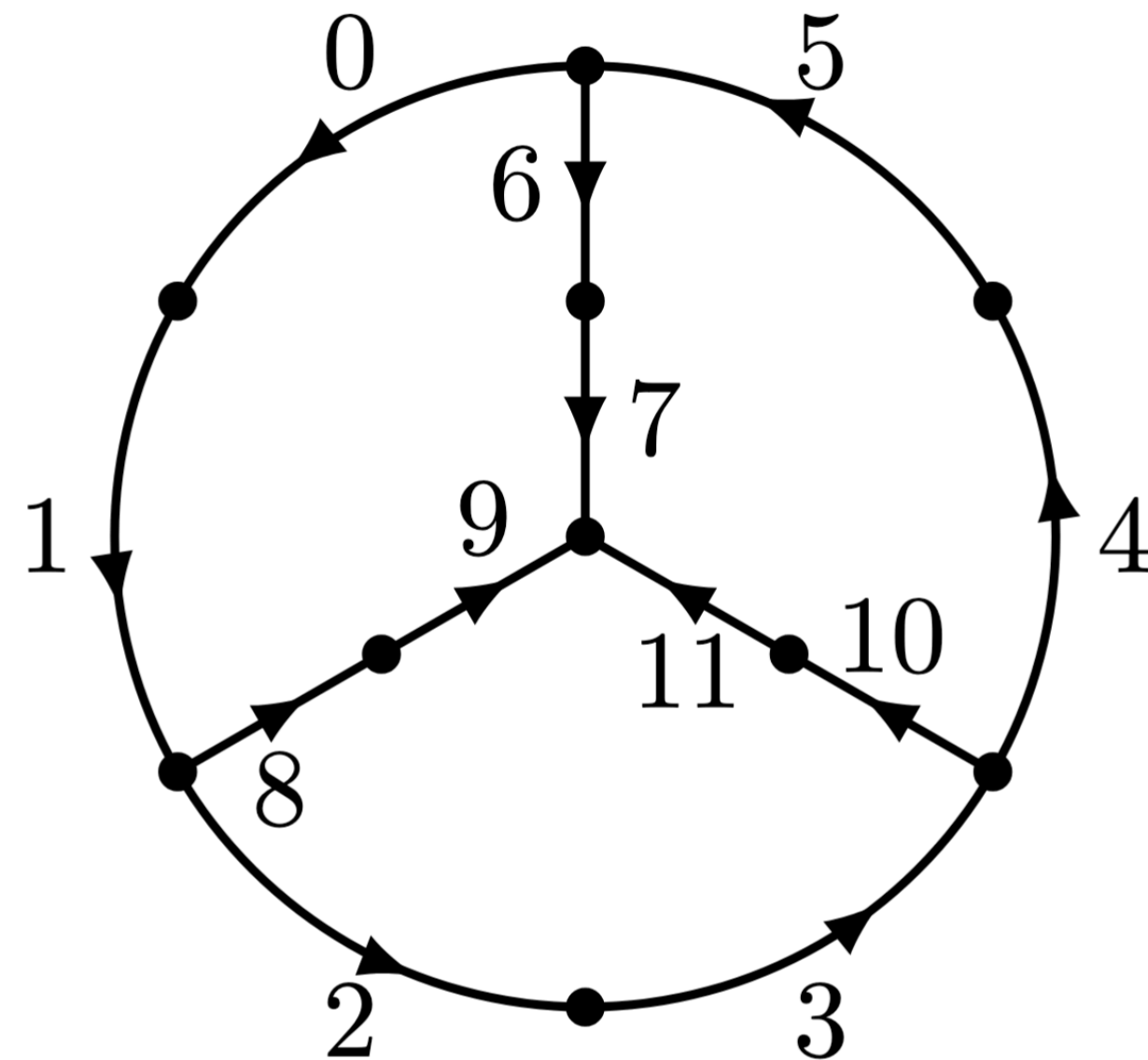
Directed Acyclic Graph

Oracle design by leveraging graph-theory principles

A **clique**, also known as a **complete subgraph**, is a subset of vertices from a specific graph in which each pair of vertices is directly connected by an edge

Minimum Clique Partition (MCP):
minimum number of cliques that generate a graph

A three-loop topology with 12 edges



$$\begin{aligned} s_0 &= e_0 \wedge e_1, & s_1 &= e_2 \wedge e_3, & s_2 &= e_4 \wedge e_5, \\ s_3 &= e_6 \wedge e_7, & s_4 &= e_8 \wedge e_9, & s_5 &= e_{10} \wedge e_{11}. \end{aligned}$$

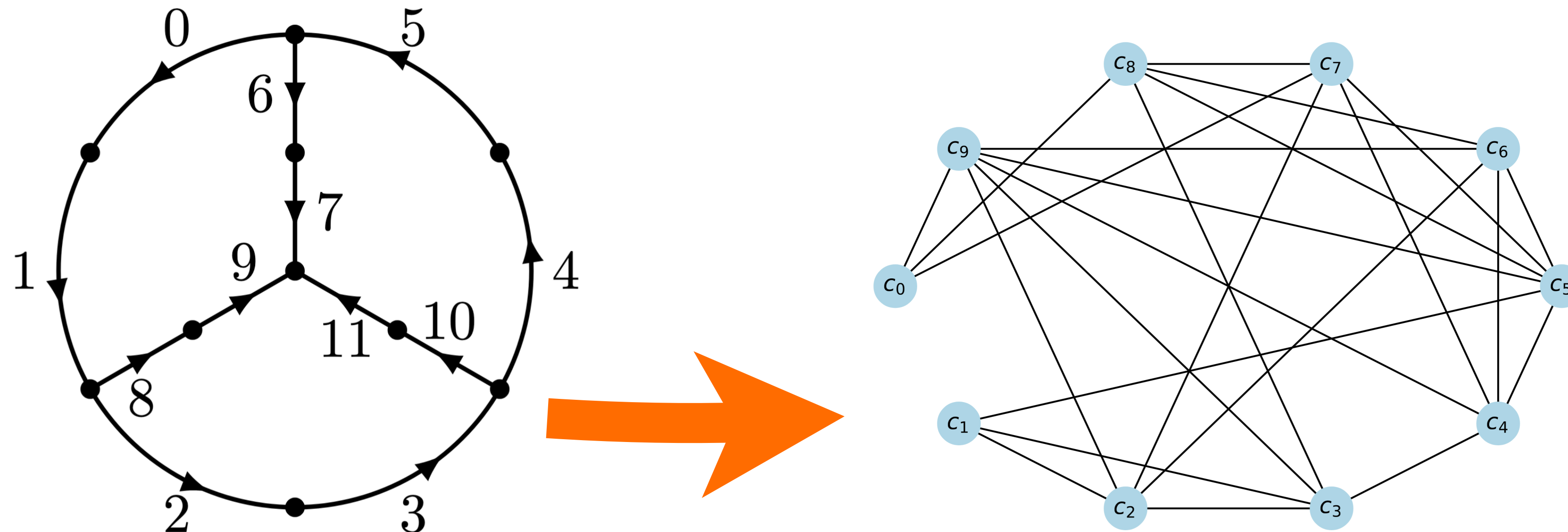
6 sets of edges
(12 qubits)

$$\begin{aligned} c_0 &= s_0 \wedge s_1 \wedge s_2, & c_1 &= s_0 \wedge \bar{s}_3 \wedge s_4, & c_2 &= s_1 \wedge \bar{s}_4 \wedge s_5, \\ c_3 &= s_2 \wedge s_3 \wedge \bar{s}_5, & c_4 &= s_0 \wedge s_1 \wedge \bar{s}_3 \wedge s_5, & c_5 &= s_1 \wedge s_2 \wedge s_3 \wedge \bar{s}_4, \\ c_6 &= s_0 \wedge s_2 \wedge s_4 \wedge \bar{s}_5, & c_7 &= \bar{c}_2, & c_8 &= \bar{c}_3, \\ & & & & c_9 &= \bar{c}_5. \end{aligned}$$

10 loop clauses
(7 ancillary qubits)

- optimise the number of ancillary qubits used to encoding the loop clauses based on the principle of “**mutually exclusive clauses**”, i.e. clauses that cannot be satisfied simultaneously: $c_i \wedge c_j = 0$
- The information on mutually exclusive clauses can be stored in a single qubit because $c_i \vee c_j = c_i \underline{\vee} c_j$

A three-loop topology with 12 edges

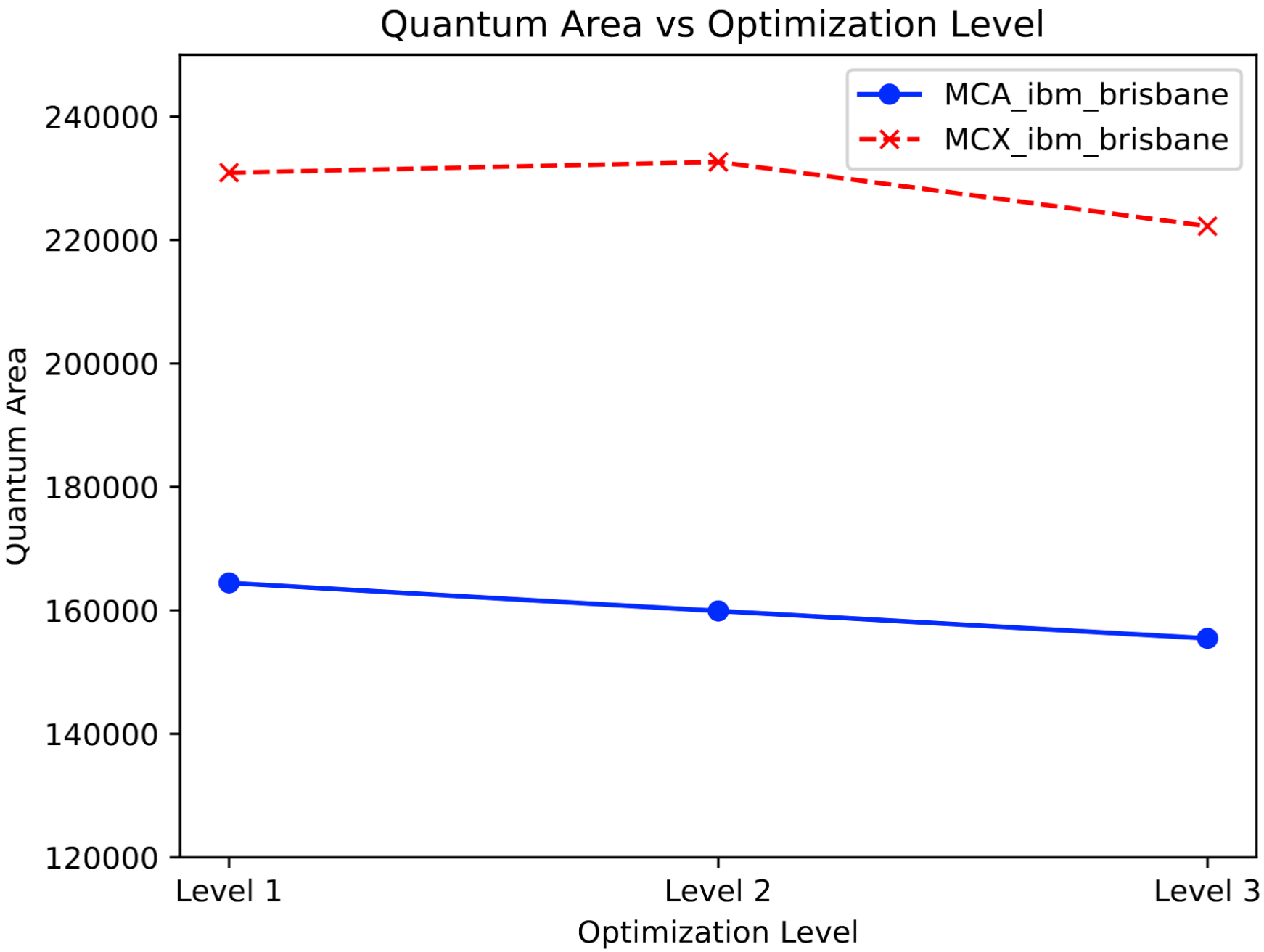
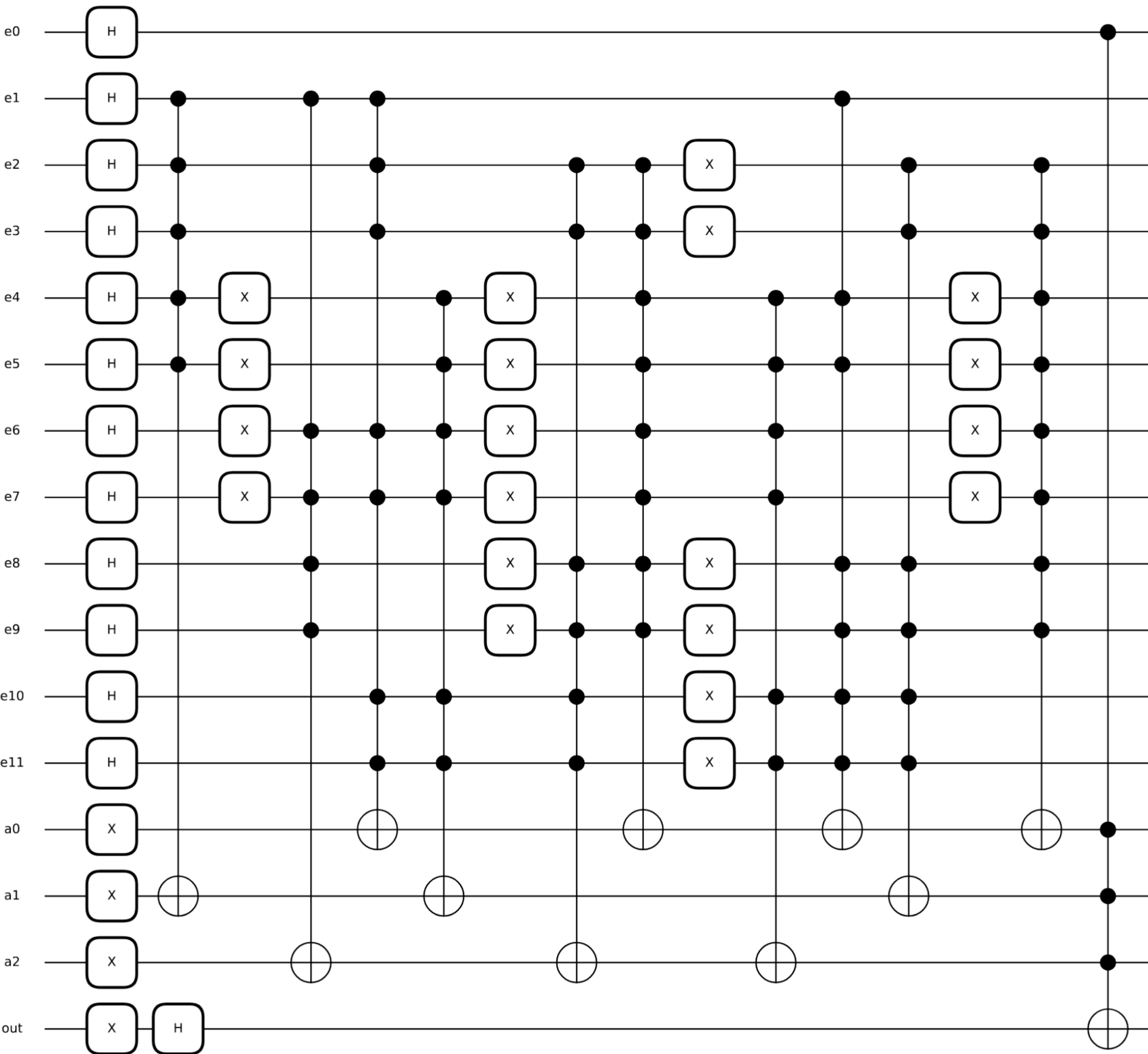


Graph representing the adjacency matrix of mutually exclusive clauses

- A first algorithm to identify MCP of the adjacency matrix
 $\text{MAUX}_{\mathbf{c}}^{(3,12)} = \{ \{c_4, c_5, \bar{c}_5, c_6\}, \{c_0, \bar{c}_2, \bar{c}_3\}, \{c_1, c_2, c_3\} \}$

mutually exclusive sets of clauses
(3 ancillary qubits instead of 7)
- A second algorithm to determine the optimal order in which the gates (Toffoli+XNOT) are applied, it impacts the quantum depth
 $\text{OMUT}_{\mathbf{c}}^{(3,12)} = \{ \{c_0\}, \{c_1, c_4, \bar{c}_3\}, \{c_2, c_5\}, \{c_3, c_6, \bar{c}_2\}, \{\bar{c}_5\} \}$

A three-loop topology with 12 edges



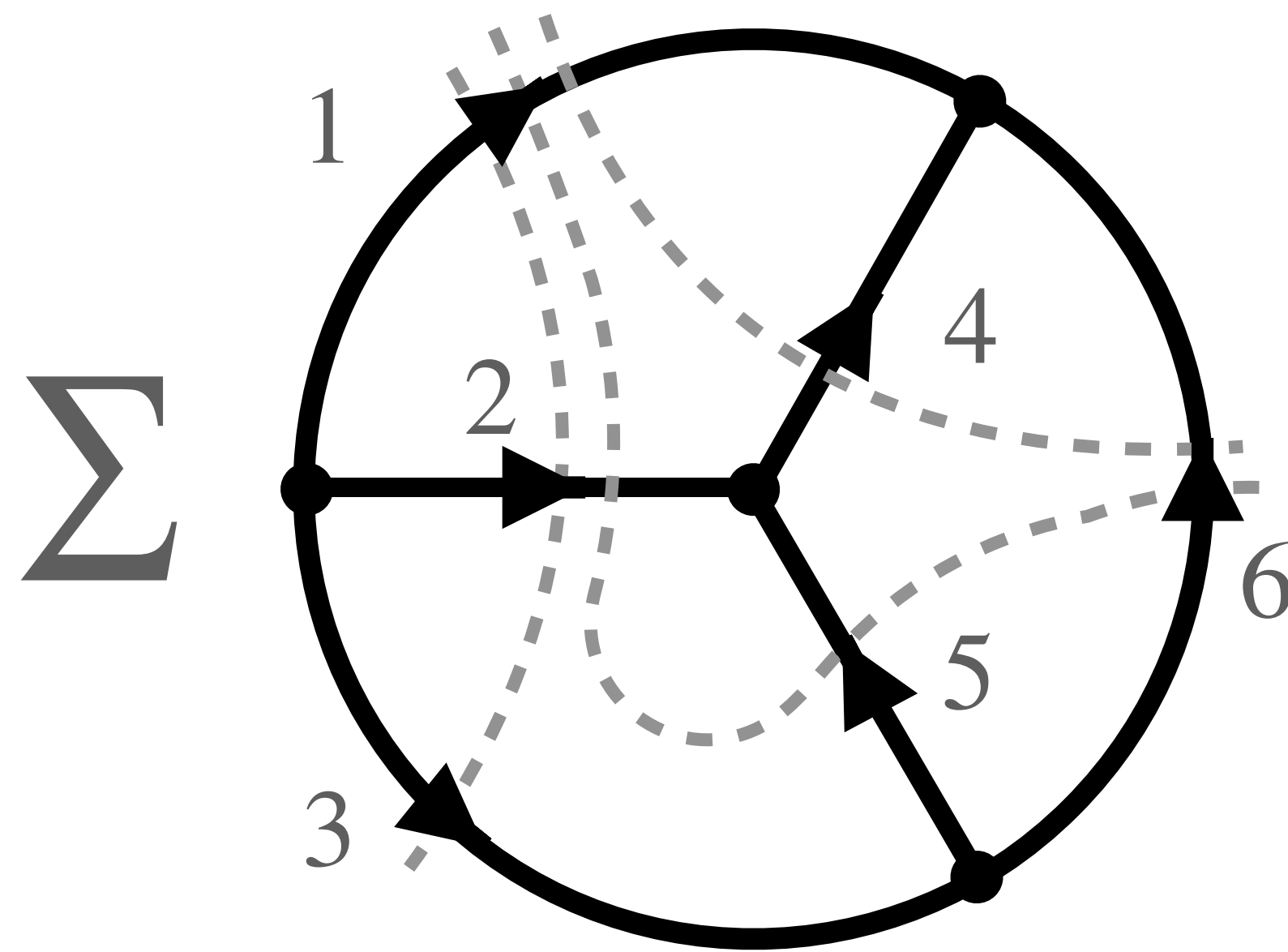
The Loop-Tree Duality LTD: vacuum amplitudes

the Euclidean space of the
loop three-momenta

$$\mathcal{A}_D^{(\Lambda)} \sim \prod \frac{1}{\lambda_{i_1 i_2 \dots i_n}} \quad \mathcal{A}^{(\Lambda)} = \int_{\vec{\ell}_1 \dots \vec{\ell}_\Lambda} \mathcal{A}_D^{(\Lambda)}$$

on-shell energies

$$\lambda_{i_1 i_2 \dots i_n} = \sum q_{i_s,0}^{(+)} = \sum \sqrt{\mathbf{q}_{i_s}^2 + m_{i_s}^2 - i0}$$



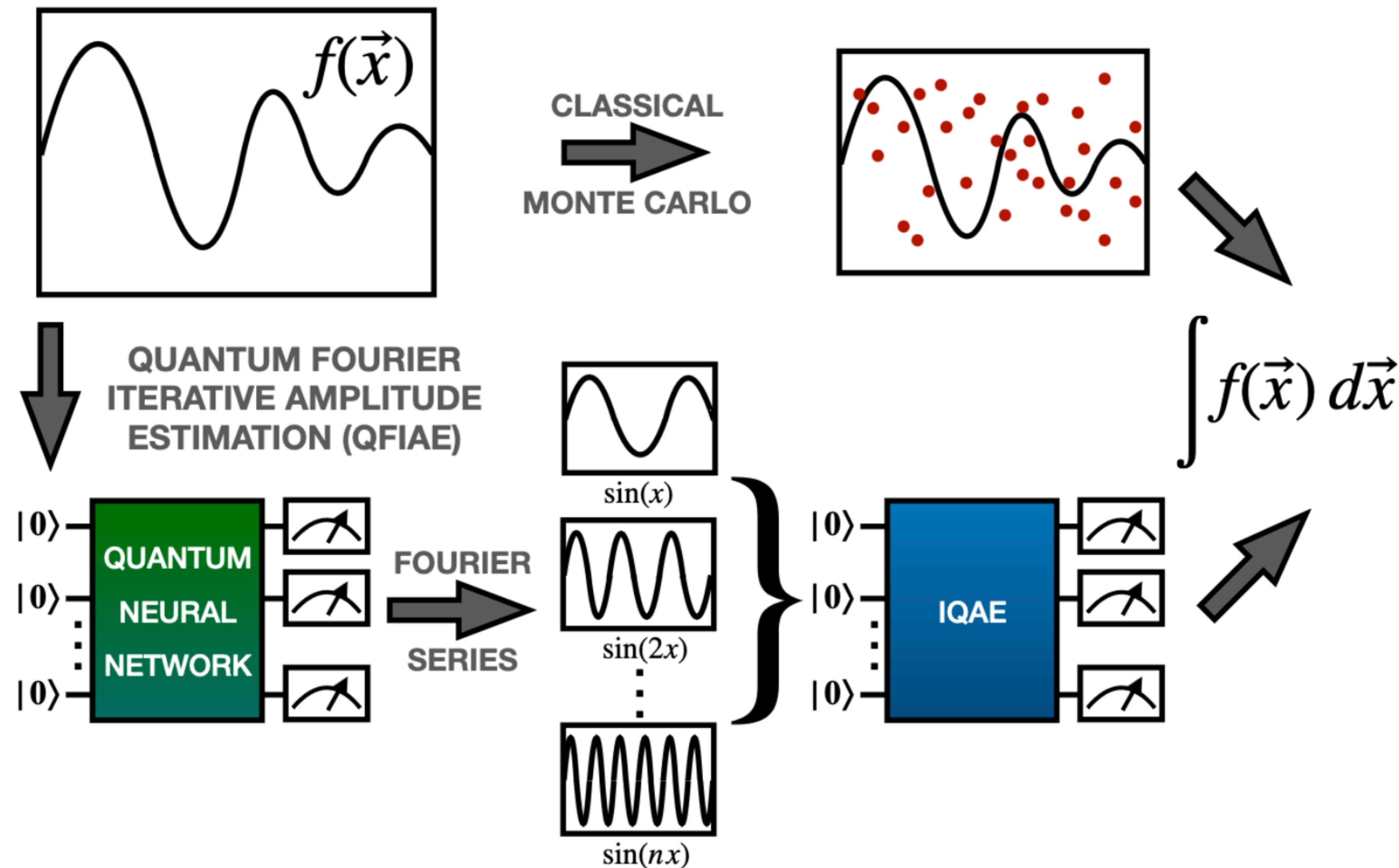
States with external particles are
generated from residues (cuts) over
causal propagators $1/\lambda_{i_1 i_2 \dots i_n}$ involving
both initial and final-state particles



Vacuum amplitudes are **intrinsically IR finite and threshold free**: local cancellation of singularities
between phase-space residues, hence **no artificial
separation** between loops and trees, and $d = 4$

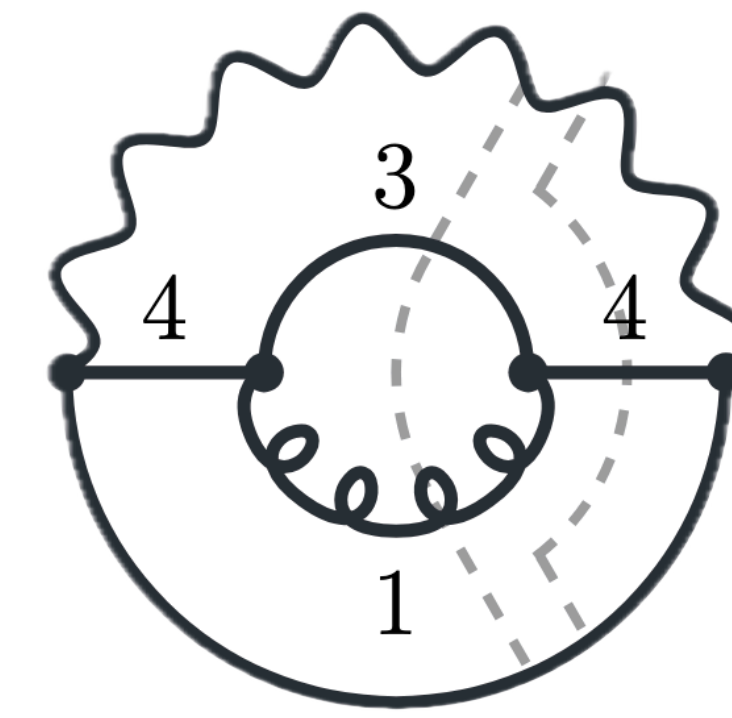
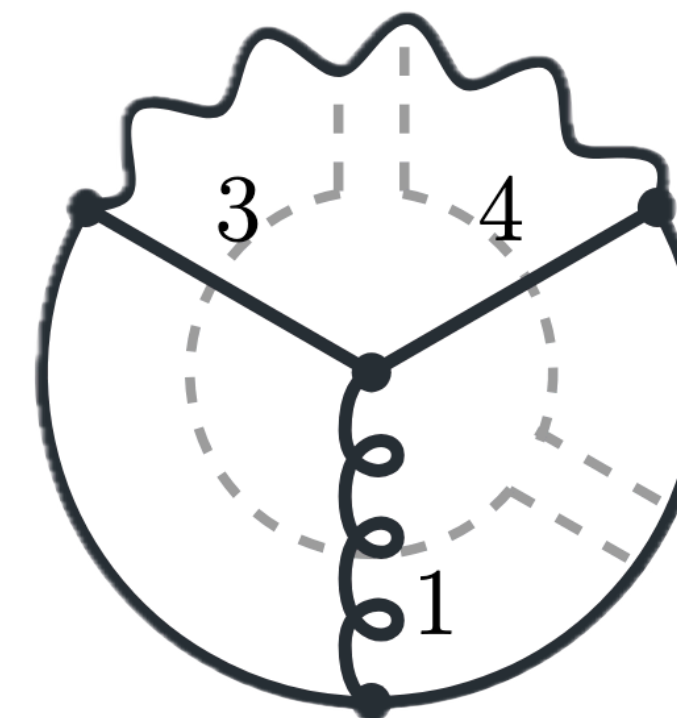
S. Ramírez Uribe, P.K. Dhani, G.F.R. Sborlini, GR,
"Rewording theoretical predictions at colliders
with vacuum amplitudes," [PRL133, 211901 \(2024\)](#)

Quantum Fourier Iterative Amplitude Estimation (QFIAE)



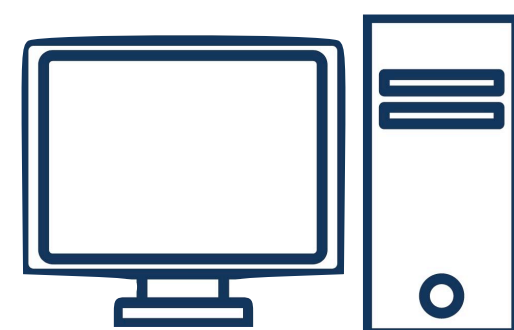
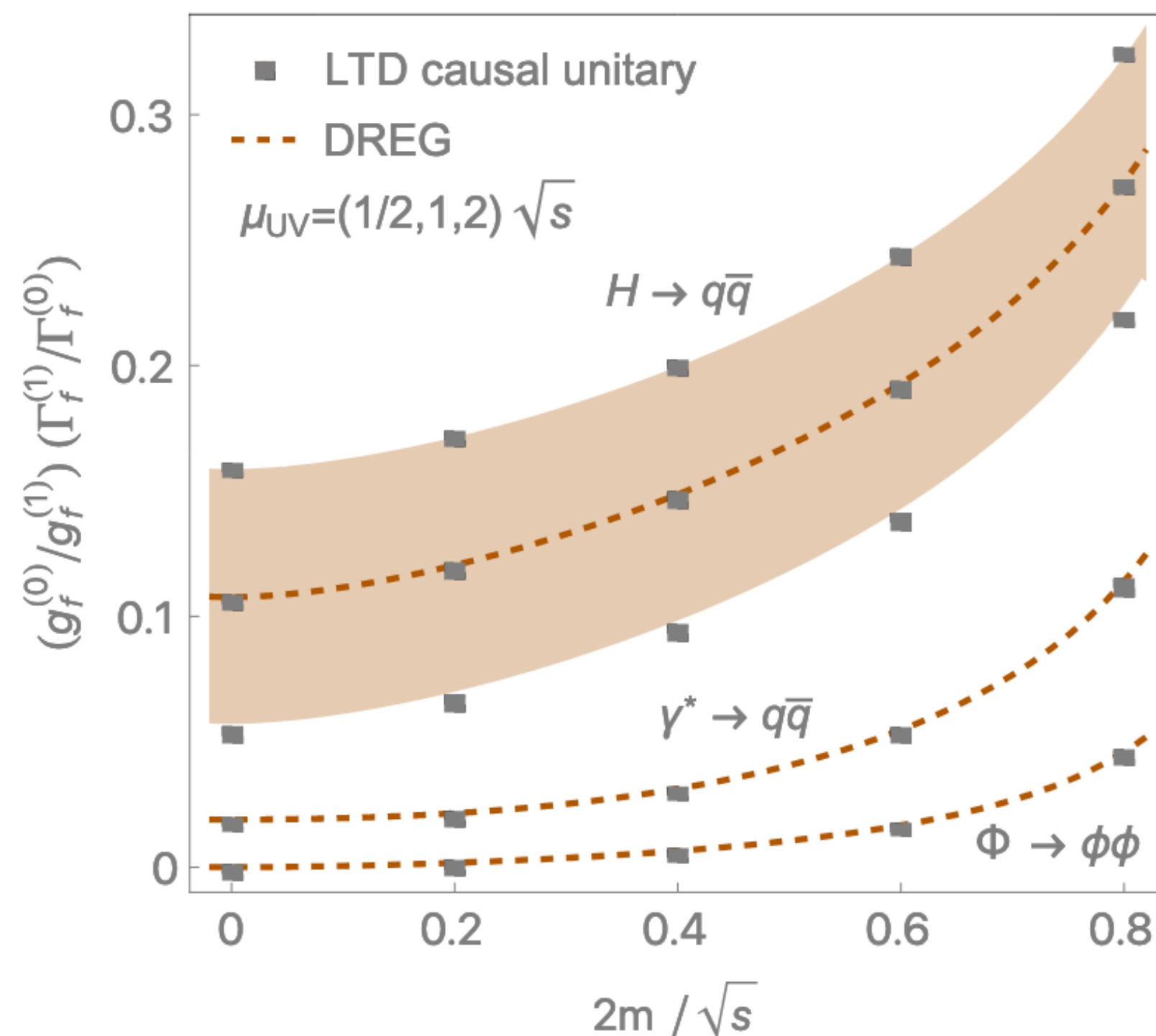
- Integration of multidimensional functions
- Quantum Machine Learning + Grover's amplification
- Fourier series using a **Quantum Neural Network (QNN)**
- Integrates each trigonometric component using Iterative Quantum Amplitude Estimation (IQAE) [Grinko, Gacon, Zoufal, Woerner, npj QI 7, 52 (2021)], a variant of Grover's algorithm

Decay rate of Higgs boson / photon / toy-scalar at NLO



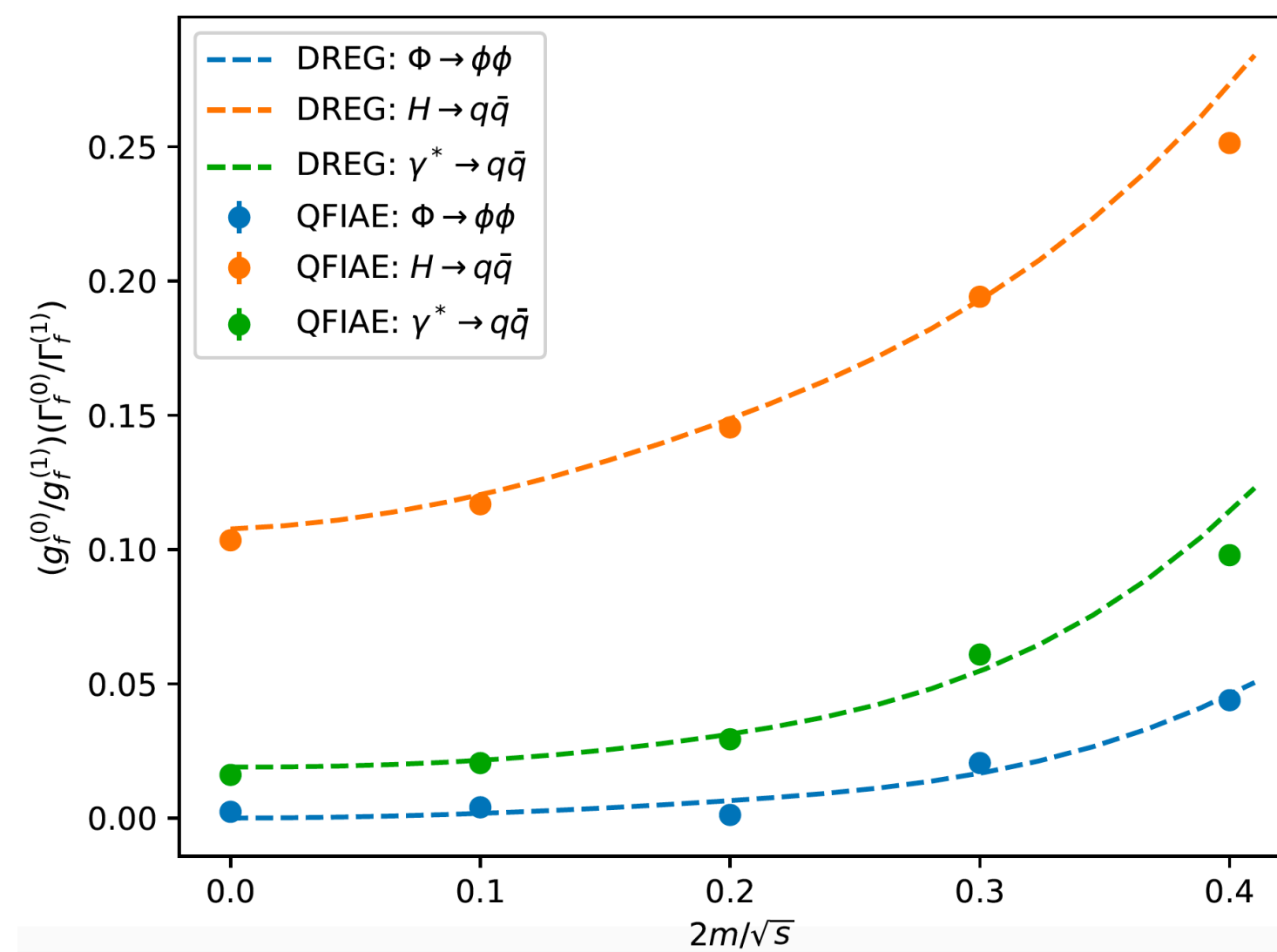
Quantum integration of decay rates at second order in perturbation theory

J.J. Martínez de Lejarza, D.F. Rentería Estrada, M. Grossi, GR, Quantum Sci.Technol. 10 (2025) 2, 025026 [2409.12236]

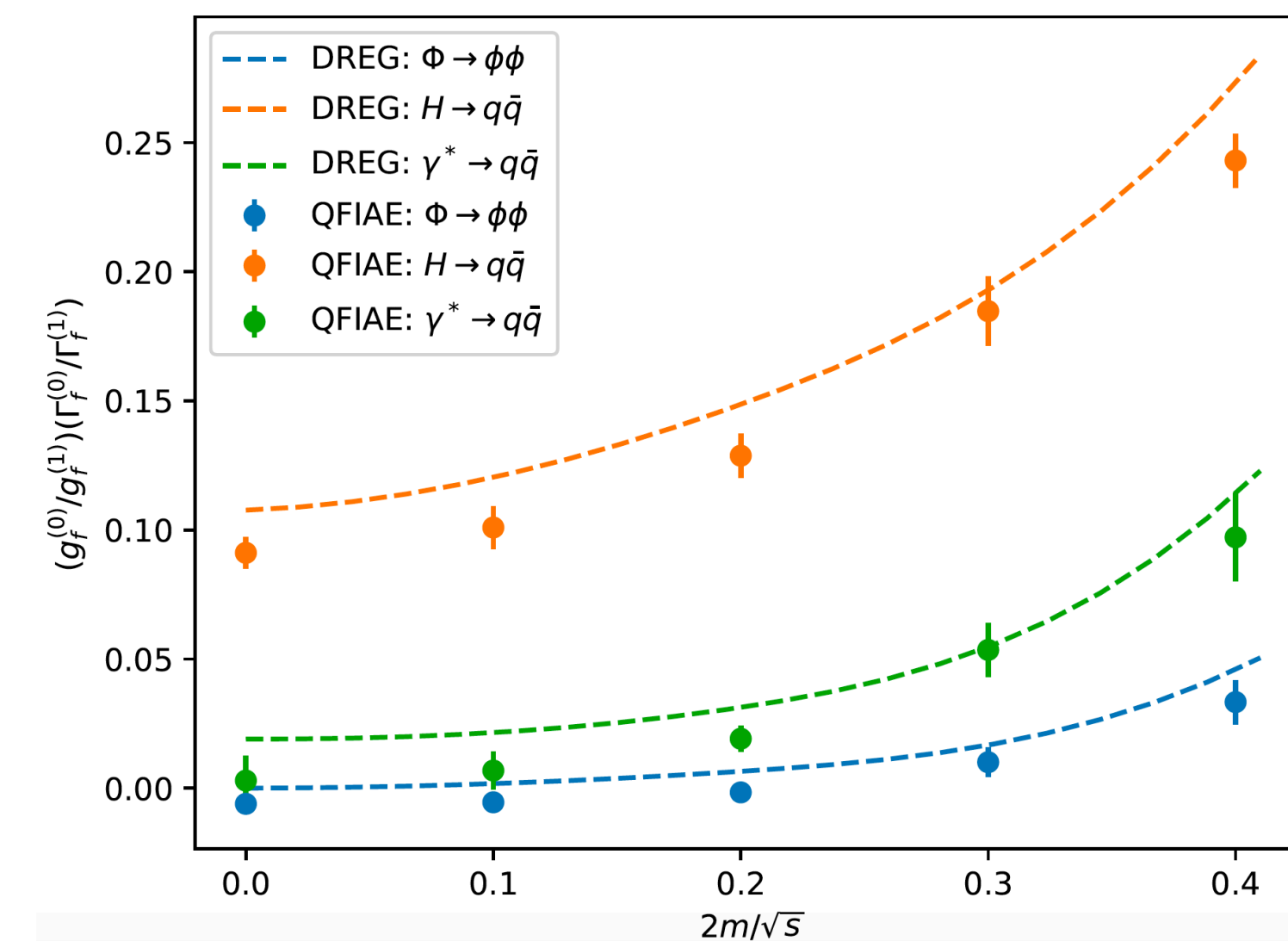


○ Classical Monte Carlo

Vacuum amplitudes and time-like causal unitary in the loop-tree duality, LTD
Collaboration, JHEP 01 (2025) 103 [2404.05492]

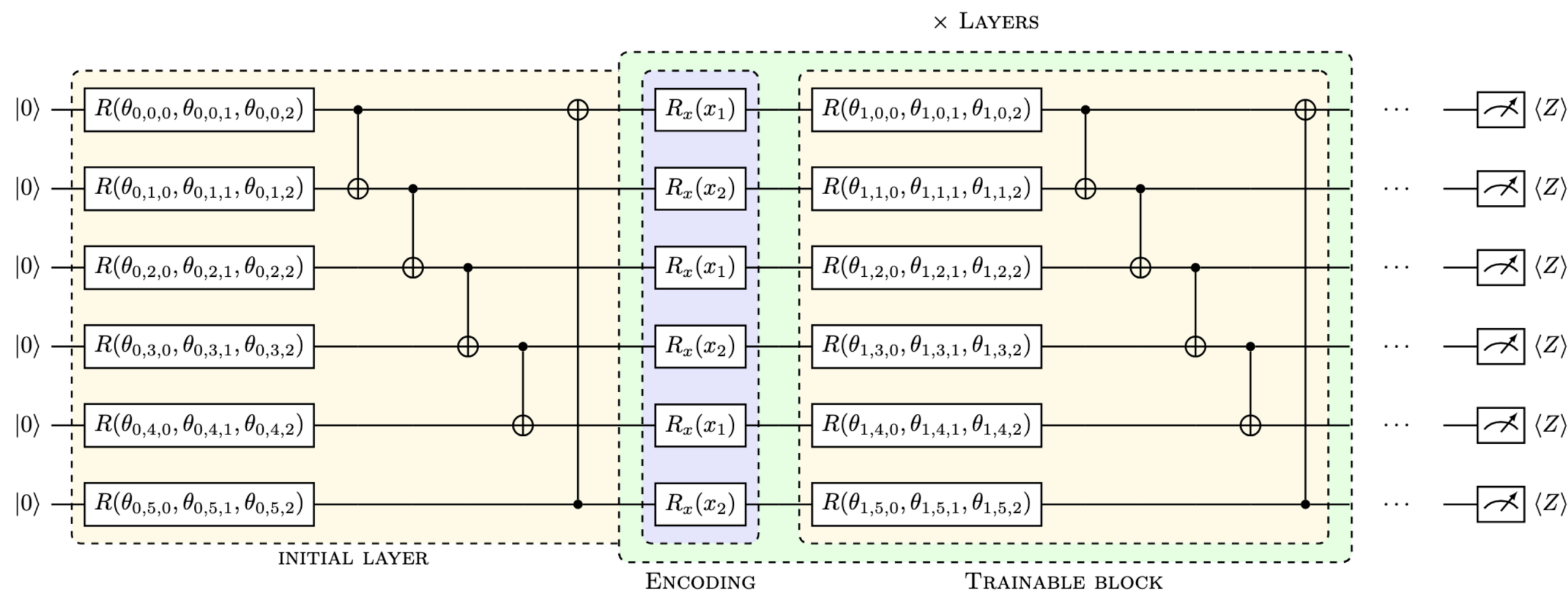


○ Quantum simulator



○ Partially in quantum hardware

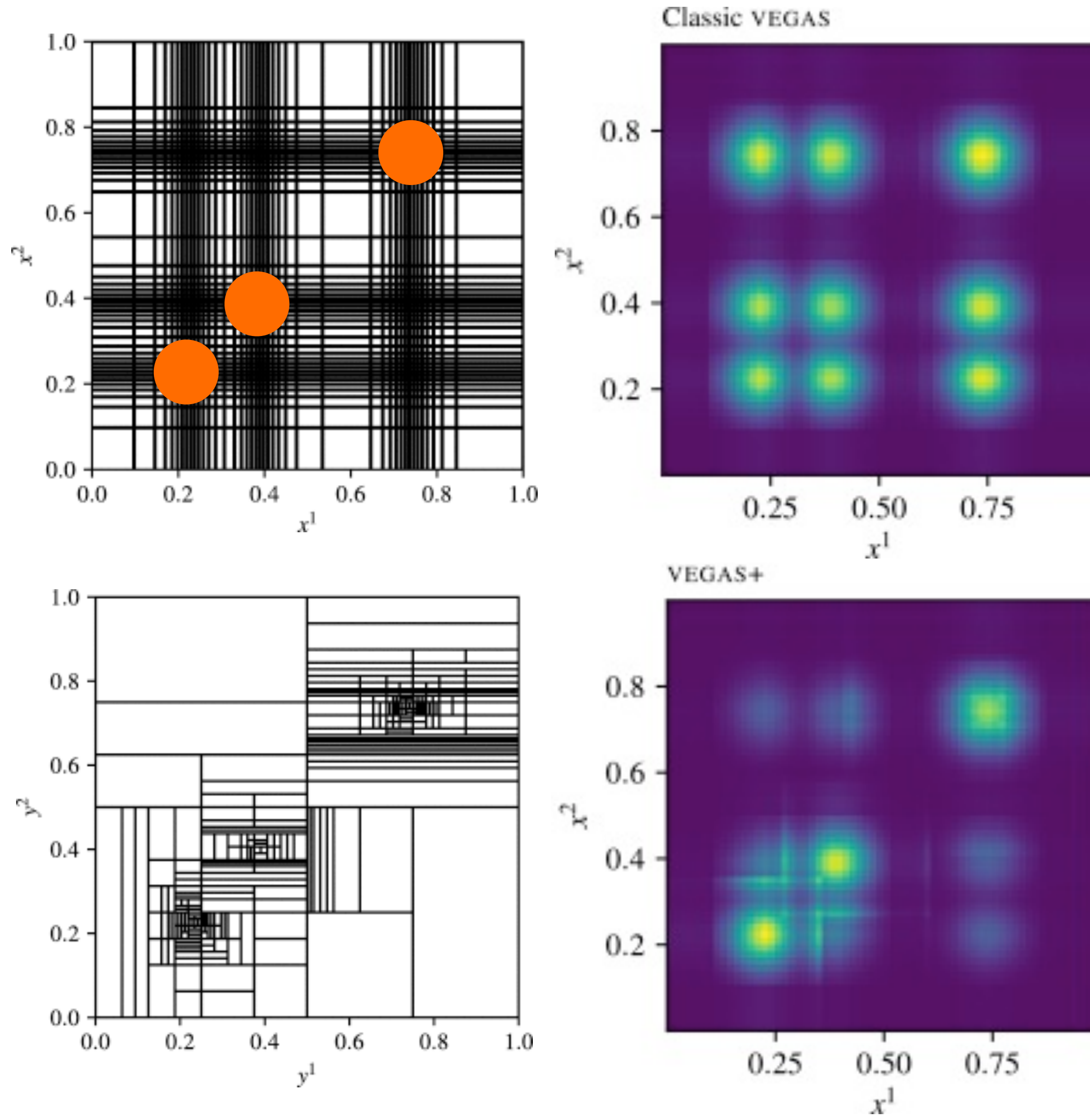
Technical details



- **PennyLane** to construct and train the QNN (6-qubit Ansatz), 20 layers
- IQAE module implemented with **Qibo** on quantum simulators, and with **Qiskit** on a real hardware (5 qubits), executed on the 27-qubit IBMQ superconducting device ibmq_mumbai
- **error mitigation**: pulse-efficient transpilation, error suppression Dynamical Decoupling (DD) within the circuit execution and error mitigation Zero Noise Extrapolation (ZNE) to the output

Decay	$2m/\sqrt{s}$	Hardware	Simulator	DREG
$\Phi \rightarrow \phi\phi(\phi)$	0.0	-0.0061(28)	0.0023(5)	0.0000
	0.1	-0.0055(31)	0.0040(6)	0.0018
	0.2	-0.0016(30)	0.0011(6)	0.0065
	0.3	0.0101(56)	0.0205(11)	0.0167
	0.4	0.0333(85)	0.0439(15)	0.0459
$H \rightarrow q\bar{q}(g)$	0.0	0.0911(61)	0.1034(13)	0.1077
	0.1	0.1009(83)	0.1169(14)	0.1204
	0.2	0.1288(85)	0.1455(14)	0.1486
	0.3	0.1847(135)	0.1941(20)	0.1928
	0.4	0.2431(104)	0.2513(30)	0.2730
$\gamma^* \rightarrow q\bar{q}(g)$	0.0	0.0029(96)	0.0161(14)	0.0190
	0.1	0.0068(74)	0.0205(13)	0.0215
	0.2	0.0191(50)	0.0293(13)	0.0313
	0.3	0.0535(103)	0.0609(20)	0.0547
	0.4	0.0971(171)	0.0979(30)	0.1140

Adaptive Importance Sampling



$$\int_{\Omega} f(x) dx \equiv \int_{[0,1]^d} J(y) f(x(y)) dy$$

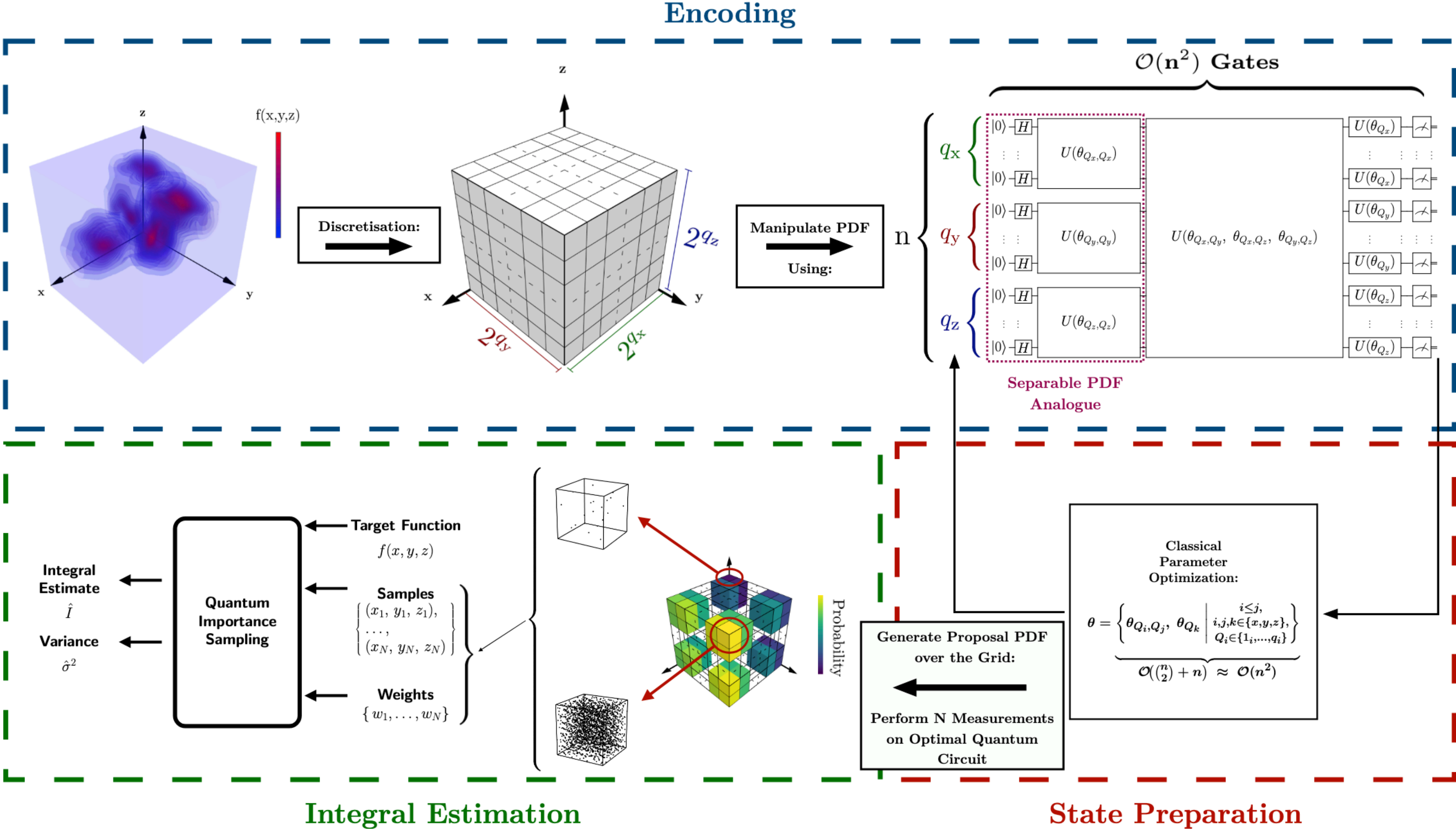
$$I_{MC}^{(VEGAS)} = \frac{1}{N} \sum_{i=1}^N J(y_i) f(x(y_i))$$

- Adapts a **grid** to concentrate samples in regions of high variance
- **The size of the grid grows** exponentially with dimensionality
- **A separable grid:** each dimension divided into intervals of varying widths independently
- Makes the grid tractable in a classical computer but generates **phantom peaks**

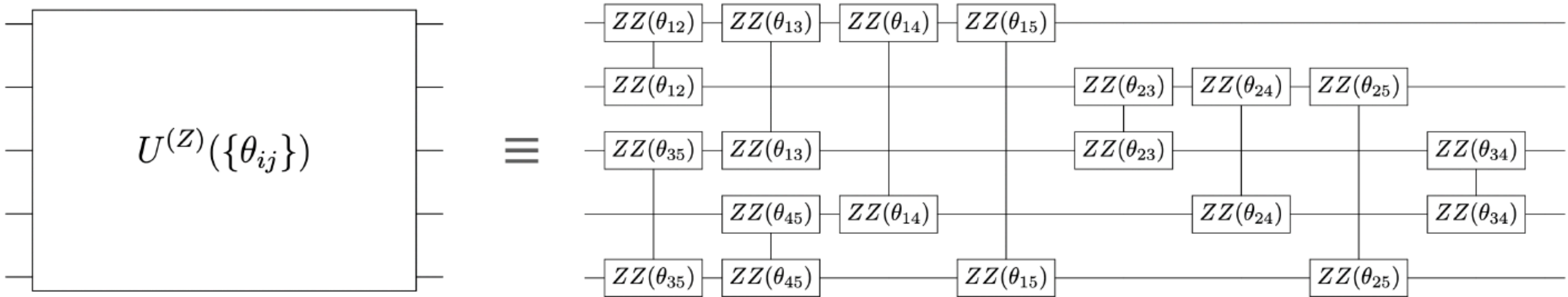
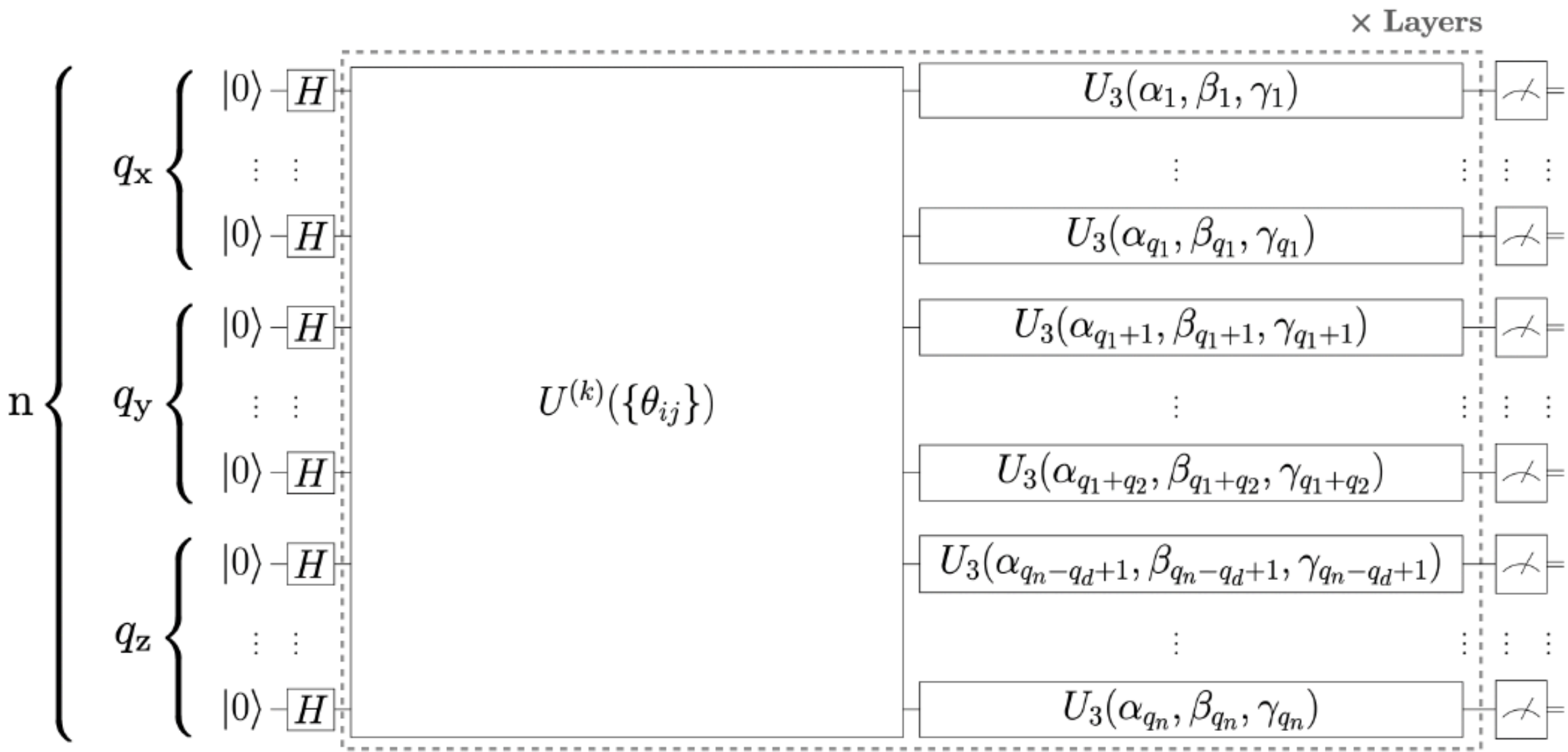
QAIS - Quantum Adaptive Importance Sampling

- Leverage a **Parametrized Quantum Circuit (PQC)** to construct a Probability Distribution Function (PDF) that accurately approximates the target function
- Reduce the number of function evaluations by **efficiently allocating samples** in the integration domain

Quantum Adaptive Importance Sampling (QAIS)

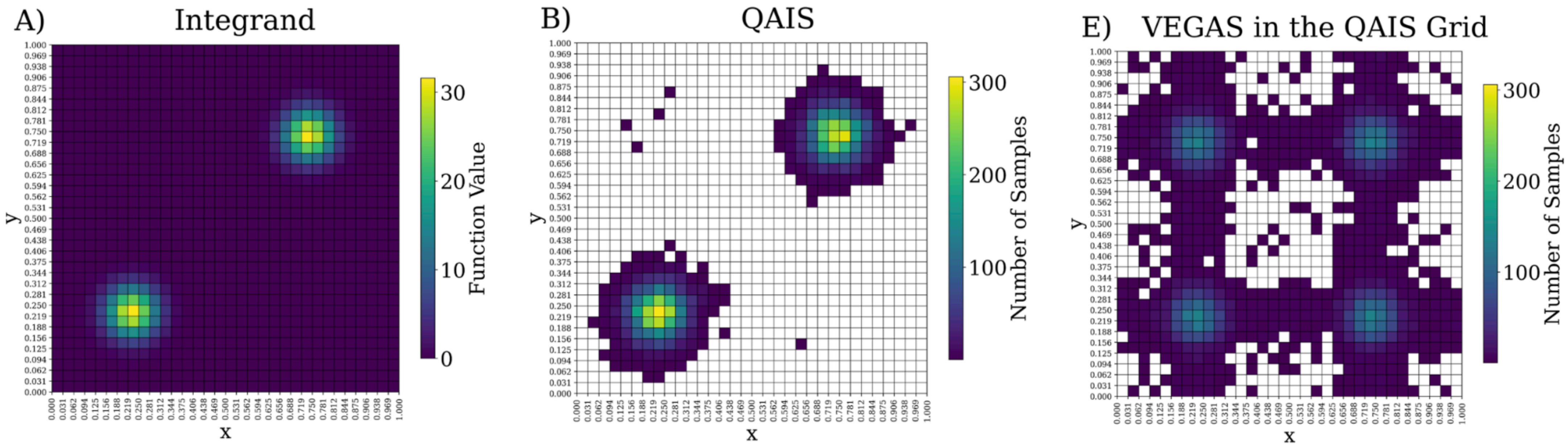


Parametrized Quantum Circuit

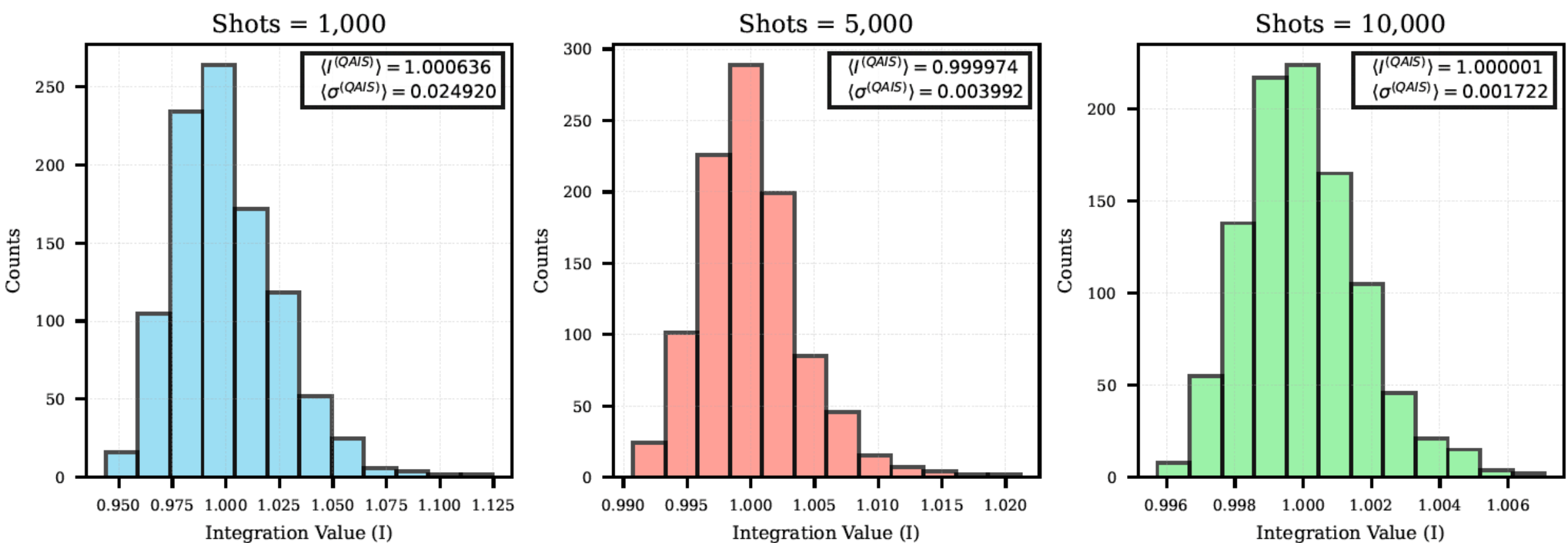


- all-to-all connectivity approach with two-qubit gates and single qubit gates
- Performs much better than Hardware Efficient Ansatz (HEA)
- Cost function: discretised version of Kullback-Leibler (KL) divergence
$$D_{\text{KL}}(P\|Q) = \sum_{i=1}^M P(\Omega^{(i)}) \log \left(\frac{P(\Omega^{(i)})}{Q(\Omega^{(i)})} \right)$$
- COBYLA optimizer

Absence of phantom peaks

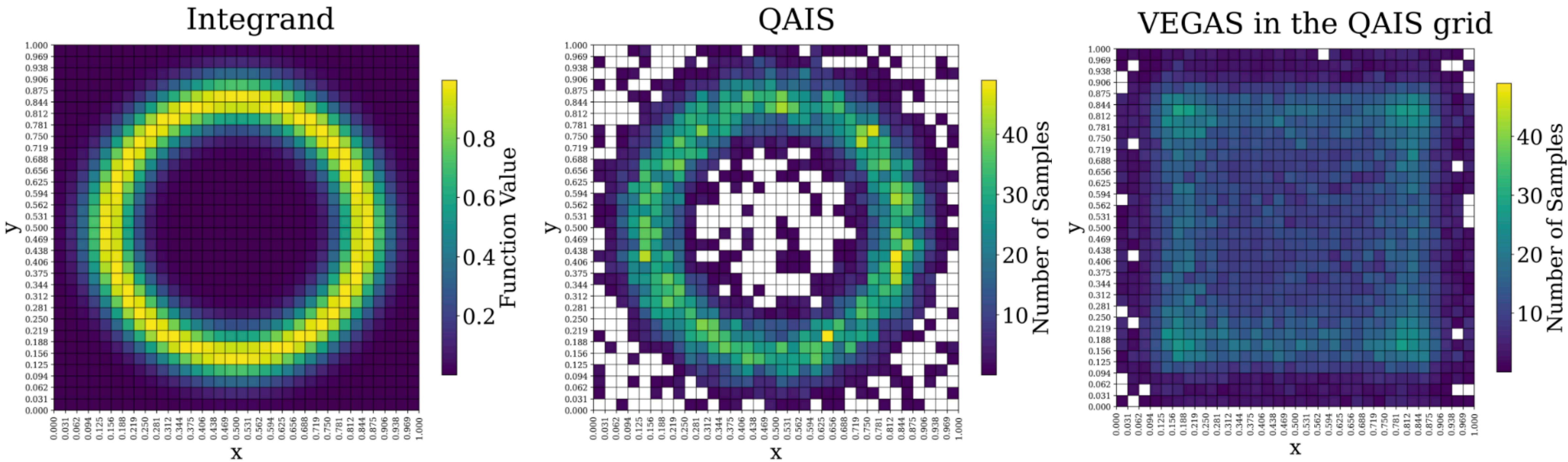


Distribution of Integration Results by Number Shots



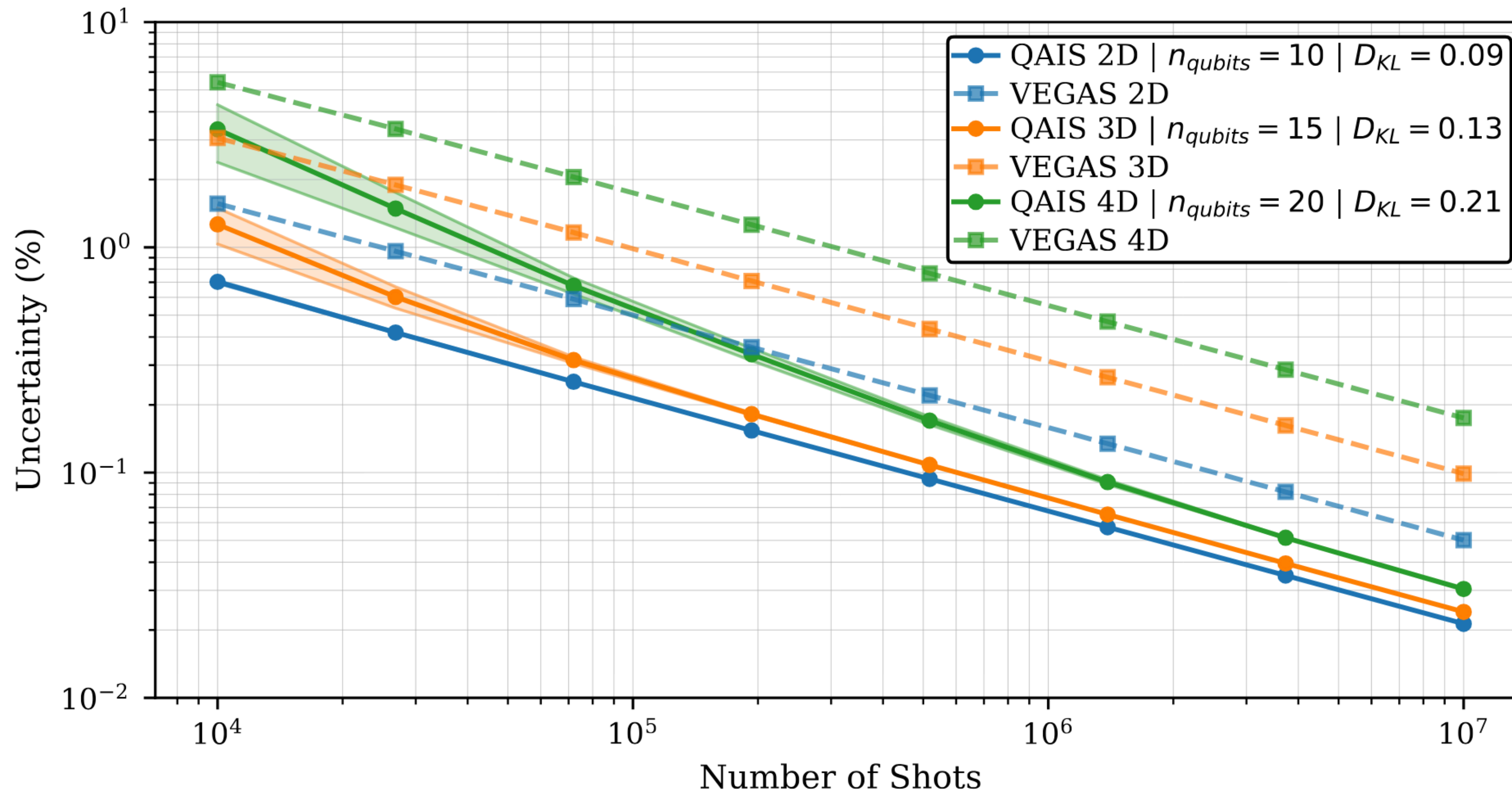
- Integral normalised to one
- 1000 independent runs with the same PDF as a function of the number of shots

A ring-shaped 2d function



Cross-dimensional Integration Comparison

$$\int_{[0,1]^d} f(\mathbf{x}) d\mathbf{x} = \int_{[0,1]^d} \left(\sum_{i=0}^2 e^{-50|\mathbf{x}-\mathbf{r}_i|} \right) d\mathbf{x}$$

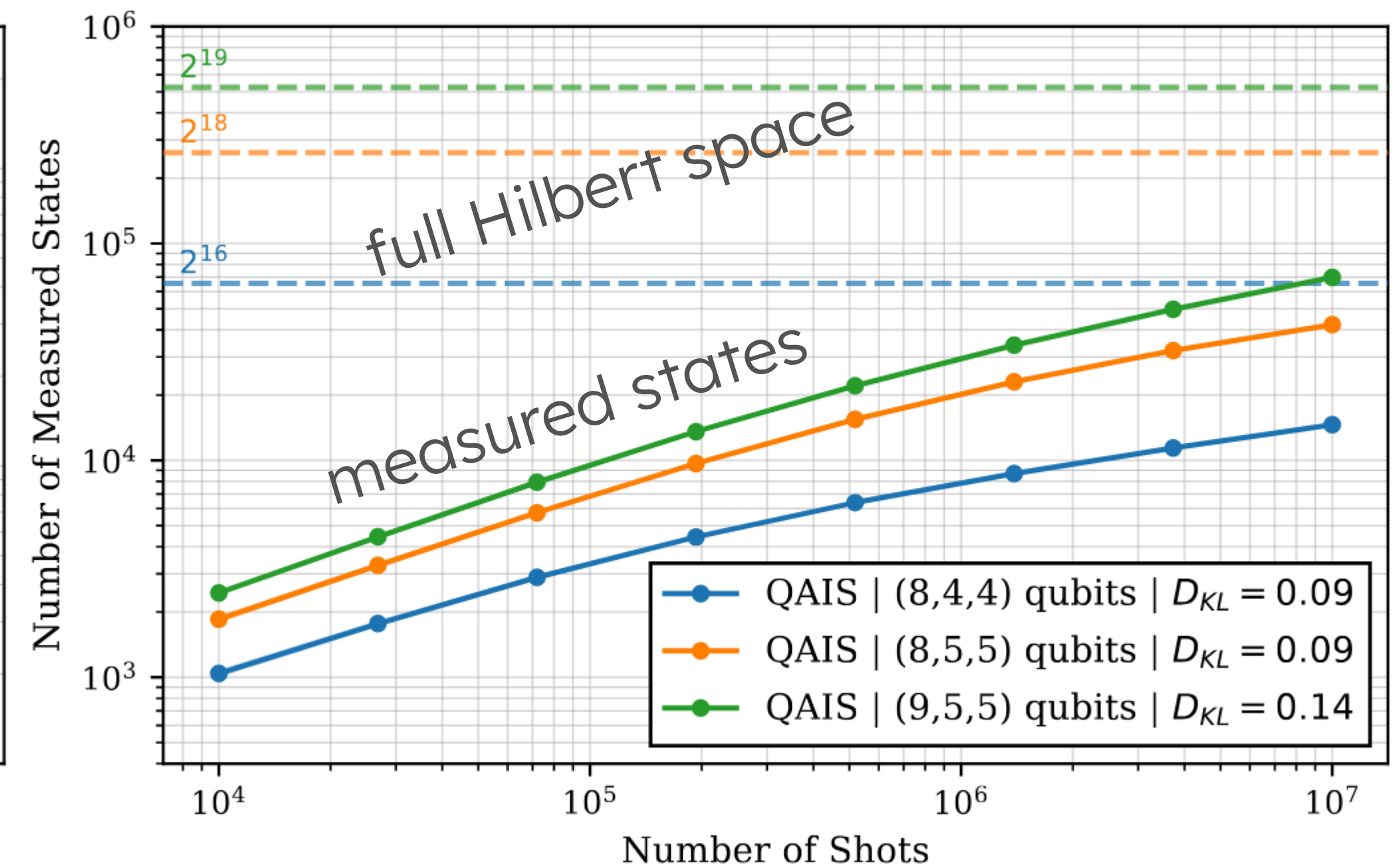
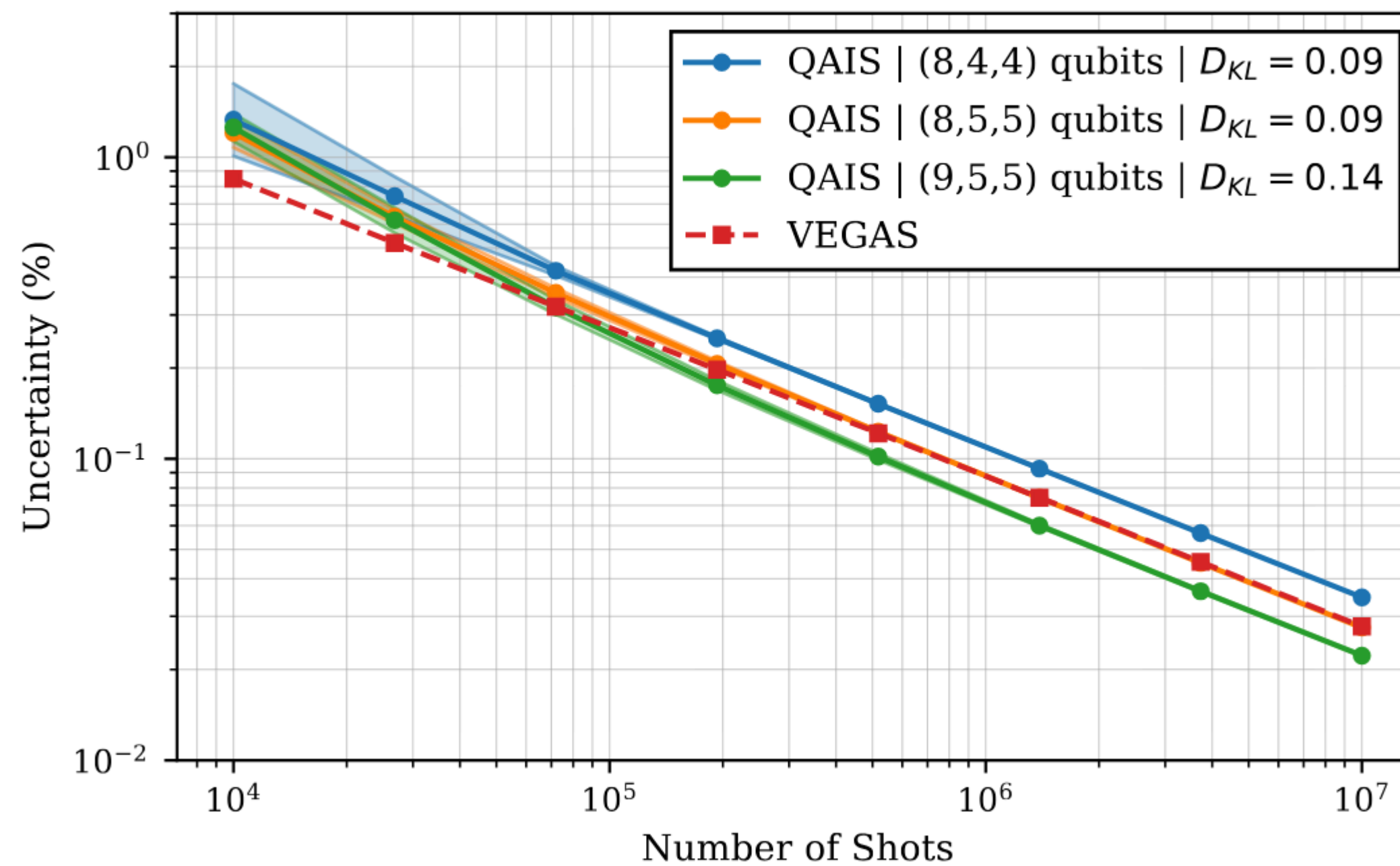
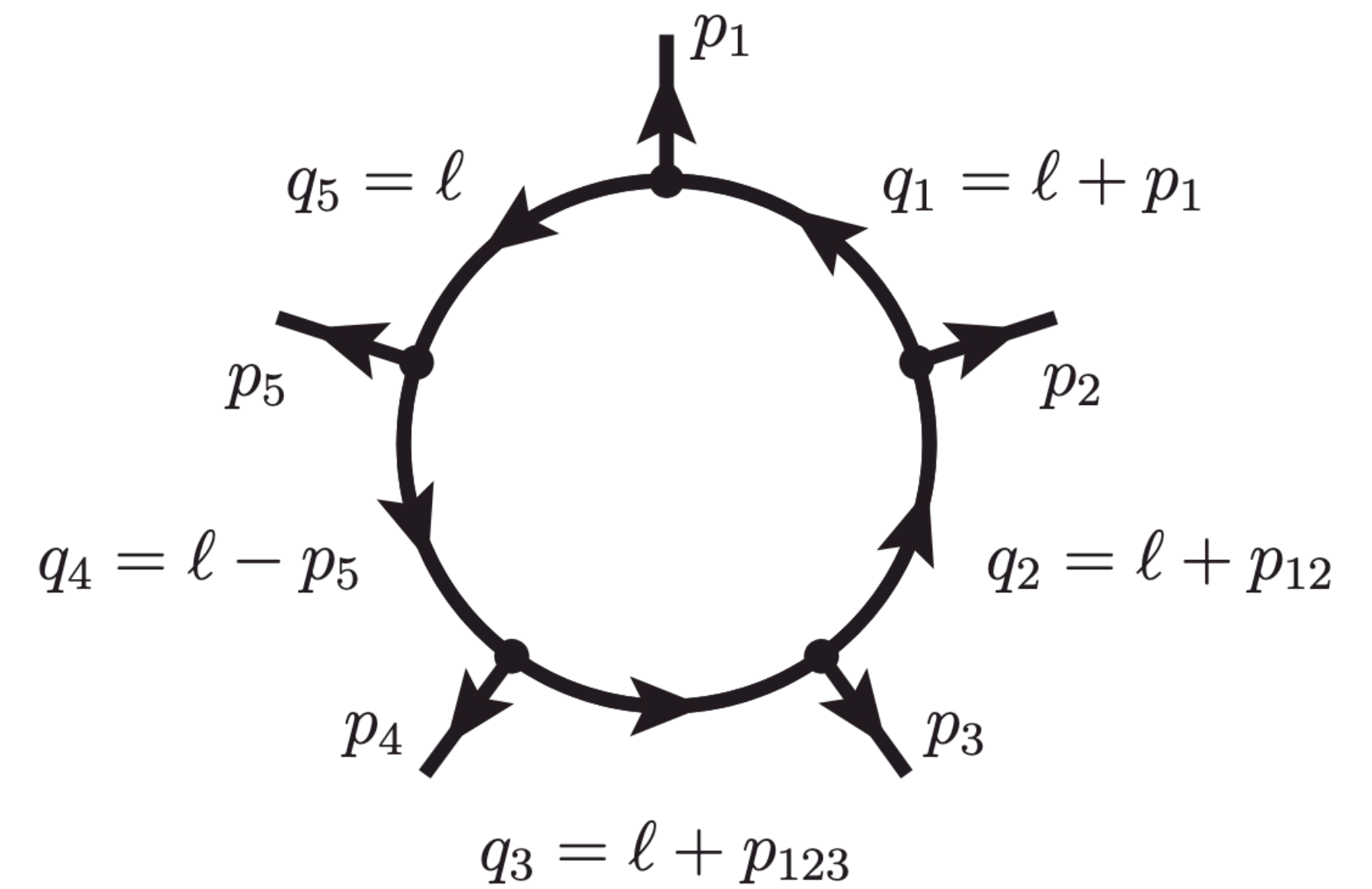


The one-loop pentagon

$$\mathcal{A}_5^{(1)} = \int_{\ell} \prod G_F(q_i) \rightarrow \int_{\vec{\ell}} \mathcal{A}_D^{(1)}$$

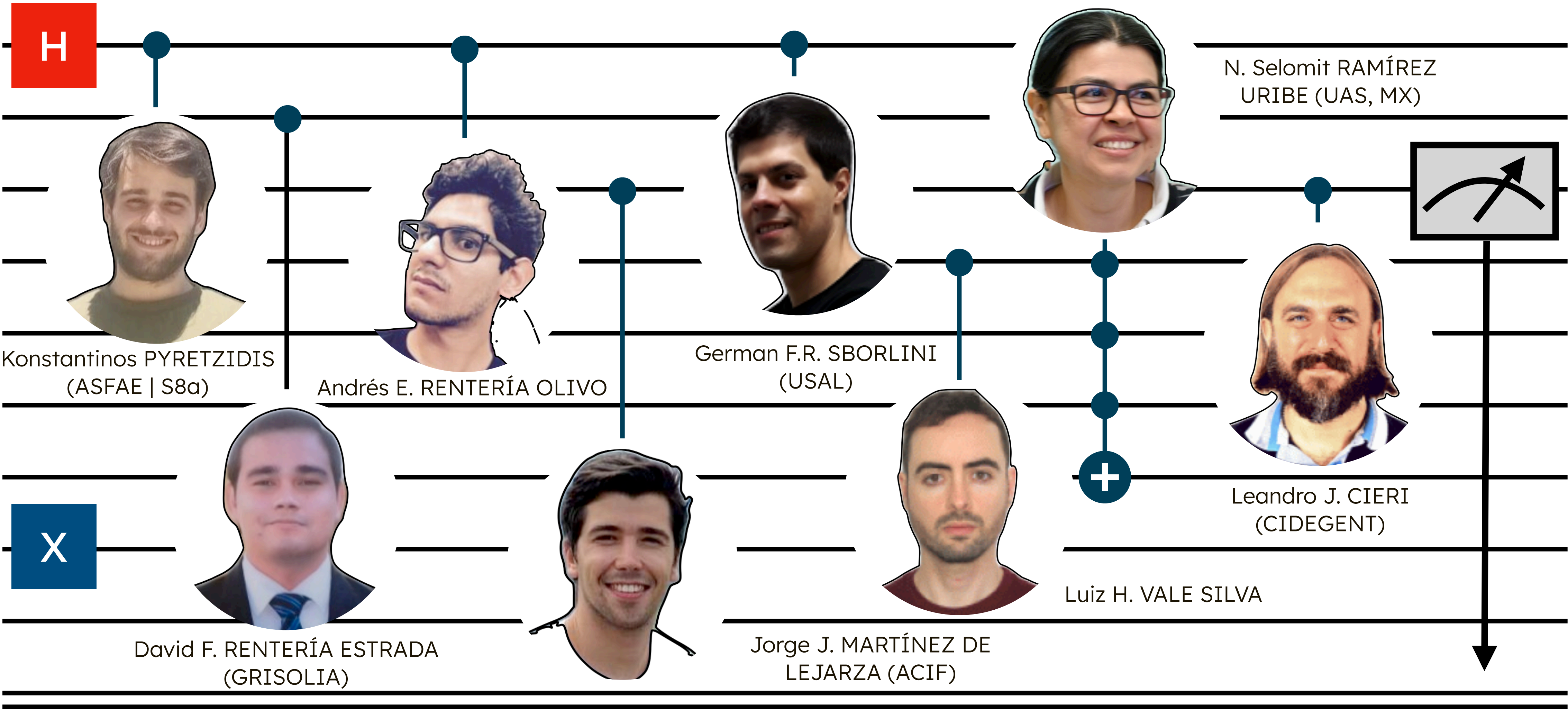
$$\lambda_i^{\pm} = q_{i,0}^{(+)} + q_{i+1,0}^{(+)} \pm p_{i,0}$$

$$\lambda_{ij}^{\pm} = q_{i,0}^{(+)} + q_{j+1,0}^{(+)} \pm (p_i + p_j)_0, \quad j = i + 1$$



○ uncertainty band from 100 independent runs with the same PDF

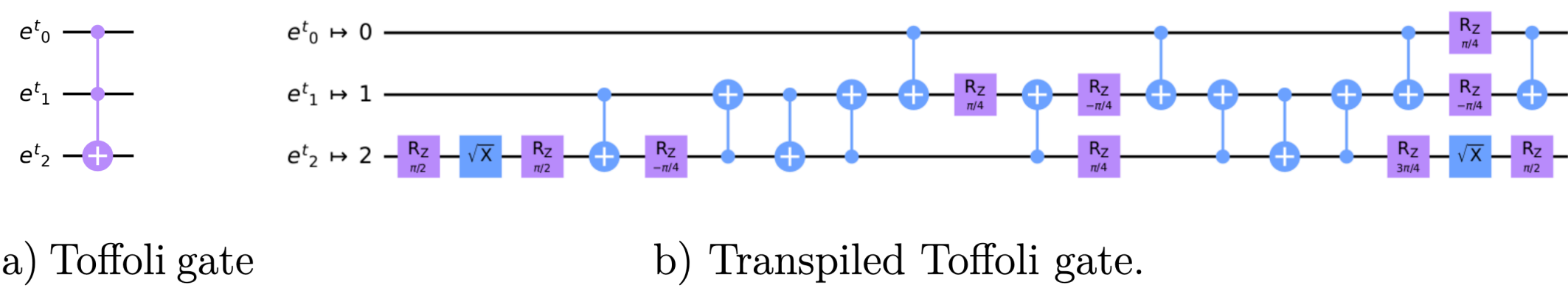
Quantum team



Conclusions

- High-energy colliders are quantum machines, and so the hard scattering processes are well suited for a quantum computing approach
- By mapping Feynman propagators to qubits and interpreting cyclic or non-causal configuration of multi-loop Feynman diagrams as multi-controlled Toffoli gates, an optimal oracle is designed through graph-theory principles.
- Motivated by **Vacuum amplitudes in LTD** as the most efficient building blocks to ensemble theoretical predictions at colliders; yielding well-behaved and manifestly causal integrand representations
- QFIAE, and in particular QAIS, to efficiently boost quantum integration and sampling of multidimensional functions
- My (long-term) dream: a **quantum event generator** with NNLO, ..., $N^k\text{LO} + N^k\text{LL}$ accuracy

The quantum area



a) Toffoli gate

b) Transpiled Toffoli gate.

- Huge difference between the theoretical quantum depth and the quantum depth of the **transpiled** quantum circuit
- **Transpilation** may involve more qubits than initially thought

- **Quantum area:** the product of the quantum depth and number of qubits required for the transpiled quantum circuit
- Hardware dependent
- Good measure of the dramatic difference in running times (**from minutes to seconds**)

