Spin effects in the tau-lepton pair induced by anomalous magnetic and electric dipole moments

A.Yu. Korchin

National Science Center Kharkiv Institute of Physics and Technology, Ukraine,
&
V.N. Karazin Kharkiv National University, Ukraine
&
Jagiellonian University, Krakow, Poland

Matter to the Deepest 2025
Recent Developments in Physics of Fundamental Interactions

Outline

- Motivation
  - ◇ electromagnetic dipole moments of τ lepton: theory and experiment
  - ◇ weak magnetic and electric dipole moments
- Spin correlations in the lepton (fermion) pair
  - ◇ electron-positron and quark-antiquark annihilation to a pair of τ leptons
  - ◇ τ-pair production in photon-photon collision for heavy-ion and *pp* scattering
  - ◇ dipole moments in the Standard Model EFT
- Examples of spin-correlation effects and signatures of the τ dipole moments at Belle and LHC

This research is done in collaboration with

Swagato Banerjee (University of Louisville, USA),
Elzbieta Richter-Was (Jagiellonian University, Poland),
Zbigniew Was (Institute of Nuclear Physics PAN, Poland)

Magnetic and electric dipole moments of the τ lepton

Electromagnetic dipole moments of a fermion f interacting with magnetic field $\vec{\mathcal{B}}$ and electric field $\vec{\mathcal{E}}$:

$$H = -\vec{\mu}_f \, \vec{\mathcal{B}} - \vec{d}_f \, \vec{\mathcal{E}}$$

In the rest frame of fermion, both 3-vectors of dipole moments can only be proportional to its spin vector

$$\vec{\mu}_f = g_f \frac{eQ_f}{2m_f} \vec{s}, \qquad a_f = \frac{g_f}{2} - 1$$

$$\vec{d}_f = \eta_f \frac{e}{2m_f} \vec{s}, \qquad d_f = \frac{\eta_f}{2} \frac{e}{2m_f}$$

where a_f is anomalous magnetic dipole moment (dimensioneless) and d_f is electric dipole moment (in units $e\,$cm).

At high energies it is convenient to use a covariant form for electromagnetic vertex γff

$$\Gamma^\mu = -ie\Big\{ F_1(q^2)\gamma^\mu + \frac{\sigma^{\mu\nu} q_\nu}{2m_f} \left[i\, F_2(q^2) + \gamma_5 \, F_3(q^2) \right] \Big\}$$

in terms of the Dirac, $F_1(q^2)$, Pauli, $F_2(q^2)$ and electric dipole, $F_3(q^2)$, form-factors which reduce to dipole moments at the real-photon point $q^2 = 0$:

$$F_2(0) = a_f \, Q_F, \qquad F_3(0) = d_f \frac{2m_f}{e}, \qquad F_1(0) = Q_f$$

Magnetic dipole moment of the τ: theory

Motivations to study magnetic moment of the τ lepton

In the Standard Model (SM) a_τ is calculated with high accuracy [S. Eidelman, M. Passera, 2007]:

$$a_{QED} = 117324(2) \times 10^{-8}, \qquad (3 \textit{ loops in QED})$$
$$a_{EW} = 47.4(5) \times 10^{-8}, \qquad (1 \textit{ loop in EW})$$
$$a_{hadron+light-by-Light} = 350.1(4.8) \times 10^{-8}$$
$$a_{SM} = a_{QED} + a_{EW} + a_{hadron+Light-by-Light} = (117721 \pm 5) \times 10^{-8}$$

The largest theoretical uncertainty comes from the hadronic contribution to vacuum polarization.

A more recent evaluation of hadronic vacuum polarization term [A. Keshavarzi, D. Nomura, T. Teubner (2020)] gives

$$a_{SM} = (117717.1 \pm 3.9) \times 10^{-8}$$

with the dominant uncertainty coming from the LbL contribution.

Magnetic dipole moment of the τ: theory

What can we say about effects of physics beyond the Standard Model (BSM)?

They can arise due to possible new heavy particles in the loops, and their effect can be written as (first noticed by [V. Beresteckii et al. (1956)]) :

$$a_{NP} \sim m_f^2, \qquad \text{or} \qquad a_{NP} = \mathcal{C}\,\frac{m_f^2}{\Lambda^2},$$

where Λ is the scale of New Physics and $\mathcal{C} \sim \mathcal{O}(1)$ (in some models $\mathcal{C} \sim \mathcal{O}(\alpha/\pi)$).

Explicit models were considered [W. Marciano (1994, 1995), A. Czarnecki and W. Marciano (2010)], in which $\mathcal{C} \sim \mathcal{O}(1)$.

Similar conclusion comes in various models, such as technicolor, multi-Higgs, SUSY, composite models, large extra dimensions, etc. In these models

$$a_{NP} \sim \mathcal{O}(1)\,\frac{m_f^2}{\Lambda^2}$$

Therefore one can expect that BSM effects for the τ lepton can be enhanced compared to the muon by a factor of $m_\tau^2/m_\mu^2 \approx 280$.

Magnetic dipole moment of the τ: experiment

Measurements for τ are extremely difficult because of its very short lifetime $2.903(5) \times 10^{-13}\,\mathrm{s}$. Therefore methods used in the electron and muon "$g-2$" experiments cannot be applied.

LEP2 (DELPHI collaboration) in 2003 obtained the limit on a_τ from $e^+e^- \to e^+e^-\tau^+\tau^-$ cross section

$$a_\tau = (-0.052, +0.013)$$

Recently ATLAS Collaboration presented new constraints [PRL 131, 151802 (2023)] from the ultraperiferal $Pb + Pb \to Pb\,(\gamma\gamma \to \tau^-\tau^+)\,Pb$ collision at $\sqrt{s_{NN}} = 5.02$ TeV:

$$-0.057 < a_\tau < 0.024$$

and also CMS Collaboration [PRL 131, 151803 (2023)] using lead-lead collision, presented the value:

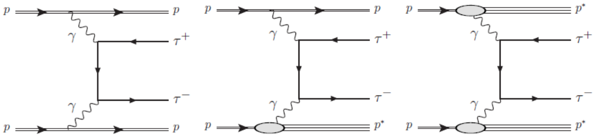
$$\begin{aligned} \sigma(\gamma\gamma \to \tau^-\tau^+) &= 4.8 \pm 0.6(stat) \pm 0.5(syst)\ \mu\mathrm{b}, \\ a_\tau &= 0.001^{+0.055}_{-0.089} \end{aligned}$$

Magnetic dipole moment of the τ: experiment

Recently there appeared the CMS result obtained in pp collisions at $\sqrt{s_{NN}} = 13$ TeV with unprecedented precision

$$a_\tau = 0.0009^{+0.0032}_{-0.0031}$$

by measuring the $\gamma\gamma \to \tau\tau$ cross section [Rept. Prog. Phys., 87(10):107801, 2024]



Experimentally here $m_{\tau\tau} > 50$ GeV, and the cross section is much lower, but the integrated luminosity is much higher than that in heavy-ion lead-lead collisions at the LHC. Also beyond the SM effects can be enhanced at high $\tau\tau$ mass.

This CMS result for the τ AMDM is in agreement with prediction of the SM, and this is the most stringent limit on the τ magnetic moment to date.

Electric dipole moment (EDM) of leptons

EDM of any lepton can take nonzero values, only if parity P, time reversal T, and CP symmetries are violated.

For lepton at rest, both $\vec{\mu}$ and $\vec{d}$ can only be proportional to the vector of spin $\vec{s}$.

Magnetic dipole moment

$$\vec{\mu} = g\frac{Qe}{2m}\vec{s}$$

η is a dimensionless constant, analogous to g

Electric dipole moment

$$\vec{d} = \eta\frac{Qe}{2m}\vec{s}$$

Hamiltonian for a fermion in B and E field

$$\hat{H} = -\vec{\mu}\cdot\vec{B} - \vec{d}\cdot\vec{E}$$

If CPT valid → EDM would violate CP

Transformation Properties

	$\vec{B}$	$\vec{E}$	$\vec{\mu}$	$\vec{d}$
C	-	-	-	-
P	+	-	+	+
T	-	+	-	-
CP	-	+	-	-
CPT	+	+	+	+

Electric dipole moment (EDM) of the τ

For the τ lepton, EDM effect can be enhanced, compared to e and μ (similarly to AMDM), and this causes great interest to EDM of the τ.

In the SM, the lepton EDM is not zero but it is extremely small because it comes from the 4-loop diagrams [I. Khriplovich, M. Pospelov (1991)], and additionally due to the smallness of *CP* violation in the CKM matrix. EDM remains small in some minimal extensions of SM [J.P. Archambault, A. Czarnecki, M. Pospelov (2004)].

Theoretical estimation of the τ EDM [Y. Yamaguchi, N. Yamanaka, PRD, 103:013001 (2021)]

$$\mathrm{SM:} \qquad d_{SM} \lesssim 10^{-41}\, e\, cm$$
$$\mathrm{SM + long-range\,correlations:} \quad d_{SM+corr.} = 7.32 \times 10^{-38}\, e\, cm$$

Experimental constraints are obtained at KEKB e^+e^- collider at $\sqrt{s} = 10.58$ GeV: [Belle collaboration, JHEP 04 (2022) 110]:

$$\begin{aligned} \mathrm{Re}(d_\tau) &= (-0.62 \pm 0.63) \times 10^{-17}\mathrm{e\,cm}, \\ \mathrm{Im}(d_\tau) &= (-0.40 \pm 0.32) \times 10^{-17}\mathrm{e\,cm} \end{aligned}$$

which are ∼20 orders of magnitude (!) larger than the SM estimate.

Electric dipole moment (EDM) of the τ

Real part of the τ EDM

direct constraint	indirect	projected for future
$< 1.7 \times 10^{-17}$ $e\,$cm	$< 1.1 \times 10^{-18}$ $e\,$cm	$\sim 10^{-19} - 10^{-20}$ $e\,$cm
Belle at $\sqrt{s} = 10.58$ GeV	estimate from ACME$^{*)}$	Belle II
	experiment for e	Belle II (polarized beam)

$^{*)}$ACME (Advanced Cold Molecule Electron) experiment uses cryogenic molecular beam of heavy polar molecule ThO to measure EDM of electron (USA, Japan).

Note also the CMS constraint from pp collisions [Rept. Prog. Phys., 87(10):107801, 2024]:

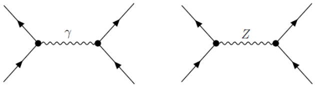
$$|d_\tau| < 2.9 \times 10^{-17}\ e\,\mathrm{cm}$$

In any case, extremely small SM value of EDM (from 10^{-41} to 10^{-38} e cm) is not reachable in experiments. Therefore any observation of τ EDM in experiments will be indication of CP violation beyond the SM.

Weak dipole moments

Weak anomalous form-factors and moments

If one goes to high energies of the LHC or future colliders, then Z-boson interaction with leptons (in general, with any fermions) becomes important



Effective $Z f \bar{f}$ vertex include SM and BSM terms

$$\Gamma^{\mu}_{Z}(q) = \Gamma^{\mu}_{Z}(q)_{SM} + \Gamma^{\mu}_{Z}(q)_{BSM} = -i\frac{g_Z}{2}\left\{\gamma^{\mu}(c_V - \gamma_5 c_A) + \frac{\sigma^{\mu\nu}q_{\nu}}{2m_{\tau}}\left[iX(s) + \gamma_5 Y(s)\right]\right\}$$

where $g_Z = e/(\sin\theta_W \cos\theta_W)$ and c_V, c_A are the vector and axial-vector couplings.

$X(s)$ is weak anomalous magnetic form-factor (CP conserving), while $Y(s)$ is weak electric form-factor (CP violating), which on the Z-boson mass shell at $s = M_Z^2$ are related to the weak dipole moments:

$$X(M_Z^2) = a^{(w)} \sin 2\theta_W, \quad Y(M_Z^2) = d^{(w)} \sin 2\theta_W$$

with weak mixing angle θ_W. Here $a^{(w)}$, $d^{(w)}$ are defined by ALEPH collaboration.

Weak dipole moments of τ lepton

What is known about weak dipole moments of the τ lepton?

Actually very little, and less than about electromagnetic moments.

Experiment ALEPH (LEP & CERN) [Eur. Phys. J. C 30, 291-304, 2003] obtained the constraints on the real and imaginary parts of the weak magnetic and electric dipole moments

$$|\mathrm{Re}(a_\tau^{(w)})|_{exp} < 0.96 \times 10^{-3}, \qquad |\mathrm{Im}(a_\tau^{(w)})|_{exp} < 2.23 \times 10^{-3},$$
$$|\mathrm{Re}(d_\tau^{(w)})|_{exp} < 0.76 \times 10^{-3}, \qquad |\mathrm{Im}(d_\tau^{(w)})|_{exp} < 1.69 \times 10^{-3}.$$

The SM prediction [J. Bernabeu et al. Nucl. Phys. B 436, 474–486, 1995] is:

$$a_\tau^{(w)}|_{SM} = -(1.77 + i\,0.51) \times 10^{-6}$$

while prediction for the weak electric moment $d_\tau^{(w)}$ is not available.

In fact, the dipole moments are encoded in the SM EFT, and are related to the corresponding Wilson coefficients in EFT Lagrangian
[B. Grzadkowski, M. Iskrzynski, M. Misiak, J. Rosiek. JHEP 10 (2010) 085]:

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM}^{(4)} + \underbrace{\Big(\frac{1}{\Lambda}\sum_i C_i^{(5)} Q_i^{(5)} + \frac{1}{\Lambda^2}\sum_i C_i^{(6)} Q_i^{(6)} + \mathcal{O}(\frac{1}{\Lambda^3}) + \mathrm{H.c.}\Big)}_{\mathcal{L}_{BSM}}$$

where Λ is the scale of new physics.
Relevant for the dipole moments terms are dimension-6 operators

$$Q_B^{(6)} = \big(\bar{L}_L\, \sigma^{\mu\nu}\, \tau_R\big)\, H\, B_{\mu\nu}, \qquad Q_W^{(6)} = \big(\bar{L}_L\, \sigma^{\mu\nu}\, \tau_R\big)\, \vec{\sigma}\, H\, \overrightarrow{W}_{\mu\nu}$$

where H is the Higgs doublet, L_L is the left doublet of the 3rd generation of leptons, τ_R is the right τ lepton singlet, $B_{\mu\nu}$ and $\overrightarrow{W}_{\mu\nu}$ are tensors of the gauge fields B^μ and $\overrightarrow{W}^\mu$.
After breaking of the $SU(2)_L \times U(1)_Y$ symmetry one can express the dipole moments through the combinations of two Wilson coefficients

$$D_{Z\tau\tau} \equiv \sin\theta_W\, C_{\tau B}^{(6)} + \cos\theta_W\, C_{\tau W}^{(6)}, \qquad D_{\gamma\tau\tau} \equiv \cos\theta_W\, C_{\tau B}^{(6)} - \sin\theta_W\, C_{\tau W}^{(6)}$$

where θ_W is the weak mixing angle ($\sin\theta_W \approx 0.48$ and $\cos\theta_W \approx 0.877$).

Dipole moments in SM Effective Field Theory

Thus, the BSM dipole moments are related to the real and imaginary parts of $D_{\gamma\tau\tau}$ and $D_{Z\tau\tau}$:

$$A(0)_{BSM} = a_\tau = \frac{\sqrt{2}\,v^2}{\Lambda^2}\,\frac{2m_\tau}{e}\,\mathrm{Re}(D_{\gamma\tau\tau}),$$

$$B(0)_{BSM} = -\frac{2m_\tau}{e}\,d_\tau = -\frac{\sqrt{2}\,v^2}{\Lambda^2}\,\frac{2m_\tau}{e}\,\mathrm{Im}(D_{\gamma\tau\tau}),$$

$$X(M_Z^2)_{BSM} = \frac{\sqrt{2}\,v^2}{\Lambda^2}\,\frac{2m_\tau}{e}\,\sin 2\theta_W\,\mathrm{Re}(D_{Z\tau\tau}) = \sin 2\theta_W\,a_\tau^{(w)},$$

$$Y(M_Z^2)_{BSM} = -\frac{\sqrt{2}\,v^2}{\Lambda^2}\,\frac{2m_\tau}{e}\,\sin 2\theta_W\,\mathrm{Im}(D_{Z\tau\tau}) = \sin 2\theta_W\,d_\tau^{(w)}$$

where $v = (\sqrt{2}G_F)^{-1/2} \approx 246$ GeV is vacuum expectation value of scalar field.

Once dipole moments are measured (or constrained), the Wilson coefficients $C_{\tau W}^{(6)}/\Lambda^2$ and $C_{\tau B}^{(6)}/\Lambda^2$ can be constrained, which will indirectly constrain an underlying high-energy theory.

Spin correlations in fermion-fermion production of τ pair

Including spin correlations in the final $\tau\tau$ pair can help obtain information of New Physics effects, in particular, on the electromagnetic and weak moments.

Consider quark-antiquark or electron-positron annihilation to a pair of **polarized** leptons

$$f(k_1) + \bar{f}(k_2) \to \tau^-(p_-) + \tau^+(p_+),$$

where f = (electron, muon, quark), and the polarization 4-vectors of the τ^- and τ^+ in their corresponding rest frames are:

$$S^-_{rest} = (0,\ \vec{S}^-), \qquad S^+_{rest} = (0,\ \vec{S}^+)$$

Then transform polarization 4-vectors to the CM system:

$$S^-_{cm} = \Big(\frac{\vec{p}\vec{S}^-}{m_\tau};\ \vec{S}^- + \frac{\vec{p}(\vec{p}\vec{S}^-)}{m_\tau(m_\tau + E)}\Big), \qquad S^+_{cm} = \Big(-\frac{\vec{p}\vec{S}^+}{m_\tau};\ \vec{S}^+ + \frac{\vec{p}(\vec{p}\vec{S}^+)}{m_\tau(m_\tau + E)}\Big)$$

where $\vec{p} = (0,\ 0,\ p)$ is 3-momentum of final τ^-, E is its energy, and the reaction plane is defined by the 3-momentum of initial lepton/quark $\vec{k}_1 = (E\sin\theta,\ 0,\ E\cos\theta)$.

Spin correlations in fermion-fermion production of τ pair

The cross section in the center-of-mass frame can be expressed through these polarizations:

$$\frac{d\sigma}{d\Omega}(f\,\bar{f} \to \tau^-\tau^+) = \frac{\beta}{64\pi^2 s}\sum_{i,j=1}^{4} R_{i,j}\,S_i^- S_j^+ = \frac{d\sigma}{d\Omega}(f\,\bar{f} \to \tau^-\tau^+)\Big|_{\mathrm{unpolar}}$$

$$\times \quad \frac{1}{4}\left(1 + \sum_{i=1}^{3} r_{i,4}\,S_i^- + \sum_{j=1}^{3} r_{4,j}\,S_j^+ + \sum_{i,j=1}^{3} r_{i,j}\,S_i^- S_j^+\right)$$

where $i,\,j = (1,2,3) \equiv (x,y,z)$, $R_{i,j} = R_{i,j}^{(\gamma)} + R_{i,j}^{(Z)} + R_{i,j}^{(\gamma Z)}$ and $r_{i,j} \equiv R_{i,j}/R_{4,4}$.

In general, 15 coefficients $r_{i,j}(s,\,\theta)$ carry information on underlying physics.

Among them there are 6 elements which are τ^- and τ^+ polarizations:

$$\vec{\mathcal{P}}_{\tau^-} = (r_{14},\, r_{24},\, r_{34}), \qquad \vec{\mathcal{P}}_{\tau^+} = (r_{41},\, r_{42},\, r_{43}) = (-r_{14},\, r_{24},\, -r_{34})$$

and 9 elements of spin-spin correlations:

$$r_{i,j} = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ -r_{12} & r_{22} & r_{23} \\ r_{13} & -r_{23} & r_{33} \end{pmatrix}$$

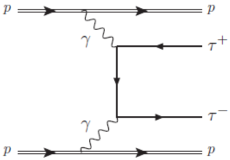
Here “1” means transverse to τ^- momentum, “2” - normal to reaction plane, “3” - longitudinal.

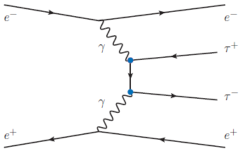
Spin correlations in $\gamma\gamma$ processes

At present it is possible to perform experiments on heavy-ion and proton-proton collisions at high energies in which τ pair is produced in $\gamma + \gamma$ process:
In particular, these are ultra-peripheral Pb+Pb scattering in which

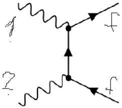
$$\text{Pb} + \text{Pb} \to \text{Pb} + \text{Pb} + \tau^- + \tau^+$$

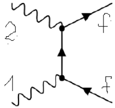
with almost real photons.
Similar experiments are also planned at the future lepton colliders, and Super Tau-Charm Facility (China) in $e^- + e^+ \to e^- + e^+ + \tau^- + \tau^+$.





On the tree level, the mechanism of $\gamma + \gamma \to \tau^- + \tau^+$ with (almost) real photons is





Spin correlations in $\gamma\gamma$ processes

One can use the formalism for polarized final τ^- and τ^+

$$\frac{d\sigma}{d\Omega}(\gamma\,\gamma \to \tau^-\tau^+) = \frac{d\sigma}{d\Omega}(\gamma\gamma \to \tau^-\tau^+)\Big|_{\rm unpol}\,\frac{1}{4}\Big(1+\sum_{i,j=1}^{3} r_{i,j}^{(\gamma\gamma)}\, S_i^-\, S_j^+\Big)$$

with 9 elements of the spin-spin correlation matrix $r_{i,j}^{(\gamma\gamma)} = R_{i,j}^{(\gamma\gamma)}/R_{4,4}^{(\gamma\gamma)}$.

Note that the symmetry relations hold:

$$r_{21}^{(\gamma\gamma)} = -r_{12}^{(\gamma\gamma)}, \qquad r_{31}^{(\gamma\gamma)} = r_{13}^{(\gamma\gamma)}, \qquad r_{32}^{(\gamma\gamma)} = -r_{23}^{(\gamma\gamma)}$$

Besides, in this case $r_{4,i}^{(\gamma\gamma)} = r_{i,4}^{(\gamma\gamma)} = 0$ for $i = 1, 2, 3$, which means $\tau^\pm$ have no polarization. Indeed, the polarization is determined by the imaginary part of dipole moments $A(0)$ or $B(0)$, which are real-valued for real photons. Each element of matrix

can be written in the form of expansion in powers of $A(0)$ and $B(0)$:

$$R_{i,j} = \frac{e^4}{(1-\beta^2\cos^2\theta)^2}\sum_{m,\, n=0\ (m+n\le 4)}^{4} A(0)^m\, B(0)^n\, c_{mn}^{(ij)}$$

with coefficients $c_{mn}^{(ij)}$ which are functions of s and θ.

Electroweak radiative corrections in the SM

Of course, if one searches for a very small effects of New Physics, the latter appear on the top of the SM calculation. Therefore the SM contributions should be included with high precision.

We included electroweak radiative corrections in the SM to the processes $q\bar{q} \to \tau^-\tau^+$ in the so-called Improved Born Approximation (IBA)

[D. Bardin, P. Christova, M. Jack et al. Comp. Phys. Comm. 133, 229 (2001);
E. Richter-Was, Z. Was. Eur. Phys. J. C74, 3177 (2014);
A. Arbuzov, S. Jadach, Z. Was et al. Comp. Phys. Comm., 260, 107734 (2021)] :

It includes:

- vacuum-polarization in the photon propagator,
- corrections to Z-boson propagator and couplings,
- WW- and ZZ-box diagrams (important above $\sqrt{s} > 140$ GeV),
- mixed $\mathcal{O}(\alpha\alpha_s, \alpha\alpha_s^2, \ldots)$ corrections from gluon insertions in the self-energy loop diagrams.

Reweighting procedure in Monte Carlo programs

It is not possible to measure directly τ polarizations, because these leptons are very short-lived with the lifetime $\sim 10^{-13}$ s. One can only use information about $\tau^{\pm}$ decay products.

The formulas for the spin-correlation matrix $R_{i,j}$ are implemented into reweighting algorithms in the Monte Carlo programs: `KKMC` for $e^- e^+ \to \tau^- \tau^+$ and `TauSpinner` for $q\bar{q} \to \tau^- \tau^+$ and $\gamma\gamma \to \tau^- \tau^+$.

The `KKMC` MC for $e^- e^+ \to \tau^- \tau^+$ was used for Belle II kinematics and is also extended for the higher energies up to and above Z boson.

As for `TauSpinner` MC, it was developed earlier for the proton-proton collisions [Z. Czyczula, T. Przedzinski, Z. Was. Eur. Phys. J. C, 72, 1988 (2012)].

$$d\sigma \sim \sum_{flavors} \int_0^1 dx_1 \int_0^1 dx_2\, f(x_1, \ldots)\, f(x_2, \ldots)\, d\Omega_{prod}^{parton\ level}\, d\Omega_{\tau^-}\, d\Omega_{\tau^+}$$
$$\times \left(\sum_{\lambda_1,\lambda_2} |\mathcal{M}^{prod}_{parton\ level}|^2\right) \left(\sum_{\lambda_1} |\mathcal{M}^{\tau^+}|^2\right) \left(\sum_{\lambda_2} |\mathcal{M}^{\tau^-}|^2\right) wt_{spin},$$

with the spin weight

$$wt_{\rm spin} = \sum_{i,j=1,2,3,4} r_{i,j}\, h_i^-\, h_j^+,$$

where $h_j^{\pm}$ are the so-called polarimetric vectors which depend on a decay mode of $\tau^{\pm}$, also $f(x_{1,2}, \ldots)$ are parton distribution functions.

Spin correlations in $e^-e^+ \to \tau^+\tau^- \to \ldots$ for Belle

Although some elements of the matrix $r_{i,j}$ strongly depend on dipole moments, it is a nontrivial task to find observables which are sensitive to dipole moments.
In addition, there are neutrinos in τ decays which are not observed, and this complicates interpretation.

The 2-body decay $\tau^\pm \to \pi^\pm \nu_\tau$ would be the simplest possibility, however, the direction of moving $\tau^\pm$ can be reconstructed only with ambiguities.

We have chosen a more complex decay mode

$$\tau^- \to \rho^- \nu_\tau \to \pi^- \pi^0 \nu_\tau, \qquad \tau^+ \to \rho^+ \bar{\nu}_\tau \to \pi^+ \pi^0 \bar{\nu}_\tau$$

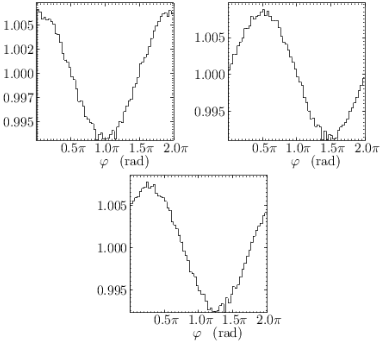
and rely only on kinematics of the secondary decays $\rho^\pm \to \pi^\pm \pi^0$.

The observable, sensitive to magnetic/electric dipole moment, is acoplanarity angle φ between the planes spanned on momenta of (π^-, π^0) and of (π^+, π^0), and defined in the $\rho^- \rho^+$ rest frame
[G.R. Bower, T. Pierzchala, Z. Was et al. (2002), K. Desch, A. Imhof, Z. Was et al (2004)].

The momenta of all 4 pions have to be reconstructed, which gives ρ^-, ρ^+ momenta, which are then boosted to the $\rho^- \rho^+$ rest frame.

Spin correlations in $e^-e^+ \to \tau^+\tau^- \to \ldots$ for Belle

Distribution of the acoplanarity angle φ at $\sqrt{s} = 10.58$ GeV



Top left: $\mathrm{Re}(A) = 0.04$, top right: $\mathrm{Re}(B) = 0.04$;
bottom: $\mathrm{Re}(A) = 0.04\cos(\pi/4)$ and $\mathrm{Re}(B) = 0.04\sin(\pi/4)$
(only events without photons, or soft photons fixed by $m^2_{\tau\tau}/s \geq 0.98$, are selected).

The constraint of the pion energies $y_1 \cdot y_2 > 0$ is also applied, with

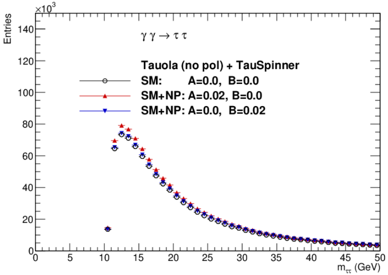
$$y_1 = (E_{\pi^-} - E_{\pi^0})/(E_{\pi^-} + E_{\pi^0}), \qquad y_2 = (E_{\pi^+} - E_{\pi^0})/(E_{\pi^+} + E_{\pi^0}).$$

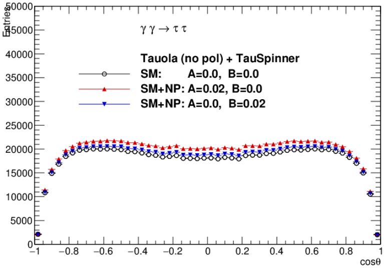
Spin correlations in $\gamma\gamma \to \tau^-\tau^+ \to \ldots$ for the LHC

We consider *pp* scattering at 13 TeV, $\tau\tau$ invariant mass range $m_{\tau\tau} = 5 - 50$ GeV and $p_{\tau\tau} > 5$ GeV. This choice roughly corresponds to the range covered by the $\gamma\gamma \to \tau\tau$ processes in PbPb collisions at the LHC.

Values for dipole moments are tentatively chosen 0.02.

Distributions of invariant mass $m_{\tau\tau}$ and $\cos\theta$ for $\gamma\gamma \to \tau\tau$ events.





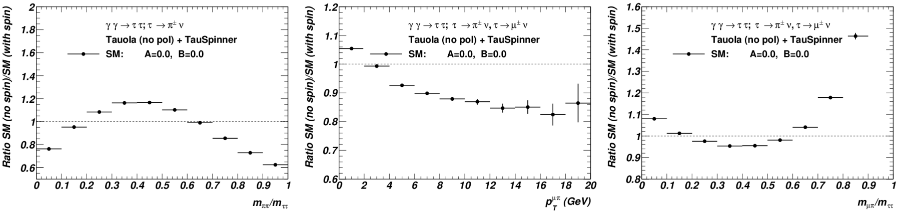
The following decay modes are summed:
(i) $\tau^\pm \to \pi^\pm + \nu_\tau$, (ii) $\tau^\pm \to \rho^\pm + \nu_\tau$, (iii) $\tau^\pm \to \mu^\pm + \nu_\tau + \nu_\mu$ and $\tau^\mp \to \pi^\mp + \nu_\tau$.
There are $\sim 0.8 \times 10^6$ events for each generated decay mode.

There is practically no effect of dipole moments, although the chosen value 0.02 is about 20 times larger than the SM value $A_{SM} \approx 0.00118$.

We choose two decay channels of $\tau^\pm$: (i) pion–pion +neutrino's and (ii) one pion–one lepton + neutrino's:

$$(i) \quad \tau^- \to \pi^- + \nu_\tau, \qquad \tau^+ \to \pi^+ + \bar{\nu}_\tau$$
$$(ii) \quad \tau^- \to \pi^- + \nu_\tau, \qquad \tau^+ \to \mu^+ + \bar{\nu}_\tau + \nu_\mu \quad (\text{+interchange } \tau^+ \leftrightarrow \tau^-)$$

Ratio of distributions $\frac{W(no\ spin-corr)}{W(with\ spin-corr)}$ in the SM.



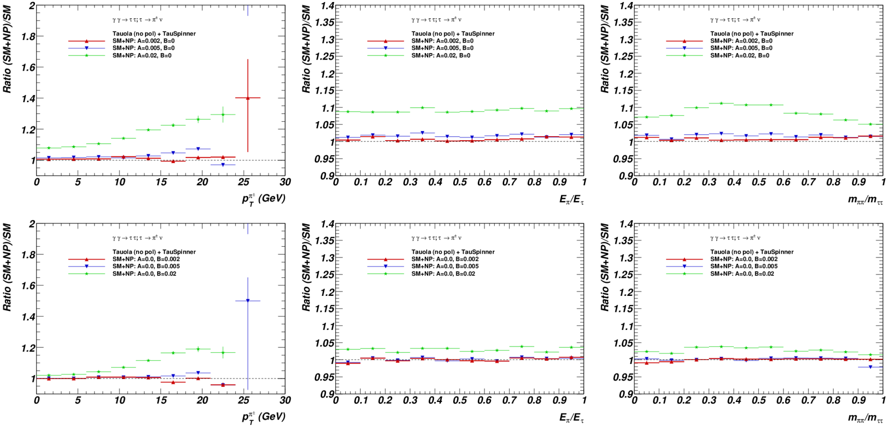
(i) left: dependence on ratio of inv. masses $m_{\pi\pi}/m_{\tau\tau}$ for $\pi^+\pi^-$ final state,
(ii) middle: on transverse momentum $p_T^{\mu\pi}$ for $\mu^\pm\pi^\mp$ final state,
(ii) right: on ratio of inv. masses $m_{\mu\pi}/m_{\tau\tau}$ for $\mu^\pm\pi^\mp$ final state.

Spin effects in $\tau\tau$ are important even in the SM (without dipole moments).

Spin correlations: $\gamma\gamma \to \tau^-\tau^+ \to \pi^-\nu_\tau\pi^+\bar{\nu}_\tau$ for the LHC

Observables sensitive to dipole moments

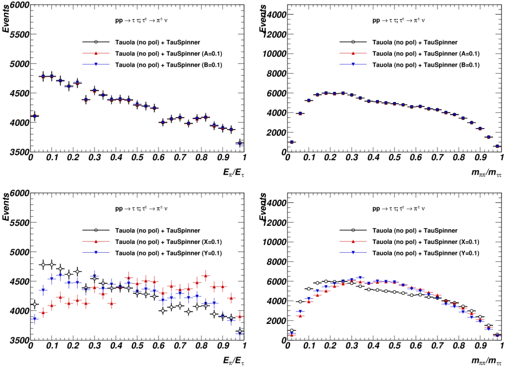
Ratio $(SM + DM)/SM$, upper plots - dependence on A, lower plots - on B.
τ leptons decay via: $\tau^- \to \pi^-\nu_\tau$, $\tau^+ \to \pi^+\bar{\nu}_\tau$. Distributions of p_T^π, E_π/E_τ and $m_{\pi\pi}/m_{\tau\tau}$.



There is effect of AMDM/EDM for the values $A(0) = 0.02$ and $B(0) = 0.02$.

Spin correlations in $q\bar{q} \to \tau^-\tau^+ \to \pi^-\nu_\tau\pi^+\bar{\nu}_\tau$ in Drell-Yan

Consider pp scattering at $\sqrt{s} = 13$ TeV, the region of $\tau^-\tau^+$ invariant masses is $65 - 150$ GeV. There are about 10^6 events for each decay mode of τ.
Decay channels are $\tau^- \to \pi^-\nu_\tau$, $\tau^+ \to \pi^+\bar{\nu}_\tau$.
Distributions of E_π/E_τ and $m_{\pi\pi}/m_{\tau\tau}$ are shown, with effect of electromagnetic and weak dipole moments. Upper plots - dependence on A, B, lower plots - on X, Y.



Weak moment contributions are visible for relatively big values of X, Y equal to 0.1 (there is no effect of magnetic and electric moments as expected).

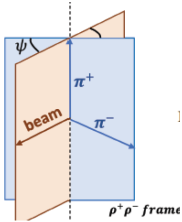
Spin correlations: $q\bar{q} \to \tau^-\tau^+ \to \rho^-\nu_\tau\rho^+\bar{\nu}_\tau$ in Drell-Yan

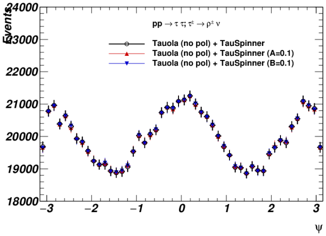
The decays of τ's to ρ mesons can also be useful to study:

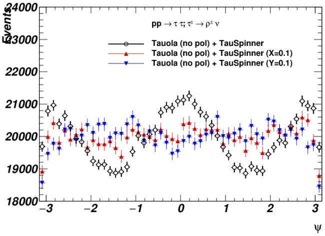
$$\tau^- \to \rho^-\nu_\tau \to \pi^-\pi^0\nu_\tau, \qquad \tau^+ \to \rho^+\bar{\nu}_\tau \to \pi^+\pi^0\bar{\nu}_\tau$$

We follow ALEPH definition of acoplanarity angle ψ between two planes: one is built on the momenta of π^+ and π^-, and another one – on the momenta of π^+ and the beam.

Distribution of acoplanarity angle ψ $(\sqrt{s} = 13$ TeV, $m_{\tau\tau} = 65 - 150$ GeV)







Middle: dependence on magnetic and electric moments;

Right: dependence on weak moments.

There is effect of weak magnetic and electric moments X, Y in this distribution.

Conclusions

- Formalism for description of spin effects in the final $\tau\tau$ pair in the processes $q\bar{q} \to \tau^-\tau^+$, $e^-e^+ \to \tau^-\tau^+$, $\gamma\gamma \to \tau^-\tau^+$ is considered.
- This approach is prepared to work with Monte Carlo generators: `KKMC` for $e^-e^+ \to \tau^-\tau^+$ and `TauSpinner` for $pp \to \tau^-\tau^+$. These MC can be used at the Belle II energies and also at higher energies of the LHC.
- Contributions from magnetic and electric dipole moments of τ are included on top of precise calculation in the Standard Model. Electroweak radiative corrections, in particular WW/ZZ box diagrams, are taken into account in Improved Born Approximation [D. Bardin et al. (2001); E. Richter-Was, Z. Was. (2014); A. Arbuzov, S. Jadach, Z. Was et al. (2021)].
- Several observables, sensitive to anomalous moments of τ-lepton, are discussed and proposed for measurements at KEKB and LHC.

Publications:

Sw. Banerjee, A.Yu. Korchin, E. Richter-Was, Z. Was, Yu. Volkotrub:

Phys. Rev. D 106 (2022) 11, 113010; Phys. Rev. D 109 (2024) 1, 013002;
Acta Phys. Pol. B Proc. Suppl. 17, 5-A20 (2024), 1-10; Phys. Rev. D 111, 013006 (2025);
e-Print: 2506.15213 [hep-ph]

Conclusions

Thank you for attention!