BabaYaga@NLO at present and future e^+e^- colliders

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INFN

in collaboration with

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What is BabaYaga@NLO?

25 years of BabaYaga

Disclaimer:
I was born after BabaYaga.
I will present what I learned
and contributed to

2000 - BabaYaga
C.M. Carloni Calame et al.,
Nucl. Phys. B 584 (2000) 459

2001 - Improved PS
C.M. Carloni Calame,
Phys. Lett. B 520 (2001) 16

2001
I was born

2004 - BabaYaga3.5
C.M. Carloni Calame et al.,
Nucl. Phys. Proc. Suppl. 131 (2004) 48

2006 - BabaYaga@NLO
for Bhabha at flavour factories
Balossini et al.,
Nucl. Phys. B758 (2006) 227

2008 - BabaYaga@NLO $e^+e^+ \to \gamma\gamma$
Balossini et al.,
Phys. Lett. 663 (2008) 209

2011 - Dark Photon
Barzè et al.,
Eur. Phys. J. C 71 (2011) 1680

2018
I started studying
Physics

2019 - NLO EW $e^+e^+ \to \gamma\gamma$
C. M. Carloni Calame et al.,
Phys.Lett.B 798 (2019)

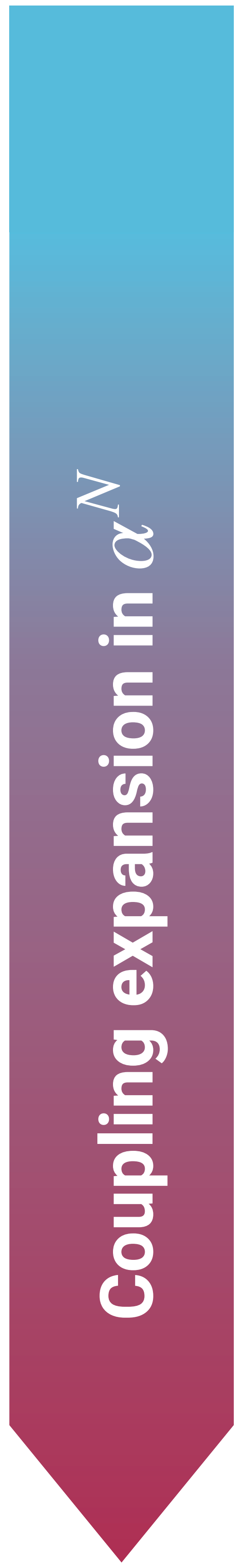
2023
I started the PhD
working on BabaYaga

2025 - $e^+e^+ \to \pi^+\pi^-$
E. Budassi et al.,
JHEP 05 (2025) 196

2025 - New Physics in SABS
M. Chiesa et al.,
Phys.Rev.D 112 (2025) 1

QED beyond fixed order

Fixed order calculations increase precision in the perturbative coupling expansion



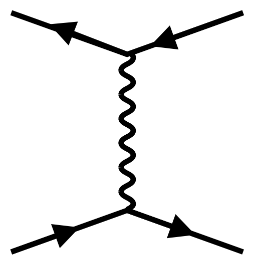
QED beyond fixed order

Logarithmic expansion in $L^N = N \log Q^2/m^2$

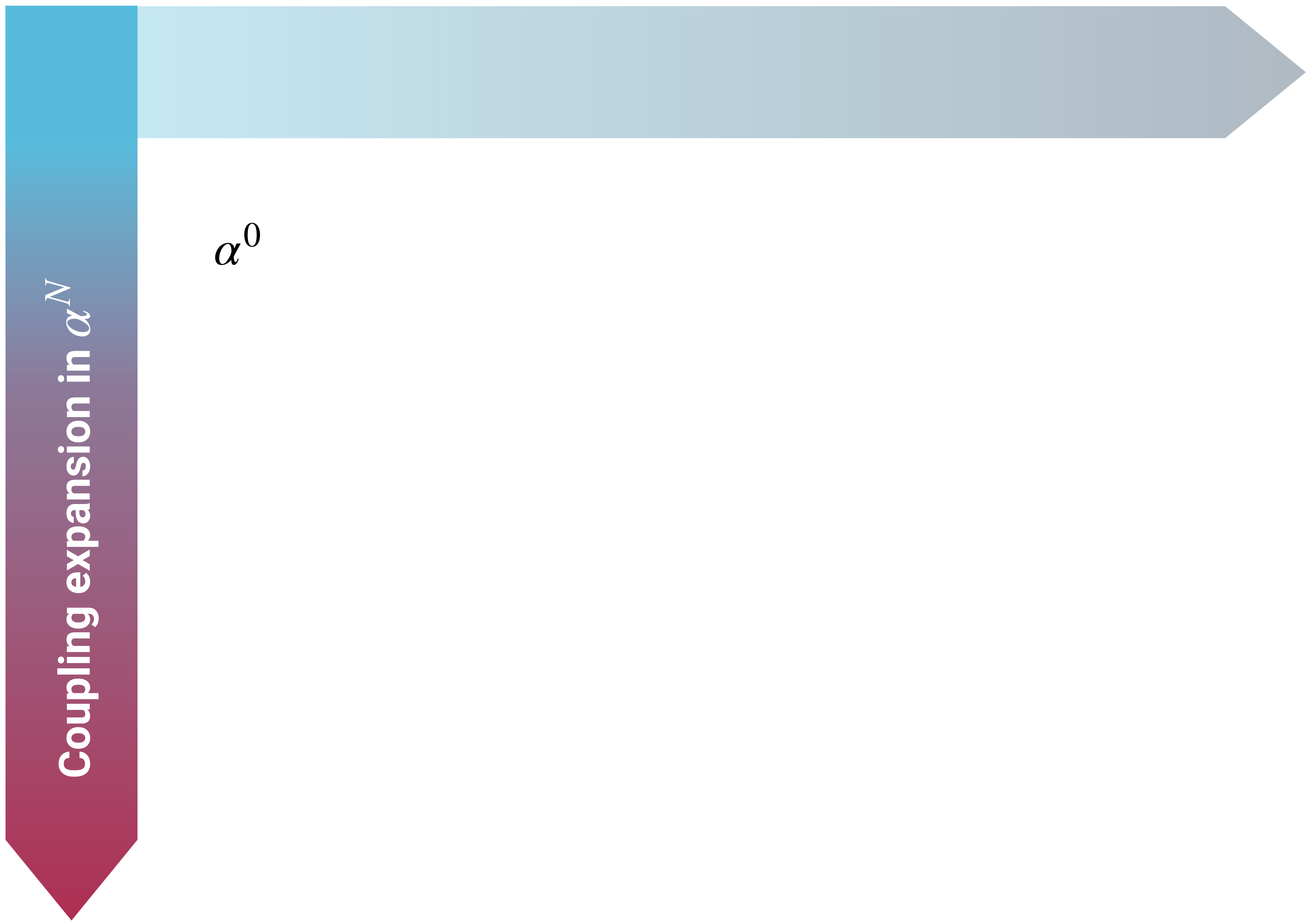
Due to the presence of **large logarithms** also the log expansion is needed for high precision

$$L = \log \frac{s}{m_e^2} \sim \mathcal{O}(10)$$

QED beyond fixed order



LO

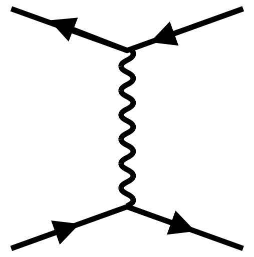
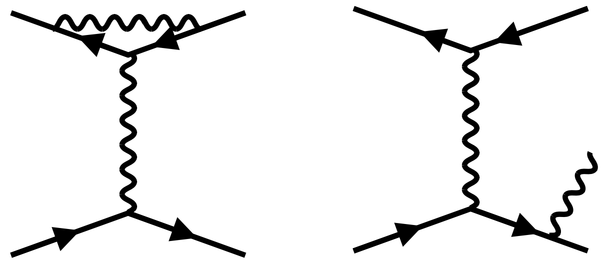


α^0

QED beyond fixed order

Logarithmic expansion in $L^N = N \log Q^2/m^2$

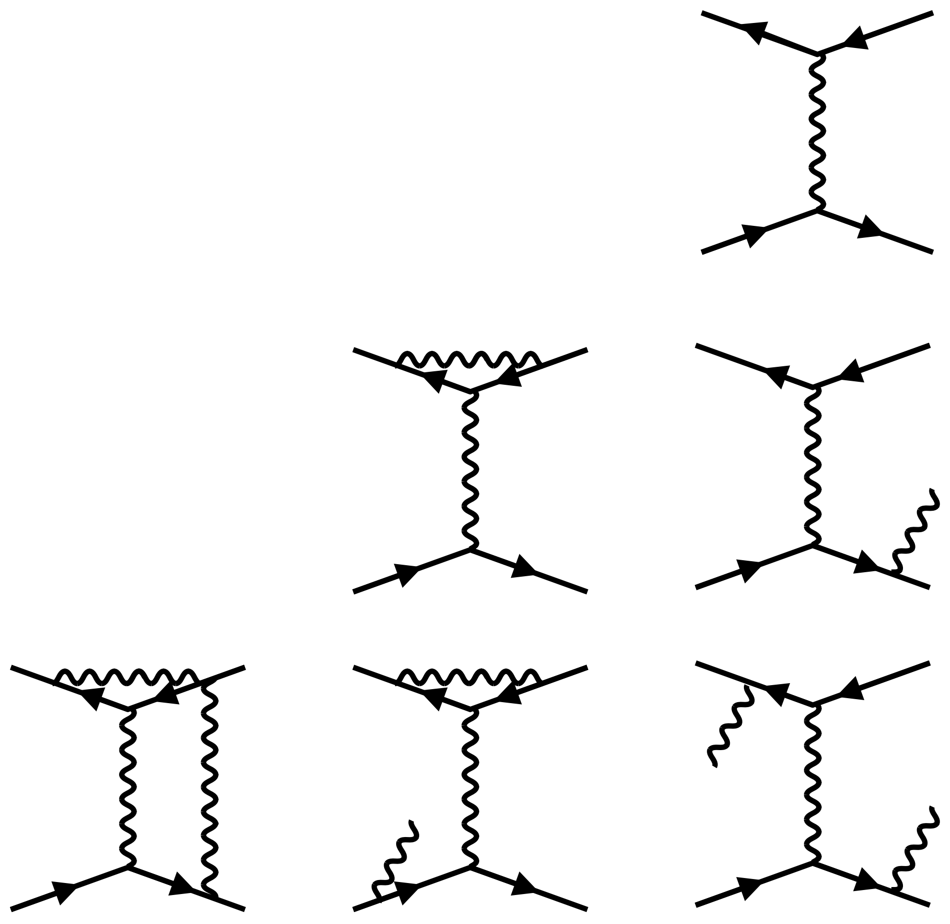
Coupling expansion in α^N

		LL	NLL	NNLL
	LO	α^0		
	NLO	αL	α	

QED beyond fixed order

Logarithmic expansion in $L^N = N \log Q^2/m^2$

Coupling expansion in α^N



	LL	NLL	NNLL
LO	α^0		
NLO	αL	α	
NNLO	$\frac{1}{2}\alpha^2 L^2$	$\frac{1}{2}\alpha^2 L$	$\frac{1}{2}\alpha^2$

QED beyond fixed order

Logarithmic expansion in $L^N = N \log Q^2/m^2$

Coupling expansion in α^N

	LL	NLL	NNLL	
LO	α^0			
NLO	αL	α		
NNLO	$\frac{1}{2}\alpha^2 L^2$	$\frac{1}{2}\alpha^2 L$	$\frac{1}{2}\alpha^2$	
H.O.	$\sum_{n=3}^{\infty} \frac{1}{n!}\alpha^n L^n$	$\sum_{n=3}^{\infty} \frac{1}{n!}\alpha^n L^{n-1}$	$\dots$	$\dots$

QED beyond fixed order

Logarithmic expansion in $L^N = N \log Q^2/m^2$

Coupling expansion in α^N

	LL	NLL	NNLL	
LO	α^0			
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NNLO	$\frac{1}{2}\alpha^2 L^2$	$\frac{1}{2}\alpha^2 L$	$\frac{1}{2}\alpha^2$	
H.O.	$\sum_{n=3}^{\infty} \frac{1}{n!}\alpha^n L^n$	$\sum_{n=3}^{\infty} \frac{1}{n!}\alpha^n L^{n-1}$	$\dots$	$\dots$

We want to stop in the α expansion at some order

QED beyond fixed order

Logarithmic expansion in $L^N = N \log Q^2/m^2$

Coupling expansion in α^N

	LL	NLL	NNLL	
LO	α^0			
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NNLO	$\frac{1}{2}\alpha^2 L^2$	$\frac{1}{2}\alpha^2 L$	$\frac{1}{2}\alpha^2$	
H.O.	$\sum_{n=3}^{\infty} \frac{1}{n!}\alpha^n L^n$	$\sum_{n=3}^{\infty} \frac{1}{n!}\alpha^n L^{n-1}$	$\dots$	$\dots$

We want to stop in the α expansion at some order

But we want to **resum** all the logarithms in a calculable way

QED beyond fixed order

Logarithmic expansion in $L^N = N \log Q^2/m^2$

Coupling expansion in α^N

	LL	NLL	NNLL	
LO	α^0			
NLO	αL	α		
NNLO	$\frac{1}{2}\alpha^2 L^2$	$\frac{1}{2}\alpha^2 L$	$\frac{1}{2}\alpha^2$	
H.O.	$\sum_{n=3}^{\infty} \frac{1}{n!}\alpha^n L^n$	$\sum_{n=3}^{\infty} \frac{1}{n!}\alpha^n L^{n-1}$	$\dots$	$\dots$

This can be done via different approaches

Structure functions
E.A. Kuraev and V.S. Fadin, Sov. J. Nucl. Phys. 41 (1985) 466.
O. Nicrosini and L. Trentadue, Phys. Lett. B 196 (1987) 551.

YFS/CEEX
S. Jadach, B.F.L. Ward and Z. Was, Phys. Rev. D 63 (2001) 113009
S. Jadach, W. Placzek and B.F.L. Ward, Phys. Lett. B 390 (1997) 298

Parton shower
C.M. Carloni Calame et al., Nucl. Phys. B 584 (2000) 459
J. Fujimoto, Y. Shimizu and T. Munehisa, Prog. Theor. Phys. 90(1993)177

QED Structure functions

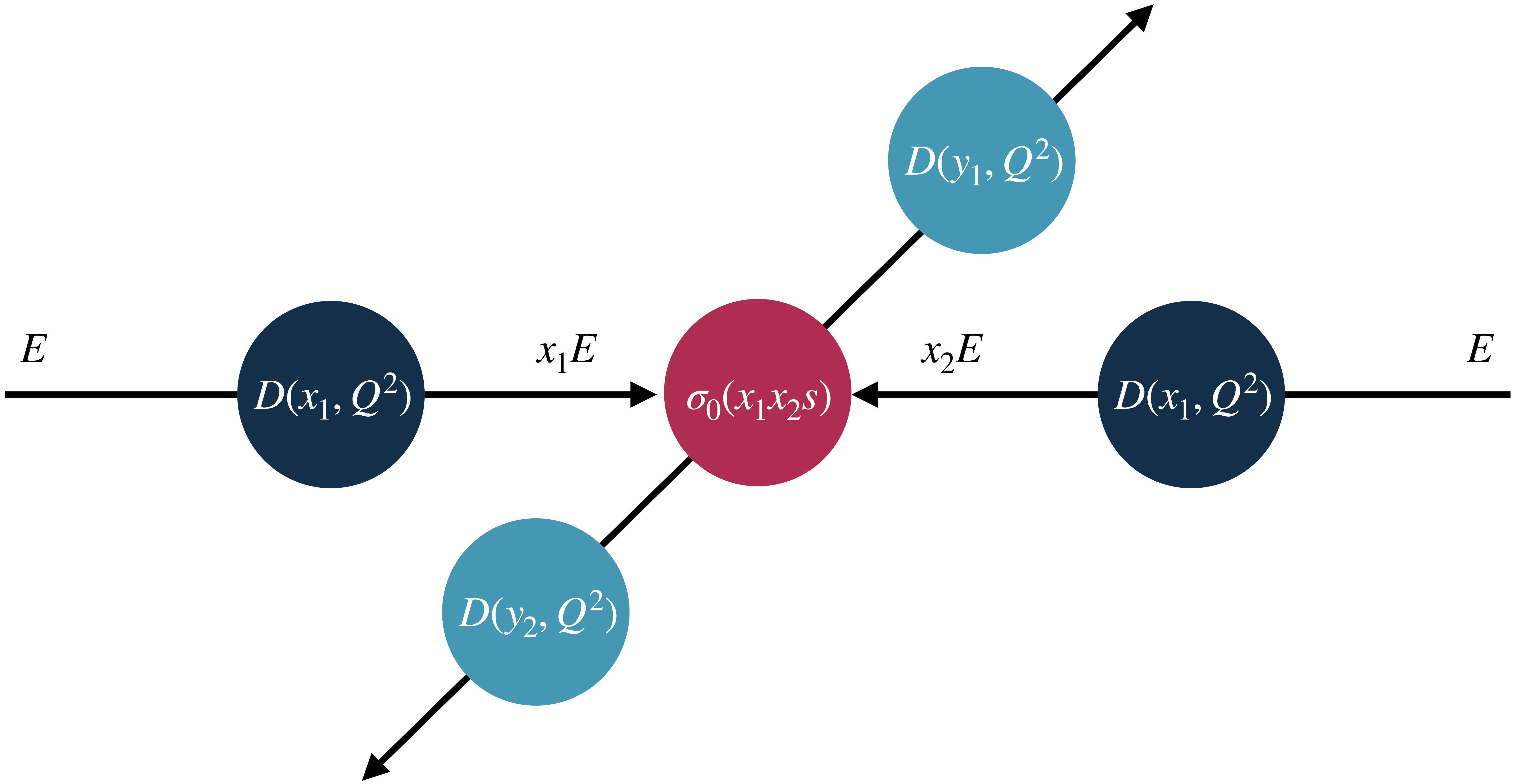
Master Formula

QED corrected cross section

$$\sigma(s) = \int dx_1 dx_2 dy_1 dy_2 \int d\Omega D(x_1, Q^2) D(x_2, Q^2) D(y_1, Q^2) D(y_2, Q^2) \frac{d\sigma_0(x_1 x_2 s)}{d\Omega} \Theta(\text{cuts})$$

Convolution of SFs

Hard-process cross section



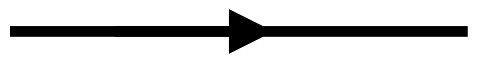
QED Structure functions

DGLAP Equation

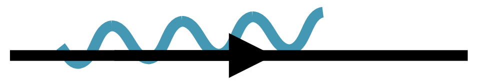
$$Q^2 \frac{\partial}{\partial Q^2} D(x, Q^2) = \frac{\alpha}{2\pi} \int_0^1 \frac{ds'}{s'} P_+(s') D\left(\frac{x}{s'}, Q^2\right)$$

Structure Functions (FS) are solutions of the DGLAP equation

$$D(x, Q^2) =$$



$$\Pi(Q^2, m^2)\delta(1-x)$$

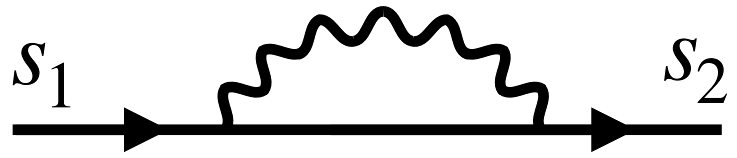


$$+\frac{\alpha}{2\pi}\int_{m^2}^{s} \Pi(Q^2, s')\frac{ds'}{s'}\Pi(s', m^2)\int_0^{x_+} dy\, P(y)\delta(x-y)$$

$$+2\ \text{photons}\ldots$$

SF generate all the emissions in collinear approximation

Sudakov form factor

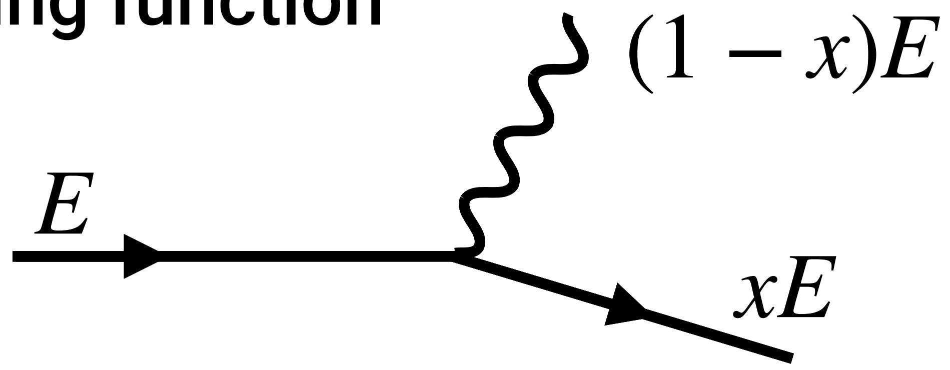


$$\Pi(s_1, s_2) = \exp\left[-\frac{\alpha}{2\pi}\int_{s_1}^{s_2}\frac{ds'}{s'}\int_0^{x_+} dz P(z)\right]$$

Probability that the particle evolves from virtuality s_1 to s_2 without emitting a photon with energy fraction bigger than $\epsilon = 1 - x_+$

Altarelli-Parisi splitting function

$$P(z) = \frac{1+z^2}{1-z}$$



Splitting of a particle of energy E in a daughter of energy xE

QED PS algorithm

The **Parton Shower (PS)** algorithm is a Monte Carlo exact solution of the DGLAP equation

$$\mathrm{d}\sigma_{\mathrm{PS}}^{\mathrm{LL}} = \Pi(\epsilon, Q^2) \sum_{n=0}^{\infty} \frac{1}{n!} \left| \mathscr{M}_n^{\mathrm{LL}} \right|^2 \mathrm{d}\Phi_n$$

$$\left| \mathscr{M}_1^{\mathrm{LL}} \right|^2 = \frac{\alpha}{2\pi} \boxed{P(z)}\ \boxed{I(k)} |\mathscr{M}_0|^2 J$$

Energy spectrum

Energy generated as the A-P splitting

$$P(z) = \frac{1+z^2}{1-z}$$

Angular spectrum

In the PS approach, you can generate the photon kinematics with $p_\perp \neq 0$

$$I(k) = \sum_{ij} \eta_i \eta_j \frac{p_i \cdot p_j}{(p_i \cdot k)(p_j \cdot k)} k_0^2$$

$$\Pi(\epsilon, Q^2) = \exp\left\{ -\frac{\alpha}{2\pi} \boxed{\int_0^{1-\epsilon} \mathrm{d}z P(z)} \boxed{\int \mathrm{d}\Omega_k I(k)} \right\}$$

Sudakov FF

The eikonal function $I(k)$ is exponentiated, as it gives the same integral as of the PS kinematics

$$= \exp\left\{ -\frac{\alpha}{2\pi} \quad I_+ \quad \log \frac{Q^2}{m^2} \right\}$$

BabaYaga@NLO master formula

BabaYaga@NLO

$$d\sigma_{\text{NLOPS}} = F_{\text{SV}}\Pi(\epsilon, Q^2)\sum_{n=0}^{\infty}\frac{1}{n!}\left(\prod_{i=0}^{n}F_{H,i}\right)\left|\mathscr{M}_n^{\text{LL}}\right|^2 d\Phi_n$$

BabaYaga@NLO master formula

BabaYaga@NLO

$$\mathrm{d}\sigma_{\mathrm{NLOPS}} = F_{\mathrm{SV}}\Pi(\epsilon, Q^2)\sum_{n=0}^{\infty}\frac{1}{n!}\left(\prod_{i=0}^{n}F_{H,i}\right)\left|\mathscr{M}_n^{\mathrm{LL}}\right|^2 \mathrm{d}\Phi_n$$

Exact NLO

virtual and real
corrections are exact

$\mathcal{O}(\alpha)$	$\mathrm{d}\sigma_\alpha$
Soft+virtual	$\left(1 + C_\alpha\right)\lvert\mathscr{M}_0\rvert^2 \mathrm{d}\Phi_0$
Real	$\lvert\mathscr{M}_1\rvert^2 \mathrm{d}\Phi_1$

BabaYaga@NLO master formula

BabaYaga@NLO

$$d\sigma_{\mathrm{NLOPS}} = F_{\mathrm{SV}} \Pi(\epsilon, Q^2) \sum_{n=0}^{\infty} \frac{1}{n!} \left(\prod_{i=0}^{n} F_{H,i} \right) \left| \mathscr{M}_n^{\mathrm{LL}} \right|^2 d\Phi_n$$

	Exact NLO virtual and real corrections are exact	**LL PS** virtual and real emissions are approximated
$\mathscr{O}(\alpha)$	$d\sigma_\alpha$	$d\sigma_\alpha^{\mathrm{LL}}$
Soft+virtual	$\left(1 + C_\alpha\right) \vert \mathscr{M}_0 \vert^2 d\Phi_0$	$\left(1 + C_\alpha^{\mathrm{LL}}\right) \vert \mathscr{M}_0 \vert^2 d\Phi_0$
Real	$\vert \mathscr{M}_1 \vert^2 d\Phi_1$	$\vert \mathscr{M}_1^{\mathrm{LL}} \vert^2 d\Phi_1$

BabaYaga@NLO master formula

BabaYaga@NLO

$$\mathrm{d}\sigma_{\mathrm{NLOPS}} = F_{\mathrm{SV}} \Pi(\epsilon, Q^2) \sum_{n=0}^{\infty} \frac{1}{n!} \left(\prod_{i=0}^{n} F_{H,i} \right) \left| \mathcal{M}_n^{\mathrm{LL}} \right|^2 \mathrm{d}\Phi_n$$

	Exact NLO	LL PS	Matched PS
	virtual and real corrections are exact	virtual and real emissions are approximated	In BabaYaga@NLO, the PS is matched with the NLO calculation via the correction factors
$\mathcal{O}(\alpha)$	$\mathrm{d}\sigma_\alpha$	$\mathrm{d}\sigma_\alpha^{\mathrm{LL}}$	$\mathrm{d}\sigma_{\mathrm{NLOPS}}^{\alpha}$
Soft+virtual	$(1 + C_\alpha) \lvert \mathcal{M}_0 \rvert^2 \mathrm{d}\Phi_0$	$(1 + C_\alpha^{\mathrm{LL}}) \lvert \mathcal{M}_0 \rvert^2 \mathrm{d}\Phi_0$	$F_{\mathrm{SV}} = 1 + (C_\alpha - C_\alpha^{\mathrm{LL}})$ affects the normalisation
Real	$\lvert \mathcal{M}_1 \rvert^2 \mathrm{d}\Phi_1$	$\lvert \mathcal{M}_1^{\mathrm{LL}} \rvert^2 \mathrm{d}\Phi_1$	$F_H = 1 + \frac{\lvert \mathcal{M}_1 \rvert^2 - \lvert \mathcal{M}_1^{\mathrm{LL}} \rvert^2}{\lvert \mathcal{M}_1^{\mathrm{LL}} \rvert^2}$ 1γ exact Matrix Element, $n\gamma$ permutations of 1γ ME

$\mathrm{d}\Phi_n$ The phase space is exact at all orders

QED beyond fixed order

Logarithmic expansion in $L^N = N \log Q^2/m^2$

Coupling expansion in α^N

	LL	NLL	NNLL	
LO	α^0			
NLO	αL	α		
NNLO	$\frac{1}{2}\alpha^2 L^2$	$\frac{1}{2}\alpha^2 L$	$\frac{1}{2}\alpha^2$	
H.O.	$\sum_{n=3}^{\infty}\frac{1}{n!}\alpha^n L^n$	$\sum_{n=3}^{\infty}\frac{1}{n!}\alpha^n L^{n-1}$	$\dots$	$\dots$

Approach	Code
SF + NLO	`MCGPJ`
YFS + matching	`BHWIDE, Sherpa`
PS + matching	`BabaYaga@NLO`
CEEX + NLO	`KKMC`

QED beyond fixed order

Logarithmic expansion in $L^N = N \log Q^2/m^2$

Coupling expansion in α^N

	LL	NLL	NNLL	
LO	α^0			
NLO	αL	α		
NNLO	$\frac{1}{2}\alpha^2 L^2$	$\boxed{\frac{1}{2}\alpha^2 L}$	$\frac{1}{2}\alpha^2$	
H.O.	$\sum_{n=3}^{\infty} \frac{1}{n!}\alpha^n L^n$	$\sum_{n=3}^{\infty} \frac{1}{n!}\alpha^n L^{n-1}$	$\dots$	$\dots$

Theoretical error = first missing term $\sim 5 \times 10^{-4} - 10^{-3}$

QED beyond fixed order

Logarithmic expansion in $L^N = N \log Q^2/m^2$

Coupling expansion in α^N

	LL	NLL	NNLL	
LO	α^0			
NLO	αL	α		
NNLO	$\frac{1}{2}\alpha^2 L^2$	$\frac{1}{2}\alpha^2 L$	$\frac{1}{2}\alpha^2$	NNLO matching
H.O.	$\sum_{n=3}^{\infty} \frac{1}{n!}\alpha^n L^n$	$\sum_{n=3}^{\infty} \frac{1}{n!}\alpha^n L^{n-1}$	$\dots$	$\dots$

PS NLL

Status of BabaYaga

	Process	Order	Accuracy
Published	$e^+e^- \to e^+e^-$	NLOPS QED LO EW LO SMEFT	$\mathcal{O}(0.1\%)$
	$e^+e^- \to \mu^+\mu^-$	NLOPS QED LO EW	$\mathcal{O}(0.1\%)$
	$e^+e^- \to \gamma\gamma$	NLOPS QED NLO EW	$\mathcal{O}(0.1\%)$
	$e^+e^- \to \pi^+\pi^-$	NLOPS QED FF (FxsQED, GVMD, FSQED)	$\mathcal{O}(0.1\%) \oplus \delta F_\pi$
Work in Progress	$e^+e^- \to e^+e^-\gamma$	NLOPS QED	
	$e^+e^- \to \mu^+\mu^-\gamma$	NLOPS QED	
	$e^+e^- \to \pi^+\pi^-\gamma$	NLOPS QED FF (FxSQED)	

BabaYaga@NLO for luminosity

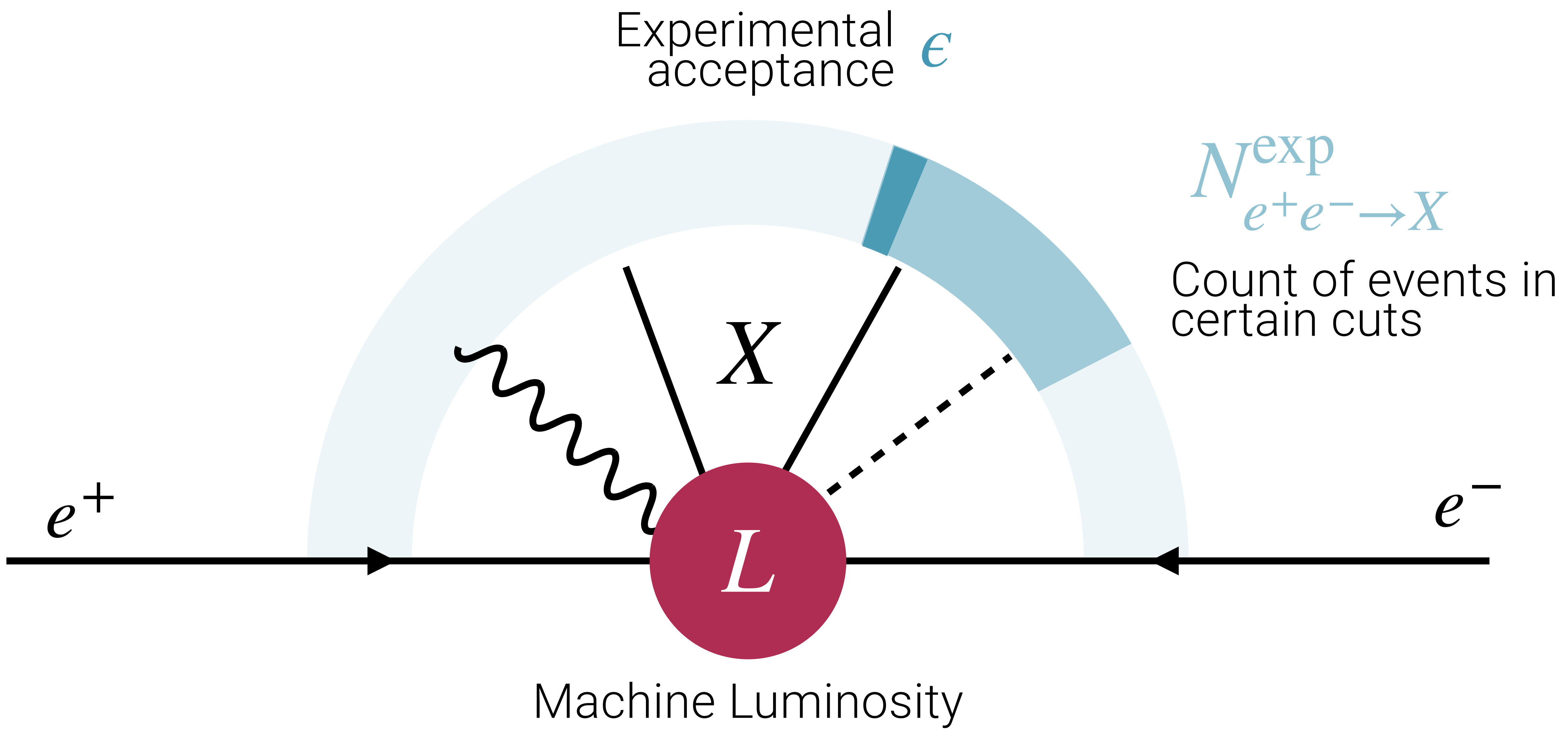
Luminosity measurements

*"**Luminosity** converts events into cross sections"*

$$\sigma^{\rm exp}_{e^+e^-\to X} = \frac{1}{\epsilon}\frac{N^{\rm exp}_{e^+e^-\to X}}{L}$$

At precision machines is important to have a small **luminosity uncertainty**

$$\frac{\delta\sigma^{\rm exp}_{e^+e^-\to X}}{\sigma^{\rm exp}_{e^+e^-\to X}} = \frac{\delta\epsilon}{\epsilon} \oplus \frac{\delta N^{\rm exp}_{e^+e^-\to X}}{N^{\rm exp}_{e^+e^-\to X}} \oplus \boxed{\frac{\delta L}{L}}$$



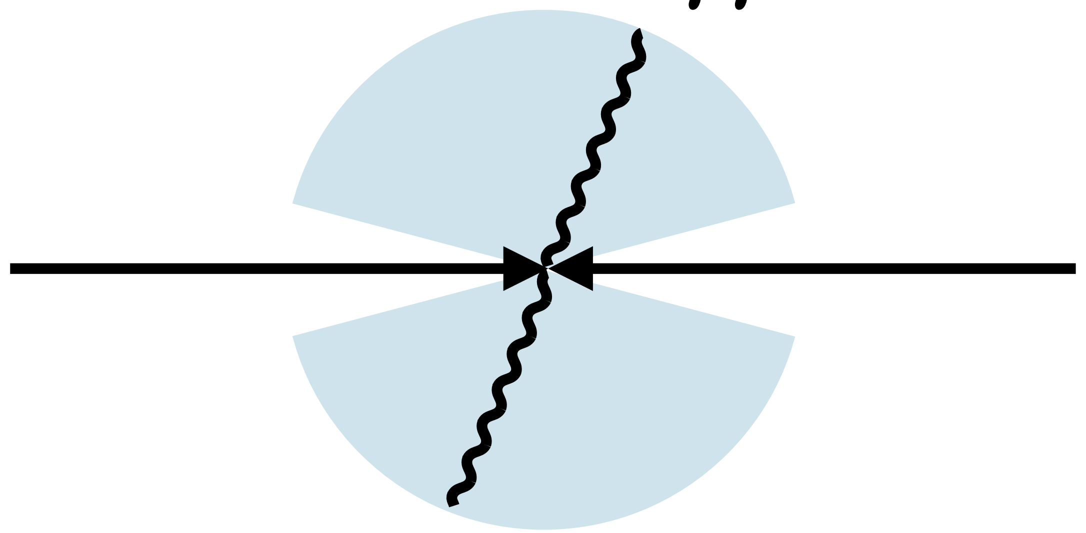
Luminosity measurements

At lepton colliders, the Luminosity is measured through a **benchmark process**

$$L = \int \mathscr{L} \mathrm{d}t = \frac{1}{\epsilon} \frac{N_0}{\sigma_0^{\mathrm{th}}}$$

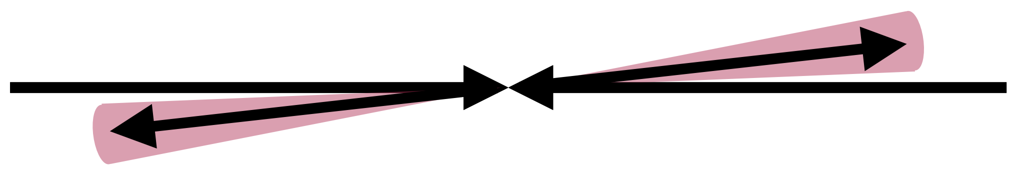
At *past*, *present* and *future* colliders: two alternative processes

$$e^+e^- \to \gamma\gamma$$



Di-photon
A pure QED process

$$e^+e^- \to e^+e^-$$



Bhabha
At small angles, a QED process

Error	Requests	FCC/CEPC
$\frac{\delta L}{L}$		$< 10^{-4} \div 10^{-5}$
$\parallel$		
$\frac{\delta \epsilon_{\mathrm{exp}}}{\epsilon_{\mathrm{exp}}}$	Low background, clear exp. signature	$\simeq 10^{-5}$
$\oplus$		
$\frac{\delta N_0}{N_0}$	$\sigma \simeq \mathcal{O}(10^2 - 10^3\,\mathrm{nb})$ Large cross section	$< 10^{-6}$
$\oplus$		
$\frac{\delta \sigma_0^{\mathrm{th}}}{\sigma_0^{\mathrm{th}}}$	$\sigma^{(n)} = \left(\frac{\alpha}{\pi}\right)^n \log^n \frac{Q^2}{m_e^2}$ Calculable at high precision	$< 10^{-4} \div 10^{-5}$

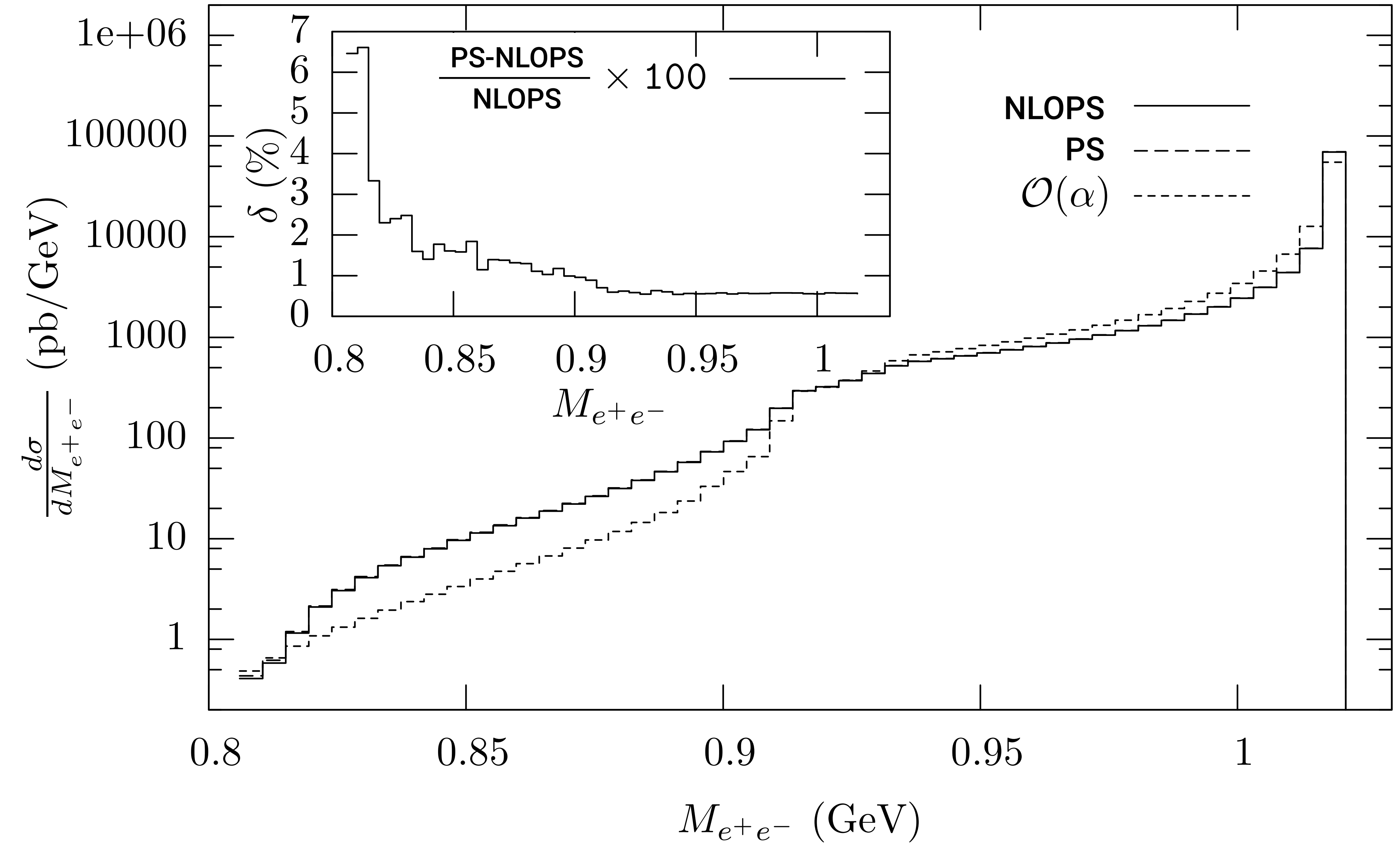
Theoretical precision

The size of radiative corrections has been studied in typical flavour factories setups

ϕ-factories
- **A** $\sqrt{s} = 1.02\ \text{GeV},\ E_{min} = 0.408\ \text{GeV},\ 20^\circ < \theta_\pm < 160^\circ,\ \xi_{max} = 10^\circ$
- **B** $\sqrt{s} = 1.02\ \text{GeV},\ E_{min} = 0.408\ \text{GeV},\ 55^\circ < \theta_\pm < 125^\circ,\ \xi_{max} = 10^\circ$

B-factories
- **C** $\sqrt{s} = 10\ \text{GeV},\ E_{min} = 4\ \text{GeV},\ 20^\circ < \theta_\pm < 160^\circ,\ \xi_{max} = 10^\circ$
- **D** $\sqrt{s} = 10\ \text{GeV},\ E_{min} = 4\ \text{GeV},\ 55^\circ < \theta_\pm < 125^\circ,\ \xi_{max} = 10^\circ$

Correction vs Setup	A	B	C	D
$\delta_\alpha = \frac{\sigma_{\text{NLO}} - \sigma_{\text{LO}}}{\sigma_{\text{LO}}}$	-11.61	-14.72	-16.03	-19.57
$\delta_{\text{HO}} = \frac{\sigma_{\text{NLOPS}} - \sigma_{\text{NLO}}}{\sigma_{\text{LO}}}$	0.39	0.82	0.73	1.44
$\delta_{\text{HO}}^{\text{PS}} = \frac{\sigma_{\text{PS}}^{\infty} - \sigma_{\text{PS}}^{\alpha}}{\sigma_{\text{LO}}}$	0.35	0.74	0.68	1.34
$\delta_{\alpha}^{\text{non-log}} = \frac{\sigma_{\text{NLO}} - \sigma_{\text{PS}}^{\alpha}}{\sigma_{\text{LO}}}$	-0.34	-0.57	-0.34	-0.56
$\delta_{\infty}^{\text{non-log}} = \frac{\sigma_{\text{NLOPS}} - \sigma_{\text{PS}}}{\sigma_{\text{LO}}}$	-0.30	-0.49	-0.29	-0.46



$\delta_{\text{HO}}^{\text{PS}} \simeq \delta_{\text{HO}}$

The higher orders are added to the NLO

$\delta_{\infty}^{\text{non-log}} \simeq \delta_{\alpha}^{\text{non-log}}$

The missing NLO finite corrections are included

Tuned comparisons

At the Φ and τ−charm factories

setup	BabaYaga@NLO	BHWIDE	MCGPJ	$\delta(\%)$
$\sqrt{s} = 1.02$ GeV, $20^\circ \le \vartheta_\mp \le 160^\circ$	6086.6(1)	6086.3(2)	—	0.005
$\sqrt{s} = 1.02$ GeV, $55^\circ \le \vartheta_\mp \le 125^\circ$	455.85(1)	455.73(1)	—	0.030
$\sqrt{s} = 3.5$ GeV, $\lvert\vartheta_+ + \vartheta_- - \pi\rvert \le 0.25$ rad	35.20(2)	—	35.181(5)	0.050

By BabaYaga group, Ping Wang and A. Sibidanov

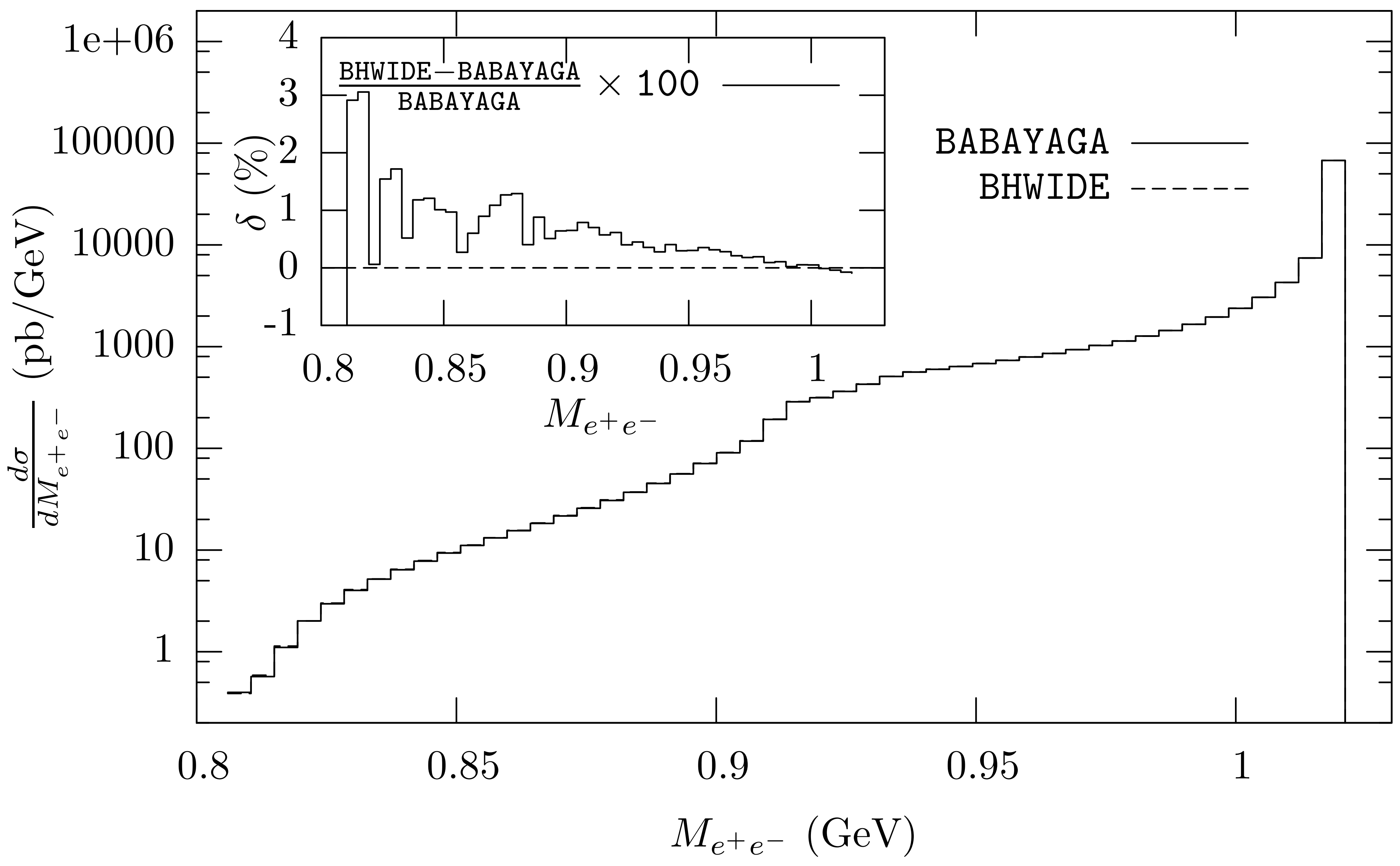
At BaBar

angular acceptance cuts	BabaYaga@NLO	BHWIDE	$\delta(\%)$
$15^\circ \div 165^\circ$	119.5(1)	119.53(8)	0.025
$40^\circ \div 140^\circ$	11.67(3)	11.660(8)	0.086
$50^\circ \div 130^\circ$	6.31(3)	6.289(4)	0.332
$60^\circ \div 120^\circ$	3.554(6)	3.549(3)	0.141

By A. Hafner and A. Denig

At KLOE

S. Actis et al. Eur. Phys. J. C 66 (2010) 585



Tests shows an agreement at $0.1\ \%$ level, which is the theoretical accuracy of BabaYaga@NLO

BabaYaga at flavour factories

Logarithmic expansion in $L^N = N \log 1\mathrm{GeV}^2/m_e^2$

Coupling expansion in α^N

	LL	NLL	NNLL
LO	$\mathcal{O}(90\%)$		
NLO	$\mathcal{O}(10\%)$	$\mathcal{O}(0.5\%)$	
NNLO	$\mathcal{O}(0.5\%)$	$\mathcal{O}(0.05\%)$	$\mathcal{O}(0.01\%)$
H.O.	$\mathcal{O}(0.01\%)$		

Size of Radiative corrections in a typical flavour-factory setup

The theoretical error starts at $\mathcal{O}(\alpha^2)$

$\mathrm{d}\sigma_{\mathrm{NNLO}} \propto$

$$\frac{1}{2} c_{\alpha^2}^{\mathrm{LL}} \alpha^2 L^2$$

LL is correct
resumed in PS approach

$$\frac{1}{2} c_{\alpha^2}^{\mathrm{NLL}} \alpha^2 L^2$$

NLL not exact
but part is captured by the product of NLO non-log and LL

$$C_\alpha^{\mathrm{LL}} \times c_\alpha^{\mathrm{NLL}} \alpha = -\frac{\alpha^2}{2\pi} I_+ \log Q^2 m^2$$

$$\frac{1}{2} c_{\alpha^2}^{\mathrm{non-log}} \alpha^2$$

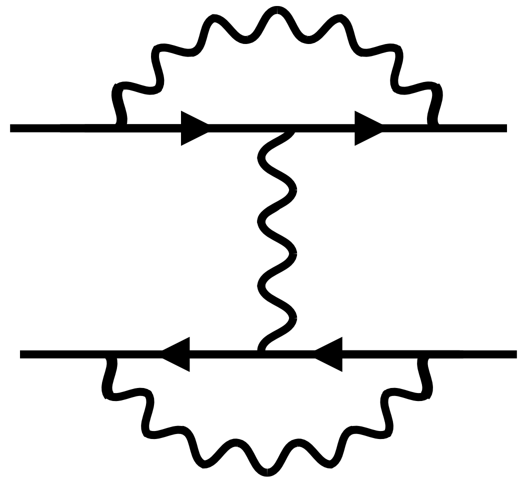
non-log not under control
There is no approximation.
NNLO matching needed

Theoretical error

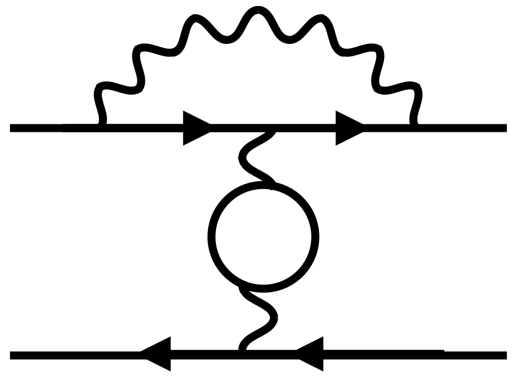
To quantify the theoretical error of BabaYaga, we can compare the exact NNLO versus the PS matched expanded

$$\sigma^{\alpha^2} = \sigma^{\alpha^2}_{\mathrm{SV}} + \sigma^{\alpha^2}_{\mathrm{SV,H}} + \sigma^{\alpha^2}_{\mathrm{HH}} \quad \text{vs} \quad \texttt{BabaYaga@NLO}\ \mathcal{O}(\alpha^2)$$

$\sigma^{\alpha^2}_{\mathrm{SV}}$

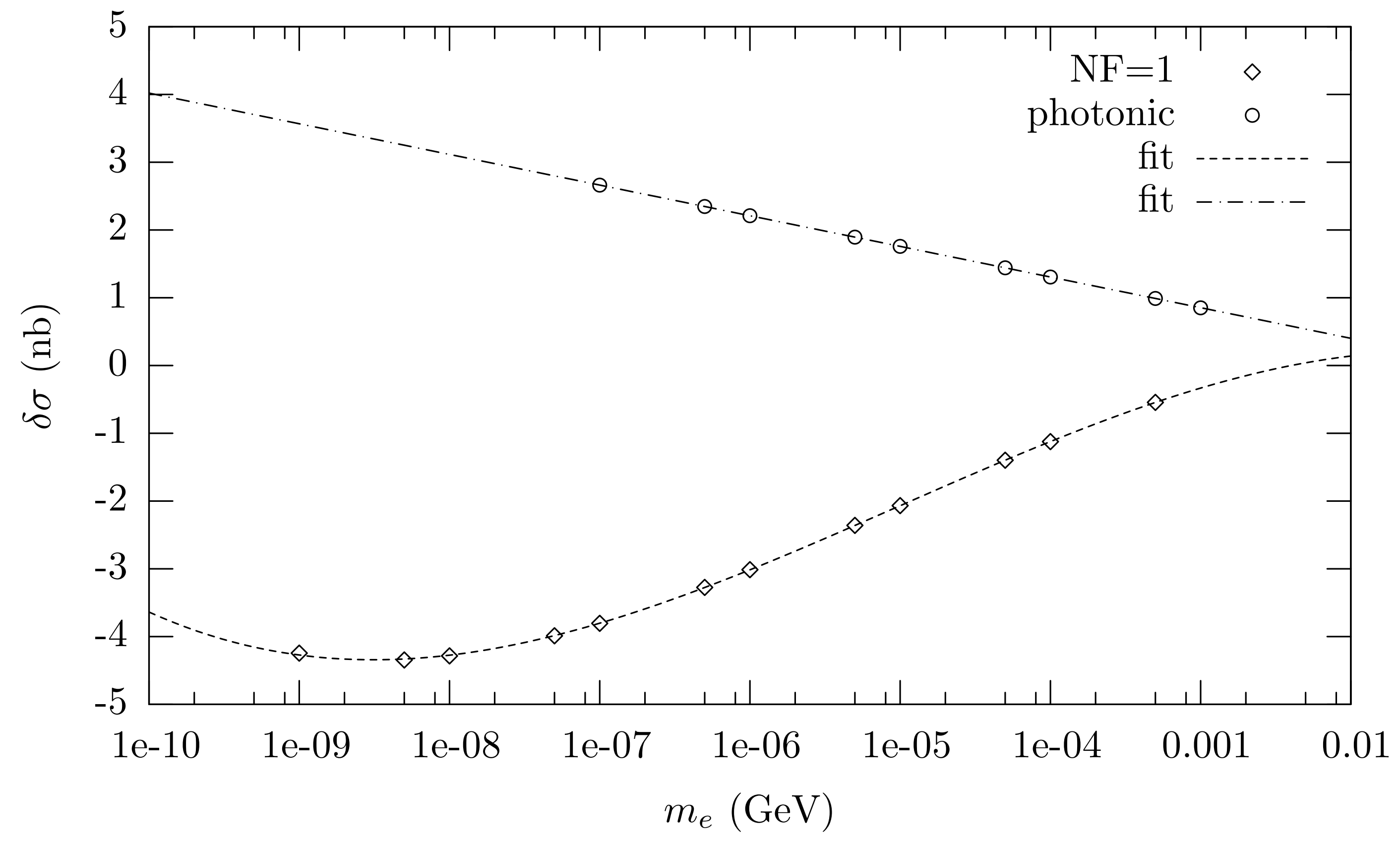


Photonic



Fermionic $N_f = 1$

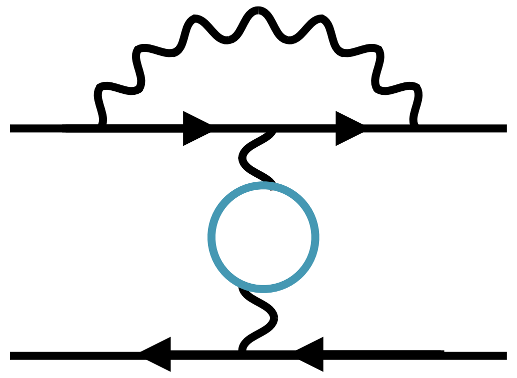
A. Penin, PRL 95 (2005) 010408
Nucl. Phys. B734 (2006) 185



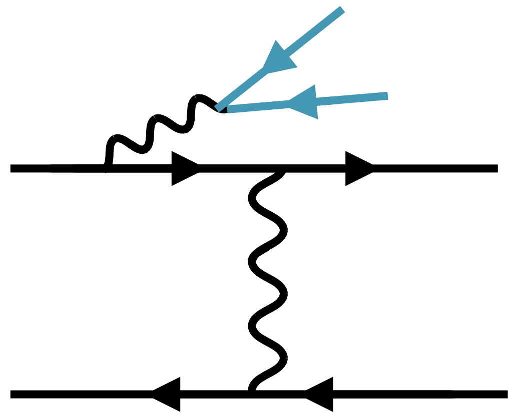
$$\frac{\delta\sigma(\mathrm{Photonic})}{\sigma_{\mathrm{LO}}} \propto \alpha^2 L$$

$$\sigma^{\alpha^2}_{\mathrm{SV}}(\mathrm{Penin}) - \sigma^{\alpha^2}_{\mathrm{SV}}(\mathrm{BY@NLO}) < 0.02\,\%\,\sigma_{\mathrm{LO}}$$

$\sigma^{\alpha^2}_{\mathrm{HH}}$



Photonic $f \neq e$



Real pairs

	$\sqrt{s}$		σ_{BY}	$S_{e^+e^-}$ [‰]	S_{lep} [‰]	S_{had} [‰]	S_{tot} [‰]
KLOE	1.020	NNLO		-3.935(4)	-4.472(4)	1.02(2)	-3.45(2)
		BB@NLO	455.71	-3.445(2)	-4.001(2)	0.876(5)	-3.126(5)
BES	3.650	NNLO		-1.469(9)	-1.913(9)	-1.3(1)	-3.2(1)
		BB@NLO	116.41	-1.521(4)	-1.971(4)	-1.071(4)	-3.042(5)
BaBar	10.56	NNLO		-1.48(2)	-2.17(2)	-1.69(8)	-3.86(8)
		BB@NLO	5.195	-1.40(1)	-2.09(1)	-1.49(1)	-3.58(2)
Belle	10.58	NNLO		-4.93(2)	-6.84(2)	-4.1(1)	-10.9(1)
		BB@NLO	5.501	-4.42(1)	-6.38(1)	-3.86(1)	-10.24(2)

$$\delta\sigma_{\mathrm{pairs}} \sim 10^{-4}$$

Carloni, Czyż, Gluza, Gunia, Montagna, Nicrosini, Piccinini, Riemann et al., JHEP 1107 (2011) 126

BabaYaga@NLO at future colliders

FCC luminosity with $e^+e^- \to \gamma\gamma$

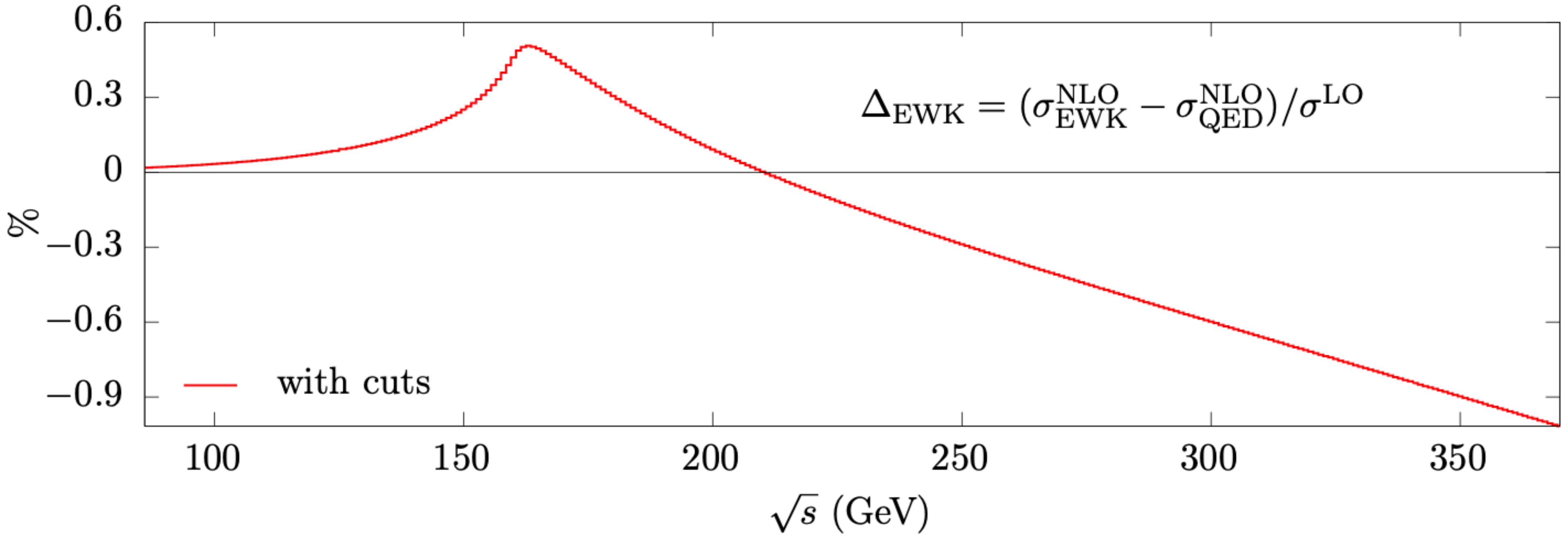
Comparison of digamma and Bhabha luminometry at FCC-ee

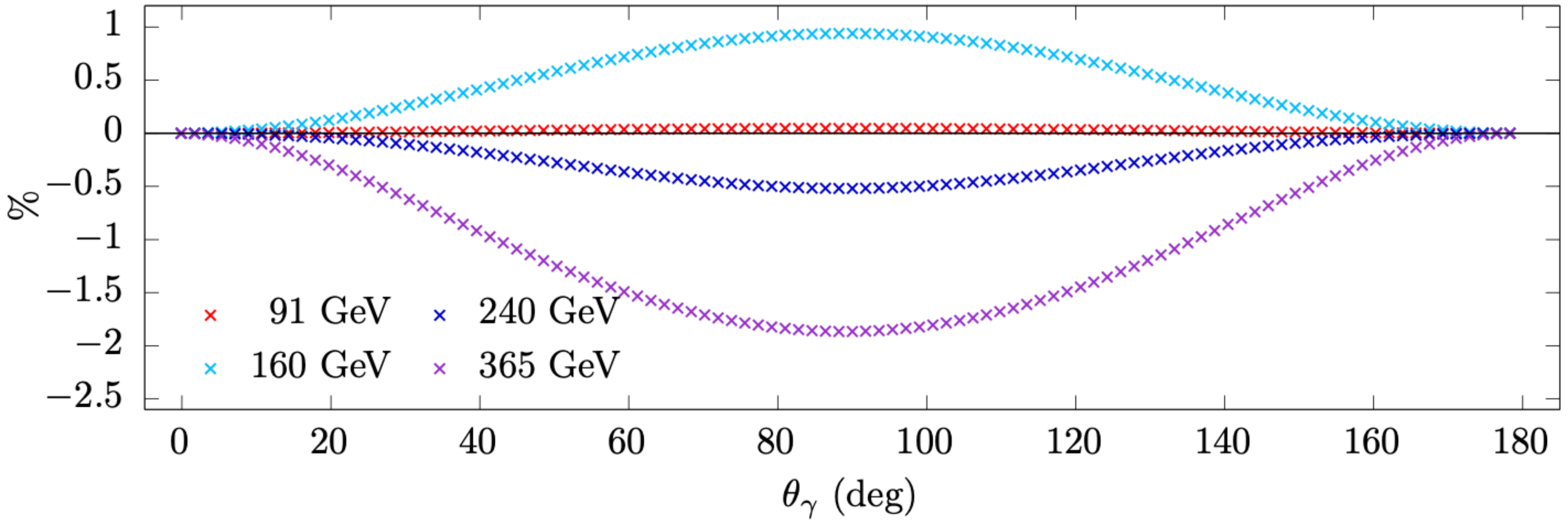
$$\sqrt{s} = 91,160,240,365\,\text{GeV}$$

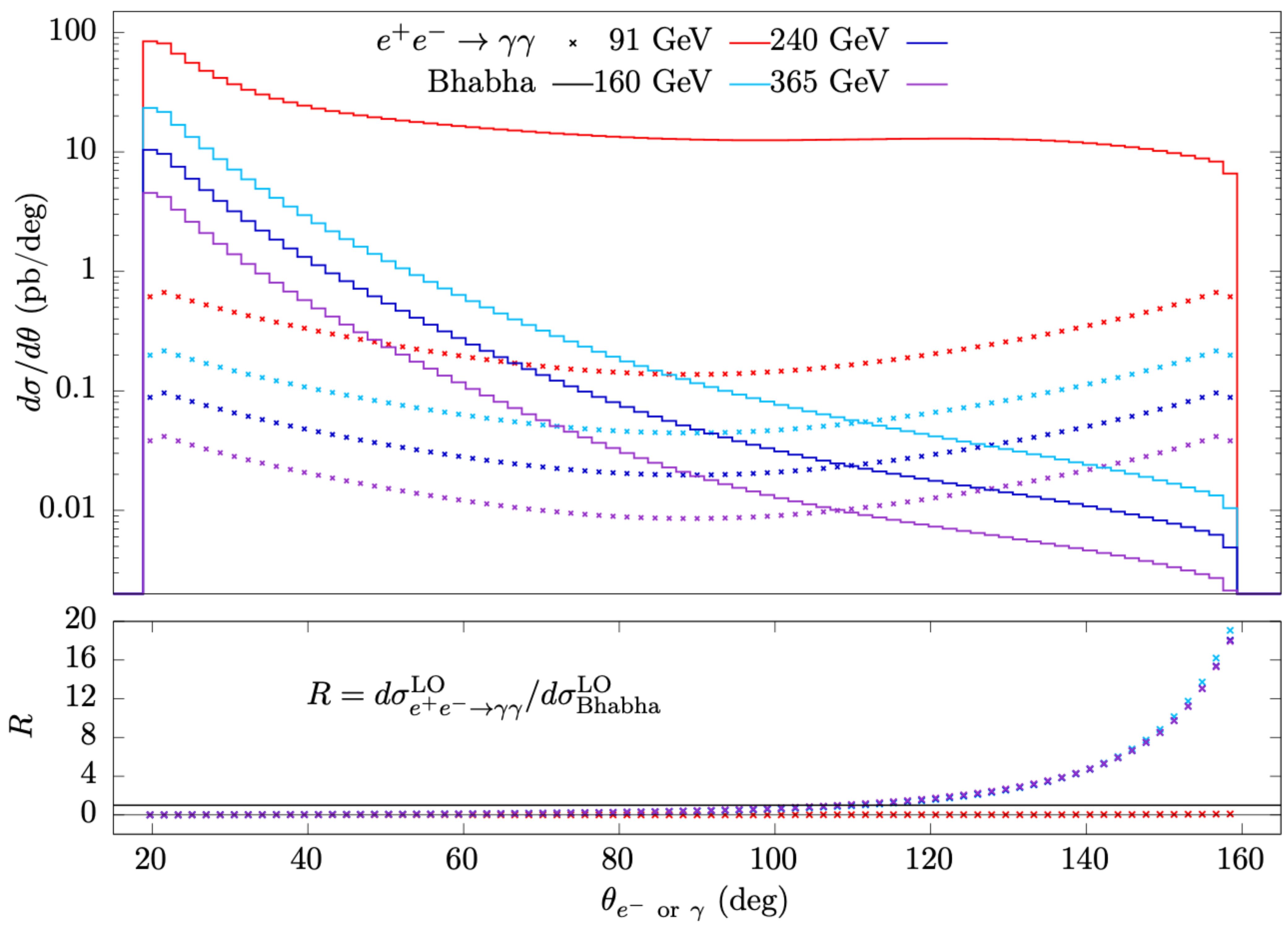
At least two photons

$$20 \le \theta_\gamma \le 160$$
$$E_\gamma \ge 0.25\sqrt{s}$$

$\sqrt{s}$ (GeV)	LO (pb)	NLO (pb)	w h.o. (pb)	Bhabha LO (pb)
91	39.821	41.043 [+3.07%]	40.870(4) [−0.43%]	2625.9
160	12.881	13.291 [+3.18%]	13.228(1) [−0.49%]	259.98
240	5.7250	5.9120 [+3.27%]	5.8812(6) [−0.54%]	115.77
365	2.4752	2.5581 [+3.35%]	2.5438(3) [−0.58%]	50.373



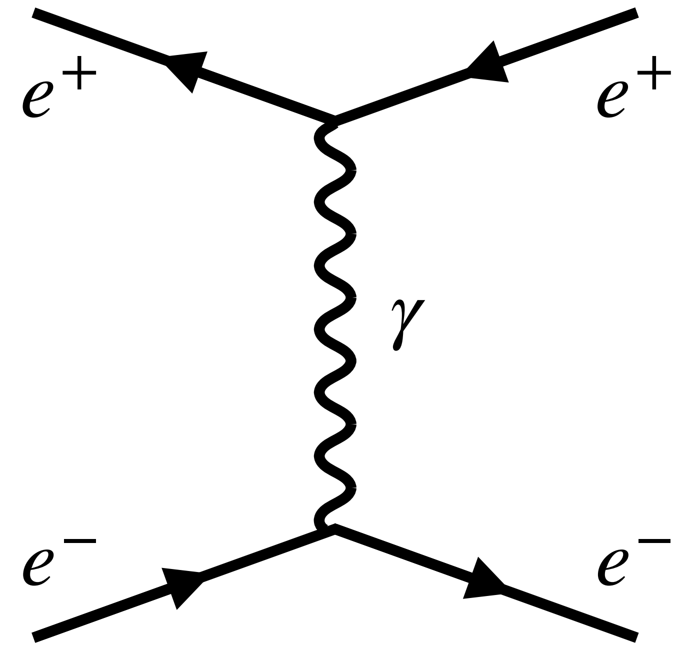




Relative effect of ElectroWeak corrections vs QED

New Physics in SABS?

Standard Model

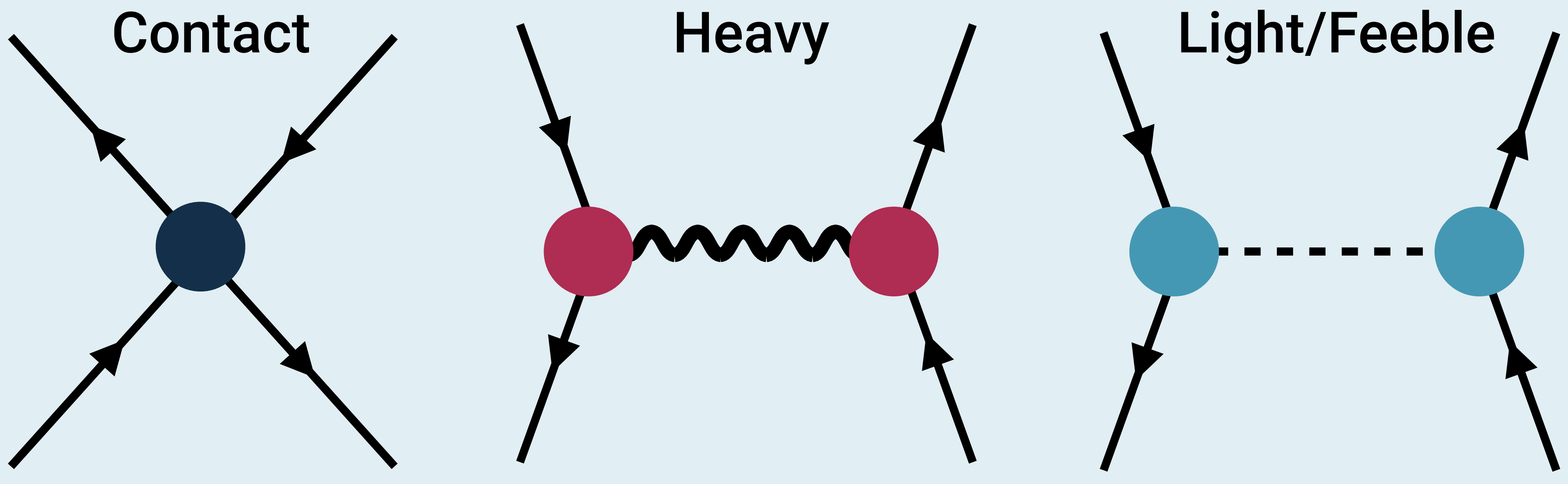


$$\sigma^{(n)} = \left(\frac{\alpha}{\pi}\right)^n \log^n \frac{Q^2}{m_e^2}$$

From the SM side you need **Monte Carlo Generators** able to simulate collisions at this level of precision

$$\left.\frac{\delta\sigma_{\rm SM}}{\sigma_{\rm SM}}\right|_{\rm th}^{\rm FCC} \leq 10^{-4}$$

New Physics



$$\frac{\delta\sigma_{\rm NP}}{\sigma_{\rm SM}} \propto \frac{2{\rm Re}\,\mathscr{M}_{\rm SM}\mathscr{M}^\dagger_{\rm NP}}{|\mathscr{M}_{\rm SM}|^2}$$

New Physics d.o.f. could contaminate the SABS

At what level?

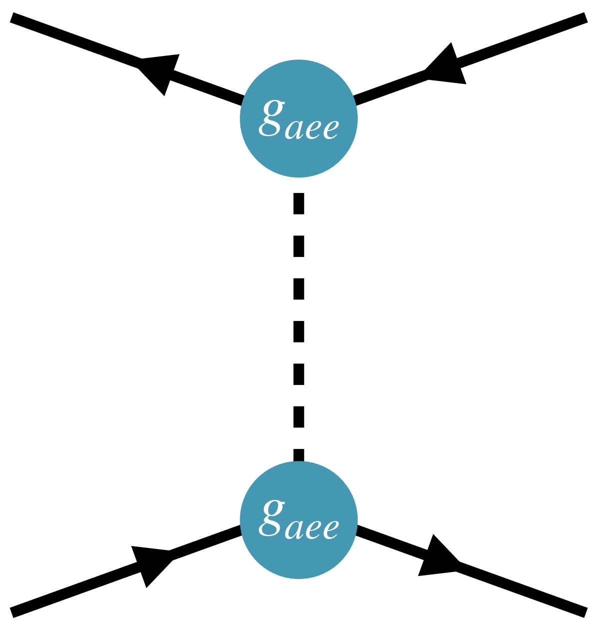
$$\frac{\delta\sigma_{\rm NP}}{\sigma_{\rm SM}} \simeq ?$$

Light New Physics $\Lambda_{NP} \ll \Lambda_{EW}$

We rely on specific model with assigned **spin** and **parity**

(Pseudo)scalar ALPs

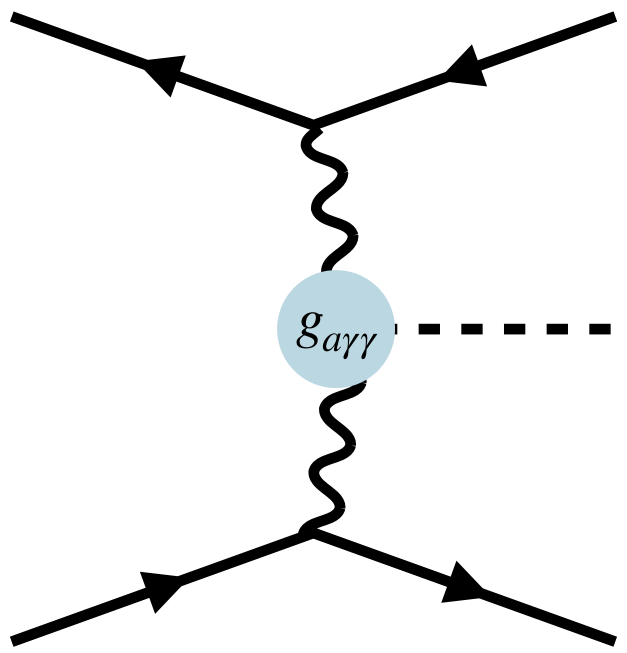
$$g_{aee}\,(\bar{e}\, i\gamma_5 e)a$$



$$(g_{aee}, m_a) \simeq (3 \times 10^{-3}, 1\,\mathrm{GeV})$$

ALP mixing with photons

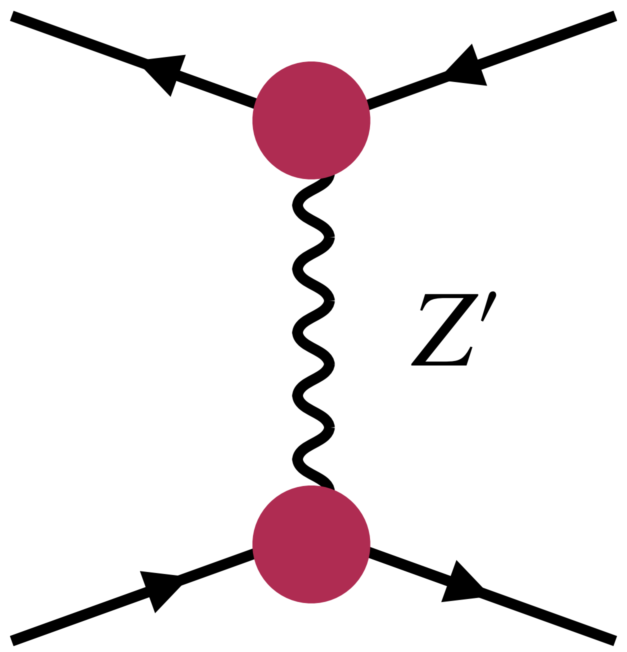
$$\frac{1}{4} g_{a\gamma\gamma}(F_{\mu\nu}\tilde{F}^{\mu\nu})a$$



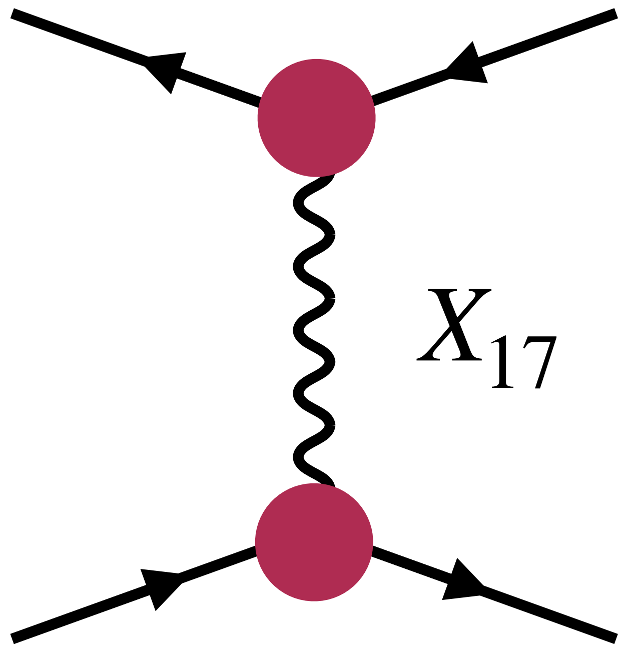
$$g_{a\gamma\gamma} \simeq 2 \times 10^{-4}\,\mathrm{GeV}^{-1}$$

Dark Vectors

$$g'_V \left(\bar{e}\,\gamma^\mu\, e\right) V_\mu + g'_A \bar{e} \left(\gamma^\mu \gamma_5\right) e\, V_\mu$$



$$(g'_V, M_V) \simeq (3 \times 10^{-4}, 1\,\mathrm{GeV})$$



$$(6 \times 10^{-4}, 17\,\mathrm{MeV})$$

The LNP contribution is **negligible** at 10^{-5} level

Heavy New Physics $\Lambda_{\mathrm{NP}} \gg \Lambda_{\mathrm{EW}}$

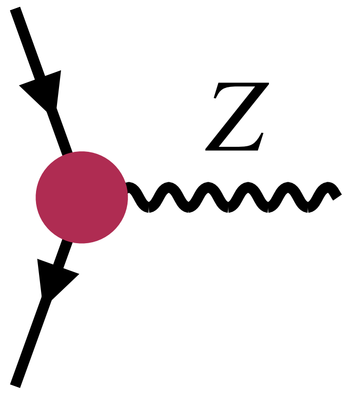
The most comprehensive way to consider HNP effects is the SMEFT framework

$$\mathscr{L}_{\mathrm{SMEFT}} = \mathscr{L}_{\mathrm{SM}} + \sum_i \frac{C_i}{\Lambda_{\mathrm{NP}}^2} \hat{O}_i^{(6)} + \mathscr{O}(\Lambda_{\mathrm{NP}}^{-4})$$

The deviation is computed taking into account **correlations** between WCs

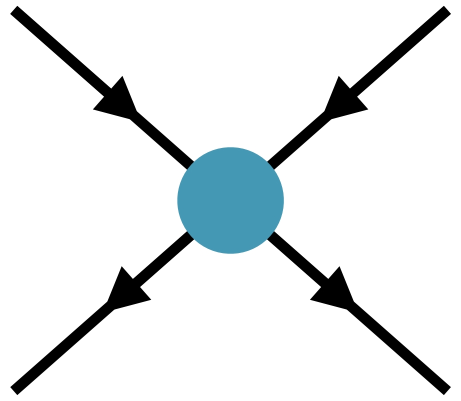
$$(\delta \pm \Delta\delta)_{\mathrm{SMEFT}} = \frac{1}{\sigma_{\mathrm{SM}}} \left(\sigma^{(6)} \pm \sqrt{\sum_{ij} \sigma_i^{(6)} V_{ij} \sigma_j^{(6)}} \right)$$

Deviations in couplings



$\Delta g_{L,R}^{Ze} \neq 0$ $\qquad g = g_{\mathrm{SM}} + \Delta g$

New interactions



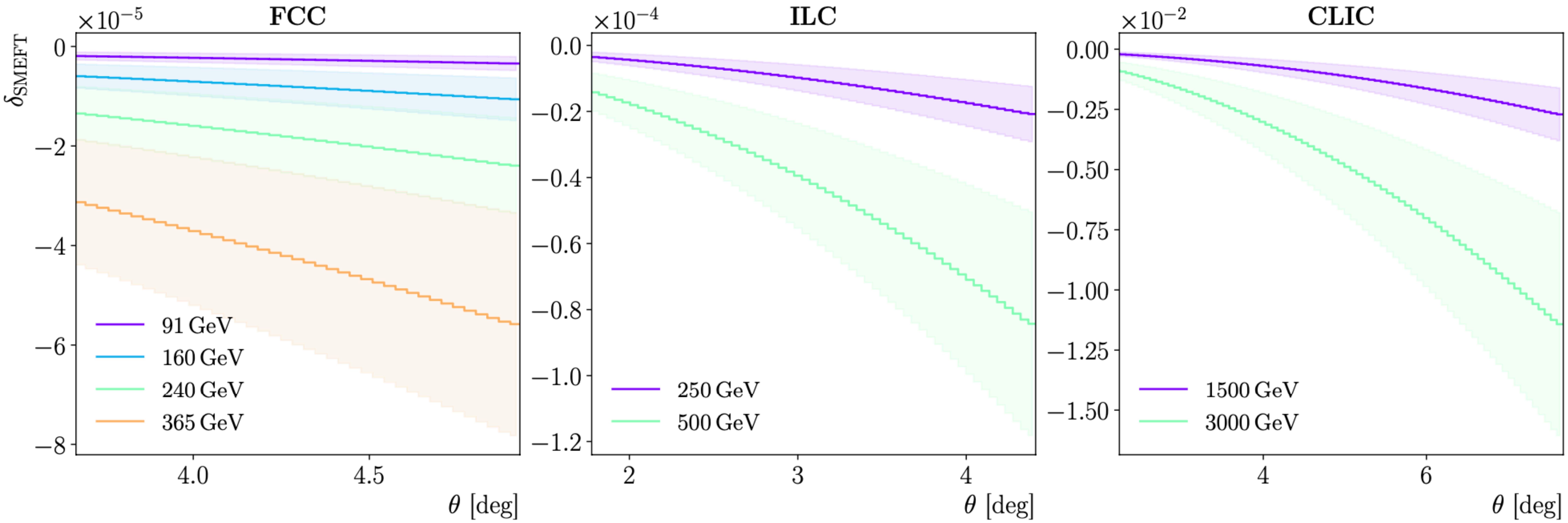
Four fermion operators absent in the SM

$$\mathscr{L}_{\mathrm{SMEFT}} \supset \sum_{ij} \frac{C_{ij}}{\Lambda_{\mathrm{NP}}^2} \left(e_i \gamma^\mu e_i\right)\left(e_j \gamma_\mu e_j\right)$$

Future Colliders scenarios taken from EPPSU inputs

Exp.	$[\theta_{\min}, \theta_{\max}]$	$\sqrt{s}$ [GeV]	$(\delta \pm \Delta\delta)_{\mathrm{SMEFT}}$	$\Delta L/L$
FCC	$[3.7°, 4.9°]$	91	$(-4.2 \pm 1.7) \times 10^{-5}$	$< 10^{-4}$
		160	$(-1.3 \pm 0.5) \times 10^{-4}$	10^{-4}
		240	$(-2.9 \pm 1.2) \times 10^{-4}$	
		365	$(-6.7 \pm 2.7) \times 10^{-4}$	
ILC	$[1.7°, 4.4°]$	250	$(-1.2 \pm 0.5) \times 10^{-4}$	$< 10^{-3}$
		500	$(-4.9 \pm 1.9) \times 10^{-4}$	
CLIC	$[2.2°, 7.7°]$	1500	$(-9.7 \pm 3.9) \times 10^{-3}$	$< 10^{-2}$
		3000	$(-4.2 \pm 1.7) \times 10^{-2}$	

Heavy New Physics: Results



$$\overrightarrow{C}_{4f} = \{C_{ll}, C_{le}, C_{ee}\}$$

4 fermions WCs impact the luminosity in a non-negligible way

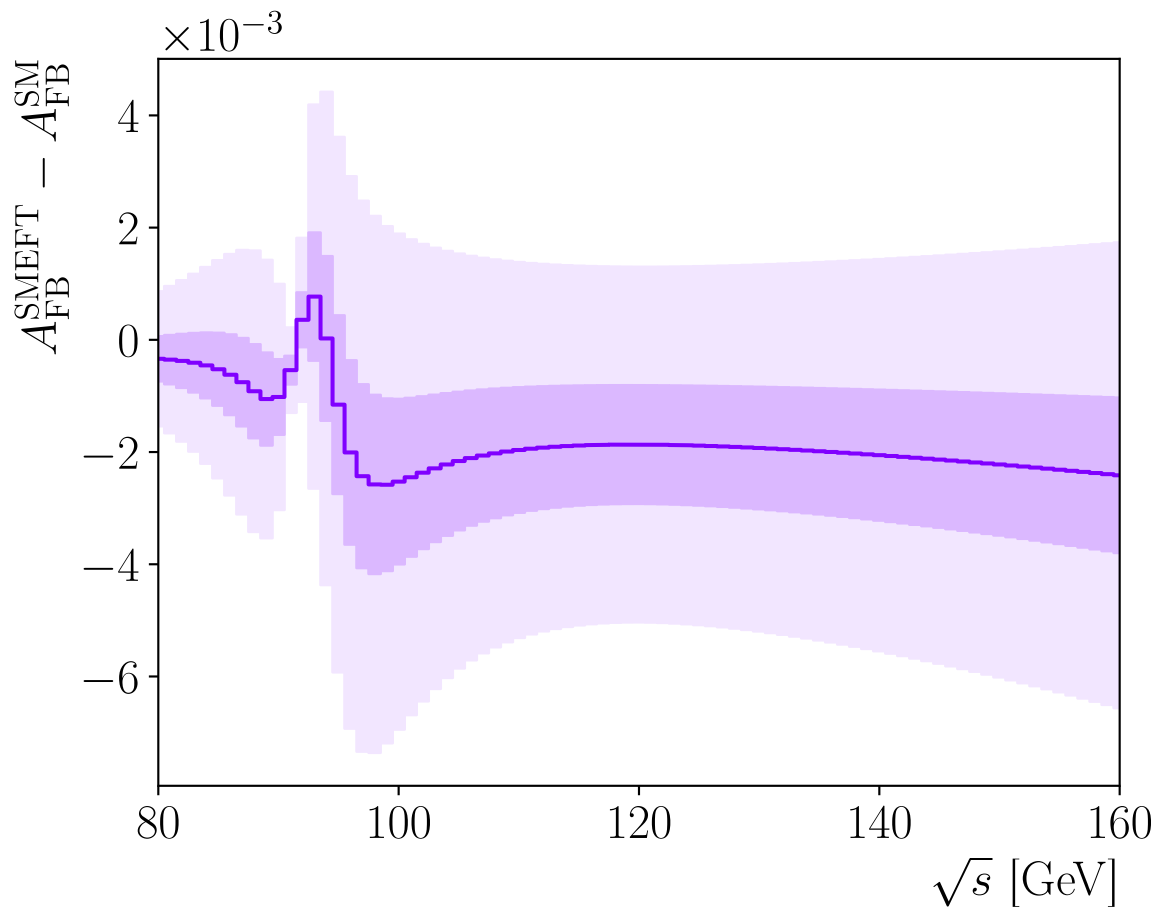
Alternatie Scenarios

Z peak runs — FCC-ee

We use the FB asymmetry as a function of $\sqrt{s_\alpha}$

$$\sum_{i\in 4f}\frac{C_i}{\Lambda_{\rm NP}^2}\left[\frac{(\sigma_{\rm F}-\sigma_{\rm B})_i^{(6)}}{(\sigma_{\rm F}-\sigma_{\rm B})_{\rm SM}}-\frac{(\sigma_{\rm F}+\sigma_{\rm B})_i^{(6)}}{(\sigma_{\rm F}+\sigma_{\rm B})_{\rm SM}}\right]_\alpha=\frac{\Delta A^0_{{\rm FB},\alpha}}{A^0_{{\rm FB},\alpha}},$$

To fit the three WCs we can use three points



$$\sqrt{s_1}=89\,{\rm GeV}$$
$$\sqrt{s_2}=93\,{\rm GeV}$$
$$\sqrt{s_3}=98\,{\rm GeV}$$

In 6 months of run on every point

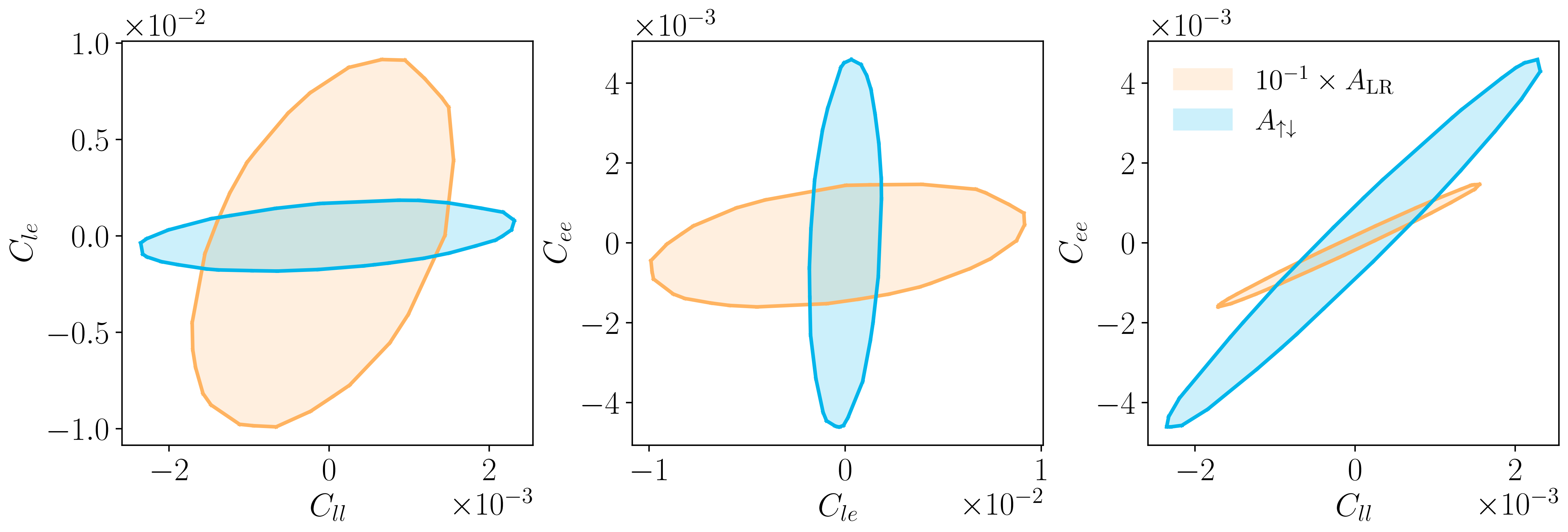
$\Delta C_{4f}<10^{-2}$ → $\delta_{\rm SMEFT}<10^{-5}$

250 GeV run — ILC

For polarised beams A_{LR} is not sensitive to all WCs.
We propose another polarisation asymmetry

$$A_{\uparrow\downarrow}^{-}(P_{e^\pm},\cos\theta)=\frac{d\sigma(P_{e^+},P_{e^-})-d\sigma(P_{e^+},-P_{e^-})}{d\sigma(P_{e^+},P_{e^-})+d\sigma(P_{e^+},-P_{e^-})}$$

Up-down asymmetry



We calculate the 68% CLs using

$$\chi^2=\sum_{\alpha=1}^{n}\frac{\left(A^0_{\rm pol}-A^{\rm th}_{\rm pol}(\vec{C}_{4f})\right)^2_\alpha}{(\Delta A^0_{\rm pol})^2_\alpha}$$

→ $\delta_{\rm SMEFT}<10^{-7}$

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BabaYaga@NLO and the pion form factor

The pion form factor

The Pion FF is a key quantity to determine the $(g-2)_\mu$

$$a_\mu^{\mathrm{HLO}} = \frac{\alpha}{\pi^2}\int_{4m_\pi^2}^{\infty}\frac{\mathrm{d}s}{s}K(s)\left(\frac{\alpha(s)}{3}\frac{\sigma(e^+e^-\to \mathrm{hadrons})}{\sigma(e^+e^-\to\mu^+\mu^-)}\right)$$

$$\simeq \frac{\alpha}{\pi^2}\int\frac{\mathrm{d}s}{s}K(s)\beta_\pi^2\,|F_\pi(s)|^2 f(s)$$

In scan experiments, the PionFF is given by the ratio

$$|F_\pi|^2 = \left(\frac{N_{\pi^+\pi^-}}{N_{e^+e^-}} - \Delta^{\mathrm{bg}}\right)\cdot\frac{\sigma^0_{e^+e^-}\cdot(1+\delta_{e^+e^-})\cdot\varepsilon_{e^+e^-}}{\sigma^0_{\pi^+\pi^-}\cdot(1+\delta_{\pi^+\pi^-})\cdot\varepsilon_{\pi^+\pi^-}}$$

$\delta_{\pi^+\pi^-}$ Radiative corrections are estimated via MC

$\epsilon_{\pi^+\pi^-}$ Also the acceptance has a MC dependence via the **charge asymmetry**

$$A_{FB}^{\mathrm{NLO}} = A_{FB}^{\mathrm{LO}} + \frac{\alpha}{\pi}A_{FB}^{\alpha} = 0 + \frac{\alpha}{\pi}\left(\frac{\sigma_B^{\mathrm{odd}} - \sigma_F^{\mathrm{odd}}}{\sigma^{\mathrm{NLO}}}\right)$$

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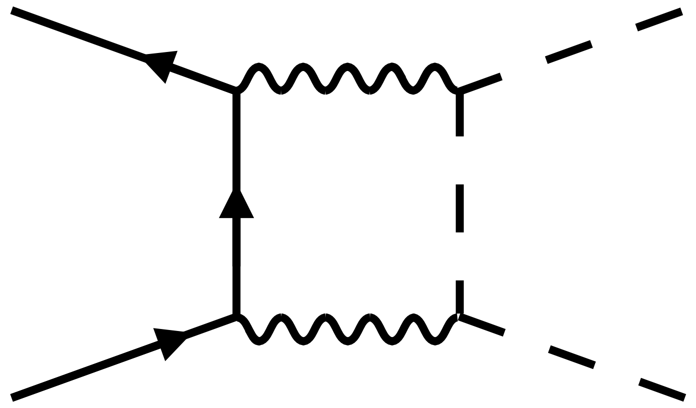
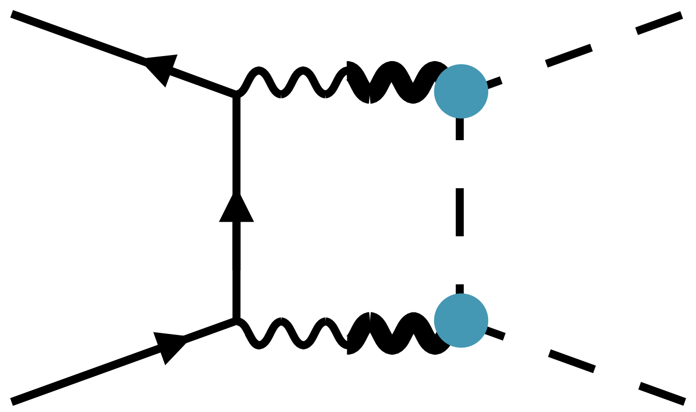
The FF has to be modelled due its non-perturbative nature

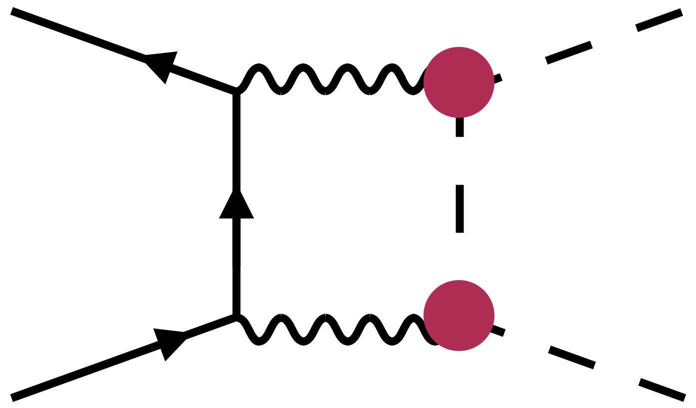


Factorised sQED — The FF is attached to virtual amplitude as to cancel infrared divergences



GVMD — A model based on the mixing of the photon with light resonances

$$F_\pi^{\mathrm{BW}}(q^2) = \sum_{v=1}^{n_r} F_{\pi,v}^{\mathrm{BW}}(q^2) = \frac{1}{c_t}\sum_{v=1}^{n_r} c_v\frac{\Lambda_v^2}{\Lambda_v^2-q^2}$$



FsQED — Relies on the analytic properties of the FF, written via a dispersion relation

$$F_\pi(q^2) = 1 + \frac{q^2}{\pi}\int_{4m_\pi^2}^{\infty}\frac{\mathrm{d}s'}{s'}\frac{\mathrm{Im}F_\pi(s')}{s'-q^2-i\varepsilon'}$$

Numerical results

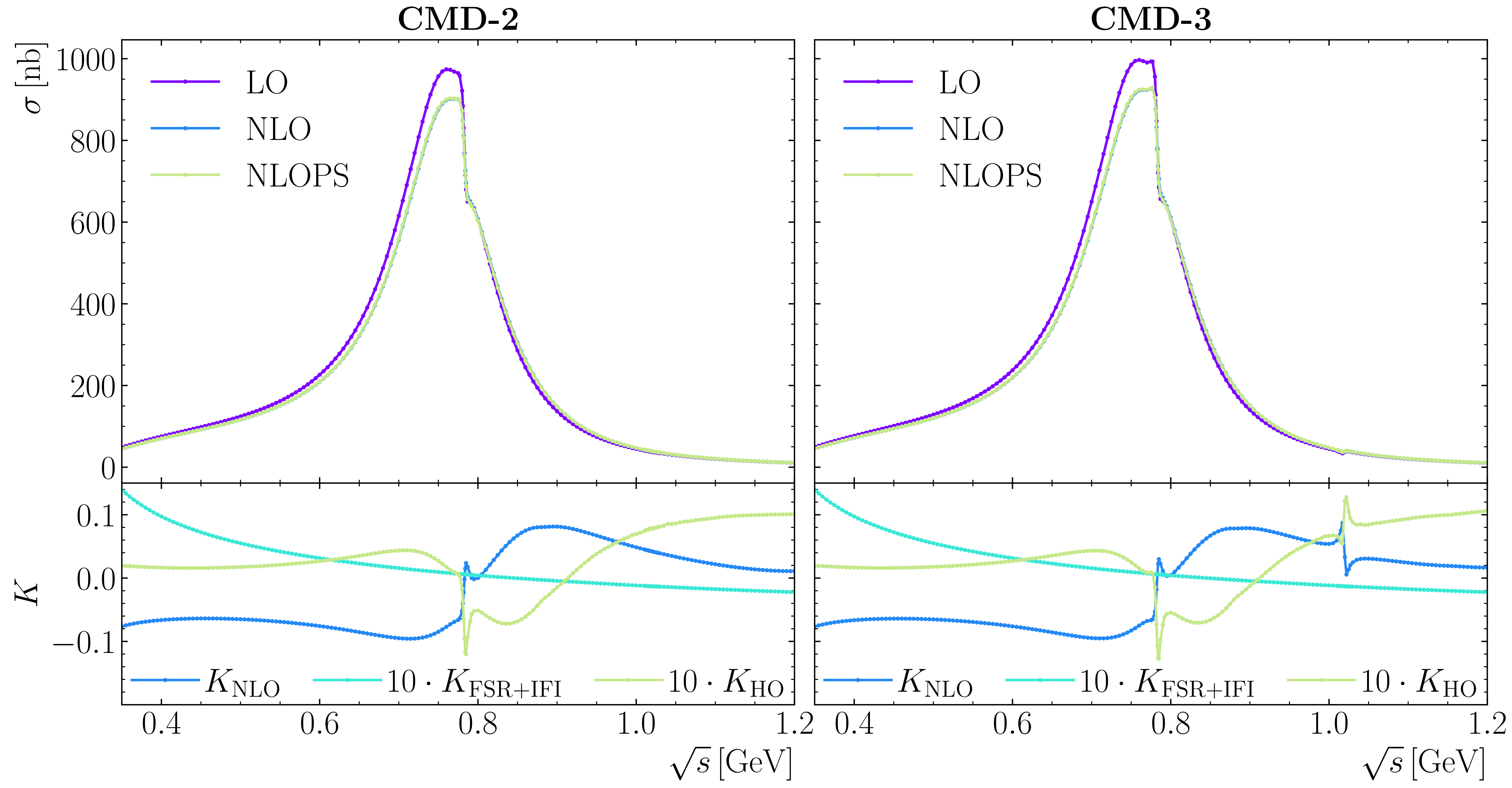
We studied the impact of Radiative corrections and of the form factor approach in a typical scenario

$$p^{\pm} \equiv |\boldsymbol{p}^{\pm}| > 0.45E\,,$$

$$\vartheta_{\rm avg} \equiv \frac{1}{2}(\pi - \vartheta^{+} + \vartheta^{-}) \in [1, \pi - 1]\,,$$

$$\delta\vartheta \equiv |\vartheta^{+} + \vartheta^{-} - \pi| < 0.25\,,$$

$$\delta\phi \equiv \big||\phi^{+} - \phi^{-}| - \pi\big| < 0.15\,,$$



Radiative corrections

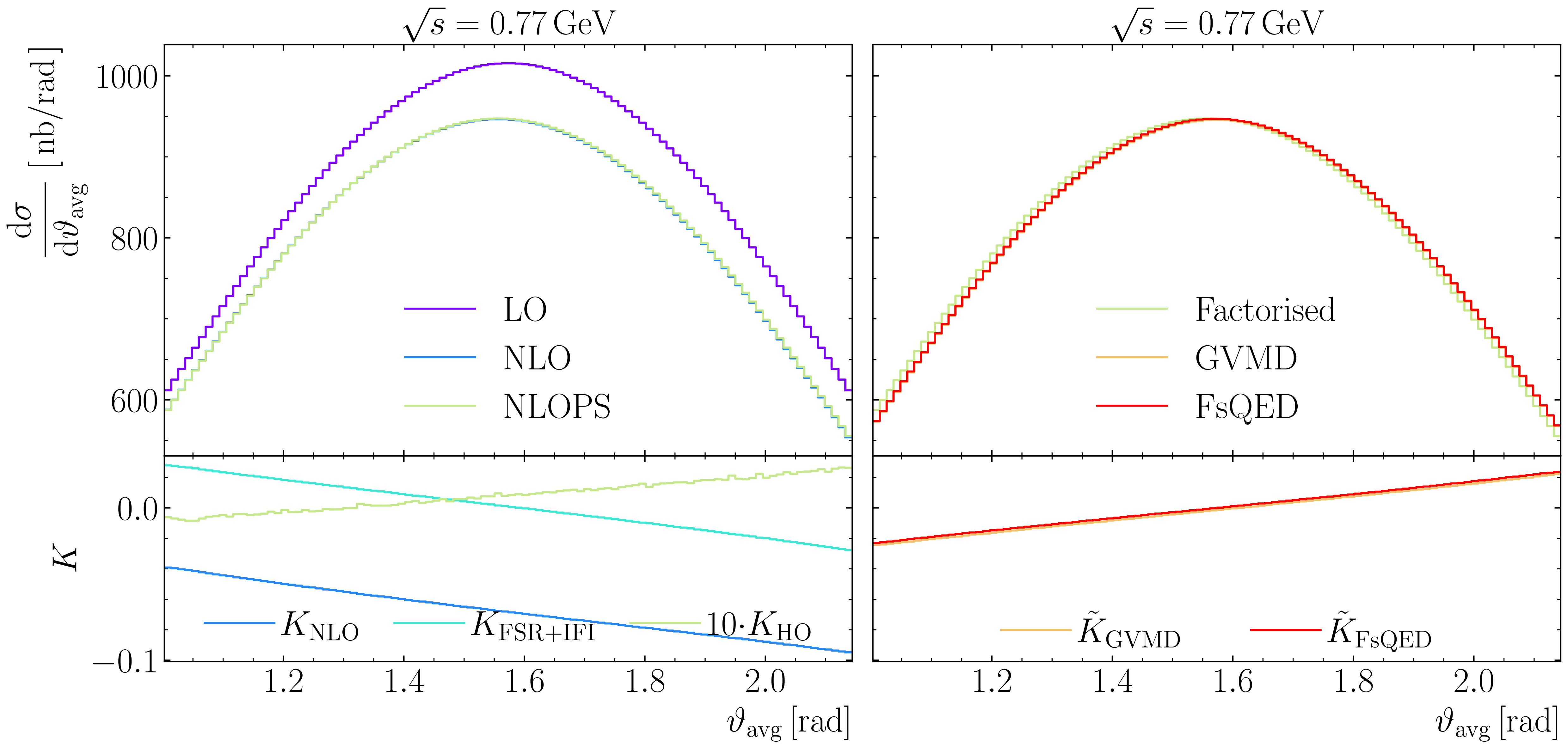
$$K_{\rm NLO} = \frac{\sigma_{\rm NLO} - \sigma_{\rm LO}}{\sigma_{\rm LO}}\,,$$

$$K_{\rm FSR+IFI} = \frac{\sigma_{\rm NLO} - \sigma_{\rm ISR}}{\sigma_{\rm LO}}\,,$$

$$K_{\rm HO} = \frac{\sigma_{\rm NLOPS} - \sigma_{\rm NLO}}{\sigma_{\rm LO}}\,,$$

FF approach

$$\tilde{K}_{\rm FF} = \left(\frac{d\sigma_{\rm FF}}{d\vartheta_{\rm avg}}\right)\left(\frac{d\sigma_{\rm Factorised}}{d\vartheta_{\rm avg}}\right)^{-1}$$

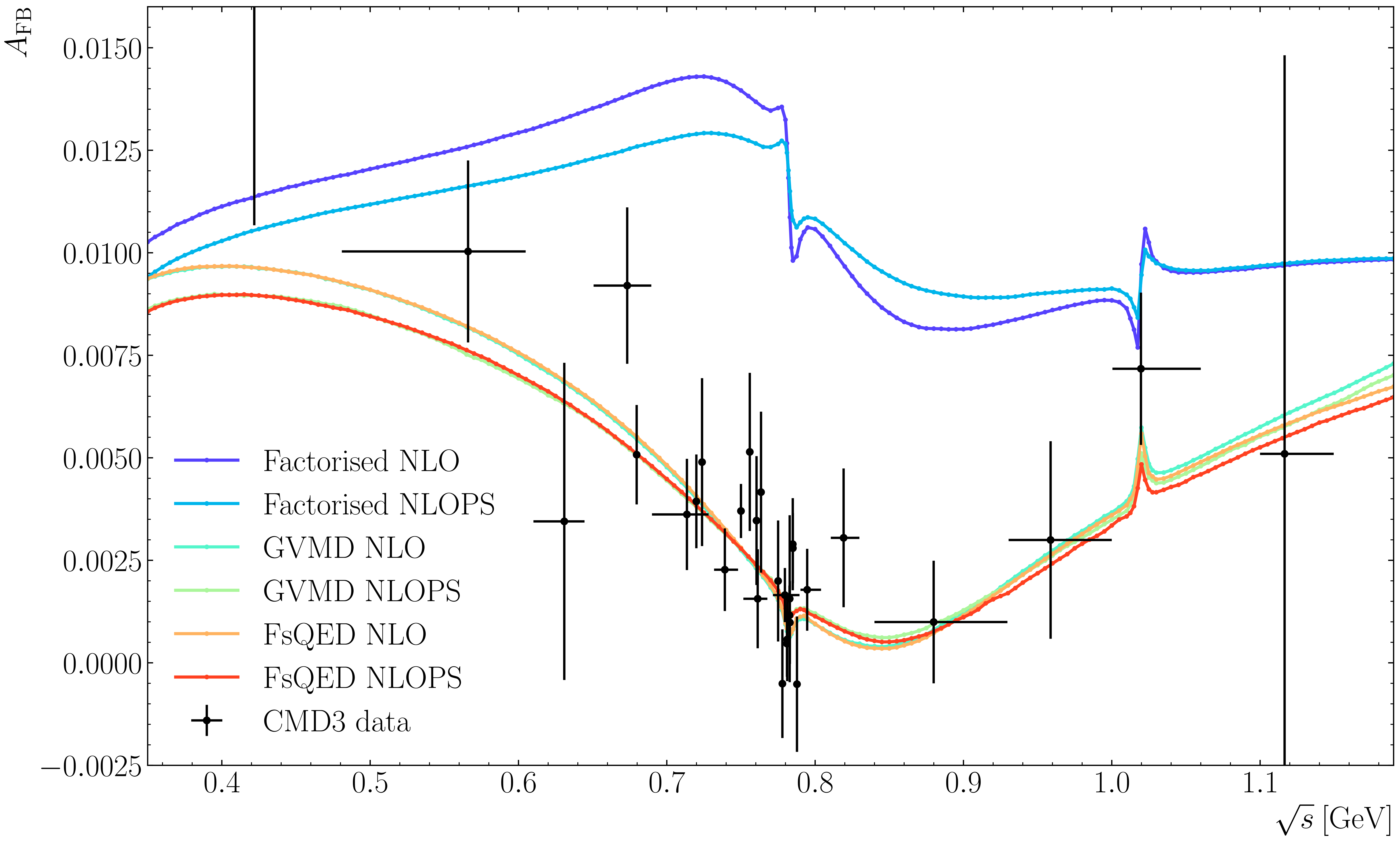


Charge asymmetry

$$A_{\mathrm{FB}} = \frac{\sigma_B - \sigma_F}{\sigma_B + \sigma_F}$$

Being a **pure NLO effect**, this observable is a test of the modelling of the FF

We proved that, up to parameterisation differences, the two approaches are **equivalent**



Radiative return

Flavour factories can measure the Pion FF also by radiative return, *i.e.* when the CMS energy is reduced by the emission of an ISR photon

There are just two MC: `Phokhara` and `AfkQED`.

Signal $e^+e^- \to \pi^+\pi^-\gamma$

Normalisation $e^+e^- \to l^+l^-\gamma$

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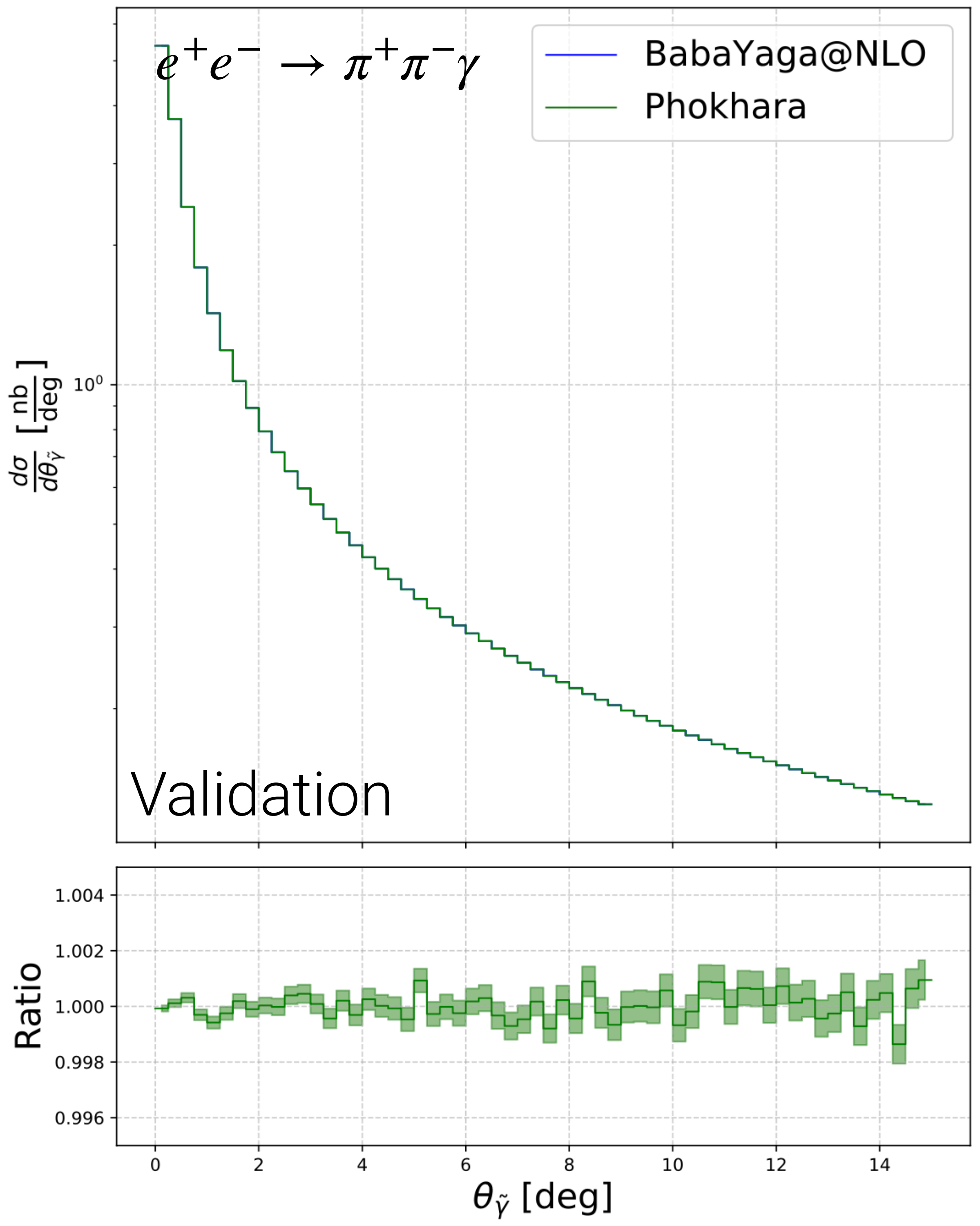
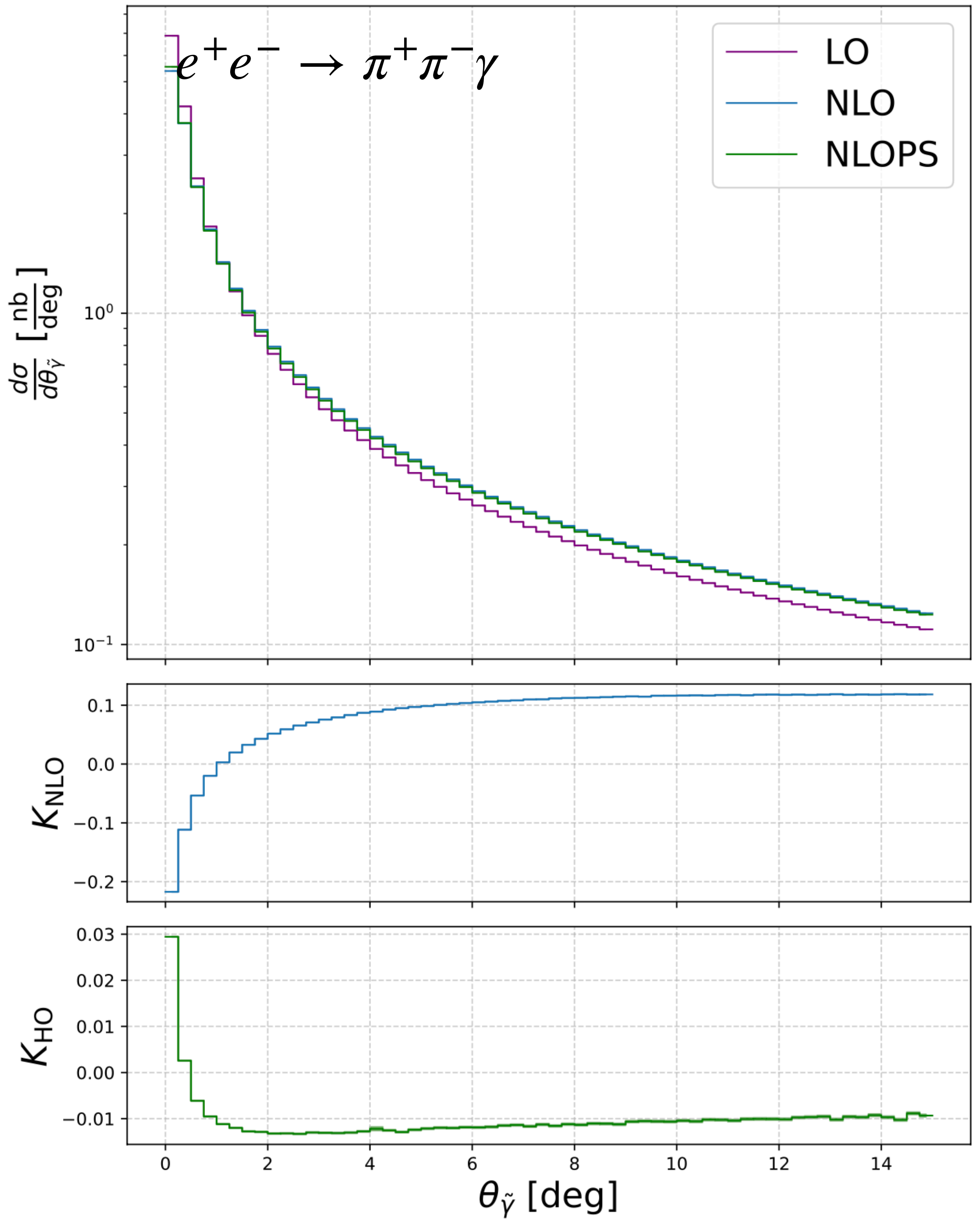
KLOE-like Small Angle

$$\theta_{\tilde{\gamma}} \leq 15^\circ \quad \text{or} \quad \theta_{\tilde{\gamma}} > 165^\circ\,,$$
$$0.35\,\text{GeV}^2 \leq M^2_{XX} \leq 0.95\,\text{GeV}^2\,,$$

$$50^\circ \leq \theta^\pm \leq 130^\circ\,,$$
$$|\mathrm{p}^z_\pm| > 90\,\text{MeV} \quad \text{or} \quad \mathrm{p}^\perp_\pm > 160\,\text{MeV}\,,$$





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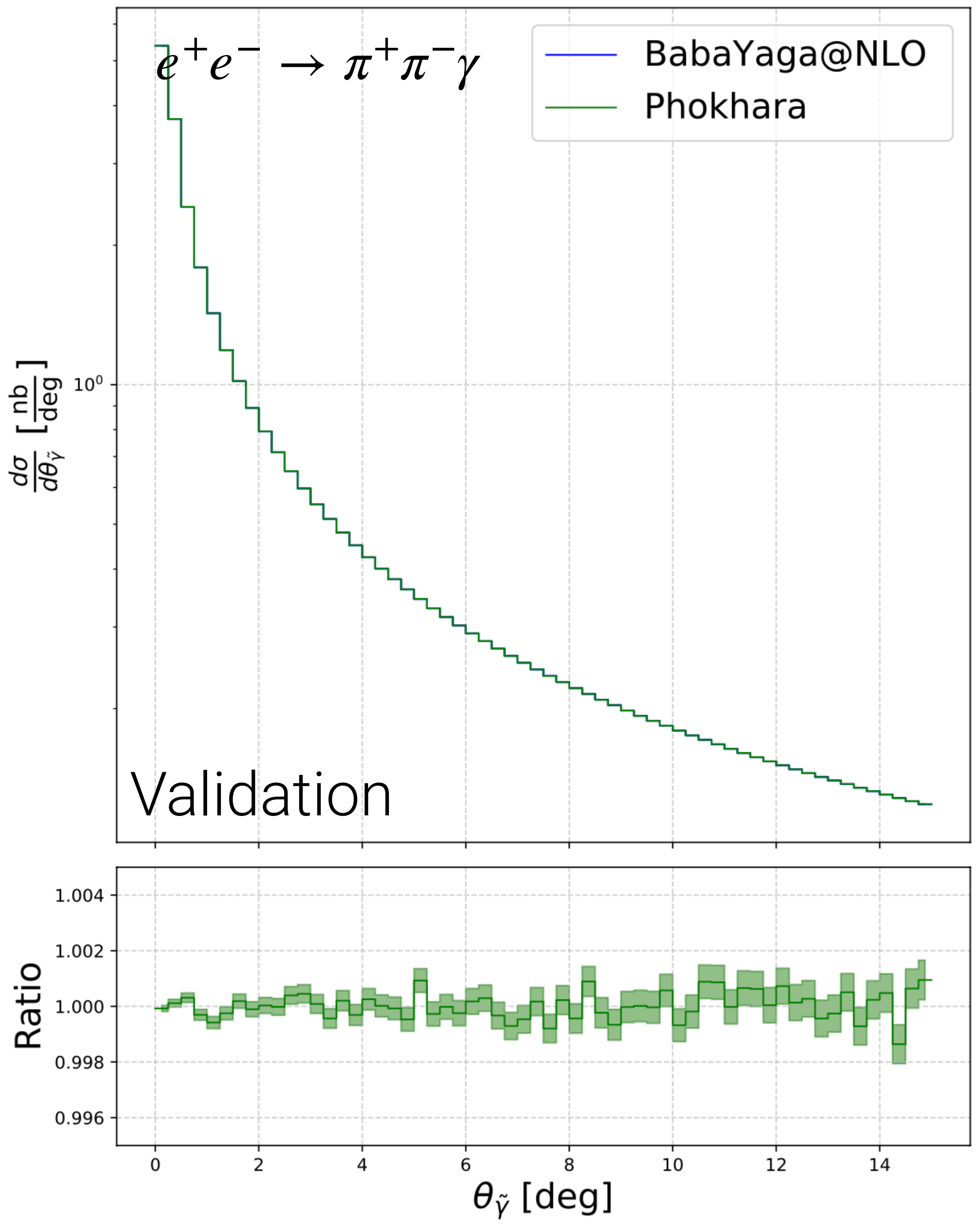
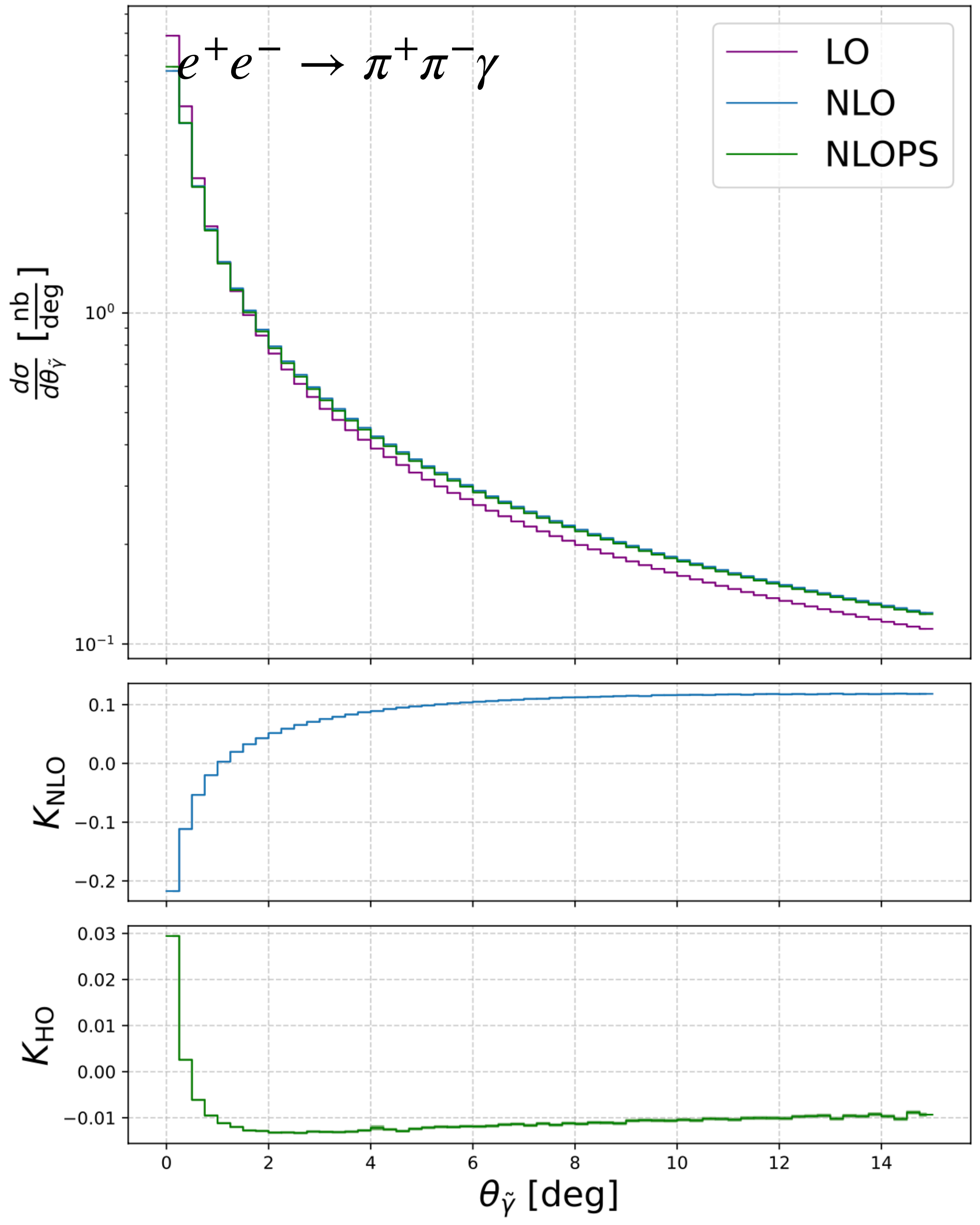
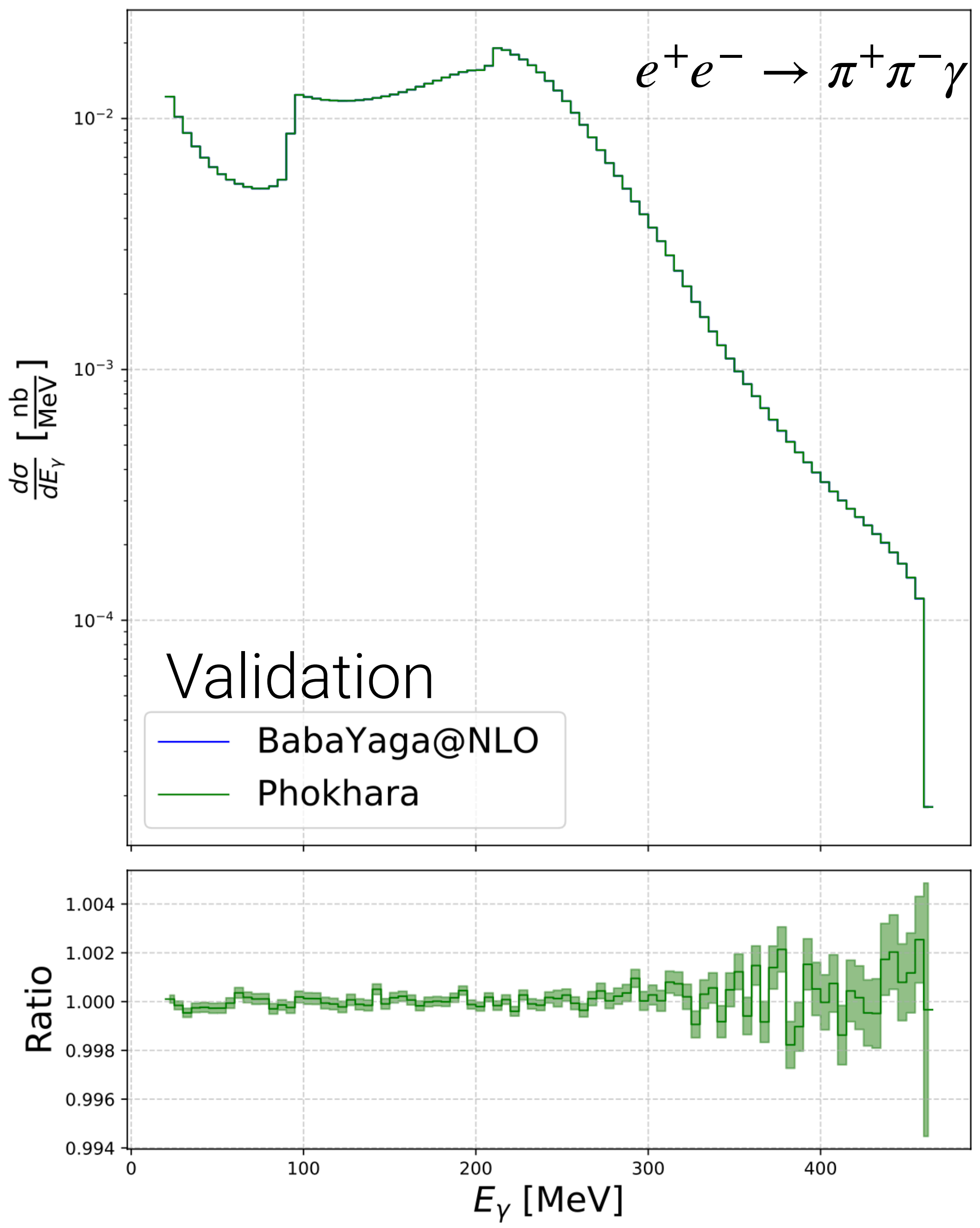
$$0.35\,\text{GeV}^2 \leq M^2_{XX} \leq 0.95\,\text{GeV}^2\,,$$
$$50^\circ \leq \theta^\pm \leq 130^\circ\,,$$
$$|\mathrm{p}^z_\pm| > 90\,\text{MeV} \quad \text{or} \quad \mathrm{p}^\perp_\pm > 160\,\text{MeV}\,,$$

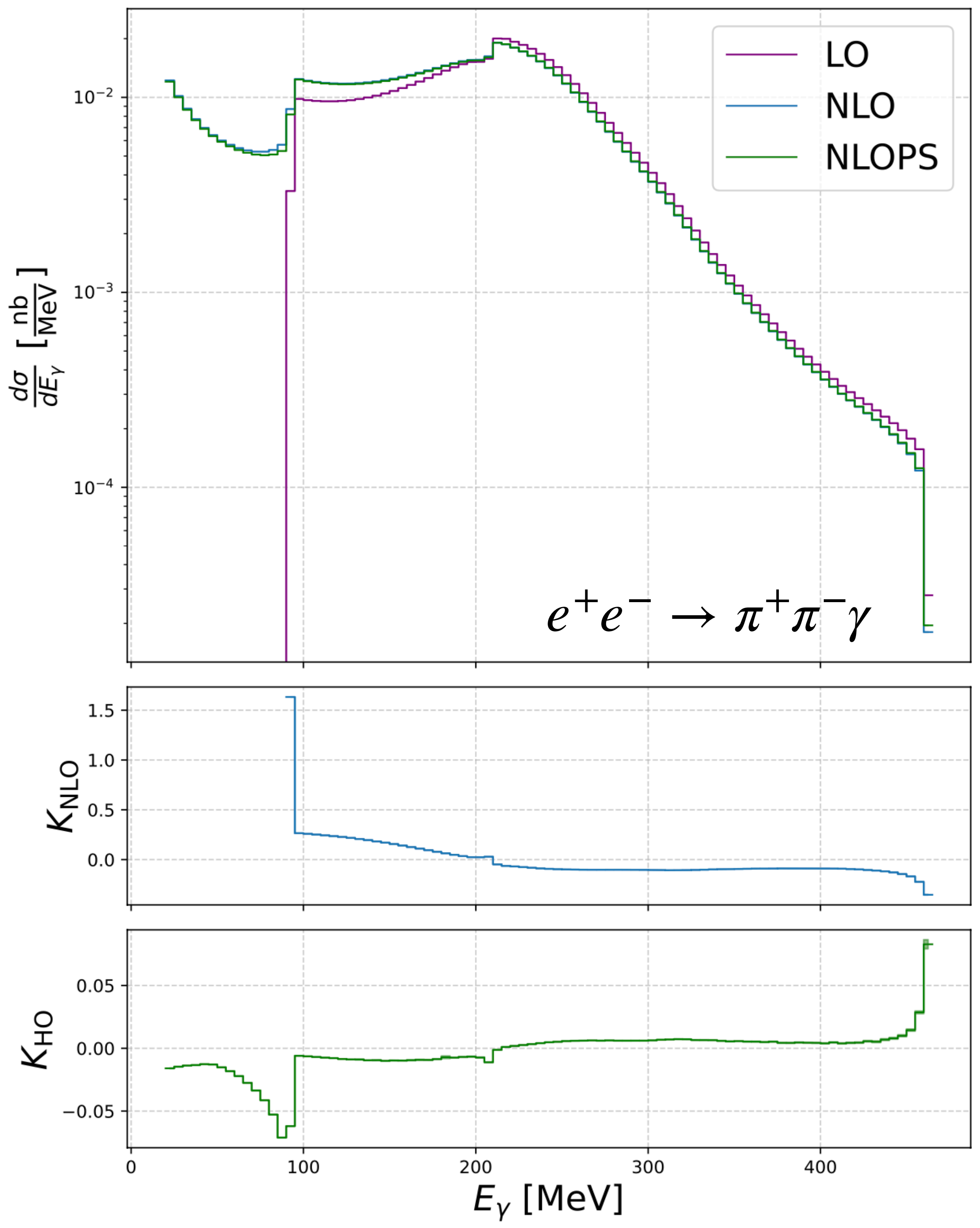
KLOE-like Large Angle

$$E_\gamma > 20\,\text{MeV} \quad \text{and} \quad 50^\circ \leq \theta_\gamma \leq 130^\circ\,,$$
$$|\mathrm{p}^z_\pm| > 90\,\text{MeV or } \mathrm{p}^\perp_\pm > 160\,\text{MeV} \quad \text{and} \quad 50^\circ \leq \theta^\pm \leq 130^\circ\,,$$
$$0.1\,\text{GeV}^2 \leq M^2_{XX} \leq 0.85\,\text{GeV}^2\,,$$









What's next?

Radiative Processes

$e^+e^- \to XX\gamma$ at NLOPS accuracy

In progress:

- NLOPS accuracy in all channels
- Pions with factorised approximation
- Improving technical performance

Goals:

- Provide an independent tool to estimate theoretical uncertainties

Next-future:

- Include GVMD and FsQED approaches in the radiative $\pi^+\pi^-\gamma$ channel
- Estimate the uncertainty due to $F_\pi(q^2)$

What's next?

Radiative Processes

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Hadronic channels

$e^+e^- \to K_L K_S, K^0\bar{K}^0$ at NLOPS accuracy

Future plans:

- Achieve NLOPS accuracy also for other relevant hadronic channels

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Luminosity

$e^+e^- \to \gamma\gamma$ NNLO pair corrections

In progress:
- Compute exact NNLO QED virtual and real pair corrections via dispersive methods
- For now, at low energies

Goals:
- Add also hadronic pairs

Next-future:
- Include Z exchange for pair corrections
- Calculate NLO SMEFT corrections for FCC-ee luminometry

$e^+e^- \to X$ for luminosity

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- NNLO matching with ISR

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+ much more for the next years!