NNLO QCD corrections to $\bar{B} \to X_s\gamma$ without interpolation in m_c

Mikołaj Misiak

University of Warsaw

"Matter To The Deepest," Katowice, September 15-19th, 2025

NNLO QCD corrections to $\bar{B} \to X_s\gamma$ without interpolation in m_c

Mikołaj Misiak

University of Warsaw

"Matter To The Deepest," Katowice, September 15-19th, 2025

The new renormalized corrections to be reported today have been calculated by M. Czaja, M. Czakon, T. Huber, MM, M. Niggetiedt, A. Rehman, K. Schönwald and M. Steinhauser [to appear in arXiv:2510.nnnnn]

NNLO QCD corrections to $\bar{B} \to X_s\gamma$ without interpolation in m_c

Mikołaj Misiak

University of Warsaw

"Matter To The Deepest," Katowice, September 15-19th, 2025

The new renormalized corrections to be reported today have been calculated by
M. Czaja, M. Czakon, T. Huber, MM, M. Niggetiedt, A. Rehman, K. Schönwald and M. Steinhauser
[to appear in arXiv:2510.nnnnn]

1. Introductory conclusions

NNLO QCD corrections to $\bar{B} \to X_s\gamma$ without interpolation in m_c

Mikołaj Misiak

University of Warsaw

“Matter To The Deepest,” Katowice, September 15-19th, 2025

The new renormalized corrections to be reported today have been calculated by
M. Czaja, M. Czakon, T. Huber, MM, M. Niggetiedt, A. Rehman, K. Schönwald and M. Steinhauser
[to appear in arXiv:2510.nnnnn]

1. Introductory conclusions
2. Perturbative amplitudes and their interference at the NNLO in QCD (new)

NNLO QCD corrections to $\bar{B} \to X_s\gamma$ without interpolation in m_c

Mikołaj Misiak

University of Warsaw

"Matter To The Deepest," Katowice, September 15-19th, 2025

The new renormalized corrections to be reported today have been calculated by
M. Czaja, M. Czakon, T. Huber, MM, M. Niggetiedt, A. Rehman, K. Schönwald and M. Steinhauser
[to appear in arXiv:2510.nnnnn]

1. Introductory conclusions
2. Perturbative amplitudes and their interference at the NNLO in QCD (new)
3. The current SM prediction for $\mathcal{B}_{s\gamma}$ and updated bounds on 2HDM-II

NNLO QCD corrections to $\bar{B} \to X_s\gamma$ without interpolation in m_c

Mikołaj Misiak
University of Warsaw

"Matter To The Deepest," Katowice, September 15-19th, 2025

The new renormalized corrections to be reported today have been calculated by
M. Czaja, M. Czakon, T. Huber, MM, M. Niggetiedt, A. Rehman, K. Schönwald and M. Steinhauser
[to appear in arXiv:2510.nnnnn]

1. Introductory conclusions
2. Perturbative amplitudes and their interference at the NNLO in QCD (new)
3. The current SM prediction for $\mathcal{B}_{s\gamma}$ and updated bounds on 2HDM-II
4. Non-perturbative QCD effects

NNLO QCD corrections to $\bar{B} \to X_s\gamma$ without interpolation in m_c

Mikołaj Misiak

University of Warsaw

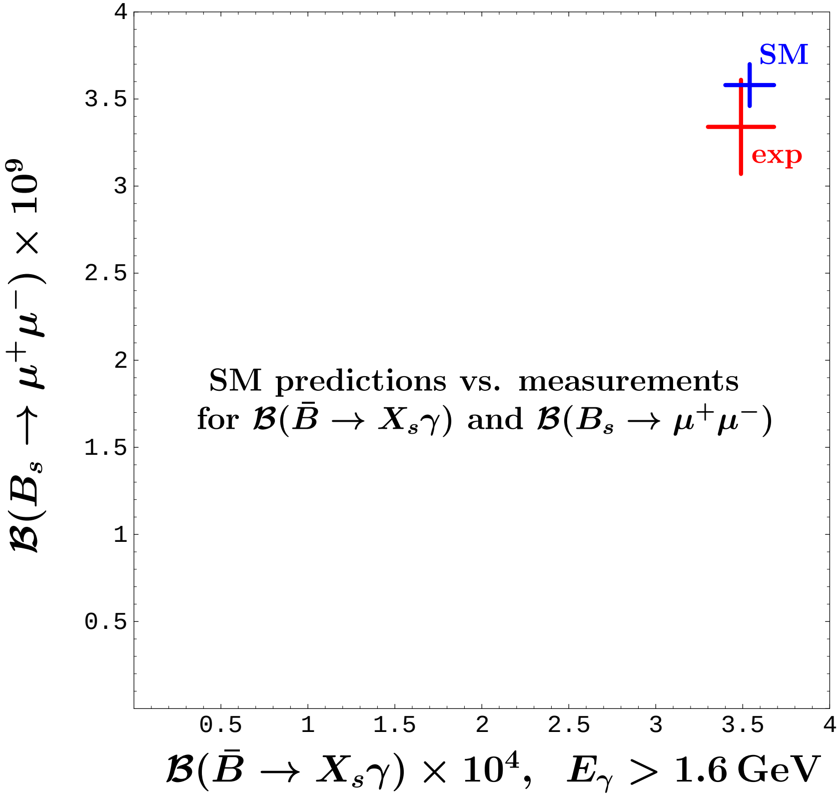
"Matter To The Deepest," Katowice, September 15-19th, 2025

The new renormalized corrections to be reported today have been calculated by
M. Czaja, M. Czakon, T. Huber, MM, M. Niggetiedt, A. Rehman, K. Schönwald and M. Steinhauser
[to appear in arXiv:2510.nnnnn]

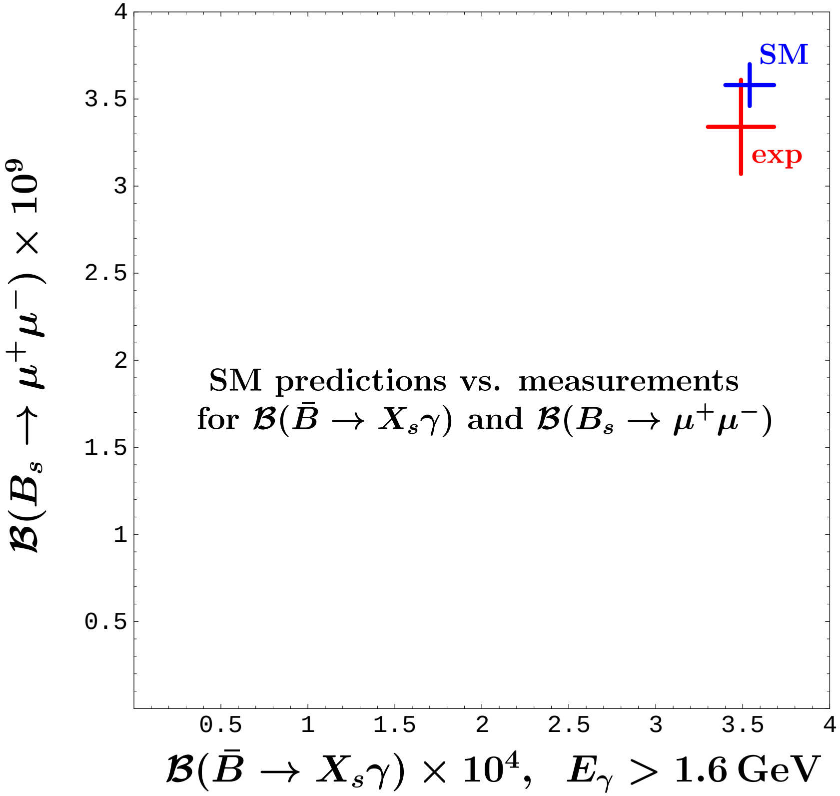
1. Introductory conclusions
2. Perturbative amplitudes and their interference at the NNLO in QCD (new)
3. The current SM prediction for $\mathcal{B}_{s\gamma}$ and updated bounds on 2HDM-II
4. Non-perturbative QCD effects
5. Summary and outlook

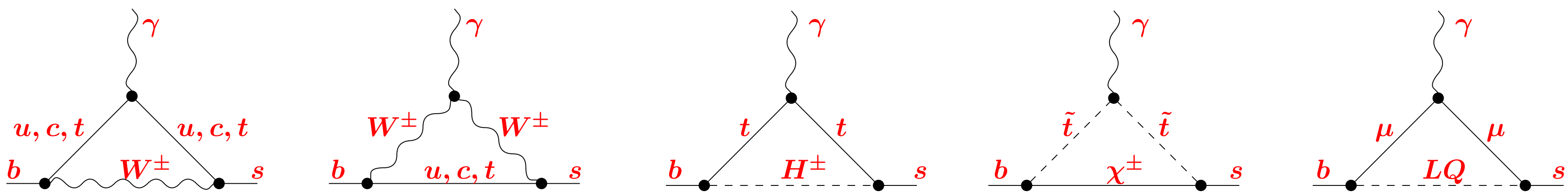
Introductory conclusions

Introductory conclusions

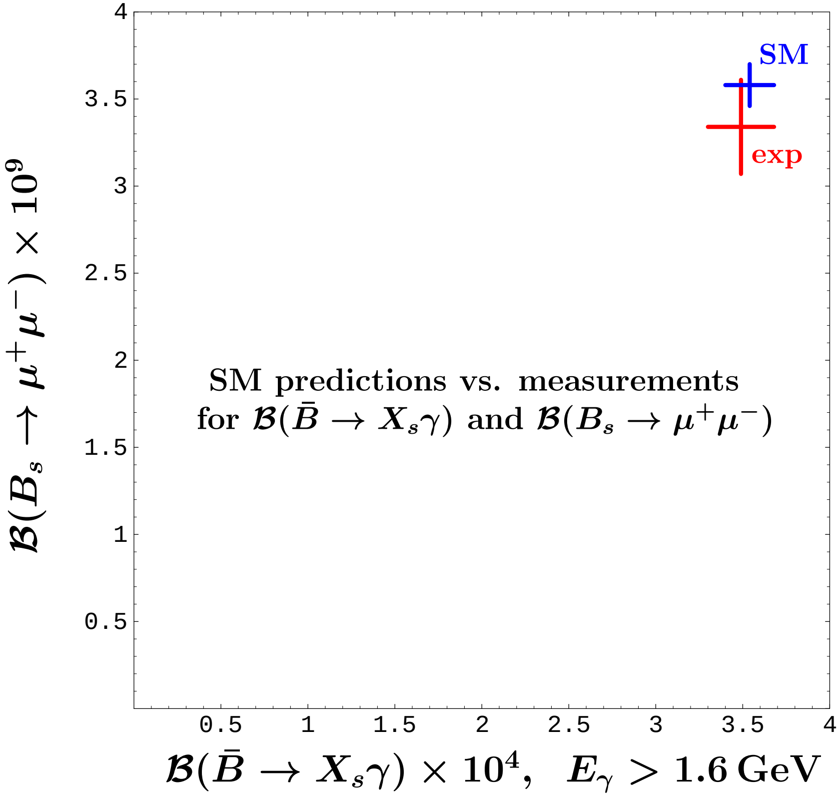


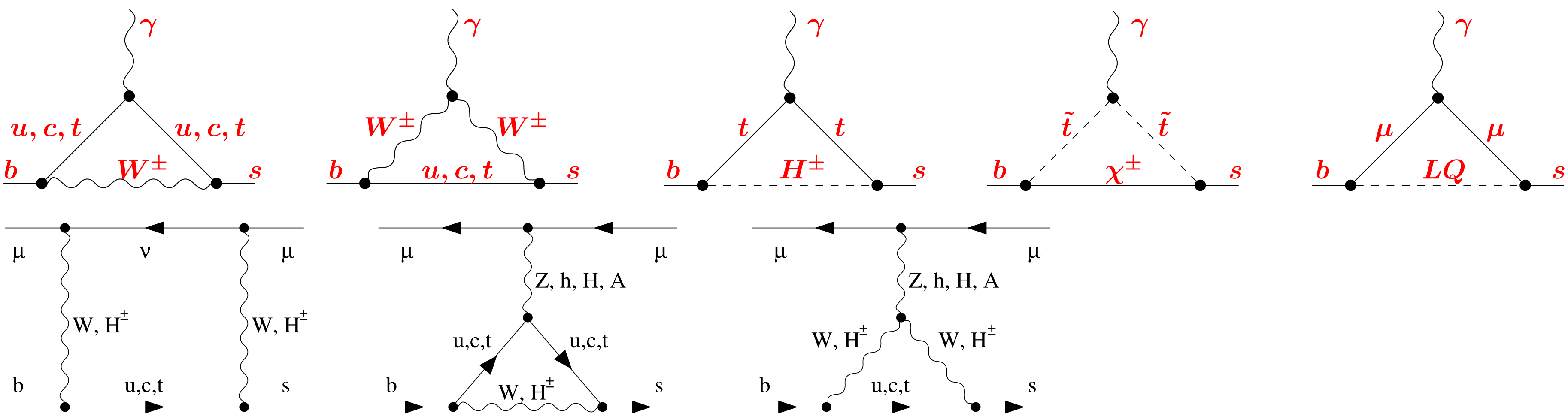
Introductory conclusions



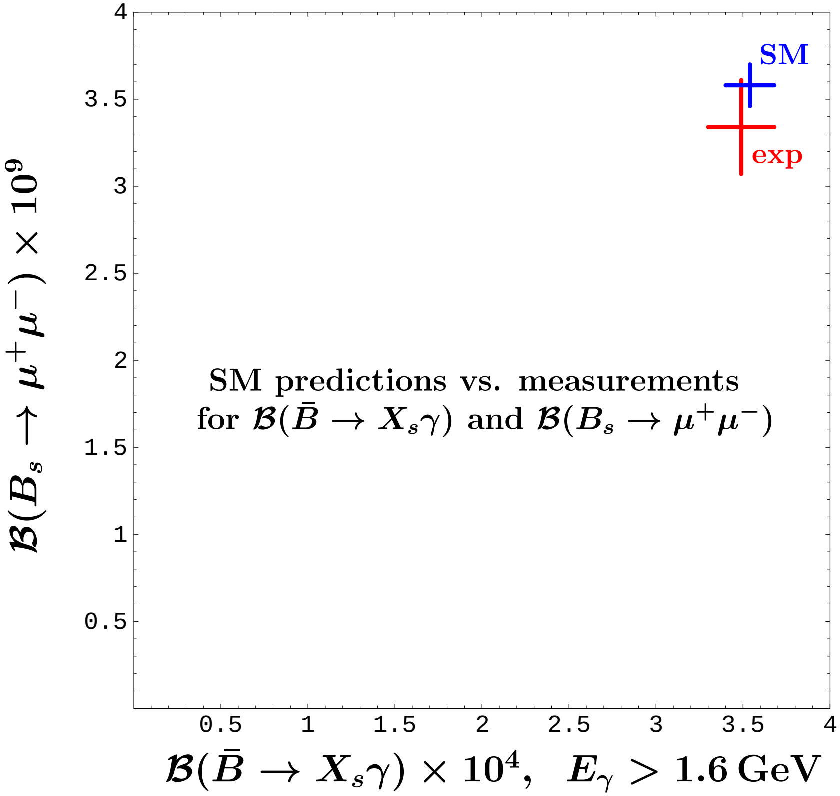


Introductory conclusions



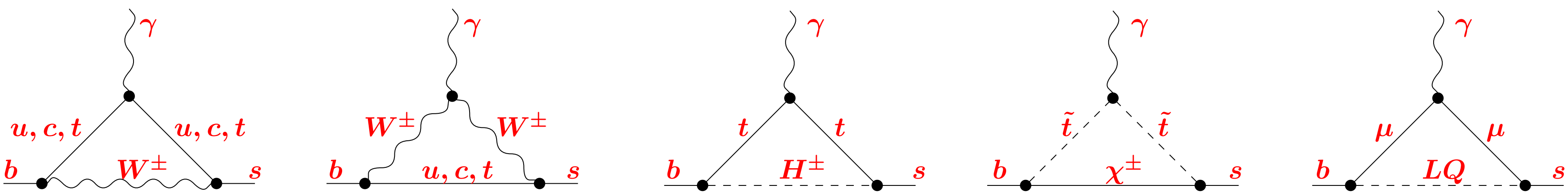


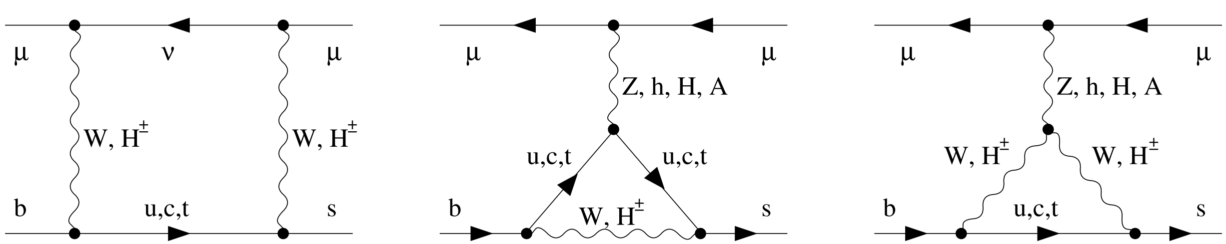
Introductory conclusions



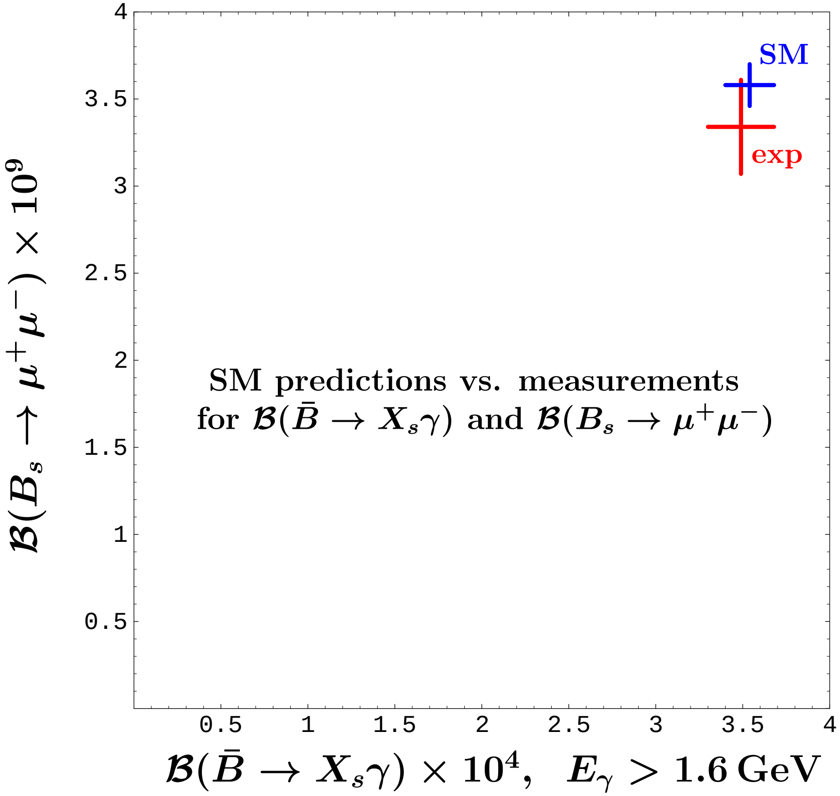
$\mathcal{B}(\bar{B} \to X_s\gamma)^{\mathrm{exp}}_{E_\gamma>1.6} \times 10^4 = 3.49 \pm 0.19 \quad (\pm 5.5\%)$

CLEO, BaBar and Belle measurements combined by PDG [2022,2024] and HFLAV [arXiv:2206.07501].





Introductory conclusions

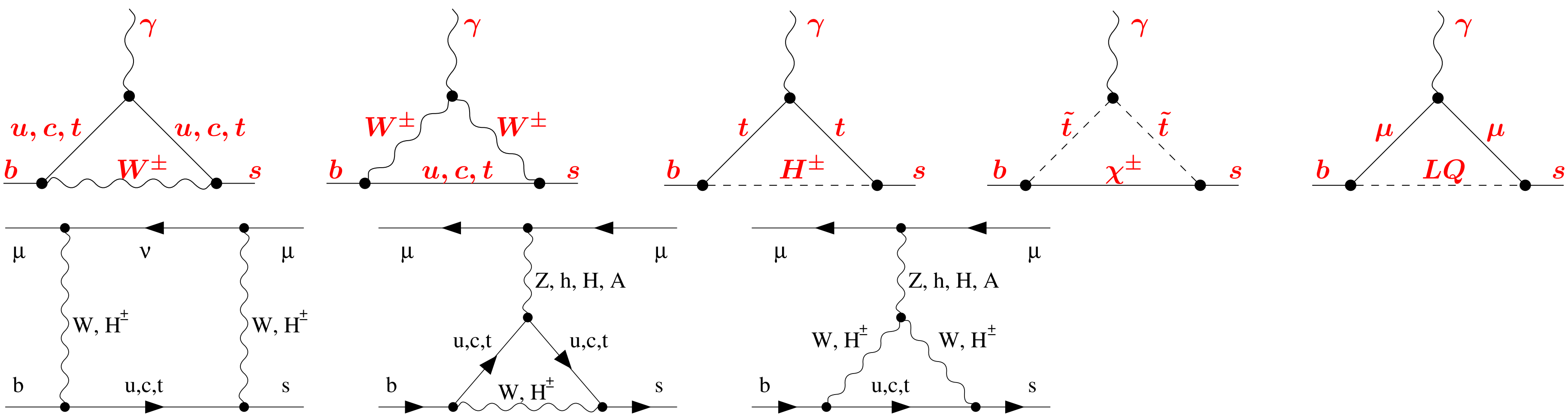


$$\mathcal{B}(\bar{B} \to X_s\gamma)^{\mathrm{exp}}_{E_\gamma>1.6} \times 10^4 = 3.49 \pm 0.19 \quad (\pm 5.5\%)$$

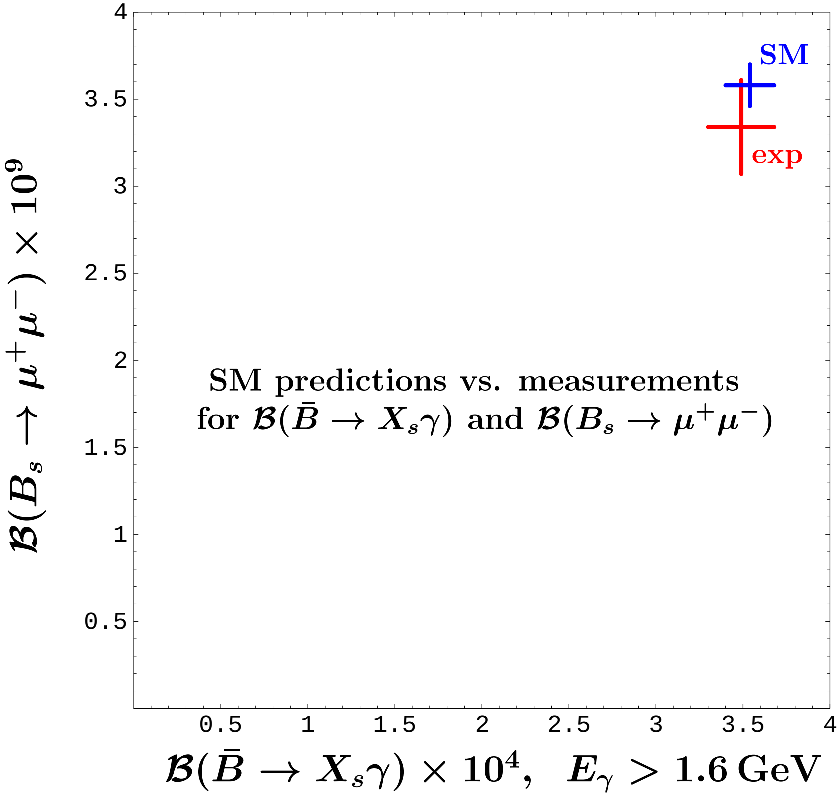
CLEO, BaBar and Belle measurements combined
by PDG [2022,2024] and HFLAV [arXiv:2206.07501].

$$\mathcal{B}(\bar{B} \to X_s\gamma)^{\mathrm{SM}}_{E_\gamma>1.6} \times 10^4 = 3.54 \pm 0.14 \quad (\pm 4.0\%)$$

Final for 2510.nnnnn.



Introductory conclusions



$$\mathcal{B}(\bar{B} \to X_s\gamma)^{\mathrm{exp}}_{E_\gamma>1.6} \times 10^4 = 3.49 \pm 0.19 \quad (\pm 5.5\%)$$

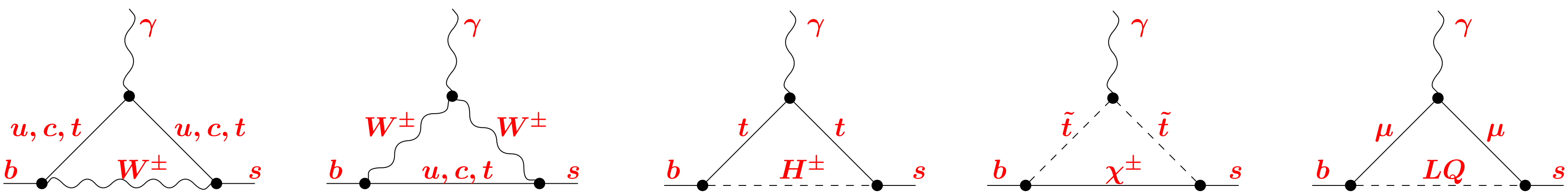
CLEO, BaBar and Belle measurements combined by PDG [2022,2024] and HFLAV [arXiv:2206.07501].

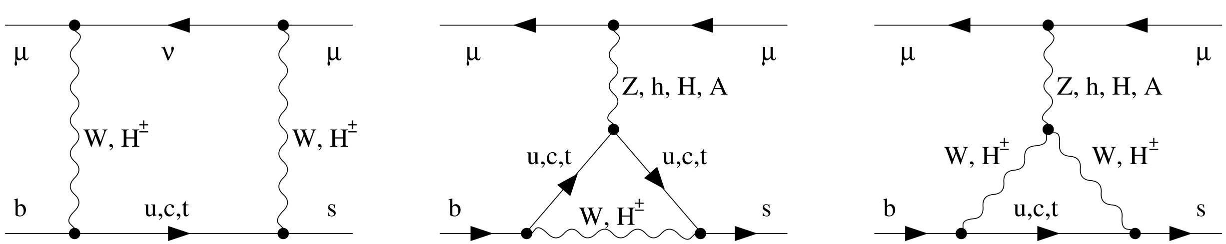
$$\mathcal{B}(\bar{B} \to X_s\gamma)^{\mathrm{SM}}_{E_\gamma>1.6} \times 10^4 = 3.54 \pm 0.14 \quad (\pm 4.0\%)$$

Final for 2510.nnnnn.

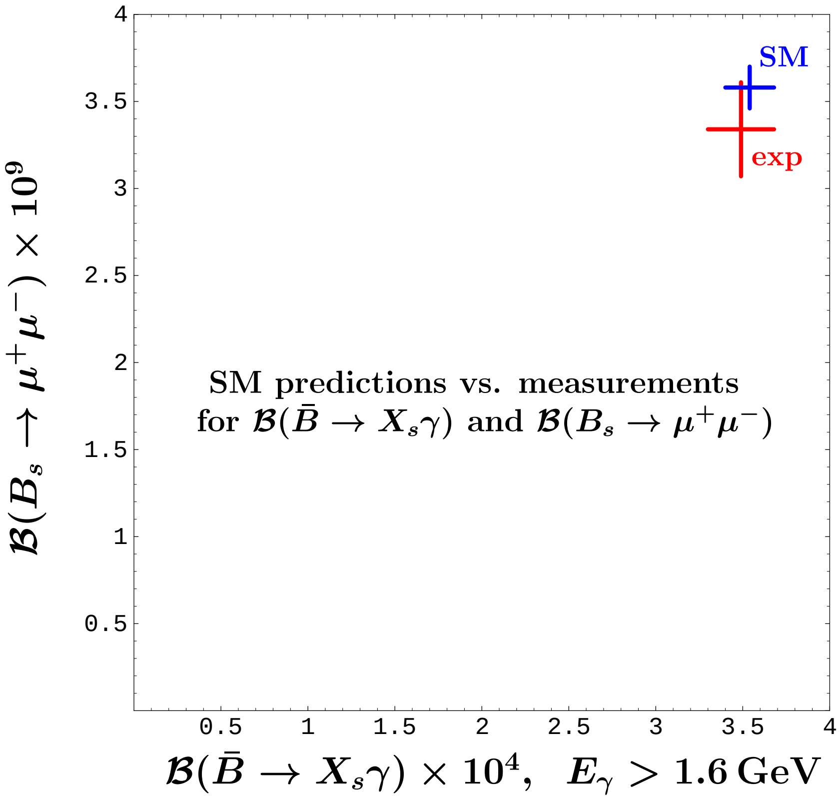
$$\mathcal{B}(B_s \to \mu^+\mu^-)^{\mathrm{exp}} \times 10^9 = 3.34 \pm 0.27 \quad (\pm 8.1\%)$$

LHCb, CMS and ATLAS measurements combined by PDG [2024].





Introductory conclusions



$\mathcal{B}(\bar{B} \to X_s\gamma)^{\mathrm{exp}}_{E_\gamma>1.6} \times 10^4 = 3.49 \pm 0.19 \quad (\pm 5.5\%)$

CLEO, BaBar and Belle measurements combined by PDG [2022,2024] and HFLAV [arXiv:2206.07501].

$\mathcal{B}(\bar{B} \to X_s\gamma)^{\mathrm{SM}}_{E_\gamma>1.6} \times 10^4 = 3.54 \pm 0.14 \quad (\pm 4.0\%)$

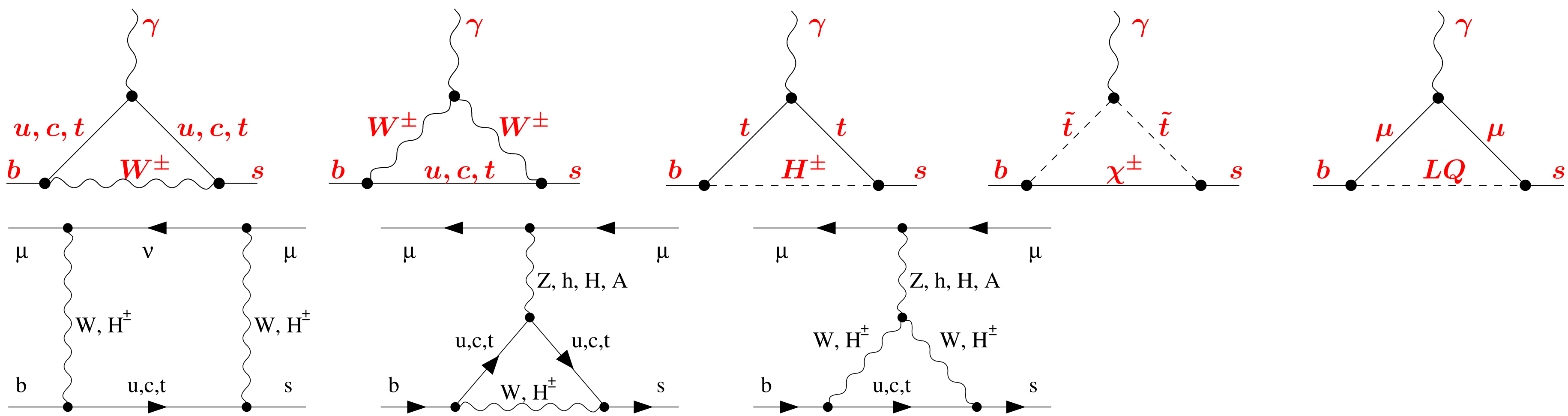
Final for 2510.nnnnn.

$\mathcal{B}(B_s \to \mu^+\mu^-)^{\mathrm{exp}} \times 10^9 = 3.34 \pm 0.27 \quad (\pm 8.1\%)$

LHCb, CMS and ATLAS measurements combined by PDG [2024].

$\mathcal{B}(B_s \to \mu^+\mu^-)^{\mathrm{SM}} \times 10^9 = 3.58 \pm 0.12 \quad (\pm 3.4\%)$

arXiv:1311.0903 by C. Bobeth, M. Gorbahn, T. Hermann, MM, E. Stamou, M. Steinhauser with parameter updates and -0.5% QED correction from arXiv:1907.07011 by M. Beneke, C. Bobeth and R. Szafron.



Determination of $\mathcal{B}(\bar{B} \to X_s\gamma)$ in the SM:

$$\mathcal{B}(\bar{B} \to X_s\gamma)_{E_\gamma > E_0} = \frac{G_F^2 \alpha_{\rm em} m_{b,\rm kin}^5}{32\pi^4 \, \Gamma_{\rm tot}} \left| V_{ts}^* V_{tb} \right|^2 \Big[\underbrace{\tilde{\rm P}({\rm E}_0)}_{\rm pert.} + \underbrace{\tilde{N}(E_0)}_{\rm non\text{-}pert.} \Big]$$

Determination of $\mathcal{B}(\bar{B} \to X_s\gamma)$ in the SM:

$$\mathcal{B}(\bar{B} \to X_s\gamma)_{E_\gamma > E_0} = \frac{G_F^2 \alpha_{\rm em} m_{b,\rm kin}^5}{32\pi^4\, \Gamma_{\rm tot}} \left|V_{ts}^* V_{tb}\right|^2 \Big[\underbrace{\tilde{\mathrm{P}}(\mathrm{E}_0)}_{\text{pert.}} + \underbrace{\tilde{N}(E_0)}_{\text{non-pert.}} \Big]$$

Conventionally $E_0 = 1.6\,\mathrm{GeV}$.

$\sim 100\%$ $\qquad$ $\mathcal{O}(3\%) - \mathcal{O}(3\%) = (0.2 \pm 1.5)\%$

Determination of $\mathcal{B}(\bar{B} \to X_s\gamma)$ in the SM:

$$\mathcal{B}(\bar{B} \to X_s\gamma)_{E_\gamma > E_0} = \frac{G_F^2 \alpha_{\rm em} m_{b,\rm kin}^5}{32\pi^4 \, \Gamma_{\rm tot}} \left|V_{ts}^* V_{tb}\right|^2 \Big[\underbrace{\tilde{\mathrm{P}}(\mathrm{E}_0)}_{\text{pert.}} + \underbrace{\tilde{N}(E_0)}_{\text{non-pert.}} \Big]$$

Conventionally $E_0 = 1.6\,\mathrm{GeV}$.

$\sim 100\%$ $\quad\quad$ $\mathcal{O}(3\%) - \mathcal{O}(3\%) = (0.2 \pm 1.5)\%$

Former normalization to $\mathrm{C} = \left|\frac{V_{ub}}{V_{cb}}\right|^2 \frac{\Gamma[\bar{B}\to X_c e\bar{\nu}]}{\Gamma[\bar{B}\to X_u e\bar{\nu}]}$ abandoned due to poor behaviour of C at $\mathcal{O}(\alpha_s^3)$.

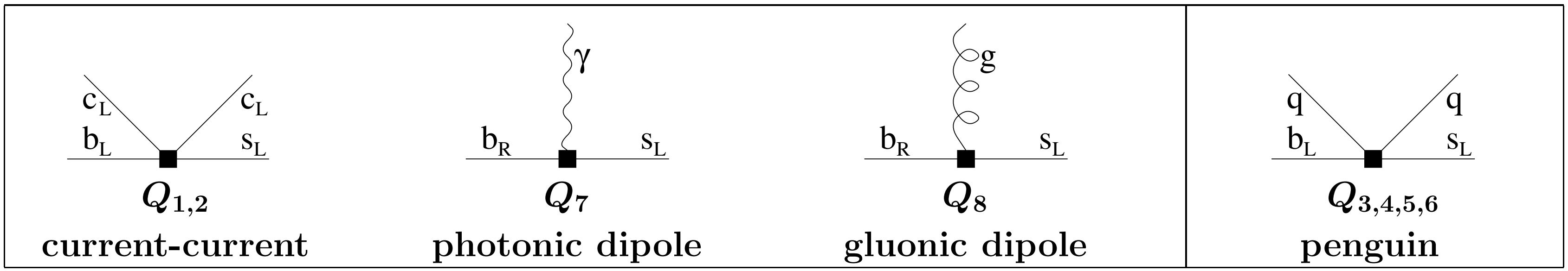
Determination of $\mathcal{B}(\bar{B} \to X_s\gamma)$ in the SM:

$$\mathcal{B}(\bar{B} \to X_s\gamma)_{E_\gamma > E_0} = \frac{G_F^2 \alpha_{\rm em} m_{b,\rm kin}^5}{32\pi^4\, \Gamma_{\rm tot}} \left| V_{ts}^* V_{tb} \right|^2 \Big[\underbrace{\tilde{P}(E_0)}_{\substack{\text{pert.} \\ \sim 100\%}} + \underbrace{\tilde{N}(E_0)}_{\substack{\text{non-pert.} \\ \mathcal{O}(3\%) - \mathcal{O}(3\%) = (0.2 \pm 1.5)\%}} \Big]$$

Conventionally $E_0 = 1.6\,\text{GeV}$.

Former normalization to $C = \left| \frac{V_{ub}}{V_{cb}} \right|^2 \frac{\Gamma[\bar{B} \to X_c e \bar{\nu}]}{\Gamma[\bar{B} \to X_u e \bar{\nu}]}$ abandoned due to poor behaviour of C at $\mathcal{O}(\alpha_s^3)$.

Eight LEFT operators Q_i matter for $\mathcal{B}_{s\gamma}^{\rm SM}$ when the NLO EW & CKM-suppressed effects are neglected:



Determination of $\mathcal{B}(\bar{B} \to X_s\gamma)$ in the SM:

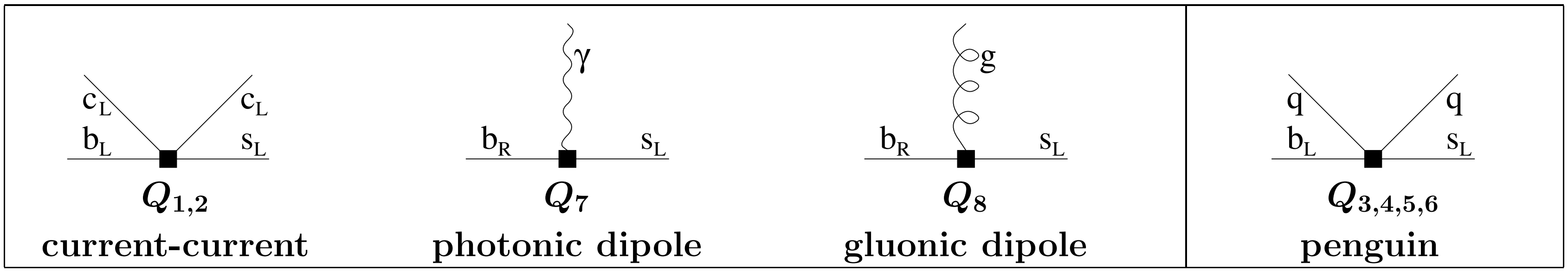
$$\mathcal{B}(\bar{B} \to X_s\gamma)_{E_\gamma > E_0} = \frac{G_F^2 \alpha_{\rm em} m_{b,\rm kin}^5}{32\pi^4\, \Gamma_{\rm tot}} \left|V_{ts}^* V_{tb}\right|^2 \Big[\underbrace{\tilde{\rm P}({\rm E}_0)}_{\text{pert.}} + \underbrace{\tilde{N}(E_0)}_{\text{non-pert.}}\Big]$$

Conventionally $E_0 = 1.6\,\mathrm{GeV}$.

pert.: $\sim 100\%$ non-pert.: $\mathcal{O}(3\%) - \mathcal{O}(3\%) = (0.2 \pm 1.5)\%$

Former normalization to $C = \left|\frac{V_{ub}}{V_{cb}}\right|^2 \frac{\Gamma[\bar{B}\to X_c e\bar{\nu}]}{\Gamma[\bar{B}\to X_u e\bar{\nu}]}$ abandoned due to poor behaviour of C at $\mathcal{O}(\alpha_s^3)$.

Eight LEFT operators Q_i matter for $\mathcal{B}_{s\gamma}^{\rm SM}$ when the NLO EW & CKM-suppressed effects are neglected:



$$\Gamma(b \to X_s^p\gamma) = \frac{G_F^2\, m_{b,\,\rm pole}^5\, \alpha_{\rm em}}{32\pi^4} \left|V_{ts}^* V_{tb}\right|^2 \sum_{i,j=1}^{8} C_i(\mu_b) C_j(\mu_b) G_{ij}, \qquad ({\rm G_{ij} = G_{ji}}, \quad \mu_{\rm b} \sim {\rm m_b})$$

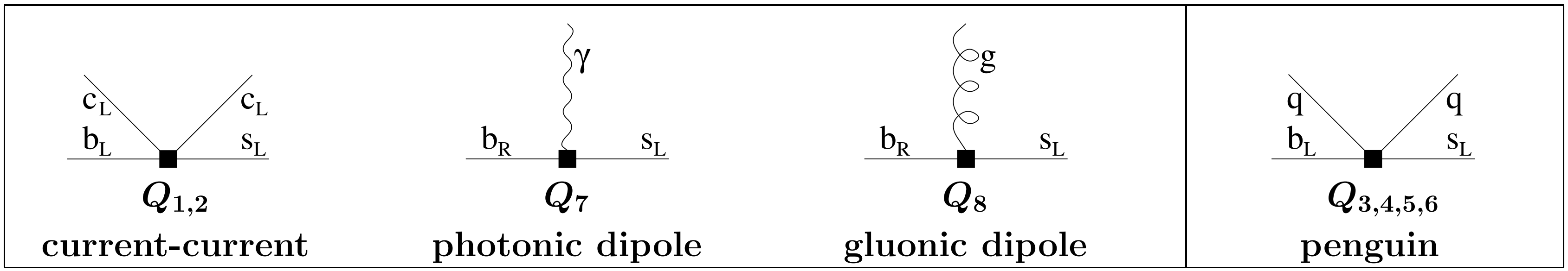
Determination of $\mathcal{B}(\bar{B} \to X_s\gamma)$ in the SM:

$$\mathcal{B}(\bar{B} \to X_s\gamma)_{E_\gamma > E_0} = \frac{G_F^2 \alpha_{\rm em} m_{b,\rm kin}^5}{32\pi^4\, \Gamma_{\rm tot}} \left|V_{ts}^* V_{tb}\right|^2 \Big[\underbrace{\tilde{\rm P}({\rm E}_0)}_{\substack{\text{pert.}\\ \sim 100\%}} + \underbrace{\tilde{N}(E_0)}_{\substack{\text{non-pert.}\\ \mathcal{O}(3\%) - \mathcal{O}(3\%) = (0.2 \pm 1.5)\%}} \Big]$$

Conventionally $E_0 = 1.6\,\mathrm{GeV}$.

Former normalization to $\mathrm{C} = \left|\frac{V_{ub}}{V_{cb}}\right|^2 \frac{\Gamma[\bar{B}\to X_c e\bar{\nu}]}{\Gamma[\bar{B}\to X_u e\bar{\nu}]}$ abandoned due to poor behaviour of C at $\mathcal{O}(\alpha_s^3)$.

Eight LEFT operators Q_i matter for $\mathcal{B}_{s\gamma}^{\rm SM}$ when the NLO EW & CKM-suppressed effects are neglected:



$$\Gamma(b \to X_s^p\gamma) = \frac{G_F^2\, m_{b,\,\rm pole}^5\, \alpha_{\rm em}}{32\pi^4} \left|V_{ts}^* V_{tb}\right|^2 \sum_{i,j=1}^{8} C_i(\mu_b) C_j(\mu_b) G_{ij}, \qquad ({\rm G_{ij} = G_{ji}}, \quad \mu_{\rm b} \sim {\rm m_b})$$

RGEs for the WCs : $\mu\frac{d}{d\mu}\vec{C}(\mu) = \hat{\gamma}^T \vec{C}(\mu),$

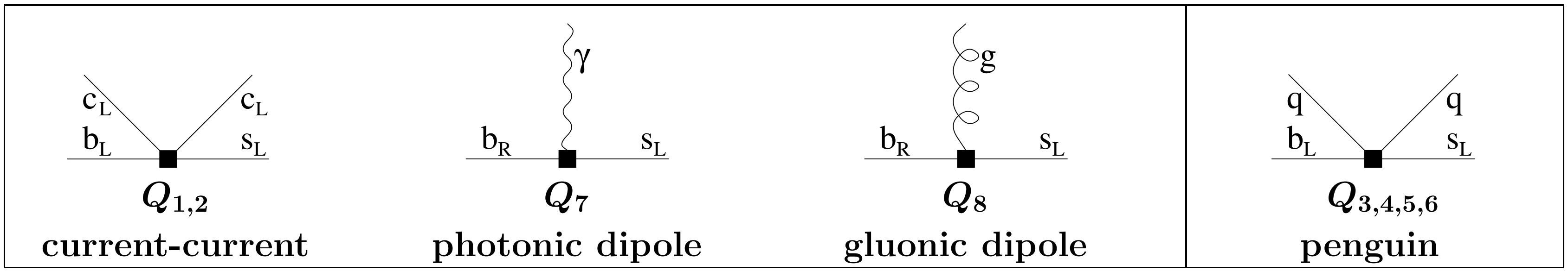
Determination of $\mathcal{B}(\bar{B} \to X_s\gamma)$ in the SM:

$$\mathcal{B}(\bar{B} \to X_s\gamma)_{E_\gamma > E_0} = \frac{G_F^2 \alpha_{\rm em} m_{b,\rm kin}^5}{32\pi^4\, \Gamma_{\rm tot}} \left|V_{ts}^* V_{tb}\right|^2 \Big[\underbrace{\tilde{\rm P}({\rm E}_0)}_{\text{pert.}} + \underbrace{\tilde{N}(E_0)}_{\text{non-pert.}}\Big]$$

Conventionally $E_0 = 1.6\,\text{GeV}$. $\qquad \sim 100\% \qquad \mathcal{O}(3\%) - \mathcal{O}(3\%) = (0.2 \pm 1.5)\%$

Former normalization to $C = \left|\frac{V_{ub}}{V_{cb}}\right|^2 \frac{\Gamma[\bar{B} \to X_c e\bar{\nu}]}{\Gamma[\bar{B} \to X_u e\bar{\nu}]}$ abandoned due to poor behaviour of C at $\mathcal{O}(\alpha_s^3)$.

Eight LEFT operators Q_i matter for $\mathcal{B}_{s\gamma}^{\rm SM}$ when the NLO EW & CKM-suppressed effects are neglected:



$$\Gamma(b \to X_s^p\gamma) = \frac{G_F^2\, m_{b,\,\rm pole}^5\, \alpha_{\rm em}}{32\pi^4} \left|V_{ts}^* V_{tb}\right|^2 \sum_{i,j=1}^{8} C_i(\mu_b) C_j(\mu_b) G_{ij}, \qquad ({\rm G_{ij} = G_{ji}}, \quad \mu_{\rm b} \sim {\rm m_b})$$

RGEs for the WCs : $\quad \mu\frac{d}{d\mu}\vec{C}(\mu) = \hat{\gamma}^T \vec{C}(\mu),$ $\qquad$ ADM: $\quad \hat{\gamma} = \frac{\alpha_s}{4\pi}\hat{\gamma}^{(0)} + \left(\frac{\alpha_s}{4\pi}\right)^2 \hat{\gamma}^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^3 \hat{\gamma}^{(2)} + \ldots\,,$

Determination of $\mathcal{B}(\bar{B} \to X_s\gamma)$ in the SM:

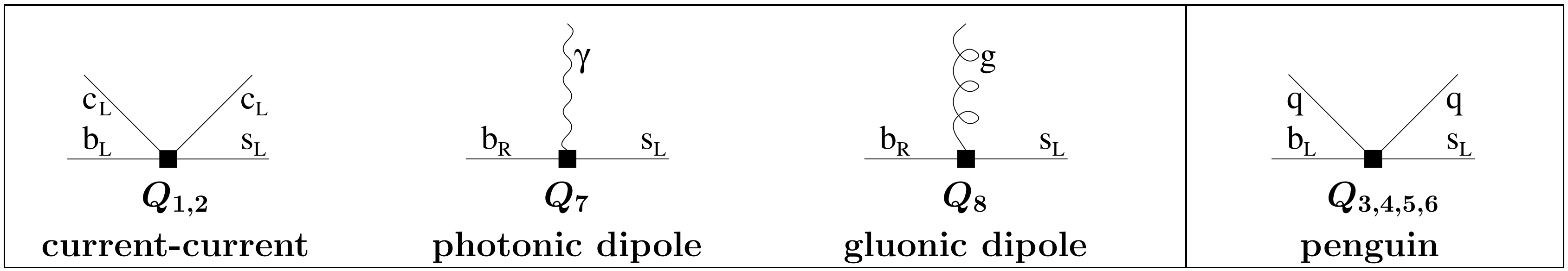
$$\mathcal{B}(\bar{B} \to X_s\gamma)_{E_\gamma > E_0} = \frac{G_F^2 \alpha_{\rm em} m_{b,\rm kin}^5}{32\pi^4\, \Gamma_{\rm tot}} \left|V_{ts}^* V_{tb}\right|^2 \Big[\underbrace{\tilde{\rm P}({\rm E}_0)}_{\rm pert.} + \underbrace{\tilde{N}(E_0)}_{\text{non-pert.}}\Big]$$

Conventionally $E_0 = 1.6\,$GeV.

pert. $\sim 100\%$ non-pert. $\mathcal{O}(3\%) - \mathcal{O}(3\%) = (0.2 \pm 1.5)\%$

Former normalization to $C = \left|\frac{V_{ub}}{V_{cb}}\right|^2 \frac{\Gamma[\bar{B}\to X_c e\bar{\nu}]}{\Gamma[\bar{B}\to X_u e\bar{\nu}]}$ abandoned due to poor behaviour of C at $\mathcal{O}(\alpha_s^3)$.

Eight LEFT operators Q_i matter for $\mathcal{B}_{s\gamma}^{\rm SM}$ when the NLO EW & CKM-suppressed effects are neglected:



$$\Gamma(b \to X_s^p\gamma) = \frac{G_F^2\, m_{b,\,\rm pole}^5\, \alpha_{\rm em}}{32\pi^4} \left|V_{ts}^* V_{tb}\right|^2 \sum_{i,j=1}^{8} C_i(\mu_b) C_j(\mu_b) G_{ij}, \qquad ({\rm G_{ij} = G_{ji}}, \quad \mu_{\rm b} \sim {\rm m_b})$$

RGEs for the WCs : $\mu\frac{d}{d\mu}\vec{C}(\mu) = \hat{\gamma}^T \vec{C}(\mu)$,   ADM: $\hat{\gamma} = \frac{\alpha_s}{4\pi}\hat{\gamma}^{(0)} + \left(\frac{\alpha_s}{4\pi}\right)^2 \hat{\gamma}^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^3 \hat{\gamma}^{(2)} + \ldots,$

Initial (matching) conditions: $\vec{C}(\mu_0) = \vec{C}^{(0)}(\mu_0) + \frac{\alpha_s}{4\pi}\vec{C}^{(1)}(\mu_0) + \left(\frac{\alpha_s}{4\pi}\right)^2 \vec{C}^{(2)}(\mu_0) + \ldots,$ $(\mu_0 \sim m_t, M_W)$

Determination of $\mathcal{B}(\bar{B} \to X_s \gamma)$ in the SM:

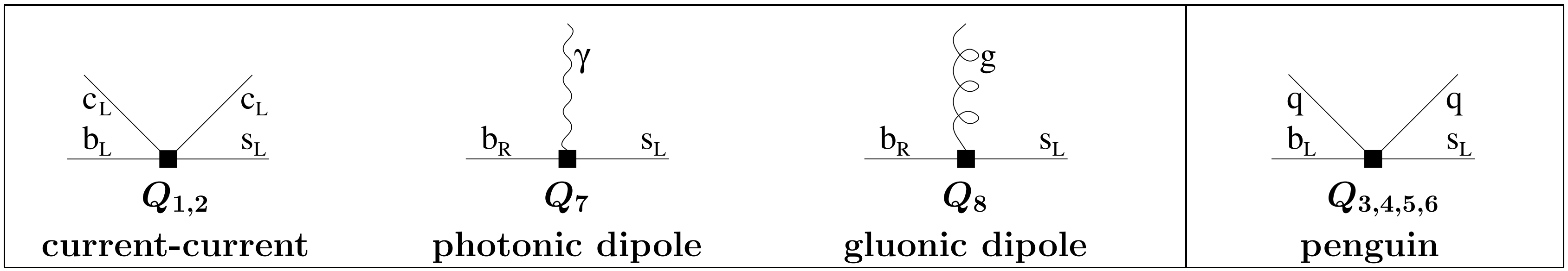
$$\mathcal{B}(\bar{B} \to X_s \gamma)_{E_\gamma > E_0} = \frac{G_F^2 \alpha_{\rm em} m_{b,\rm kin}^5}{32\pi^4\, \Gamma_{\rm tot}} \left| V_{ts}^* V_{tb} \right|^2 \Big[\underbrace{\tilde{\rm P}({\rm E}_0)}_{\text{pert.}} + \underbrace{\tilde{N}(E_0)}_{\text{non-pert.}} \Big]$$

Conventionally $E_0 = 1.6\,$GeV.

pert.: $\sim 100\%$; non-pert.: $\mathcal{O}(3\%) - \mathcal{O}(3\%) = (0.2 \pm 1.5)\%$

Former normalization to $\mathrm{C} = \left| \frac{V_{ub}}{V_{cb}} \right|^2 \frac{\Gamma[\bar{B} \to X_c e \bar{\nu}]}{\Gamma[\bar{B} \to X_u e \bar{\nu}]}$ abandoned due to poor behaviour of C at $\mathcal{O}(\alpha_s^3)$.

Eight LEFT operators Q_i matter for $\mathcal{B}_{s\gamma}^{\rm SM}$ when the NLO EW & CKM-suppressed effects are neglected:



$$\Gamma(b \to X_s^p \gamma) = \frac{G_F^2\, m_{b,\,\rm pole}^5\, \alpha_{\rm em}}{32\pi^4} \left| V_{ts}^* V_{tb} \right|^2 \sum_{i,j=1}^{8} C_i(\mu_b) C_j(\mu_b) G_{ij}, \qquad ({\rm G_{ij} = G_{ji}}, \quad \mu_{\rm b} \sim {\rm m_b})$$

RGEs for the WCs : $\mu \frac{d}{d\mu} \vec{C}(\mu) = \hat{\gamma}^T \vec{C}(\mu)$, ADM: $\hat{\gamma} = \frac{\alpha_s}{4\pi} \hat{\gamma}^{(0)} + \left(\frac{\alpha_s}{4\pi}\right)^2 \hat{\gamma}^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^3 \hat{\gamma}^{(2)} + \ldots$,

Initial (matching) conditions: $\vec{C}(\mu_0) = \vec{C}^{(0)}(\mu_0) + \frac{\alpha_s}{4\pi} \vec{C}^{(1)}(\mu_0) + \left(\frac{\alpha_s}{4\pi}\right)^2 \vec{C}^{(2)}(\mu_0) + \ldots,$ $(\mu_0 \sim m_t, M_W)$

Perturbative expansion of $\hat{G}$: $\hat{G} = \hat{G}^{(0)} + \frac{\alpha_s}{4\pi} \hat{G}^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 \hat{G}^{(2)} + \ldots,$

Determination of $\mathcal{B}(\bar{B} \to X_s\gamma)$ in the SM:

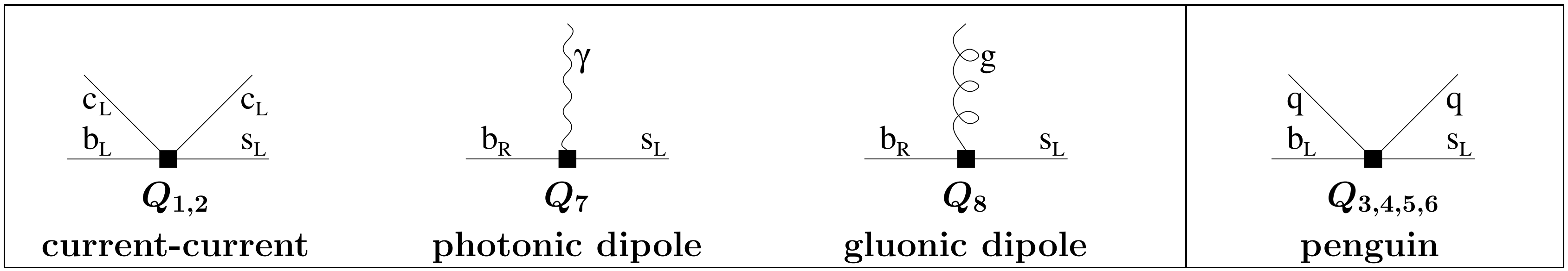
$$\mathcal{B}(\bar{B} \to X_s\gamma)_{E_\gamma > E_0} = \frac{G_F^2 \alpha_{\rm em} m_{b,\rm kin}^5}{32\pi^4\, \Gamma_{\rm tot}} \left|V_{ts}^* V_{tb}\right|^2 \Big[\underbrace{\tilde{\rm P}({\rm E_0})}_{\rm pert.} + \underbrace{\tilde{N}(E_0)}_{\text{non-pert.}} \Big]$$

Conventionally $E_0 = 1.6\,\mathrm{GeV}$.

pert. $\sim 100\%$ non-pert. $\mathcal{O}(3\%) - \mathcal{O}(3\%) = (0.2 \pm 1.5)\%$

Former normalization to $\mathrm{C} = \left|\frac{V_{ub}}{V_{cb}}\right|^2 \frac{\Gamma[\bar{B}\to X_c e\bar{\nu}]}{\Gamma[\bar{B}\to X_u e\bar{\nu}]}$ abandoned due to poor behaviour of C at $\mathcal{O}(\alpha_s^3)$.

Eight LEFT operators Q_i matter for $\mathcal{B}_{s\gamma}^{\rm SM}$ when the NLO EW & CKM-suppressed effects are neglected:



$$\Gamma(b \to X_s^p\gamma) = \frac{G_F^2\, m_{b,\,\rm pole}^5\, \alpha_{\rm em}}{32\pi^4} \left|V_{ts}^* V_{tb}\right|^2 \sum_{i,j=1}^{8} C_i(\mu_b) C_j(\mu_b) G_{ij}, \qquad ({\rm G_{ij} = G_{ji}}, \quad \mu_{\rm b} \sim {\rm m_b})$$

RGEs for the WCs : $\mu \frac{d}{d\mu} \vec{C}(\mu) = \hat{\gamma}^T \vec{C}(\mu)$, ADM: $\hat{\gamma} = \frac{\alpha_s}{4\pi}\hat{\gamma}^{(0)} + \left(\frac{\alpha_s}{4\pi}\right)^2 \hat{\gamma}^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^3 \hat{\gamma}^{(2)} + \ldots$,

Initial (matching) conditions: $\vec{C}(\mu_0) = \vec{C}^{(0)}(\mu_0) + \frac{\alpha_s}{4\pi}\vec{C}^{(1)}(\mu_0) + \left(\frac{\alpha_s}{4\pi}\right)^2 \vec{C}^{(2)}(\mu_0) + \ldots$, $(\mu_0 \sim m_t, M_W)$

Perturbative expansion of $\hat{G}$: $\hat{G} = \hat{G}^{(0)} + \frac{\alpha_s}{4\pi}\hat{G}^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 \hat{G}^{(2)} + \ldots$,

In the following, focus on G_{77}, G_{78} and G_{27} (the latter stands for "G_{17} and G_{27}").

Calculations of G_{77}, G_{78} and G_{27} up to the NNLO in QCD.

Calculations of G_{77}, G_{78} and G_{27} up to the NNLO in QCD.

$$G_{77} = 1 + \frac{\alpha_s}{4\pi} G_{77}^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 G_{77}^{(2)} + \ldots,$$

$$G_{78} = \frac{\alpha_s}{4\pi} G_{78}^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 G_{78}^{(2)} + \ldots,$$

$$G_{27} = \frac{\alpha_s}{4\pi} G_{27}^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 G_{27}^{(2)} + \ldots,$$

Calculations of G_{77}, G_{78} and G_{27} up to the NNLO in QCD.

$$G_{77} = 1 + \frac{\alpha_s}{4\pi} \mathrm{G}_{77}^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 \mathrm{G}_{77}^{(2)} + \ldots,$$

$$G_{78} = \frac{\alpha_s}{4\pi} \mathrm{G}_{78}^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 \mathrm{G}_{78}^{(2)} + \ldots,$$

$$G_{27} = \frac{\alpha_s}{4\pi} \mathrm{G}_{27}^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 \mathrm{G}_{27}^{(2)} + \ldots,$$

<u>NLO</u>

A. Ali and C. Greub, ZPC 49 (1991) 431, PLB 259 (1991) 182, PLB 361 (1995) 146 [hep-ph/9506374],
C. Greub, T. Hurth and D. Wyler, hep-ph/9602281, hep-ph/9603404.

Calculations of G_{77}, G_{78} and G_{27} up to the NNLO in QCD.

$$G_{77} = 1 + \frac{\alpha_s}{4\pi} \mathbf{G}_{77}^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 \mathbf{G}_{77}^{(2)} + \ldots,$$

$$G_{78} = \frac{\alpha_s}{4\pi} \mathbf{G}_{78}^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 \mathbf{G}_{78}^{(2)} + \ldots,$$

$$G_{27} = \frac{\alpha_s}{4\pi} \mathbf{G}_{27}^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 \mathbf{G}_{27}^{(2)} + \ldots,$$

NLO

A. Ali and C. Greub, ZPC 49 (1991) 431, PLB 259 (1991) 182, PLB 361 (1995) 146 [hep-ph/9506374], C. Greub, T. Hurth and D. Wyler, hep-ph/9602281, hep-ph/9603404.

NNLO

Calculations of G_{77}, G_{78} and G_{27} up to the NNLO in QCD.

$$G_{77} = 1 + \frac{\alpha_s}{4\pi} \mathbf{G}_{77}^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 \mathbf{G}_{77}^{(2)} + \ldots,$$

$$G_{78} = \frac{\alpha_s}{4\pi} \mathbf{G}_{78}^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 \mathbf{G}_{78}^{(2)} + \ldots,$$

$$G_{27} = \frac{\alpha_s}{4\pi} \mathbf{G}_{27}^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 \mathbf{G}_{27}^{(2)} + \ldots,$$

NLO

A. Ali and C. Greub, ZPC 49 (1991) 431, PLB 259 (1991) 182, PLB 361 (1995) 146 [hep-ph/9506374], C. Greub, T. Hurth and D. Wyler, hep-ph/9602281, hep-ph/9603404.

NNLO

$G_{77}^{(2)}$: K. Melnikov and A. Mitov, hep-ph/0505097,
I. Blokland, A. Czarnecki, MM, M. Ślusarczyk and F. Tkachov, hep-ph/0506055,
H. M. Asatrian, A. Hovhannisyan, V. Poghosyan, T. Ewerth, C. Greub and T. Hurth, hep-ph/0605009,
H. M. Asatrian, T. Ewerth, A. Ferroglia, P. Gambino and C. Greub, hep-ph/0607316,
H. M. Asatrian, T. Ewerth, H. Gabrielyan and C. Greub, hep-ph/0611123.

Calculations of G_{77}, G_{78} and G_{27} up to the NNLO in QCD.

$$G_{77} = 1 + \frac{\alpha_s}{4\pi} \mathrm{G}_{77}^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 \mathrm{G}_{77}^{(2)} + \ldots,$$

$$G_{78} = \frac{\alpha_s}{4\pi} \mathrm{G}_{78}^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 \mathrm{G}_{78}^{(2)} + \ldots,$$

$$G_{27} = \frac{\alpha_s}{4\pi} \mathrm{G}_{27}^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 \mathrm{G}_{27}^{(2)} + \ldots,$$

NLO

A. Ali and C. Greub, ZPC 49 (1991) 431, PLB 259 (1991) 182, PLB 361 (1995) 146 [hep-ph/9506374], C. Greub, T. Hurth and D. Wyler, hep-ph/9602281, hep-ph/9603404.

NNLO

$G_{77}^{(2)}$: K. Melnikov and A. Mitov, hep-ph/0505097,
I. Blokland, A. Czarnecki, MM, M. Ślusarczyk and F. Tkachov, hep-ph/0506055,
H. M. Asatrian, A. Hovhannisyan, V. Poghosyan, T. Ewerth, C. Greub and T. Hurth, hep-ph/0605009,
H. M. Asatrian, T. Ewerth, A. Ferroglia, P. Gambino and C. Greub, hep-ph/0607316,
H. M. Asatrian, T. Ewerth, H. Gabrielyan and C. Greub, hep-ph/0611123.

$G_{78}^{(2)}$: H. M. Asatrian, T. Ewerth, A. Ferroglia, C. Greub and G. Ossola, arXiv:1005.5587.

Calculations of G_{77}, G_{78} and G_{27} up to the NNLO in QCD.

$$G_{77} = 1 + \frac{\alpha_s}{4\pi} G^{(1)}_{77} + \left(\frac{\alpha_s}{4\pi}\right)^2 G^{(2)}_{77} + \ldots,$$

$$G_{78} = \frac{\alpha_s}{4\pi} G^{(1)}_{78} + \left(\frac{\alpha_s}{4\pi}\right)^2 G^{(2)}_{78} + \ldots,$$

$$G_{27} = \frac{\alpha_s}{4\pi} G^{(1)}_{27} + \left(\frac{\alpha_s}{4\pi}\right)^2 G^{(2)}_{27} + \ldots,$$

NLO

A. Ali and C. Greub, ZPC 49 (1991) 431, PLB 259 (1991) 182, PLB 361 (1995) 146 [hep-ph/9506374],
C. Greub, T. Hurth and D. Wyler, hep-ph/9602281, hep-ph/9603404.

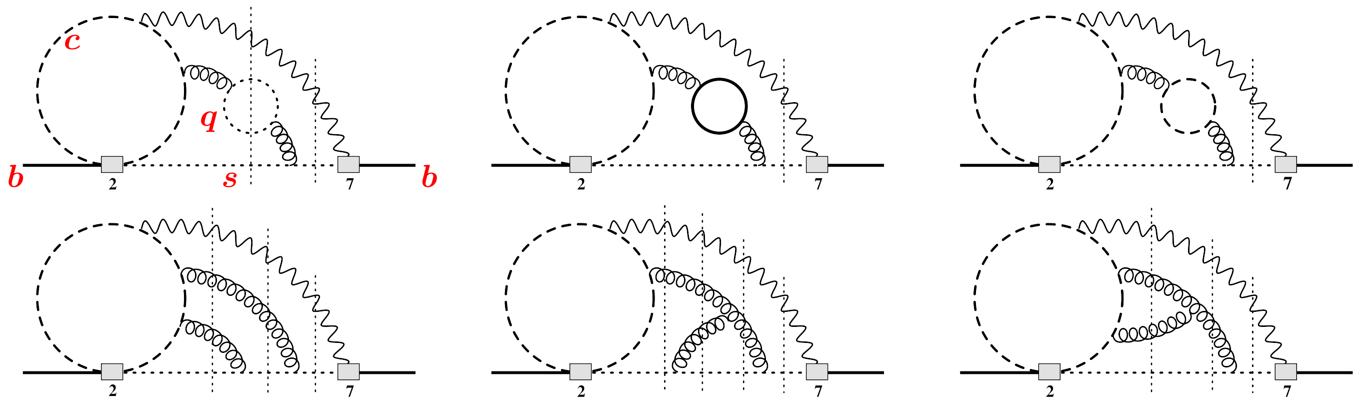
NNLO

$G^{(2)}_{77}$: K. Melnikov and A. Mitov, hep-ph/0505097,
I. Blokland, A. Czarnecki, MM, M. Ślusarczyk and F. Tkachov, hep-ph/0506055,
H. M. Asatrian, A. Hovhannisyan, V. Poghosyan, T. Ewerth, C. Greub and T. Hurth, hep-ph/0605009,
H. M. Asatrian, T. Ewerth, A. Ferroglia, P. Gambino and C. Greub, hep-ph/0607316,
H. M. Asatrian, T. Ewerth, H. Gabrielyan and C. Greub, hep-ph/0611123.

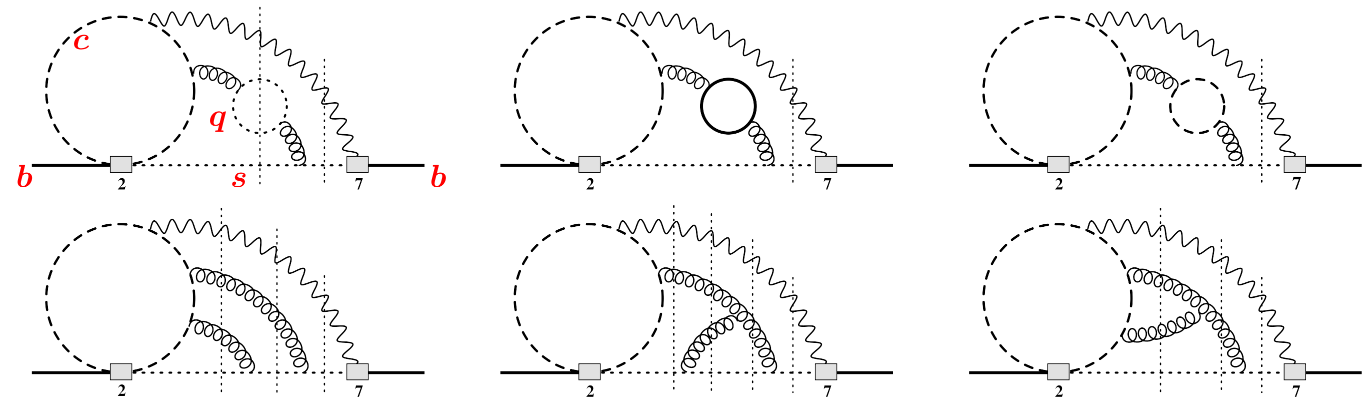
$G^{(2)}_{78}$: H. M. Asatrian, T. Ewerth, A. Ferroglia, C. Greub and G. Ossola, arXiv:1005.5587.

$G^{(2)}_{27}$: Z. Ligeti, M.E. Luke, A.V. Manohar and M.B. Wise, hep-ph/9903305, large-β_0, 4-body,
K. Bieri, C. Greub and M. Steinhauser, hep-ph/0302051, large-β_0, 2-body,
MM and M. Steinhauser, hep-ph/0609241, arXiv:1005.1173, $m_c \gg m_b$,
R. Boughezal, M. Czakon and T. Schutzmeier, arXiv:0707.3090, large-β_0 and massive loops, 2-body,
M. Czakon, P. Fiedler, T. Huber, MM, T. Schutzmeier and M. Steinhauser, arXiv:1503.01791, $m_c = 0$,
MM, A. Rehman and M. Steinhauser, arXiv:1702.07674, arXiv:2002.01548, counterterms, large-β_0 and massive loops,
C. Greub, H. M. Asatrian, F. Saturnino and C. Wiegand, arXiv:2303.01714, physical m_c, partial 2-body,
M. Fael, F. Lange, K. Schönwald and M. Steinhauser, arXiv:2309.14706, physical m_c, full 2-body,
M. Czaja, M. Czakon, T. Huber, MM, M. Niggetiedt, A. Rehman,
K. Schönwald and M. Steinhauser, arXiv:2309.14707, physical m_c, full 2-body,
C. Greub, H. M. Asatrian, H. H. Asatryan, L. Born and J. Eicher, arXiv:2407.17270, physical m_c, full 2-body,
M. Czaja, M. Czakon, T. Huber, MM, M. Niggetiedt, A. Rehman,
K. Schönwald and M. Steinhauser, arXiv:2510.nnnnn, physical m_c, fully inclusive, renormalized result.

Sample propagator diagrams with cuts contributing to G_{27} @ NNLO:

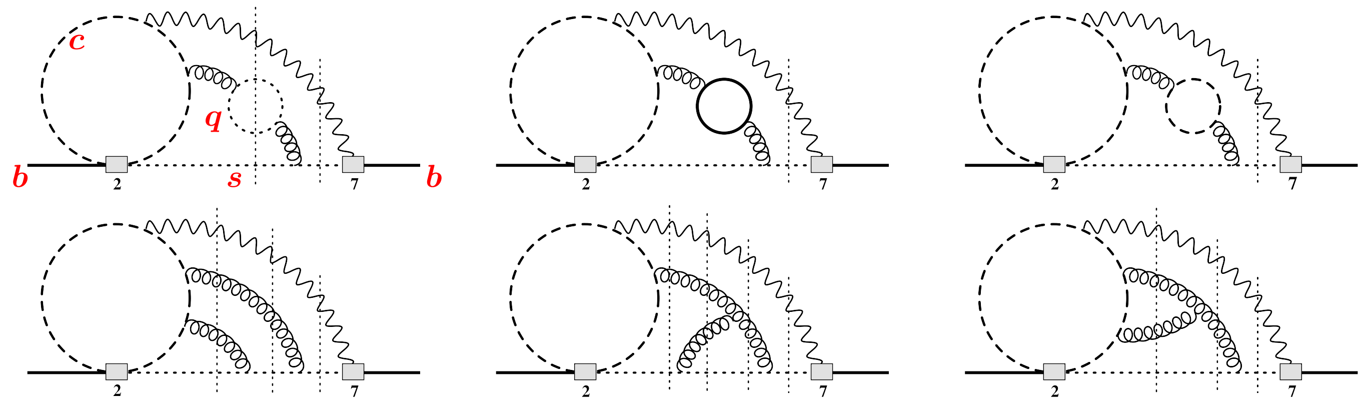


Sample propagator diagrams with cuts contributing to G_{27} @ NNLO:



1. **Generation of diagrams and performing the Dirac algebra to express everything in terms of (a few) $\times 10^5$ four-loop two-scale scalar integrals with unitarity cuts ($\mathcal{O}(500)$ families).**

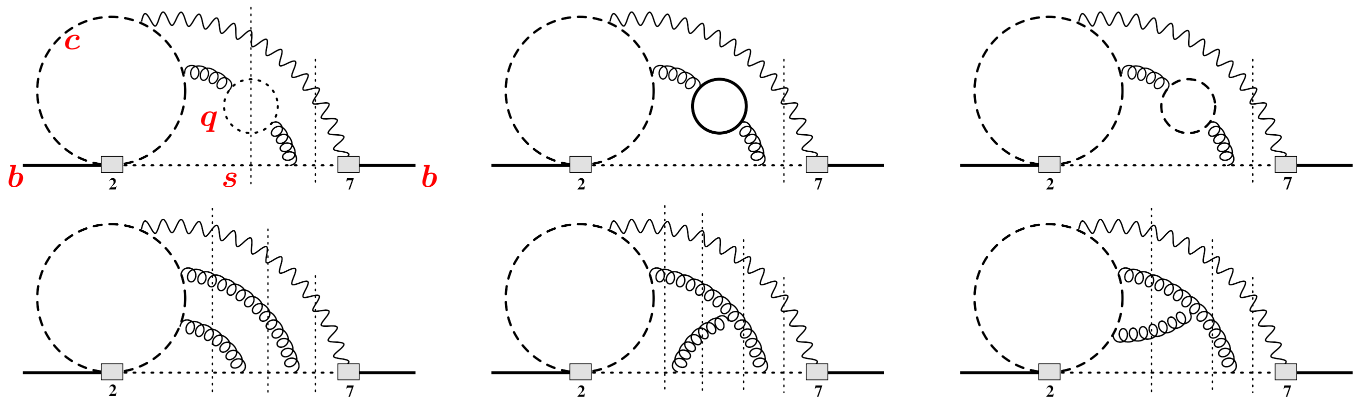
Sample propagator diagrams with cuts contributing to G_{27} @ NNLO:



1. Generation of diagrams and performing the Dirac algebra to express everything in terms of (a few) $\times 10^5$ four-loop two-scale scalar integrals with unitarity cuts ($\mathcal{O}(500)$ families).

2. Reduction to master integrals (MIs) with the help of Integration By Parts (IBP) [KIRA]. $\mathcal{O}(1\,\mathrm{TB})$ RAM and weeks of CPU needed for the most complicated families.

Sample propagator diagrams with cuts contributing to G_{27} @ NNLO:



1. Generation of diagrams and performing the Dirac algebra to express everything in terms of (a few) $\times 10^5$ four-loop two-scale scalar integrals with unitarity cuts ($\mathcal{O}(500)$ families).

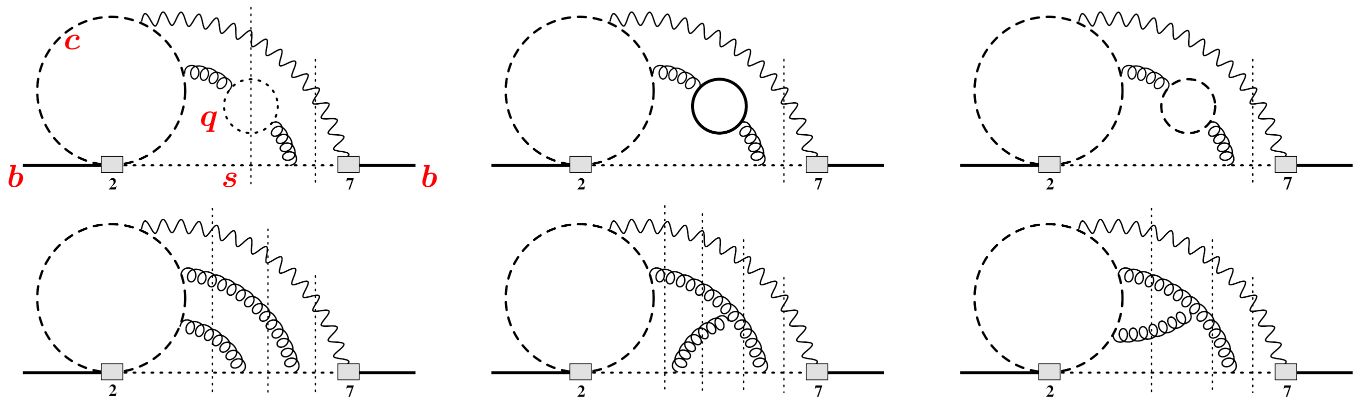
2. Reduction to master integrals (MIs) with the help of Integration By Parts (IBP) [`KIRA`]. $\mathcal{O}(1\,\mathrm{TB})$ RAM and weeks of CPU needed for the most complicated families.

3. Extending the set of master integrals M_k so that it closes under differentiation with respect to $z = m_c^2/m_b^2$. This way one obtains a system of differential equations

$$\frac{d}{dz} M_k(z,\epsilon) = \sum_l R_{kl}(z,\epsilon)\, M_l(z,\epsilon), \qquad (*)$$

where R_{kl} are rational functions of their arguments.

Sample propagator diagrams with cuts contributing to G_{27} @ NNLO:



1. Generation of diagrams and performing the Dirac algebra to express everything in terms of (a few) $\times 10^5$ four-loop two-scale scalar integrals with unitarity cuts ($\mathcal{O}(500)$ families).

2. Reduction to master integrals (MIs) with the help of Integration By Parts (IBP) [KIRA]. $\mathcal{O}(1\,\mathrm{TB})$ RAM and weeks of CPU needed for the most complicated families.

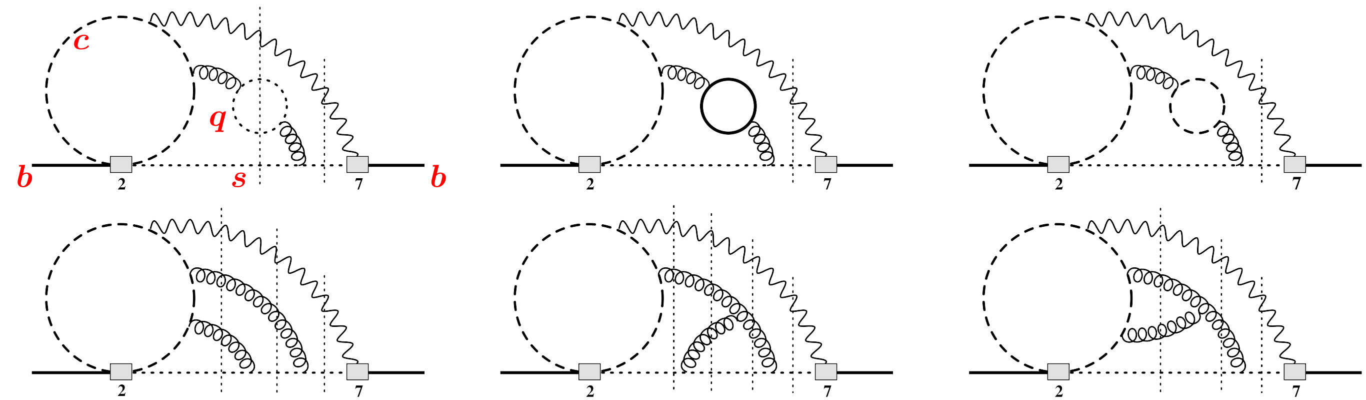
3. Extending the set of master integrals M_k so that it closes under differentiation with respect to $z = m_c^2/m_b^2$. This way one obtains a system of differential equations

$$\frac{d}{dz} M_k(z,\epsilon) = \sum_l R_{kl}(z,\epsilon)\, M_l(z,\epsilon), \qquad (*)$$

where R_{kl} are rational functions of their arguments.

4. Calculating boundary conditions for $(*)$ using automatized asymptotic expansions at $m_c \gg m_b$.

Sample propagator diagrams with cuts contributing to G_{27} @ NNLO:



1. Generation of diagrams and performing the Dirac algebra to express everything in terms of (a few) $\times 10^5$ four-loop two-scale scalar integrals with unitarity cuts ($\mathcal{O}(500)$ families).

2. Reduction to master integrals (MIs) with the help of Integration By Parts (IBP) [KIRA]. $\mathcal{O}(1\,\mathrm{TB})$ RAM and weeks of CPU needed for the most complicated families.

3. Extending the set of master integrals M_k so that it closes under differentiation with respect to $z = m_c^2/m_b^2$. This way one obtains a system of differential equations

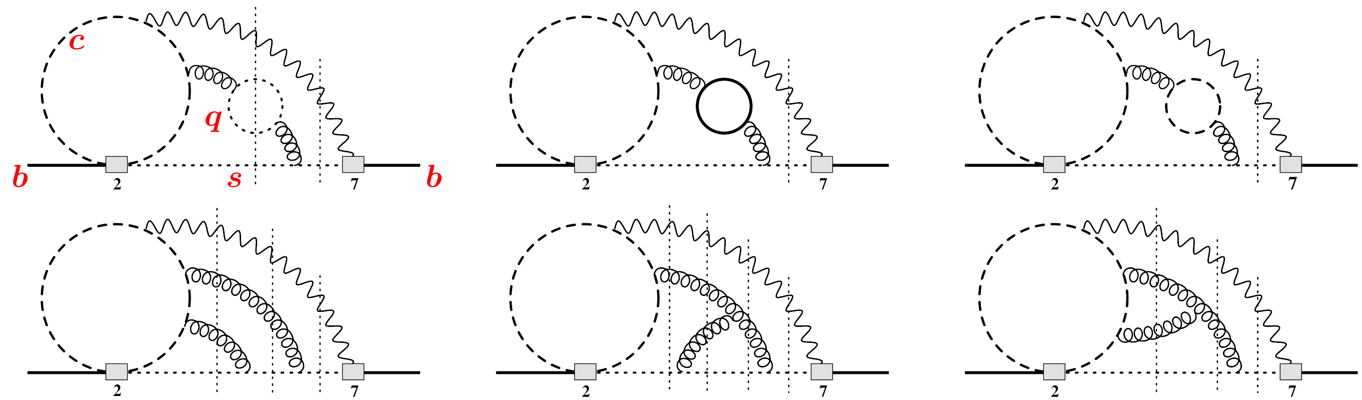
$$\frac{d}{dz} M_k(z,\epsilon) = \sum_l R_{kl}(z,\epsilon)\, M_l(z,\epsilon), \qquad (*)$$

where R_{kl} are rational functions of their arguments.

4. Calculating boundary conditions for $(*)$ using automatized asymptotic expansions at $m_c \gg m_b$.

5. Calculating three-loop single-scale master integrals for the boundary conditions.

Sample propagator diagrams with cuts contributing to G_{27} @ NNLO:



1. Generation of diagrams and performing the Dirac algebra to express everything in terms of (a few) $\times 10^5$ four-loop two-scale scalar integrals with unitarity cuts ($\mathcal{O}(500)$ families).

2. Reduction to master integrals (MIs) with the help of Integration By Parts (IBP) [KIRA]. $\mathcal{O}(1\,\mathrm{TB})$ RAM and weeks of CPU needed for the most complicated families.

3. Extending the set of master integrals M_k so that it closes under differentiation with respect to $z = m_c^2/m_b^2$. This way one obtains a system of differential equations

$$\frac{d}{dz} M_k(z,\epsilon) = \sum_l R_{kl}(z,\epsilon)\, M_l(z,\epsilon), \qquad (*)$$

where R_{kl} are rational functions of their arguments.

4. Calculating boundary conditions for $(*)$ using automatized asymptotic expansions at $m_c \gg m_b$.

5. Calculating three-loop single-scale master integrals for the boundary conditions.

6. Solving the system $(*)$ numerically [e.g., A.C. Hindmarsch, http://www.netlib.org/odepack] along an ellipse in the complex z plane.

Fully inclusive (2-, 3- and 4-body), renormalized results for $G_{17}^{(2)}$ and $G_{27}^{(2)}$.

M. Czaja, M. Czakon, T. Huber, MM, M. Niggetiedt, A. Rehman, K. Schönwald and M. Steinhauser, arXiv:2510.nnnnn.

Fully inclusive (2-, 3- and 4-body), renormalized results for $G_{17}^{(2)}$ and $G_{27}^{(2)}$.

M. Czaja, M. Czakon, T. Huber, MM, M. Niggetiedt, A. Rehman, K. Schönwald and M. Steinhauser, arXiv:2510.nnnnn.

All the $\frac{1}{\epsilon^n}$ poles have cancelled with better than 10^{-55} accuracy.

Fully inclusive (2-, 3- and 4-body), renormalized results for $G_{17}^{(2)}$ and $G_{27}^{(2)}$.

M. Czaja, M. Czakon, T. Huber, MM, M. Niggetiedt, A. Rehman, K. Schönwald and M. Steinhauser, arXiv:2510.nnnnn.

All the $\frac{1}{\epsilon^n}$ poles have cancelled with better than 10^{-55} accuracy.

Comparison to the interpolated NNLO correction in arXiv:1503.01791.

$$\frac{\Delta \mathcal{B}_{s\gamma}}{\mathcal{B}_{s\gamma}} \simeq U(z,\delta) \equiv \frac{\alpha_s^2(\mu_b)}{8\pi^2} \; \frac{C_1^{(0)}(\mu_b) F_1(z,\delta) + \left(C_2^{(0)}(\mu_b) - \frac{1}{6} C_1^{(0)}(\mu_b) \right) F_2(z,\delta)}{C_7^{(0)\mathrm{eff}}(\mu_b)} \qquad \left(z = \frac{m_c^2}{m_b^2} \right)$$

Fully inclusive (2-, 3- and 4-body), renormalized results for $G_{17}^{(2)}$ and $G_{27}^{(2)}$.

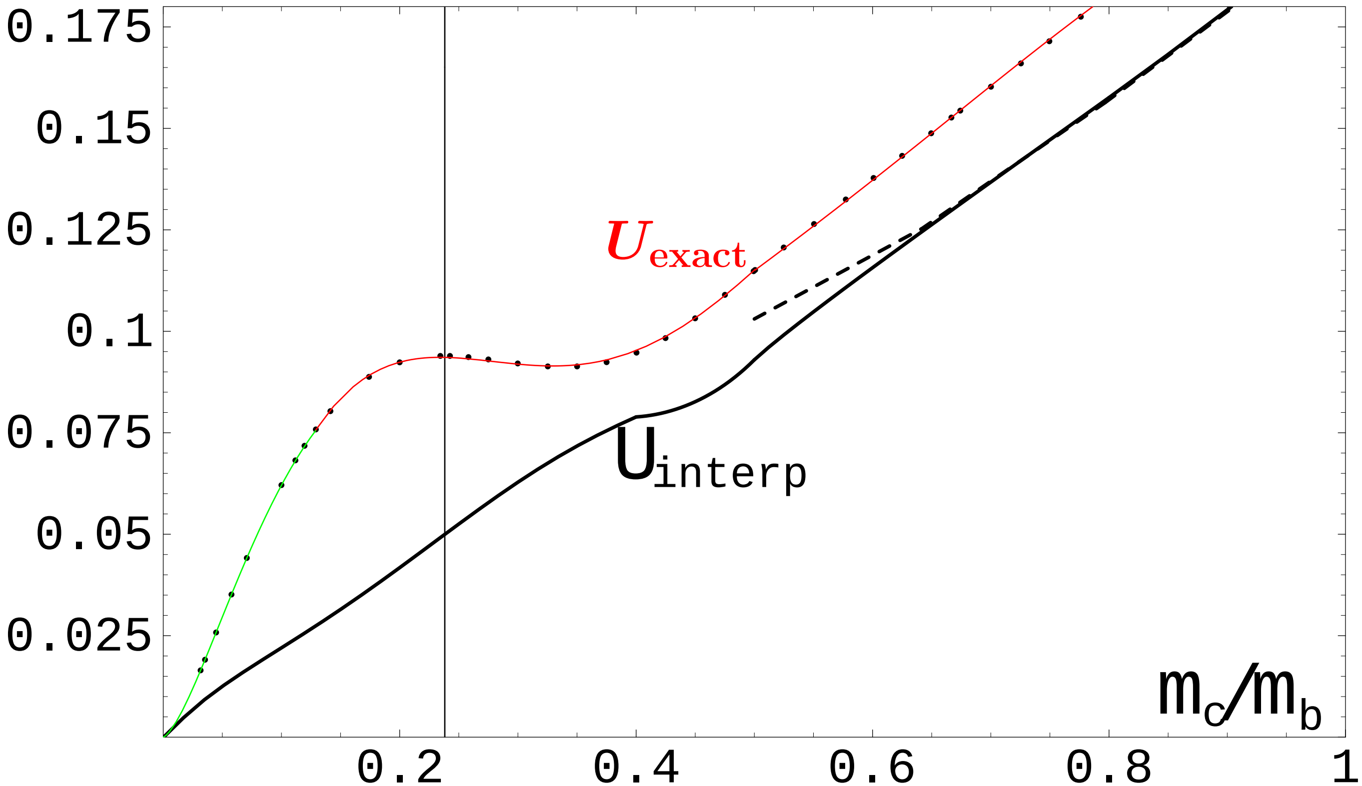
M. Czaja, M. Czakon, T. Huber, MM, M. Niggetiedt, A. Rehman, K. Schönwald and M. Steinhauser, arXiv:2510.nnnnn.

All the $\frac{1}{\epsilon^n}$ poles have cancelled with better than 10^{-55} accuracy.

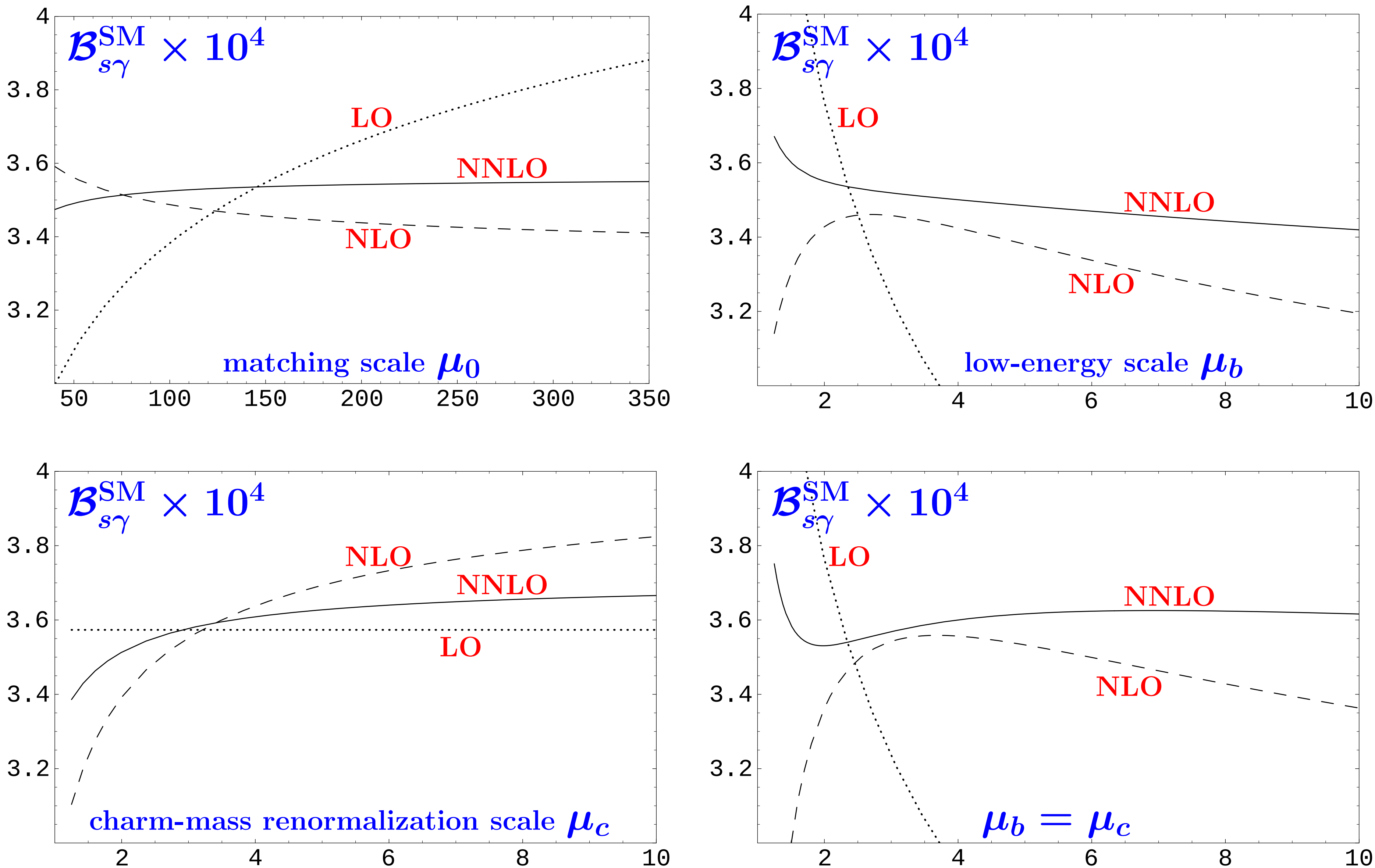
Comparison to the interpolated NNLO correction in arXiv:1503.01791.

$$\frac{\Delta \mathcal{B}_{s\gamma}}{\mathcal{B}_{s\gamma}} \simeq U(z,\delta) \equiv \frac{\alpha_s^2(\mu_b)}{8\pi^2} \frac{C_1^{(0)}(\mu_b) F_1(z,\delta) + \left(C_2^{(0)}(\mu_b) - \frac{1}{6} C_1^{(0)}(\mu_b) \right) F_2(z,\delta)}{C_7^{(0)\text{eff}}(\mu_b)} \qquad \left(z = \frac{m_c^2}{m_b^2} \right)$$

Interpolated and exact results for $\delta = 1$ (no cut on E_γ):



Renormalization scale dependence of $\mathcal{B}_{s\gamma}^{\rm SM}$ for $E_\gamma > 1.6$ GeV



"Central" values: $\mu_0 = 160$ GeV, $\mu_b = \mu_c = \frac{1}{2} m_{b,kin}(1\,\text{GeV}) \simeq 2.29\,\text{GeV}$.

Updated SM prediction:

$$\mathcal{B}_{s\gamma}^{\mathrm{SM}} = (3.54 \pm 0.14) \times 10^{-4}$$

Updated SM prediction:

$$\mathcal{B}_{s\gamma}^{\rm SM} = (3.54 \pm 0.14) \times 10^{-4}$$

No interpolation in m_c.

Updated SM prediction:

$$\mathcal{B}_{s\gamma}^{\rm SM} = (3.54 \pm 0.14) \times 10^{-4}$$

No interpolation in m_c. Non-perturbative uncertainty treatment as in arXiv:2002.01548.

Updated SM prediction:

$$\mathcal{B}_{s\gamma}^{\rm SM} = (3.54 \pm 0.14) \times 10^{-4} \quad \left[\pm 4.0\% \simeq \pm\sqrt{(3.0\%)^2_{\rm h.o.} + (2.7\%)^2_{\rm param}} \,\right].$$

No interpolation in m_c. Non-perturbative uncertainty treatment as in arXiv:2002.01548.

Updated SM prediction:

$$\mathcal{B}_{s\gamma}^{\rm SM} = (3.54 \pm 0.14) \times 10^{-4} \quad \left[\pm 4.0\% \simeq \pm\sqrt{(3.0\%)^2_{\rm h.o.} + (2.7\%)^2_{\rm param}}\,\right].$$

No interpolation in m_c. Non-perturbative uncertainty treatment as in arXiv:2002.01548.

The perturbative NLO corrections are now formally complete thanks to including formerly missing 4-body and 5-body contributions from arXiv:2510.nnnnn [K. M. Brune, T. Huber, L-T. Moos].

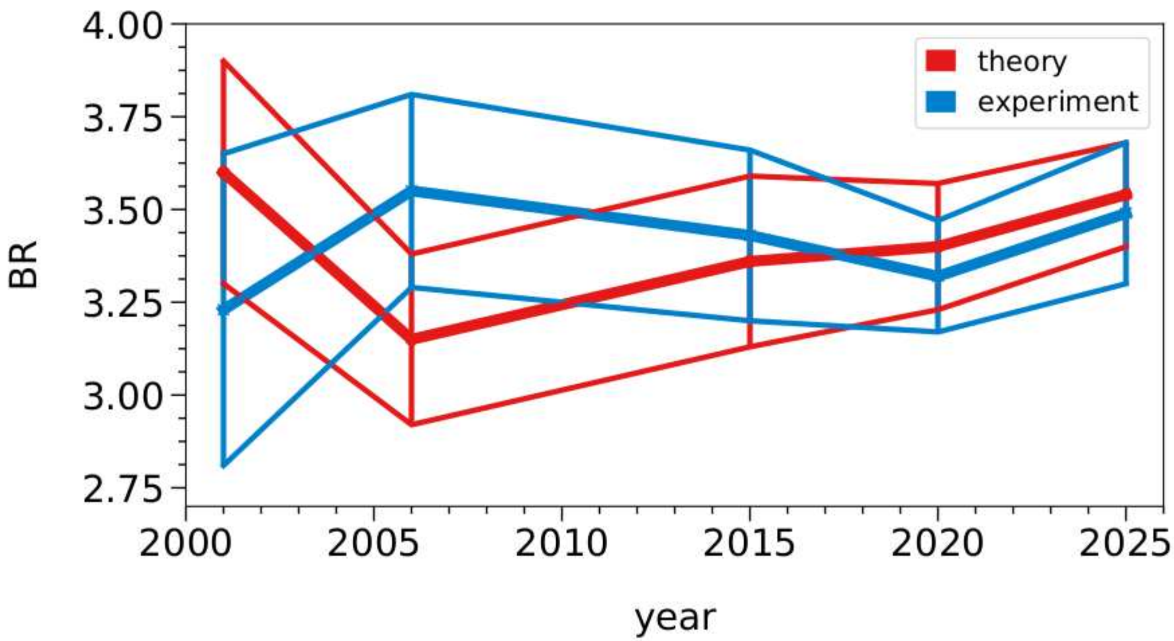
Updated SM prediction:

$$\mathcal{B}_{s\gamma}^{\rm SM} = (3.54 \pm 0.14) \times 10^{-4} \quad \left[\pm 4.0\% \simeq \pm\sqrt{(3.0\%)^2_{\rm h.o.} + (2.7\%)^2_{\rm param}}\, \right].$$

No interpolation in m_c. Non-perturbative uncertainty treatment as in arXiv:2002.01548.

The perturbative NLO corrections are now formally complete thanks to including formerly missing 4-body and 5-body contributions from arXiv:2510.nnnnn [K. M. Brune, T. Huber, L-T. Moos].

Time evolution of selected SM predictions for $\mathcal{B}_{s\gamma}$ and experimental averages:



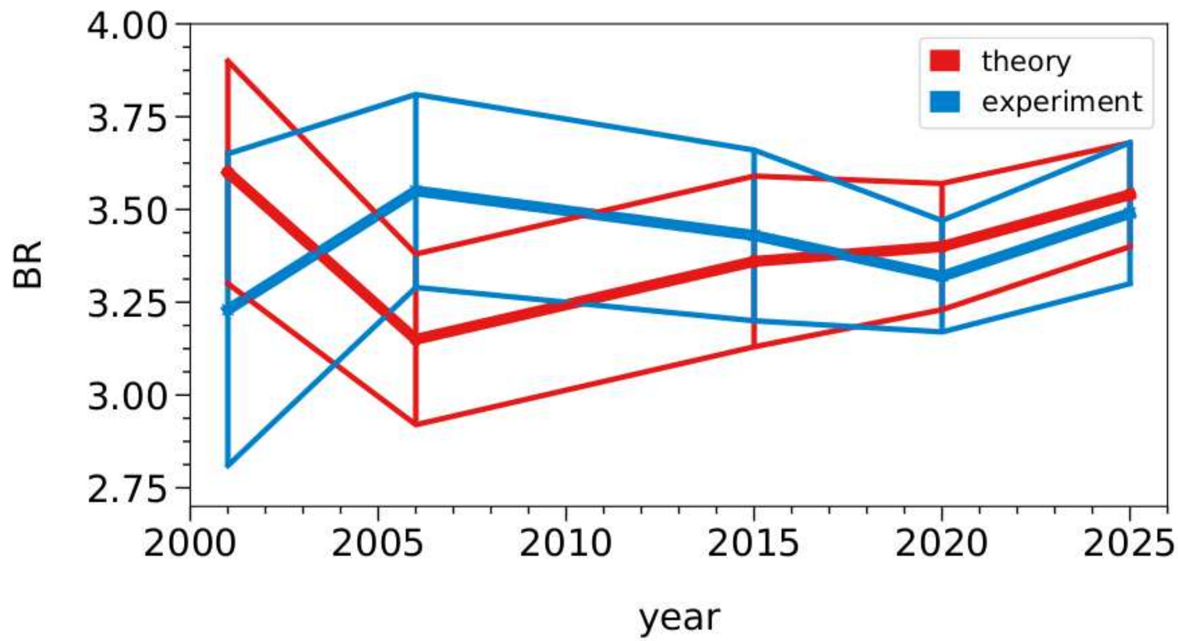
Updated SM prediction:

$$\mathcal{B}_{s\gamma}^{\rm SM} = (3.54 \pm 0.14) \times 10^{-4} \quad \left[\pm 4.0\% \simeq \pm\sqrt{(3.0\%)^2_{\rm h.o.} + (2.7\%)^2_{\rm param}}\ \right].$$

No interpolation in m_c. Non-perturbative uncertainty treatment as in arXiv:2002.01548.

The perturbative NLO corrections are now formally complete thanks to including formerly missing 4-body and 5-body contributions from arXiv:2510.nnnnn [K. M. Brune, T. Huber, L-T. Moos].

Time evolution of selected SM predictions for $\mathcal{B}_{s\gamma}$ and experimental averages:



Improvements in G_{27}

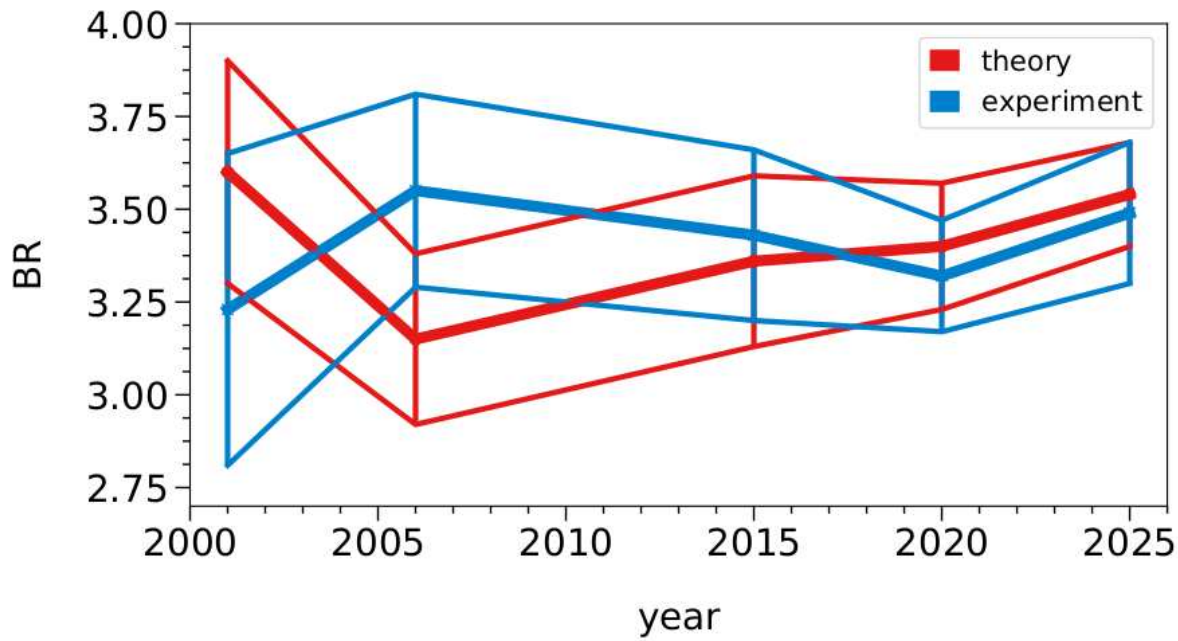
Updated SM prediction:

$$\mathcal{B}_{s\gamma}^{\rm SM} = (3.54 \pm 0.14) \times 10^{-4} \qquad \left[\pm 4.0\% \simeq \pm\sqrt{(3.0\%)^2_{\rm h.o.} + (2.7\%)^2_{\rm param}}\,\right].$$

No interpolation in m_c. Non-perturbative uncertainty treatment as in arXiv:2002.01548.

The perturbative NLO corrections are now formally complete thanks to including formerly missing 4-body and 5-body contributions from arXiv:2510.nnnnn [K. M. Brune, T. Huber, L-T. Moos].

Time evolution of selected SM predictions for $\mathcal{B}_{s\gamma}$ and experimental averages:



2000: NLO only,

Improvements in G_{27}

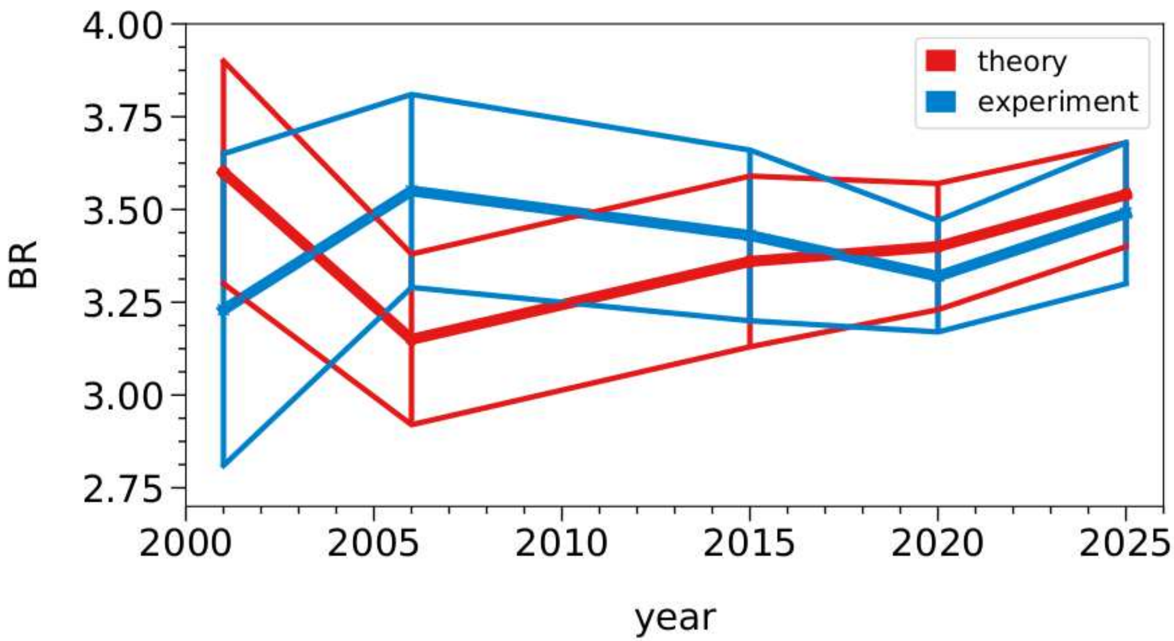
Updated SM prediction:

$$\mathcal{B}_{s\gamma}^{\rm SM} = (3.54 \pm 0.14) \times 10^{-4} \qquad \left[\pm 4.0\% \simeq \pm\sqrt{(3.0\%)^2_{\rm h.o.} + (2.7\%)^2_{\rm param}}\, \right].$$

No interpolation in m_c. Non-perturbative uncertainty treatment as in arXiv:2002.01548.

The perturbative NLO corrections are now formally complete thanks to including formerly missing 4-body and 5-body contributions from arXiv:2510.nnnnn [K. M. Brune, T. Huber, L-T. Moos].

Time evolution of selected SM predictions for $\mathcal{B}_{s\gamma}$ and experimental averages:



2000: NLO only,

2006: extrapolation of NNLO from the $m_c \gg m_b$ limit,

Improvements in G_{27}

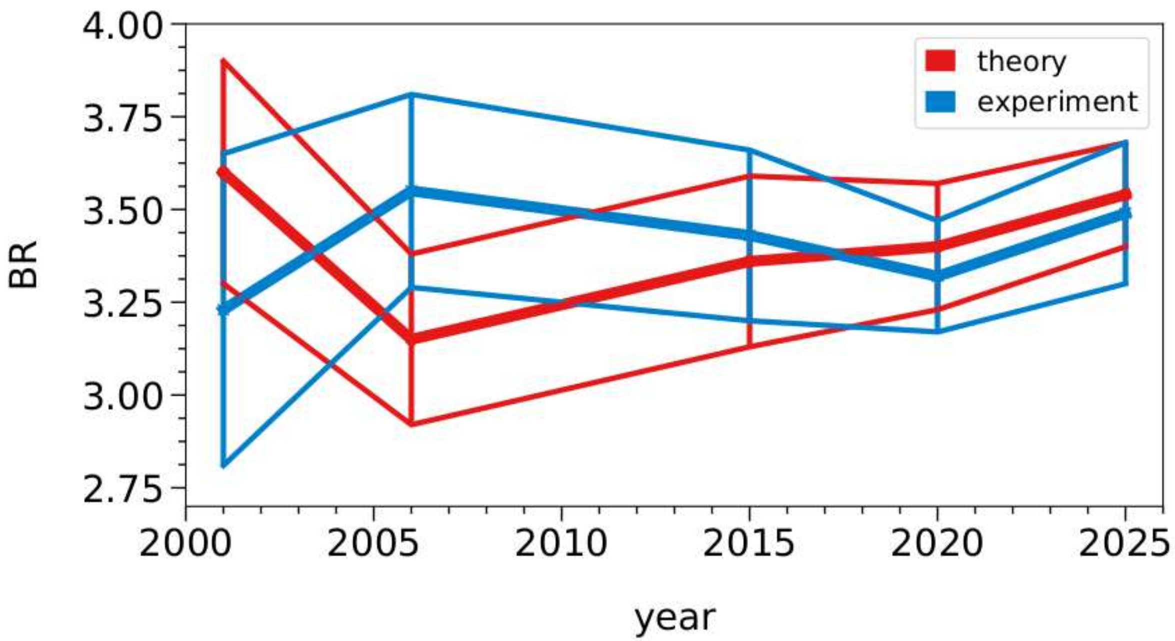
Updated SM prediction:

$$\mathcal{B}_{s\gamma}^{\rm SM} = (3.54 \pm 0.14) \times 10^{-4} \quad \left[\pm 4.0\% \simeq \pm\sqrt{(3.0\%)^2_{\rm h.o.} + (2.7\%)^2_{\rm param}}\,\right].$$

No interpolation in m_c. Non-perturbative uncertainty treatment as in arXiv:2002.01548.

The perturbative NLO corrections are now formally complete thanks to including formerly missing 4-body and 5-body contributions from arXiv:2510.nnnnn [K. M. Brune, T. Huber, L-T. Moos].

Time evolution of selected SM predictions for $\mathcal{B}_{s\gamma}$ and experimental averages:



Improvements in G_{27}

2000: NLO only,

2006: extrapolation of NNLO from the $m_c \gg m_b$ limit,

2015: interpolation of NNLO between the $m_c = 0$ and $m_c \gg m_b$ limits,

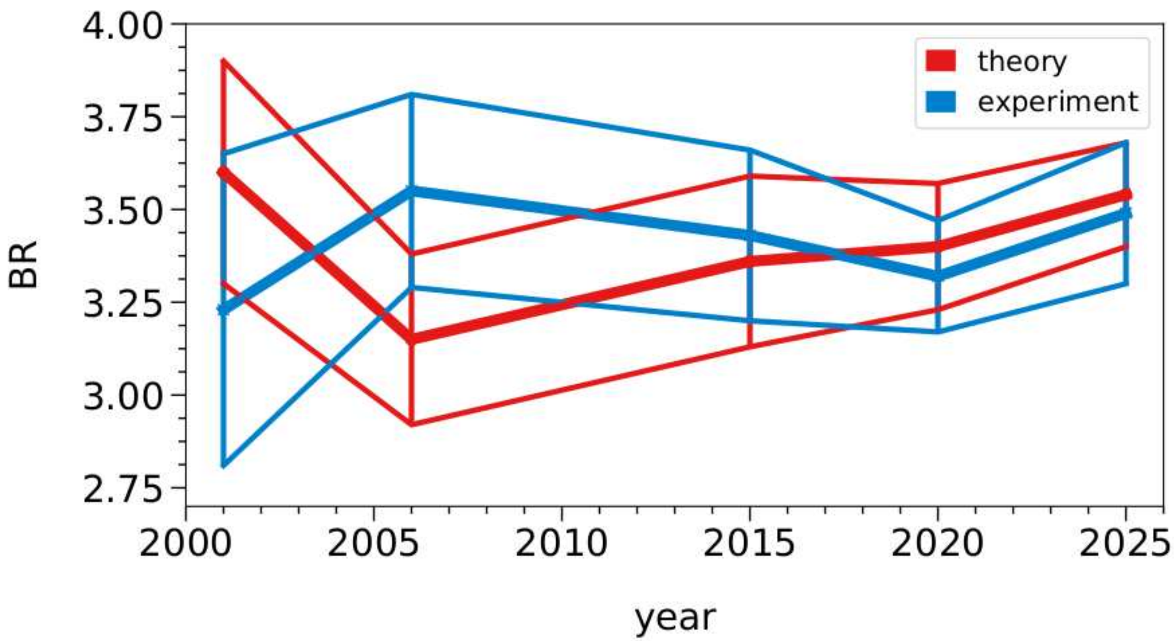
Updated SM prediction:

$$\mathcal{B}_{s\gamma}^{\rm SM} = (3.54 \pm 0.14) \times 10^{-4} \qquad \left[\pm 4.0\% \simeq \pm\sqrt{(3.0\%)^2_{\rm h.o.} + (2.7\%)^2_{\rm param}} \,\right].$$

No interpolation in m_c. Non-perturbative uncertainty treatment as in arXiv:2002.01548.

The perturbative NLO corrections are now formally complete thanks to including formerly missing 4-body and 5-body contributions from arXiv:2510.nnnnn [K. M. Brune, T. Huber, L-T. Moos].

Time evolution of selected SM predictions for $\mathcal{B}_{s\gamma}$ and experimental averages:



Improvements in G_{27}

2000: NLO only,
2006: extrapolation of NNLO from the $m_c \gg m_b$ limit,
2015: interpolation of NNLO between the $m_c = 0$ and $m_c \gg m_b$ limits,
2025: complete NNLO.

$\mathcal{B}_{s\gamma}$ in the Two-Higgs-Doublet Model II

$\mathcal{B}_{s\gamma}$ in the Two-Higgs-Doublet Model II

$$\mathcal{L}_{\text{Yukawa}} = -\bar{q}_i Y^u_{ij} u_j H_2 - \bar{q}_i Y^d_{ij} d_j H_1 - \bar{l}_i Y^e_{ij} e_j H_1 + \text{h.c.},$$

$\mathcal{B}_{s\gamma}$ in the Two-Higgs-Doublet Model II

$$\mathcal{L}_{\text{Yukawa}} = -\bar{q}_i Y^u_{ij} u_j H_2 - \bar{q}_i Y^d_{ij} d_j H_1 - \bar{l}_i Y^e_{ij} e_j H_1 + \text{h.c.}, \qquad \langle H_2 \rangle = \frac{v \sin\beta}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \qquad \langle H_1 \rangle = \frac{v \cos\beta}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

$\mathcal{B}_{s\gamma}$ in the Two-Higgs-Doublet Model II

$$\mathcal{L}_{\text{Yukawa}} = -\bar{q}_i Y^u_{ij} u_j H_2 - \bar{q}_i Y^d_{ij} d_j H_1 - \bar{l}_i Y^e_{ij} e_j H_1 + \text{h.c.}, \qquad \langle H_2 \rangle = \frac{v\sin\beta}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \qquad \langle H_1 \rangle = \frac{v\cos\beta}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

$$\mathcal{L}_{H^\pm \bar{q} q} = \frac{g_{\text{ew}}}{M_W\sqrt{2}} \left[H^+ \bar{u}_i \left(m_{u_i} V_{ij} P_L \cot\beta + V_{ij} m_{d_j} P_R \tan\beta \right) d_j + \text{h.c.} \right].$$

$\mathcal{B}_{s\gamma}$ in the Two-Higgs-Doublet Model II

$$\mathcal{L}_{\text{Yukawa}} = -\bar{q}_i Y^u_{ij} u_j H_2 - \bar{q}_i Y^d_{ij} d_j H_1 - \bar{l}_i Y^e_{ij} e_j H_1 + \text{h.c.}, \qquad \langle H_2 \rangle = \frac{v \sin\beta}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \qquad \langle H_1 \rangle = \frac{v \cos\beta}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

$$\mathcal{L}_{H^\pm \bar{q} q} = \frac{g_{\text{ew}}}{M_W \sqrt{2}} \left[H^+ \bar{u}_i \left(m_{u_i} V_{ij} P_L \cot\beta + V_{ij} m_{d_j} P_R \tan\beta \right) d_j + \text{h.c.} \right].$$

$$\delta C_7(\mu_0) \quad = \quad F_1\left(\frac{m_t^2}{M^2_{H^\pm}}\right) \; + \; F_2\left(\frac{m_t^2}{M^2_{H^\pm}}\right) \cot^2\beta$$

$\mathcal{B}_{s\gamma}$ in the Two-Higgs-Doublet Model II

$\mathcal{L}_{\rm Yukawa} = -\bar{q}_i Y^u_{ij} u_j H_2 - \bar{q}_i Y^d_{ij} d_j H_1 - \bar{l}_i Y^e_{ij} e_j H_1 + \text{h.c.},\qquad \langle H_2 \rangle = \frac{v \sin\beta}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \qquad \langle H_1 \rangle = \frac{v \cos\beta}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$

$\mathcal{L}_{H^\pm \bar{q} q} = \frac{g_{\rm ew}}{M_W \sqrt{2}} \left[H^+ \bar{u}_i \left(m_{u_i} V_{ij} P_L \cot\beta + V_{ij} m_{d_j} P_R \tan\beta \right) d_j + \text{h.c.} \right].$

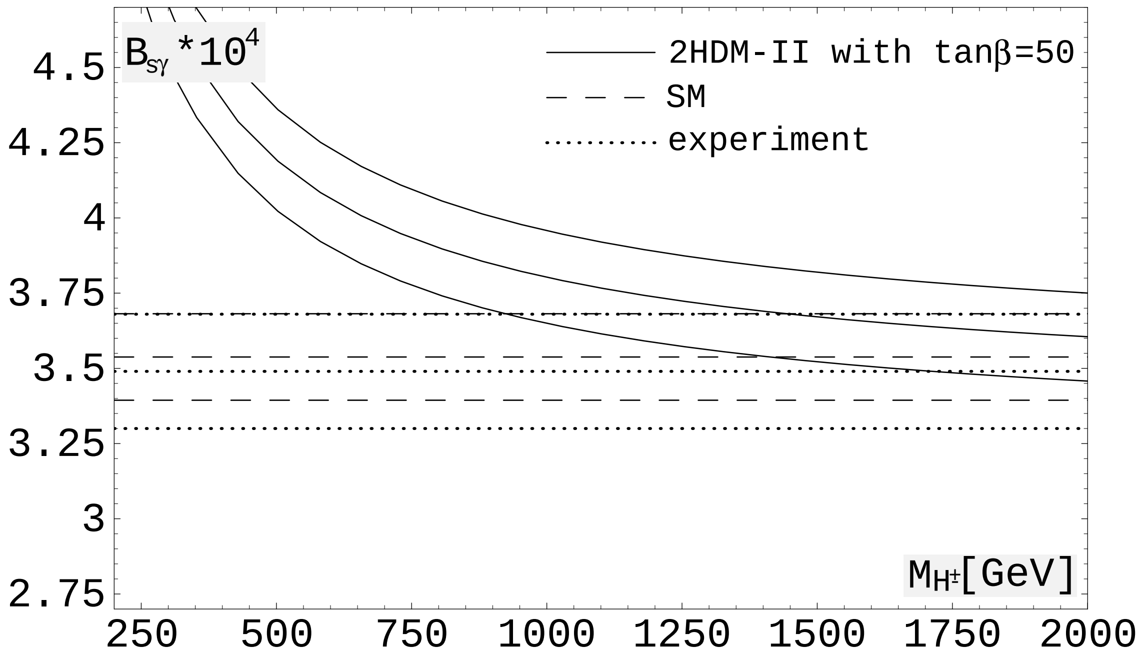
$$\delta C_7(\mu_0) = \underbrace{F_1\left(\frac{m_t^2}{M_{H^\pm}^2}\right)}_{\mathcal{B}_{s\gamma}\,\uparrow} + \underbrace{F_2\left(\frac{m_t^2}{M_{H^\pm}^2}\right)\cot^2\beta}_{\mathcal{B}_{s\gamma}\,\uparrow}$$

$\mathcal{B}_{s\gamma}$ in the Two-Higgs-Doublet Model II

$$\mathcal{L}_{\rm Yukawa} = -\bar{q}_i Y^u_{ij} u_j H_2 - \bar{q}_i Y^d_{ij} d_j H_1 - \bar{l}_i Y^e_{ij} e_j H_1 + \text{h.c.}, \qquad \langle H_2 \rangle = \frac{v \sin\beta}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \qquad \langle H_1 \rangle = \frac{v \cos\beta}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

$$\mathcal{L}_{H^\pm \bar{q} q} = \frac{g_{\rm ew}}{M_W \sqrt{2}} \left[H^+ \bar{u}_i \left(m_{u_i} V_{ij} P_L \cot\beta + V_{ij} m_{d_j} P_R \tan\beta \right) d_j + \text{h.c.} \right].$$

$$\delta C_7(\mu_0) = \underbrace{F_1\left(\frac{m_t^2}{M_{H^\pm}^2}\right)}_{\mathcal{B}_{s\gamma}\uparrow} + \underbrace{F_2\left(\frac{m_t^2}{M_{H^\pm}^2}\right) \cot^2\beta}_{\mathcal{B}_{s\gamma}\uparrow}$$

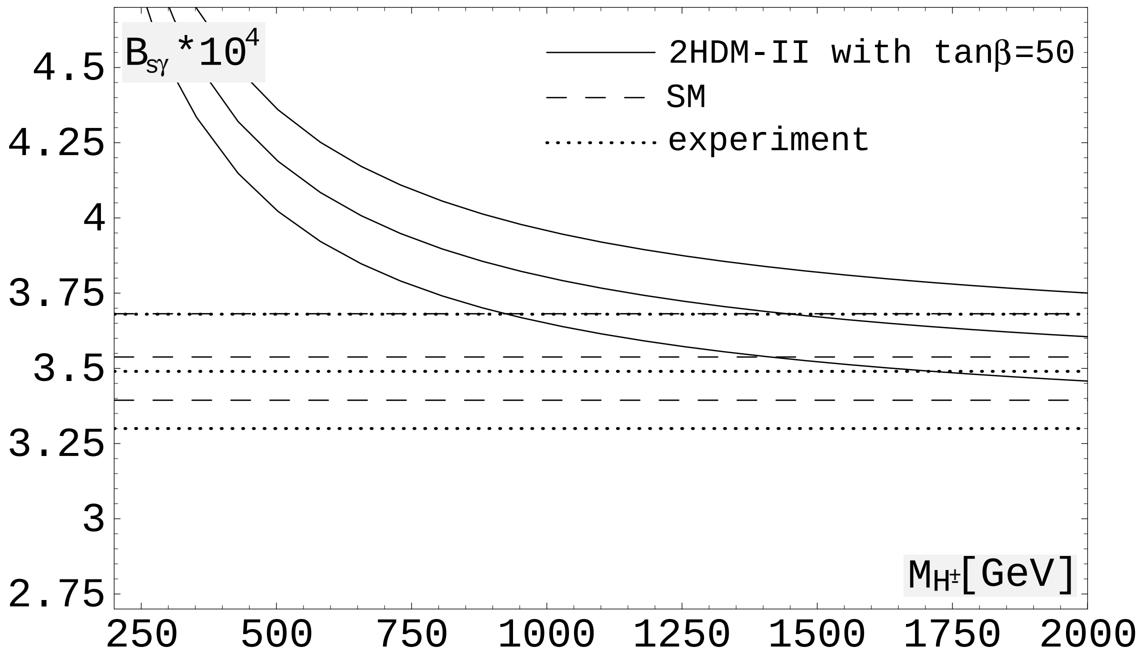


$\mathcal{B}_{s\gamma}$ in the Two-Higgs-Doublet Model II

$$\mathcal{L}_{\text{Yukawa}} = -\bar{q}_i Y^u_{ij} u_j H_2 - \bar{q}_i Y^d_{ij} d_j H_1 - \bar{l}_i Y^e_{ij} e_j H_1 + \text{h.c.}, \qquad \langle H_2 \rangle = \frac{v \sin\beta}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \qquad \langle H_1 \rangle = \frac{v \cos\beta}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

$$\mathcal{L}_{H^\pm \bar{q} q} = \frac{g_{\text{ew}}}{M_W \sqrt{2}} \left[H^+ \bar{u}_i \left(m_{u_i} V_{ij} P_L \cot\beta + V_{ij} m_{d_j} P_R \tan\beta \right) d_j + \text{h.c.} \right].$$

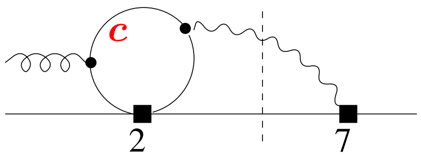
$$\delta C_7(\mu_0) \quad = \quad \underbrace{F_1\left(\frac{m_t^2}{M_{H^\pm}^2}\right)}_{\mathcal{B}_{s\gamma} \uparrow} \; + \; \underbrace{F_2\left(\frac{m_t^2}{M_{H^\pm}^2}\right) \cot^2\beta}_{\mathcal{B}_{s\gamma} \uparrow}$$



$$M_{H^\pm} > 675\,\text{GeV} \quad @\ 95\%\text{C.L.}$$

Resolved photon contribution to the Q_7-$Q_{1,2}$ interference.

M.B. Voloshin, hep-ph/9612483; A. Khodjamirian, R. Rückl, G. Stoll and D. Wyler, hep-ph/9702318; Z. Ligeti, L. Randall and M.B. Wise, hep-ph/9702322; G. Buchalla, G. Isidori, G. Rey, hep-ph/9705253; M. Benzke, S.J. Lee, M. Neubert, G. Paz, arXiv:1003.5012; A. Gunawardana, G. Paz, arXiv:1908.02812; M. Benzke, T. Hurth, arXiv:2006.00624; R. Bartocci, P. Böer, T. Hurth, arXiv:2411.16634.

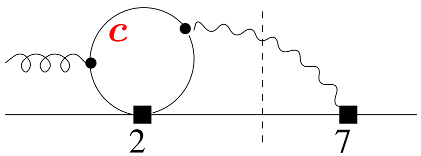
$\langle \bar{B} |$  $| \bar{B} \rangle$

$$\delta \mathrm{N}(\mathrm{E}_0) = (C_2 - \tfrac{1}{6} C_1) C_7 \underbrace{\left[-\frac{\mu_{\mathrm{G}}^2}{27 \mathrm{m}_{\mathrm{c}}^2} + \frac{\Lambda_{17}}{\mathrm{m}_{\mathrm{b}}} \right]}_{-\frac{\kappa_{\mathrm{V}} \mu_G^2}{27 m_c^2}}$$

Resolved photon contribution to the Q_7-$Q_{1,2}$ interference.

M.B. Voloshin, hep-ph/9612483; A. Khodjamirian, R. Rückl, G. Stoll and D. Wyler, hep-ph/9702318;
Z. Ligeti, L. Randall and M.B. Wise, hep-ph/9702322; G. Buchalla, G. Isidori, G. Rey, hep-ph/9705253;
M. Benzke, S.J. Lee, M. Neubert, G. Paz, arXiv:1003.5012; A. Gunawardana, G. Paz, arXiv:1908.02812;
M. Benzke, T. Hurth, arXiv:2006.00624; R. Bartocci, P. Böer, T. Hurth, arXiv:2411.16634.

$\langle \bar{B}|$



$|\bar{B}\rangle$

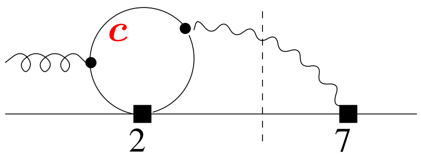
$$\delta \mathrm{N}(\mathrm{E}_0) = \left(C_2 - \tfrac{1}{6} C_1\right) C_7 \underbrace{\left[-\frac{\mu_G^2}{27 m_c^2} + \frac{\Lambda_{17}}{m_b} \right]}_{-\frac{\kappa_V \mu_G^2}{27 m_c^2}}$$

$$\Lambda_{17} = \tfrac{2}{3} \mathrm{Re} \int_{-\infty}^{\infty} \frac{d\omega_1}{\omega_1} \left[1 - F\left(\frac{m_c^2 - i\varepsilon}{m_b \omega_1} \right) + \frac{m_b \omega_1}{12 m_c^2} \right] h_{17}(\omega_1, \mu)$$

$\omega_1 \leftrightarrow$ gluon momentum, $\quad F(x) = 4x \arctan^2\left(1/\sqrt{4x-1}\right)$

Resolved photon contribution to the Q_7-$Q_{1,2}$ interference.

M.B. Voloshin, hep-ph/9612483; A. Khodjamirian, R. Rückl, G. Stoll and D. Wyler, hep-ph/9702318;
Z. Ligeti, L. Randall and M.B. Wise, hep-ph/9702322; G. Buchalla, G. Isidori, G. Rey, hep-ph/9705253;
M. Benzke, S.J. Lee, M. Neubert, G. Paz, arXiv:1003.5012; A. Gunawardana, G. Paz, arXiv:1908.02812;
M. Benzke, T. Hurth, arXiv:2006.00624; R. Bartocci, P. Böer, T. Hurth, arXiv:2411.16634.

$\langle \bar{B}|$  $|\bar{B}\rangle$

$$\delta \mathrm{N}(\mathrm{E}_0) = (C_2 - \tfrac{1}{6}C_1)C_7 \underbrace{\left[-\frac{\mu_{\mathrm{G}}^2}{27\mathrm{m}_{\mathrm{c}}^2} + \frac{\Lambda_{17}}{\mathrm{m}_{\mathrm{b}}} \right]}_{-\frac{\kappa_{\mathrm{V}}\mu_G^2}{27 m_c^2}}$$

$$\Lambda_{17} = \tfrac{2}{3}\mathrm{Re}\int_{-\infty}^{\infty} \frac{d\omega_1}{\omega_1} \left[1 - F\left(\frac{m_c^2 - i\varepsilon}{m_b\omega_1}\right) + \frac{m_b\omega_1}{12 m_c^2}\right] h_{17}(\omega_1, \mu)$$

$\omega_1 \leftrightarrow$ gluon momentum, $\quad F(x) = 4x \arctan^2\left(1/\sqrt{4x-1}\right)$

The soft function h_{17}:

$$h_{17}(\omega_1,\mu) = \int \frac{dr}{4\pi M_B} e^{-i\omega_1 r} \langle \bar{B}|(\bar{h}S_{\bar{n}})(0)\not{\bar{n}}\, i\gamma_\alpha^\perp \bar{n}_\beta (S_{\bar{n}}^\dagger g G_s^{\alpha\beta} S_{\bar{n}})(r\bar{n})(S_{\bar{n}}^\dagger h)(0)|\bar{B}\rangle \qquad (m_b - 2E_0 \gg \Lambda_{\mathrm{QCD}})$$

A class of models for h_{17}:

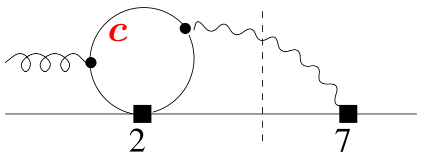
$$h_{17}(\omega_1,\mu) = e^{-\frac{\omega_1^2}{2\sigma^2}} \sum_n a_{2n} H_{2n}\left(\frac{\omega_1}{\sigma\sqrt{2}}\right), \qquad \sigma < 1\ \mathrm{GeV}$$

Hermite polynomials

Constraints on moments (e.g.): $\int d\omega_1 h_{17} = \frac{2}{3}\mu_G^2, \qquad \int d\omega_1 \omega_1^2 h_{17} = \frac{2}{15}(5m_5 + 3m_6 - 2m_9).$

Resolved photon contribution to the Q_7-$Q_{1,2}$ interference.

M.B. Voloshin, hep-ph/9612483; A. Khodjamirian, R. Rückl, G. Stoll and D. Wyler, hep-ph/9702318; Z. Ligeti, L. Randall and M.B. Wise, hep-ph/9702322; G. Buchalla, G. Isidori, G. Rey, hep-ph/9705253; M. Benzke, S.J. Lee, M. Neubert, G. Paz, arXiv:1003.5012; A. Gunawardana, G. Paz, arXiv:1908.02812; M. Benzke, T. Hurth, arXiv:2006.00624; R. Bartocci, P. Böer, T. Hurth, arXiv:2411.16634.

$\langle \bar{B} |$  $| \bar{B} \rangle$

$$\delta \mathrm{N}(\mathrm{E}_0) = \left(C_2 - \tfrac{1}{6} C_1\right) C_7 \underbrace{\left[-\frac{\mu_G^2}{27 m_c^2} + \frac{\Lambda_{17}}{m_b} \right]}_{-\frac{\kappa_V \mu_G^2}{27 m_c^2}}$$

$$\Lambda_{17} = \tfrac{2}{3} \mathrm{Re} \int_{-\infty}^{\infty} \frac{d\omega_1}{\omega_1} \left[1 - F\left(\frac{m_c^2 - i\varepsilon}{m_b \omega_1} \right) + \frac{m_b \omega_1}{12 m_c^2} \right] h_{17}(\omega_1, \mu)$$

$\omega_1 \leftrightarrow$ gluon momentum, $\quad F(x) = 4x \arctan^2\left(1/\sqrt{4x-1}\right)$

The soft function h_{17}:

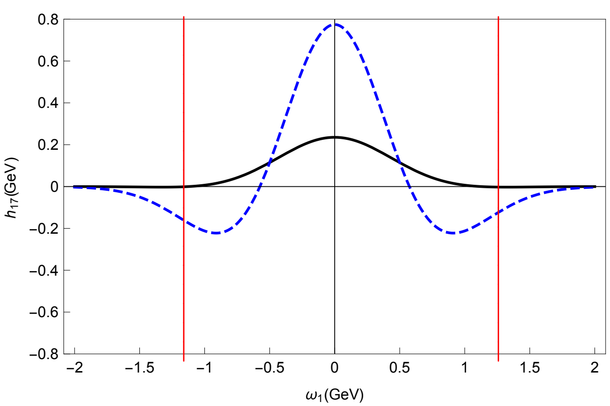
$$h_{17}(\omega_1, \mu) = \int \frac{dr}{4\pi M_B} e^{-i\omega_1 r} \langle \bar{B} | (\bar{h} S_{\bar{n}})(0) \not{\bar{n}}\, i \gamma_\alpha^\perp \bar{n}_\beta (S_{\bar{n}}^\dagger g G_s^{\alpha\beta} S_{\bar{n}})(r\bar{n}) (S_{\bar{n}}^\dagger h)(0) | \bar{B} \rangle \qquad (m_b - 2E_0 \gg \Lambda_{\mathrm{QCD}})$$

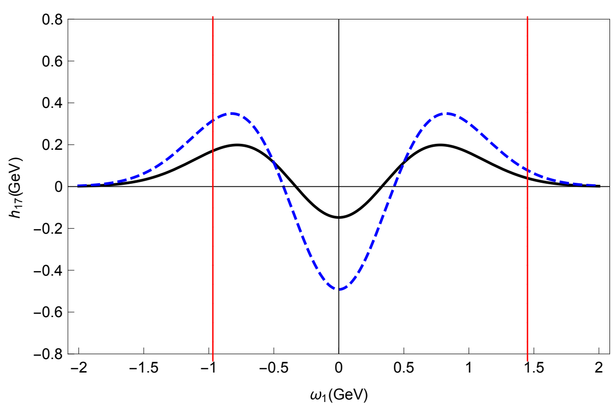
A class of models for h_{17}:

$$h_{17}(\omega_1, \mu) = e^{-\frac{\omega_1^2}{2\sigma^2}} \sum_n a_{2n} H_{2n}\left(\frac{\omega_1}{\sigma\sqrt{2}} \right), \qquad \sigma < 1\ \mathrm{GeV}$$

Hermite polynomials

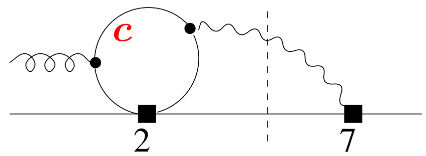
Constraints on moments (e.g.): $\int d\omega_1 h_{17} = \tfrac{2}{3} \mu_G^2, \qquad \int d\omega_1 \omega_1^2 h_{17} = \tfrac{2}{15}(5 m_5 + 3 m_6 - 2 m_9).$





Resolved photon contribution to the Q_7-$Q_{1,2}$ interference.

M.B. Voloshin, hep-ph/9612483; A. Khodjamirian, R. Rückl, G. Stoll and D. Wyler, hep-ph/9702318;
Z. Ligeti, L. Randall and M.B. Wise, hep-ph/9702322; G. Buchalla, G. Isidori, G. Rey, hep-ph/9705253;
M. Benzke, S.J. Lee, M. Neubert, G. Paz, arXiv:1003.5012; A. Gunawardana, G. Paz, arXiv:1908.02812;
M. Benzke, T. Hurth, arXiv:2006.00624; R. Bartocci, P. Böer, T. Hurth, arXiv:2411.16634.

$\langle \bar{B}|$  $|\bar{B}\rangle$

$$\delta \mathrm{N}(\mathrm{E}_0) = (C_2 - \tfrac{1}{6}C_1)C_7 \underbrace{\left[-\frac{\mu_{\mathrm{G}}^2}{27\mathrm{m}_{\mathrm{c}}^2} + \frac{\Lambda_{17}}{\mathrm{m}_{\mathrm{b}}} \right]}_{-\frac{\kappa_{\mathrm{V}}\mu_G^2}{27m_c^2}}$$

$$\Lambda_{17} = \tfrac{2}{3}\mathrm{Re}\int_{-\infty}^{\infty} \frac{d\omega_1}{\omega_1} \left[1 - F\left(\frac{m_c^2 - i\varepsilon}{m_b\omega_1} \right) + \frac{m_b\omega_1}{12m_c^2} \right] h_{17}(\omega_1, \mu)$$

$\omega_1 \leftrightarrow$ gluon momentum, $\qquad F(x) = 4x \arctan^2\left(1/\sqrt{4x-1}\right)$

The soft function h_{17}:

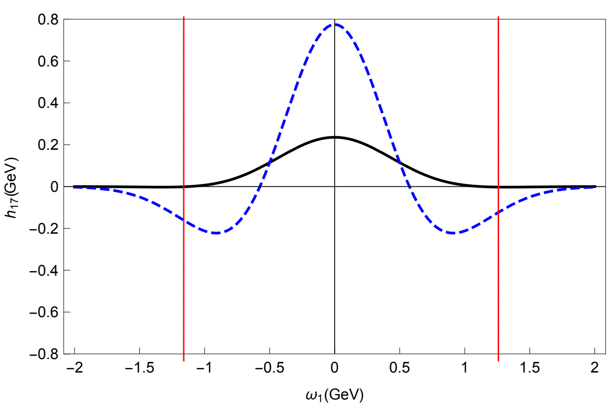
$$h_{17}(\omega_1,\mu) = \int \frac{dr}{4\pi M_B} e^{-i\omega_1 r} \langle \bar{B}| (\bar{h}S_{\bar{n}})(0) \not{\bar{n}}\, i\gamma_\alpha^\perp \bar{n}_\beta (S_{\bar{n}}^\dagger g G_s^{\alpha\beta} S_{\bar{n}})(r\bar{n}) (S_{\bar{n}}^\dagger h)(0) |\bar{B}\rangle \qquad (m_b - 2E_0 \gg \Lambda_{\mathrm{QCD}})$$

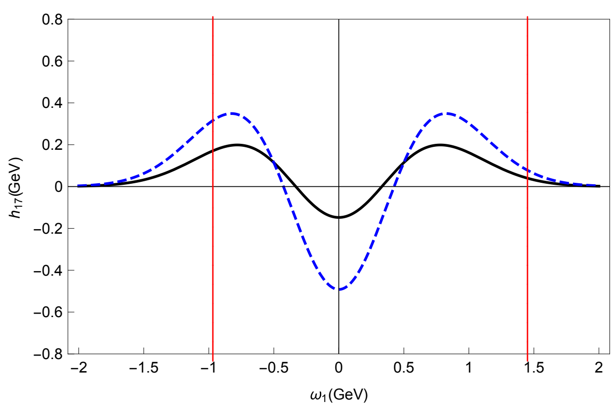
A class of models for h_{17}:

$$h_{17}(\omega_1,\mu) = e^{-\frac{\omega_1^2}{2\sigma^2}} \sum_n a_{2n} H_{2n}\left(\frac{\omega_1}{\sigma\sqrt{2}} \right), \qquad \sigma < 1\ \mathrm{GeV}$$

Hermite polynomials

Constraints on moments (e.g.): $\quad \int d\omega_1 h_{17} = \tfrac{2}{3}\mu_G^2, \qquad \int d\omega_1 \omega_1^2 h_{17} = \tfrac{2}{15}(5m_5 + 3m_6 - 2m_9).$





G+P numerically:
$\Lambda_{17} \in [-24, 5]$ MeV for $m_c = 1.17$ GeV.

In our code: $\kappa_V = 1.2 \pm 0.3$.
Warning: scheme for m_c!

Moment constraints vs. models of h_{17}

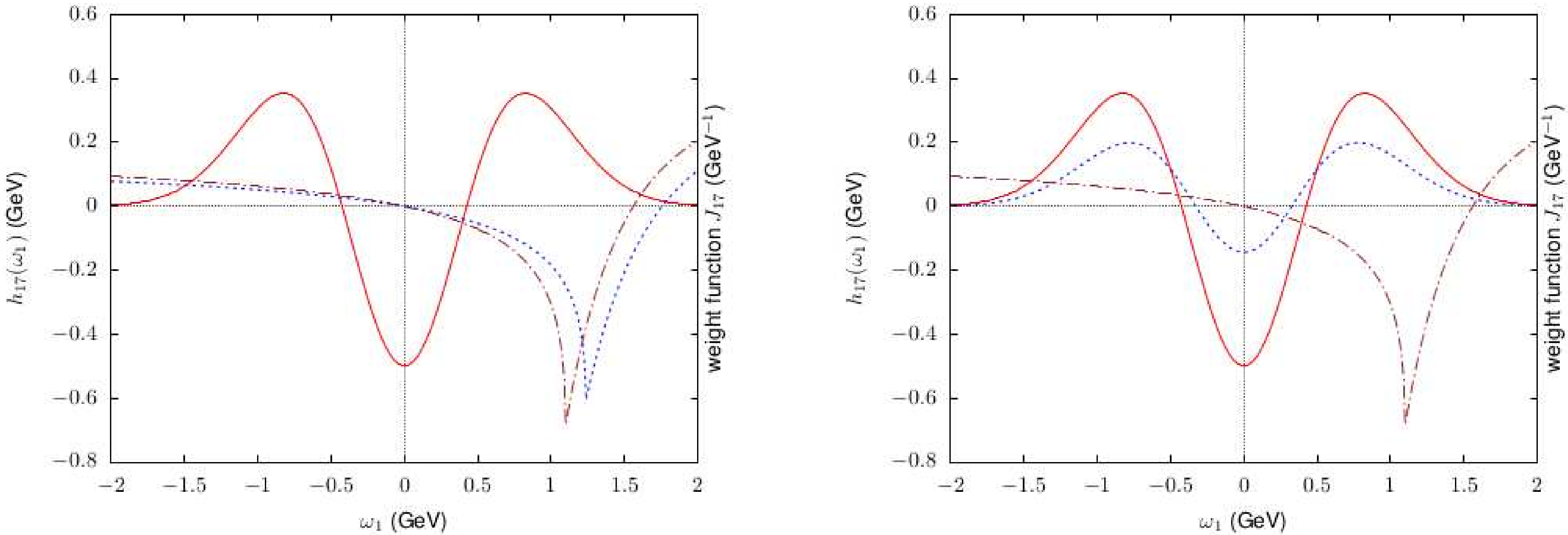
M. Benzke, S.J. Lee, M. Neubert, G. Paz, arXiv:1003.5012 – only the leading moment included.

A. Gunawardana, G. Paz, arXiv:1908.02812 – estimates of the subleading moments from LLSA included.

M. Benzke, T. Hurth, arXiv:2006.00624 – as above but with more generous modeling and partial $1/m_b^2$ corrections.

R. Bartocci, P. Böer, T. Hurth, arXiv:2411.16634 – RG evolution of $h_{17}(\omega_1, \mu)$.

Plots from arXiv:2006.00624:



Another recent contribution: T. Hurth and R. Szafron, arXiv:2301.01739 – clarifying the SCET treatment of resolved photons in the Q_8-Q_8 interference.

Summary and outlook

- **Updated SM prediction:**

$$\mathcal{B}_{s\gamma}^{\text{SM}} = (3.54 \pm 0.14) \times 10^{-4} \quad (\pm 4.0\%).$$

  No interpolation in m_c, non-perturbative uncertainty treatment as in arXiv:2002.01548.
  The perturbative NLO corrections are now formally complete.

-

-

-

Summary and outlook

- **Updated SM prediction:**

$$\mathcal{B}_{s\gamma}^{\rm SM} = (3.54 \pm 0.14) \times 10^{-4} \quad (\pm 4.0\%).$$

No interpolation in m_c, non-perturbative uncertainty treatment as in arXiv:2002.01548.
The perturbative NLO corrections are now formally complete.

- **Current experimental world average (PDG 2024, HFLAV 2024):**

$$\mathcal{B}_{s\gamma}^{\rm SM} = (3.49 \pm 0.19) \times 10^{-4} \quad (\pm 5.5\%).$$

Belle II prospects: ±2.6%, arXiv:1808.10567.

-

-

Summary and outlook

- **Updated SM prediction:**

$$\mathcal{B}_{s\gamma}^{\rm SM} = (3.54 \pm 0.14) \times 10^{-4} \qquad (\pm 4.0\%).$$

  No interpolation in m_c, non-perturbative uncertainty treatment as in arXiv:2002.01548.
  The perturbative NLO corrections are now formally complete.

- **Current experimental world average (PDG 2024, HFLAV 2024):**

$$\mathcal{B}_{s\gamma}^{\rm SM} = (3.49 \pm 0.19) \times 10^{-4} \qquad (\pm 5.5\%).$$

  Belle II prospects: $\pm 2.6\%$, arXiv:1808.10567.

- **Perturbative outlook:**

  Include $E_\gamma > E_0$ in $G_{17}^{(2)}$ and $G_{27}^{(2)}$.

  Calculate other than 2-body contributions in $G_{11}^{(2)}$, $G_{12}^{(2)}$, $G_{22}^{(2)}$, $G_{18}^{(2)}$ and $G_{28}^{(2)}$.

  Calculate $G_{ij}^{(2)}$ without neglecting Q_3-Q_6.

  N^3LO ?

-

Summary and outlook

- **Updated SM prediction:**

$$\mathcal{B}_{s\gamma}^{\mathrm{SM}} = (3.54 \pm 0.14) \times 10^{-4} \quad (\pm 4.0\%).$$

No interpolation in m_c, non-perturbative uncertainty treatment as in arXiv:2002.01548.
The perturbative NLO corrections are now formally complete.

- **Current experimental world average (PDG 2024, HFLAV 2024):**

$$\mathcal{B}_{s\gamma}^{\mathrm{SM}} = (3.49 \pm 0.19) \times 10^{-4} \quad (\pm 5.5\%).$$

Belle II prospects: $\pm 2.6\%$, arXiv:1808.10567.

- **Perturbative outlook:**

Include $E_\gamma > E_0$ in $G_{17}^{(2)}$ and $G_{27}^{(2)}$.

Calculate other than 2-body contributions in $G_{11}^{(2)}$, $G_{12}^{(2)}$, $G_{22}^{(2)}$, $G_{18}^{(2)}$ and $G_{28}^{(2)}$.

Calculate $G_{ij}^{(2)}$ without neglecting Q_3-Q_6.

N^3LO ?

- **Non-perturbative outlook:**

Complete the $\mathcal{O}\left(\frac{1}{m_b^2}\right)$ and $\mathcal{O}(\alpha_s)$ resolved-photon corrections in the $Q_{1,2}$-Q_7 interference.

Use arXiv:2301.01739 to update the Q_8-Q_8 interference.

Use arXiv:2211.07663 [B. Dehnadi, I. Novikov, F. J. Tackmann] and the SIMBA analysis in arXiv:2007.04320 to improve HFLAV/PDG averaging and extrapolation to $E_0 = 1.6\,\mathrm{GeV}$.

BACKUP SLIDES

The "hard" contribution to $\bar{B} \to X_s\gamma$

J. Chay, H. Georgi, B. Grinstein PLB 247 (1990) 399.
A.F. Falk, M. Luke, M. Savage, PRD 49 (1994) 3367.

The "hard" contribution to $\bar{B} \to X_s\gamma$

J. Chay, H. Georgi, B. Grinstein PLB 247 (1990) 399.
A.F. Falk, M. Luke, M. Savage, PRD 49 (1994) 3367.

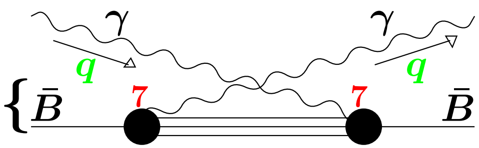
Goal: calculate the inclusive sum $\Sigma_{X_s} \left| C_7(\mu_b)\langle X_s\gamma|O_7|\bar{B}\rangle + C_2(\mu_b)\langle X_s\gamma|O_2|\bar{B}\rangle + ... \right|^2$

The "hard" contribution to $\bar{B} \to X_s\gamma$

J. Chay, H. Georgi, B. Grinstein PLB 247 (1990) 399.
A.F. Falk, M. Luke, M. Savage, PRD 49 (1994) 3367.

Goal: calculate the inclusive sum $\Sigma_{X_s} \left| C_7(\mu_b)\langle X_s\gamma|O_7|\bar{B}\rangle + C_2(\mu_b)\langle X_s\gamma|O_2|\bar{B}\rangle + ... \right|^2$

The "77" term in this sum is "hard". It is related via the optical theorem to the imaginary part of the elastic forward scattering amplitude $\bar{B}(\vec{p}=0)\gamma(\vec{q}) \to \bar{B}(\vec{p}=0)\gamma(\vec{q})$:

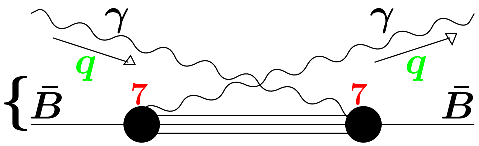
$\mathrm{Im}\{$  $\} \equiv \mathrm{Im}\, A$

The "hard" contribution to $\bar{B} \to X_s\gamma$

J. Chay, H. Georgi, B. Grinstein PLB 247 (1990) 399.
A.F. Falk, M. Luke, M. Savage, PRD 49 (1994) 3367.

Goal: calculate the inclusive sum $\Sigma_{X_s} \left| C_7(\mu_b)\langle X_s\gamma|O_7|\bar{B}\rangle + C_2(\mu_b)\langle X_s\gamma|O_2|\bar{B}\rangle + \ldots \right|^2$

The "77" term in this sum is "hard". It is related via the optical theorem to the imaginary part of the elastic forward scattering amplitude $\bar{B}(\vec{p}=0)\gamma(\vec{q}) \to \bar{B}(\vec{p}=0)\gamma(\vec{q})$:

$\mathrm{Im}\{$ 

$\} \equiv \mathrm{Im}\, A$

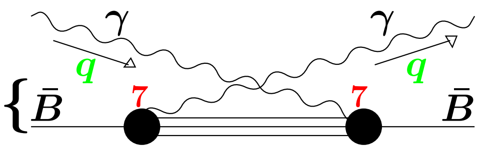
When the photons are soft enough, $m^2_{X_s} = |m_B(m_B - 2E_\gamma)| \gg \Lambda^2$ $\Rightarrow$ **Short-distance dominance** $\Rightarrow$ **OPE.**

The "hard" contribution to $\bar{B} \to X_s\gamma$

J. Chay, H. Georgi, B. Grinstein PLB 247 (1990) 399.
A.F. Falk, M. Luke, M. Savage, PRD 49 (1994) 3367.

Goal: calculate the inclusive sum $\Sigma_{X_s} \left| C_7(\mu_b)\langle X_s\gamma|O_7|\bar{B}\rangle + C_2(\mu_b)\langle X_s\gamma|O_2|\bar{B}\rangle + \ldots \right|^2$

The "77" term in this sum is "hard". It is related via the optical theorem to the imaginary part of the elastic forward scattering amplitude $\bar{B}(\vec{p}=0)\gamma(\vec{q}) \to \bar{B}(\vec{p}=0)\gamma(\vec{q})$:

$\mathrm{Im}\{$  $\} \equiv \mathrm{Im}\, A$

When the photons are soft enough, $m_{X_s}^2 = |m_B(m_B - 2E_\gamma)| \gg \Lambda^2 \Rightarrow$ **Short-distance dominance** $\Rightarrow$ **OPE.**

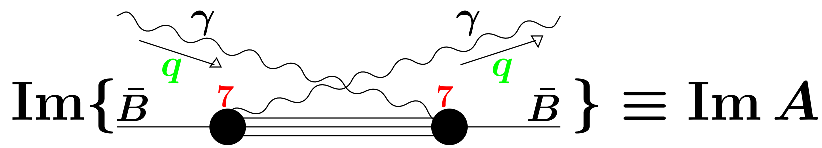
However, the $\bar{B} \to X_s\gamma$ **photon spectrum is dominated by hard photons** $E_\gamma \sim m_b/2$ **.**

The "hard" contribution to $\bar{B} \to X_s \gamma$

J. Chay, H. Georgi, B. Grinstein PLB 247 (1990) 399.
A.F. Falk, M. Luke, M. Savage, PRD 49 (1994) 3367.

Goal: calculate the inclusive sum $\Sigma_{X_s} \left| C_7(\mu_b) \langle X_s \gamma | O_7 | \bar{B} \rangle + C_2(\mu_b) \langle X_s \gamma | O_2 | \bar{B} \rangle + \ldots \right|^2$

The "77" term in this sum is "hard". It is related via the optical theorem to the imaginary part of the elastic forward scattering amplitude $\bar{B}(\vec{p}=0)\gamma(\vec{q}) \to \bar{B}(\vec{p}=0)\gamma(\vec{q})$:

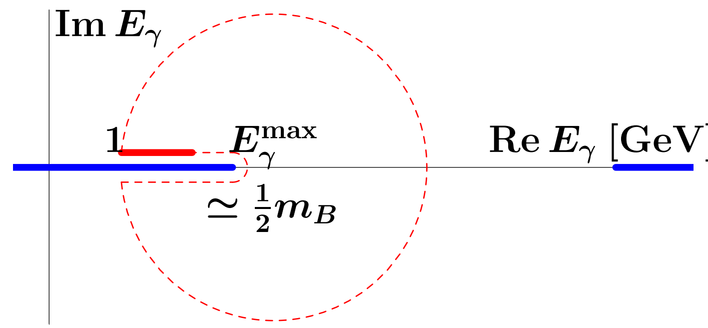


When the photons are soft enough, $m_{X_s}^2 = |m_B(m_B - 2E_\gamma)| \gg \Lambda^2$ $\Rightarrow$ Short-distance dominance $\Rightarrow$ OPE.

However, the $\bar{B} \to X_s \gamma$ photon spectrum is dominated by hard photons $E_\gamma \sim m_b/2$.

Once $A(E_\gamma)$ is considered as a function of arbitrary complex E_γ , ImA turns out to be proportional to the discontinuity of A at the physical cut. Consequently,

$$\int_{1\,\mathrm{GeV}}^{E_\gamma^{\max}} dE_\gamma \, \mathrm{Im}\, A(E_\gamma) \sim \oint_{\mathrm{circle}} dE_\gamma \, A(E_\gamma).$$

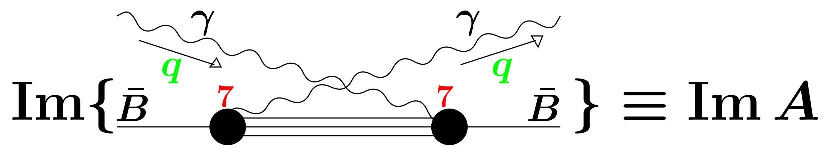


The "hard" contribution to $\bar{B} \to X_s\gamma$

J. Chay, H. Georgi, B. Grinstein PLB 247 (1990) 399.
A.F. Falk, M. Luke, M. Savage, PRD 49 (1994) 3367.

Goal: calculate the inclusive sum $\Sigma_{X_s} \left| C_7(\mu_b)\langle X_s\gamma|O_7|\bar{B}\rangle + C_2(\mu_b)\langle X_s\gamma|O_2|\bar{B}\rangle + \ldots \right|^2$

The "77" term in this sum is "hard". It is related via the optical theorem to the imaginary part of the elastic forward scattering amplitude $\bar{B}(\vec{p}=0)\gamma(\vec{q}) \to \bar{B}(\vec{p}=0)\gamma(\vec{q})$:

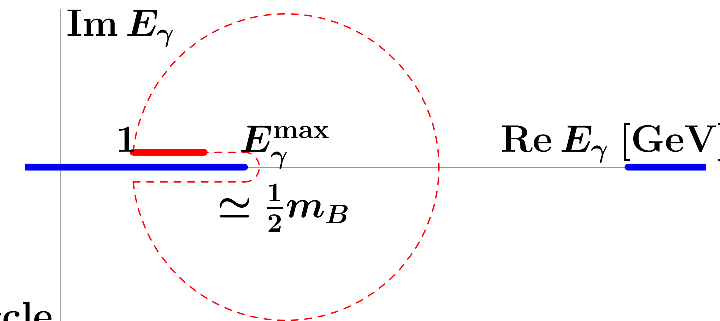


When the photons are soft enough, $m_{X_s}^2 = |m_B(m_B - 2E_\gamma)| \gg \Lambda^2$ $\Rightarrow$ Short-distance dominance $\Rightarrow$ OPE.

However, the $\bar{B} \to X_s\gamma$ photon spectrum is dominated by hard photons $E_\gamma \sim m_b/2$.

Once $A(E_\gamma)$ is considered as a function of arbitrary complex E_γ , $\mathrm{Im}A$ turns out to be proportional to the discontinuity of A at the physical cut. Consequently,

$$\int_{1\ \mathrm{GeV}}^{E_\gamma^{\max}} dE_\gamma\ \mathrm{Im}\, A(E_\gamma) \sim \oint_{\mathrm{circle}} dE_\gamma\ A(E_\gamma).$$



Since the condition $|m_B(m_B - 2E_\gamma)| \gg \Lambda^2$ is fulfilled along the circle, the OPE coefficients can be calculated perturbatively, which gives

$$A(E_\gamma)\big|_{\mathrm{circle}} \simeq \sum_j \left[\frac{F^{(j)}_{\mathrm{polynomial}}(2E_\gamma/m_b)}{m_b^{n_j}(1-2E_\gamma/m_b)^{k_j}} + \mathcal{O}\left(\alpha_s(\mu_{\mathrm{hard}})\right) \right] \langle \bar{B}(\vec{p}=0)|Q^{(j)}_{\mathrm{local\ operator}}|\bar{B}(\vec{p}=0)\rangle.$$

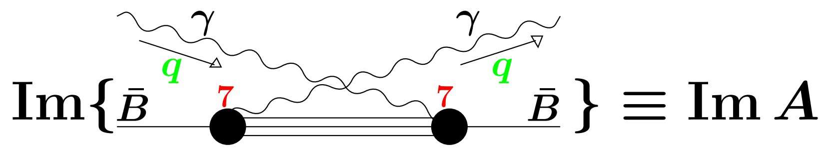
Thus, contributions from higher-dimensional operators are suppressed by powers of Λ/m_b.

The "hard" contribution to $\bar{B} \to X_s\gamma$

J. Chay, H. Georgi, B. Grinstein PLB 247 (1990) 399.
A.F. Falk, M. Luke, M. Savage, PRD 49 (1994) 3367.

Goal: calculate the inclusive sum $\Sigma_{X_s} \left| C_7(\mu_b)\langle X_s\gamma|O_7|\bar{B}\rangle + C_2(\mu_b)\langle X_s\gamma|O_2|\bar{B}\rangle + \ldots \right|^2$

The "77" term in this sum is "hard". It is related via the optical theorem to the imaginary part of the elastic forward scattering amplitude $\bar{B}(\vec{p}=0)\gamma(\vec{q}) \to \bar{B}(\vec{p}=0)\gamma(\vec{q})$:

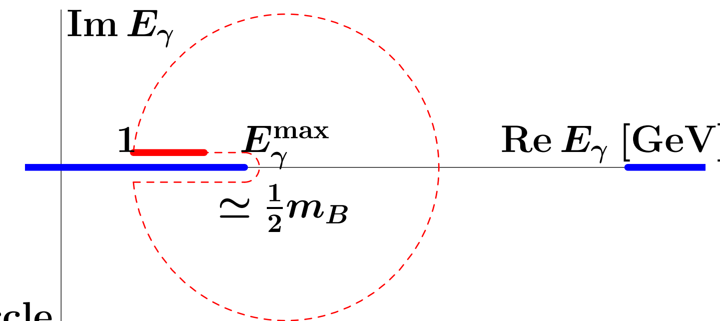


When the photons are soft enough, $m^2_{X_s} = |m_B(m_B - 2E_\gamma)| \gg \Lambda^2$ $\Rightarrow$ Short-distance dominance $\Rightarrow$ OPE.

However, the $\bar{B} \to X_s\gamma$ photon spectrum is dominated by hard photons $E_\gamma \sim m_b/2$.

Once $A(E_\gamma)$ is considered as a function of arbitrary complex E_γ , $\mathrm{Im}A$ turns out to be proportional to the discontinuity of A at the physical cut. Consequently,

$$\int_{1\,\mathrm{GeV}}^{E_\gamma^{\max}} dE_\gamma \; \mathrm{Im}\, A(E_\gamma) \sim \oint_{\mathrm{circle}} dE_\gamma \; A(E_\gamma).$$



Since the condition $|m_B(m_B - 2E_\gamma)| \gg \Lambda^2$ is fulfilled along the circle, the OPE coefficients can be calculated perturbatively, which gives

$$A(E_\gamma)\big|_{\mathrm{circle}} \simeq \sum_j \left[\frac{F^{(j)}_{\mathrm{polynomial}}(2E_\gamma/m_b)}{m_b^{n_j}(1-2E_\gamma/m_b)^{k_j}} + \mathcal{O}\left(\alpha_s(\mu_{\mathrm{hard}})\right) \right] \langle \bar{B}(\vec{p}=0)|Q^{(j)}_{\mathrm{local\ operator}}|\bar{B}(\vec{p}=0)\rangle.$$

Thus, contributions from higher-dimensional operators are suppressed by powers of Λ/m_b.

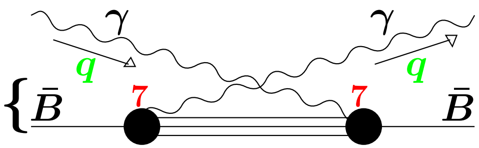
At $(\Lambda/m_b)^0$: $\quad \langle \bar{B}(\vec{p})|\bar{b}\gamma^\mu b|\bar{B}(\vec{p})\rangle = 2p^\mu \quad \Rightarrow \quad \Gamma(\bar{B} \to X_s\gamma) = \Gamma(b \to X_s^{\mathrm{parton}}\gamma) + \mathcal{O}(\Lambda/m_b).$

The “hard” contribution to $\bar{B} \to X_s \gamma$

J. Chay, H. Georgi, B. Grinstein PLB 247 (1990) 399.
A.F. Falk, M. Luke, M. Savage, PRD 49 (1994) 3367.

Goal: calculate the inclusive sum $\Sigma_{X_s} \left| C_7(\mu_b) \langle X_s \gamma | O_7 | \bar{B} \rangle + C_2(\mu_b) \langle X_s \gamma | O_2 | \bar{B} \rangle + \ldots \right|^2$

The “77” term in this sum is “hard”. It is related via the optical theorem to the imaginary part of the elastic forward scattering amplitude $\bar{B}(\vec{p}=0)\gamma(\vec{q}) \to \bar{B}(\vec{p}=0)\gamma(\vec{q})$:

$\mathrm{Im}\{$ 

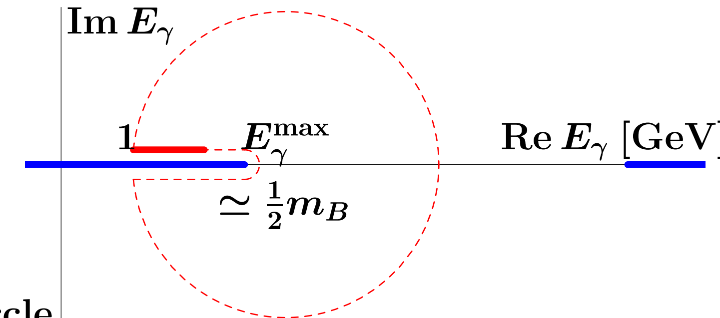
$\} \equiv \mathrm{Im}\, A$

When the photons are soft enough, $m_{X_s}^2 = |m_B(m_B - 2E_\gamma)| \gg \Lambda^2 \;\Rightarrow$ Short-distance dominance $\Rightarrow$ OPE.

However, the $\bar{B} \to X_s \gamma$ photon spectrum is dominated by hard photons $E_\gamma \sim m_b/2$.

Once $A(E_\gamma)$ is considered as a function of arbitrary complex E_γ , $\mathrm{Im}A$ turns out to be proportional to the discontinuity of A at the physical cut. Consequently,

$$\int_{1\,\mathrm{GeV}}^{E_\gamma^{\max}} dE_\gamma \; \mathrm{Im}\, A(E_\gamma) \sim \oint_{\mathrm{circle}} dE_\gamma \; A(E_\gamma).$$



Since the condition $|m_B(m_B - 2E_\gamma)| \gg \Lambda^2$ is fulfilled along the circle, the OPE coefficients can be calculated perturbatively, which gives

$$A(E_\gamma)\big|_{\mathrm{circle}} \simeq \sum_j \left[\frac{F^{(j)}_{\mathrm{polynomial}}(2E_\gamma/m_b)}{m_b^{n_j}(1-2E_\gamma/m_b)^{k_j}} + \mathcal{O}\left(\alpha_s(\mu_{\mathrm{hard}})\right) \right] \langle \bar{B}(\vec{p}=0) | Q^{(j)}_{\mathrm{local\ operator}} | \bar{B}(\vec{p}=0) \rangle.$$

Thus, contributions from higher-dimensional operators are suppressed by powers of Λ/m_b.

At $(\Lambda/m_b)^0$: $\quad \langle \bar{B}(\vec{p}) | \bar{b}\gamma^\mu b | \bar{B}(\vec{p}) \rangle = 2p^\mu \quad \Rightarrow \quad \Gamma(\bar{B} \to X_s \gamma) = \Gamma(b \to X_s^{\mathrm{parton}} \gamma) + \mathcal{O}(\Lambda/m_b)$.

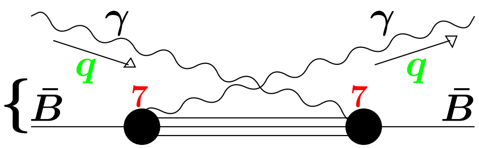
At $(\Lambda/m_b)^1$: $\quad$ Nothing! All the possible operators vanish by the equations of motion.

The "hard" contribution to $\bar{B} \to X_s\gamma$

J. Chay, H. Georgi, B. Grinstein PLB 247 (1990) 399.
A.F. Falk, M. Luke, M. Savage, PRD 49 (1994) 3367.

Goal: calculate the inclusive sum $\Sigma_{X_s} \left| C_7(\mu_b)\langle X_s\gamma|O_7|\bar{B}\rangle + C_2(\mu_b)\langle X_s\gamma|O_2|\bar{B}\rangle + \ldots \right|^2$

The "77" term in this sum is "hard". It is related via the optical theorem to the imaginary part of the elastic forward scattering amplitude $\bar{B}(\vec{p}=0)\gamma(\vec{q}) \to \bar{B}(\vec{p}=0)\gamma(\vec{q})$:

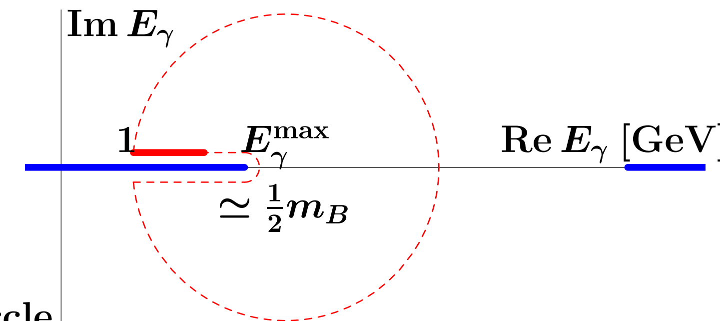
Im{  } $\equiv$ Im A

When the photons are soft enough, $m^2_{X_s} = |m_B(m_B - 2E_\gamma)| \gg \Lambda^2$ $\Rightarrow$ Short-distance dominance $\Rightarrow$ OPE.

However, the $\bar{B} \to X_s\gamma$ photon spectrum is dominated by hard photons $E_\gamma \sim m_b/2$.

Once $A(E_\gamma)$ is considered as a function of arbitrary complex E_γ , ImA turns out to be proportional to the discontinuity of A at the physical cut. Consequently,

$$\int_{1\ \mathrm{GeV}}^{E_\gamma^{\max}} dE_\gamma \ \mathrm{Im}\, A(E_\gamma) \sim \oint_{\mathrm{circle}} dE_\gamma \ A(E_\gamma).$$



Since the condition $|m_B(m_B - 2E_\gamma)| \gg \Lambda^2$ is fulfilled along the circle, the OPE coefficients can be calculated perturbatively, which gives

$$A(E_\gamma)\big|_{\mathrm{circle}} \simeq \sum_j \left[\frac{F^{(j)}_{\mathrm{polynomial}}(2E_\gamma/m_b)}{m_b^{n_j}(1-2E_\gamma/m_b)^{k_j}} + \mathcal{O}\left(\alpha_s(\mu_{\mathrm{hard}})\right) \right] \langle \bar{B}(\vec{p}=0)|Q^{(j)}_{\mathrm{local\ operator}}|\bar{B}(\vec{p}=0)\rangle.$$

Thus, contributions from higher-dimensional operators are suppressed by powers of Λ/m_b.

At $(\Lambda/m_b)^0$: $\quad \langle \bar{B}(\vec{p})|\bar{b}\gamma^\mu b|\bar{B}(\vec{p})\rangle = 2p^\mu \quad \Rightarrow \quad \Gamma(\bar{B} \to X_s\gamma) = \Gamma(b \to X_s^{\mathrm{parton}}\gamma) + \mathcal{O}(\Lambda/m_b)$.

At $(\Lambda/m_b)^1$: Nothing! All the possible operators vanish by the equations of motion.

At $(\Lambda/m_b)^2$: $\quad \langle \bar{B}(\vec{p})|\bar{b}_v D^\mu D_\mu b_v|\bar{B}(\vec{p})\rangle \sim m_B\,\mu_\pi^2$,

$\quad \langle \bar{B}(\vec{p})|\bar{b}_v g_s G_{\mu\nu}\sigma^{\mu\nu} b_v|\bar{B}(\vec{p})\rangle \sim m_B\,\mu_G^2$,

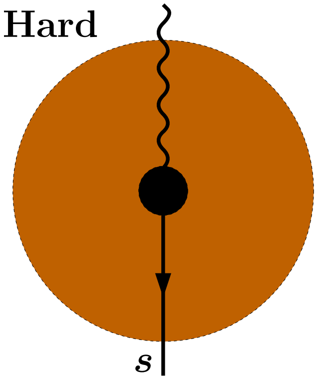
The HQET heavy-quark field: $b_v(x) = \frac{1}{2}(1+\not{v})b(x)\exp(im_b\, v\cdot x)$ with $v = p/m_B$.

Energetic photon production in charmless decays of the $\bar{B}$-meson

$(E_\gamma \gtrsim \frac{m_b}{3} \simeq 1.6\,\mathrm{GeV})$

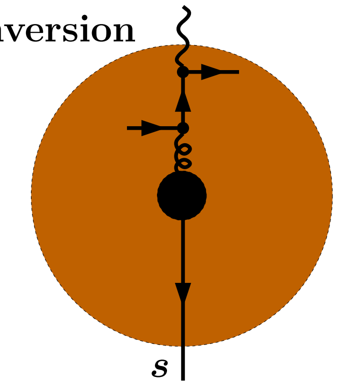
A. Without long-distance charm loops:

1. **Hard**



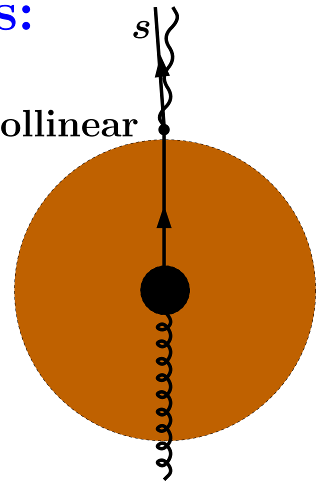
Dominant, well-controlled.

2. **Conversion**



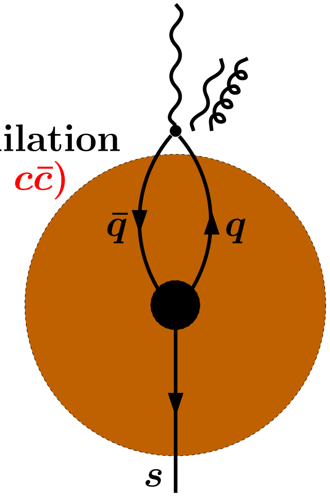
$\mathcal{O}(\alpha_s \Lambda / m_b)$,
$\leftrightarrow$ izospin asymmetry.

3. **Collinear**



Perturbatively $< 1\%$,
$\leftrightarrow$ fragmentation functions.

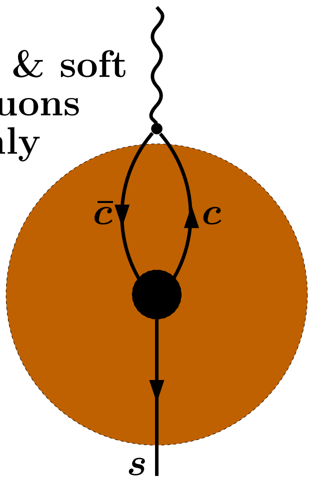
4. **Annihilation** $(q\bar{q} \neq c\bar{c})$



Exp. π^0, η, η', ω subtracted,
perturbatively $\sim 0.1\%$.

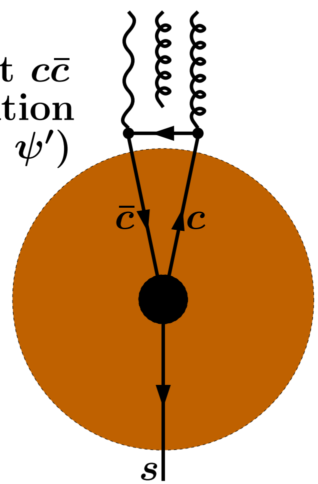
B. With long-distance charm loops:

5. $c\bar{c}$ & soft gluons only



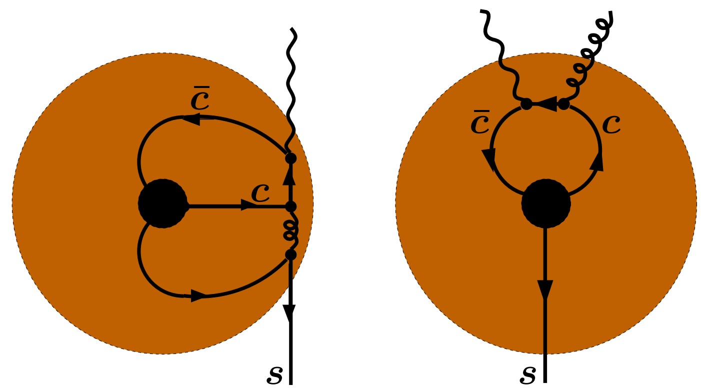
HQET & SCET.

6. Boosted light $c\bar{c}$ state annihilation (e.g. η_c, J/ψ, ψ')



Exp. J/ψ subtracted ($< 1\%$).

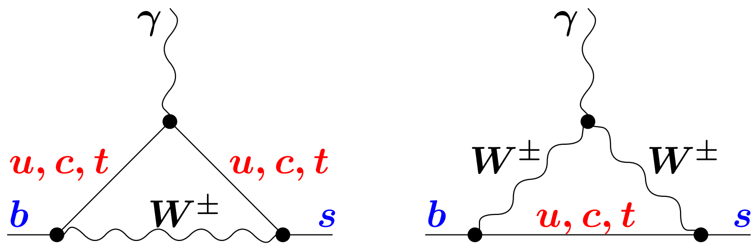
7. Annihilation of $c\bar{c}$ in a heavy $(\bar{c}s)(\bar{q}c)$ state



Examples of SM diagrams for the matching of $C_7(\mu_0)$ $(\mu_0 \sim M_W, m_t)$

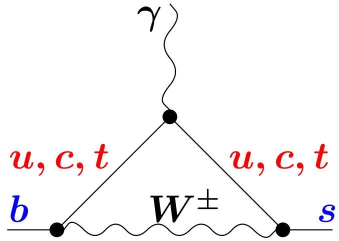
Examples of SM diagrams for the matching of $C_7(\mu_0)$ $(\mu_0 \sim M_W, m_t)$

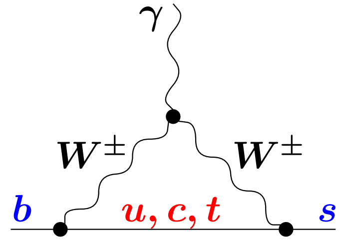
LO:
[Inami, Lim, 1981]



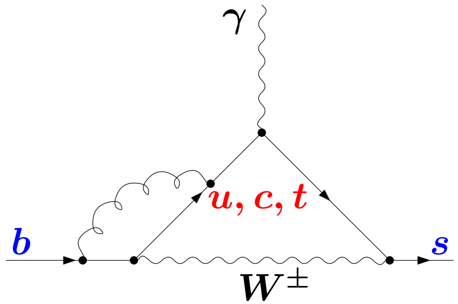
Examples of SM diagrams for the matching of $C_7(\mu_0)$ $(\mu_0 \sim M_W, m_t)$

LO:
[Inami, Lim, 1981]



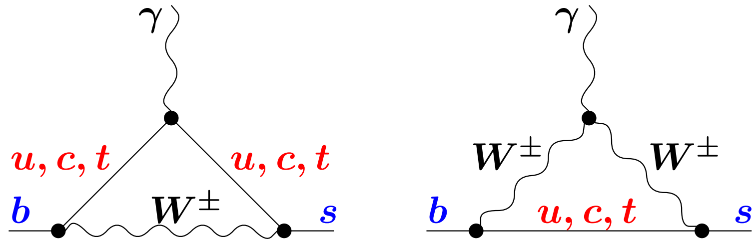


NLO:
[Adel, Yao, 1993]

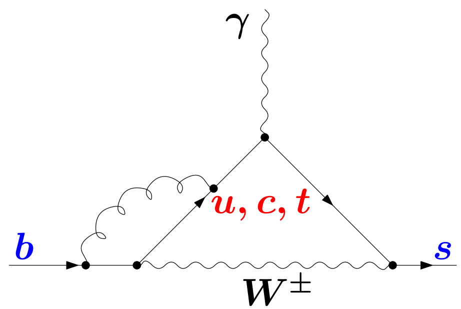


Examples of SM diagrams for the matching of $C_7(\mu_0)$ $(\mu_0 \sim M_W, m_t)$

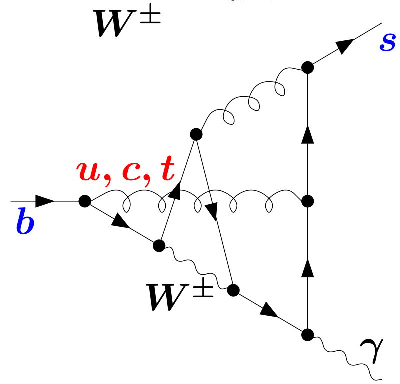
LO:
[Inami, Lim, 1981]



NLO:
[Adel, Yao, 1993]

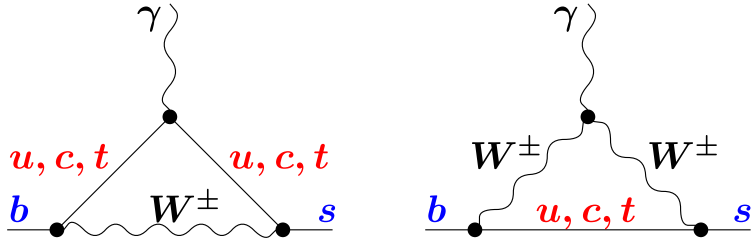


NNLO:
[Steinhauser, MM, 2004]

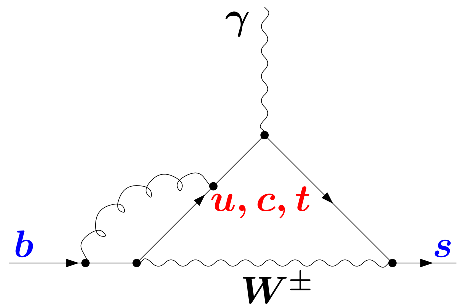


Examples of SM diagrams for the matching of $C_7(\mu_0)$ $(\mu_0 \sim M_W, m_t)$

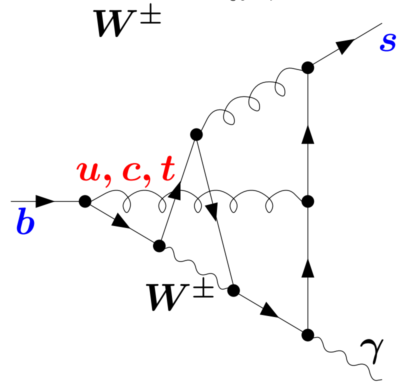
LO:
[Inami, Lim, 1981]



NLO:
[Adel, Yao, 1993]



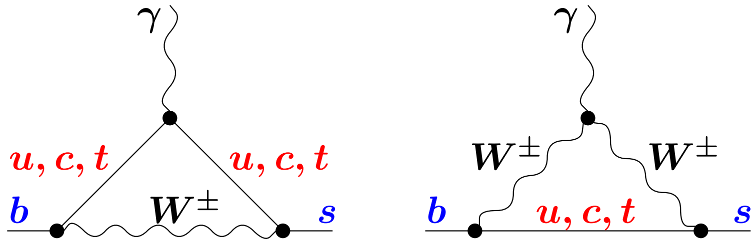
NNLO:
[Steinhauser, MM, 2004]



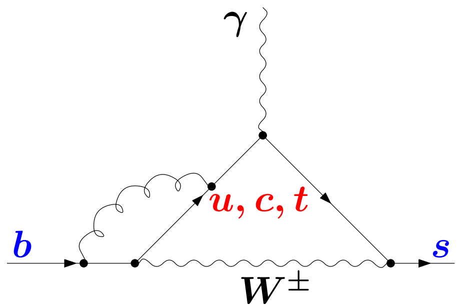
NNLO method:

Examples of SM diagrams for the matching of $C_7(\mu_0)$ $(\mu_0 \sim M_W, m_t)$

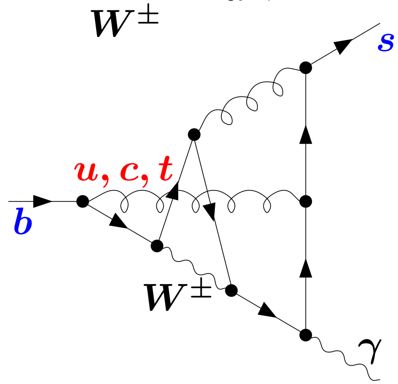
LO:
[Inami, Lim, 1981]



NLO:
[Adel, Yao, 1993]



NNLO:
[Steinhauser, MM, 2004]

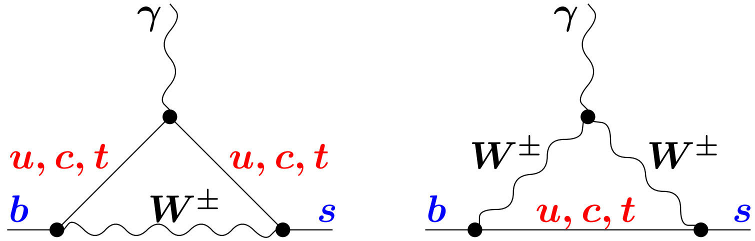


NNLO method:

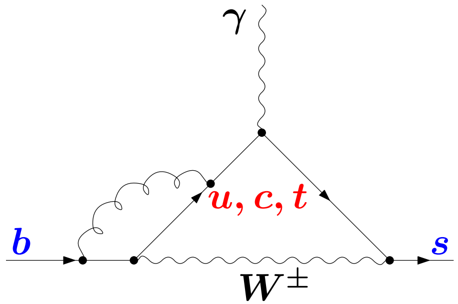
- Taylor expansion in the off-shell external momenta is applied prior to loop integration.

Examples of SM diagrams for the matching of $C_7(\mu_0)$ $(\mu_0 \sim M_W, m_t)$

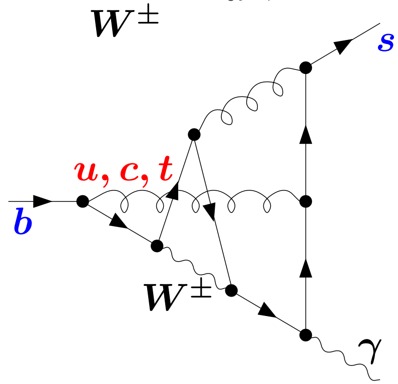
LO:
[Inami, Lim, 1981]



NLO:
[Adel, Yao, 1993]



NNLO:
[Steinhauser, MM, 2004]

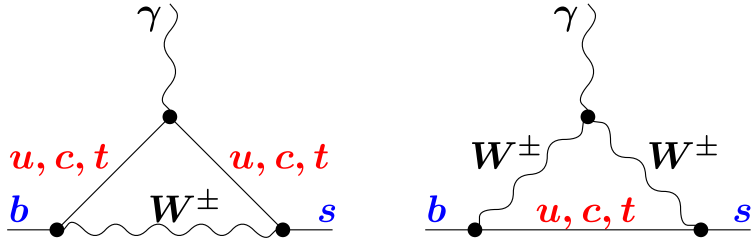


NNLO method:

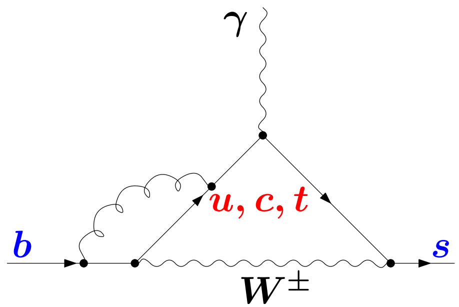
- Taylor expansion in the off-shell external momenta is applied prior to loop integration.
- The UV and spurious IR divergences are regulated dimensionally.

Examples of SM diagrams for the matching of $C_7(\mu_0)$ $(\mu_0 \sim M_W, m_t)$

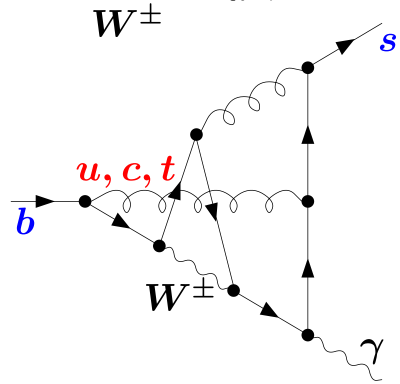
LO:
[Inami, Lim, 1981]



NLO:
[Adel, Yao, 1993]



NNLO:
[Steinhauser, MM, 2004]

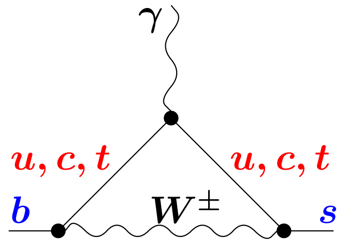


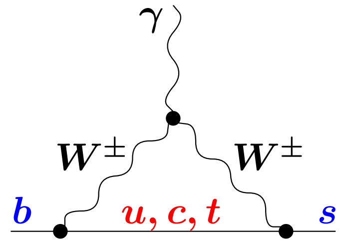
NNLO method:

- Taylor expansion in the off-shell external momenta is applied prior to loop integration.
- The UV and spurious IR divergences are regulated dimensionally.
- $\Rightarrow$ In the effective theory, only tree-level diagrams survive (tree vertices and UV counterterms). The UV renormalization constants are known from former anomalous-dimension calculations.

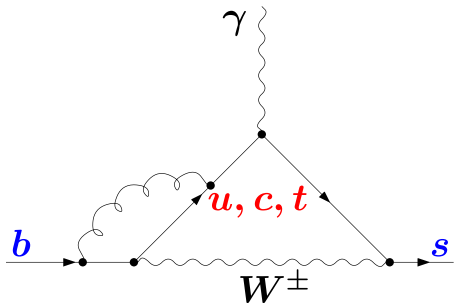
Examples of SM diagrams for the matching of $C_7(\mu_0)$ $(\mu_0 \sim M_W, m_t)$

LO:
[Inami, Lim, 1981]

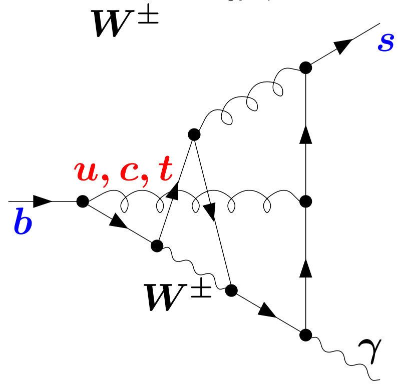




NLO:
[Adel, Yao, 1993]



NNLO:
[Steinhauser, MM, 2004]

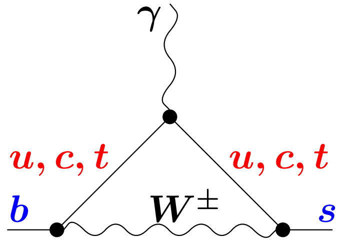


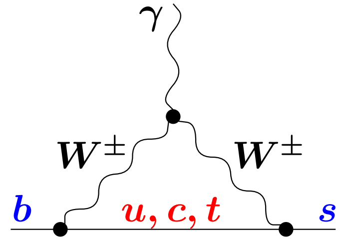
NNLO method:

- Taylor expansion in the off-shell external momenta is applied prior to loop integration.
- The UV and spurious IR divergences are regulated dimensionally.
- $\Rightarrow$ In the effective theory, only tree-level diagrams survive (tree vertices and UV counterterms). The UV renormalization constants are known from former anomalous-dimension calculations.
- All the $1/\epsilon$ poles cancel in the matching equation, i.e. in the difference between the effective theory and the full SM Green functions.

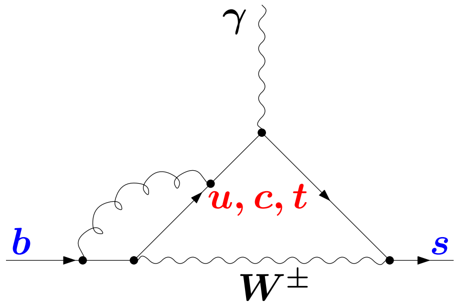
Examples of SM diagrams for the matching of $C_7(\mu_0)$ $\quad (\mu_0 \sim M_W, m_t)$

LO:
[Inami, Lim, 1981]

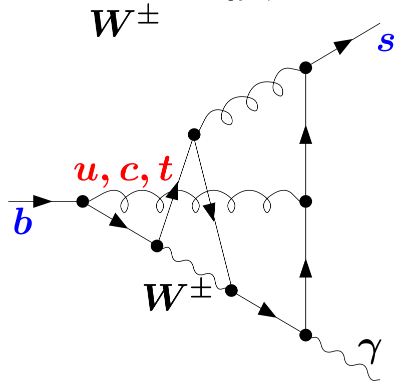




NLO:
[Adel, Yao, 1993]



NNLO:
[Steinhauser, MM, 2004]



NNLO method:

- Taylor expansion in the off-shell external momenta is applied prior to loop integration.
- The UV and spurious IR divergences are regulated dimensionally.
- $\Rightarrow$ In the effective theory, only tree-level diagrams survive (tree vertices and UV counterterms). The UV renormalization constants are known from former anomalous-dimension calculations.
- All the $1/\epsilon$ poles cancel in the matching equation, i.e. in the difference between the effective theory and the full SM Green functions.
- At the 3-loop level, the difference $m_t - M_W$ is taken into account with the help of expansions in y^n and $(1-y^2)^n$ up to $n = 8$, where $y = M_W/m_t$.

Resummation of large logarithms $\left(\alpha_s \ln \frac{\mu_0^2}{\mu_b^2}\right)^n$ in the $b \to s\gamma$ amplitude.

Resummation of large logarithms $\left(\alpha_s \ln \frac{\mu_0^2}{\mu_b^2}\right)^n$ in the $b \to s\gamma$ amplitude.

RGE for the Wilson coefficients: $$\mu \frac{d}{d\mu} C_j(\mu) = C_i(\mu) \gamma_{ij}(\mu)$$

Resummation of large logarithms $\left(\alpha_s \ln \frac{\mu_0^2}{\mu_b^2}\right)^n$ in the $b \to s\gamma$ amplitude.

RGE for the Wilson coefficients:

$$\mu \frac{d}{d\mu} C_j(\mu) = C_i(\mu) \gamma_{ij}(\mu)$$

Evolution from $\mu_0 \sim M_W, m_t$ down to $\mu_b \sim m_b$.

Resummation of large logarithms $\left(\alpha_s \ln \frac{\mu_0^2}{\mu_b^2}\right)^n$ in the $b \to s\gamma$ amplitude.

RGE for the Wilson coefficients: $$\mu \frac{d}{d\mu} C_j(\mu) = C_i(\mu) \gamma_{ij}(\mu)$$ Evolution from $\mu_0 \sim M_W, m_t$ down to $\mu_b \sim m_b$.

The anomalous dimension matrix γ_{ij} is found from the effective theory renormalization constants, e.g.:

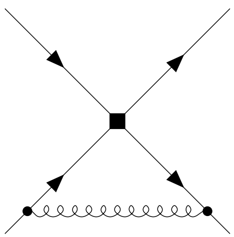
Resummation of large logarithms $\left(\alpha_s \ln \frac{\mu_0^2}{\mu_b^2}\right)^n$ in the $b \to s\gamma$ amplitude.

RGE for the Wilson coefficients: $$\mu \frac{d}{d\mu} C_j(\mu) = C_i(\mu)\gamma_{ij}(\mu)$$ Evolution from $\mu_0 \sim M_W, m_t$ down to $\mu_b \sim m_b$.

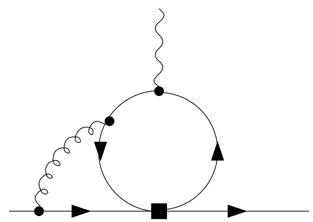
The anomalous dimension matrix γ_{ij} is found from the effective theory renormalization constants, e.g.:

Z_{22} Z_{27} Z_{87}

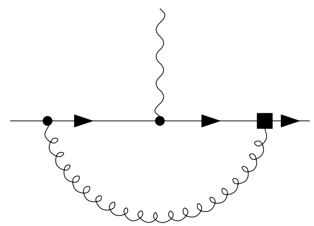
LO



[Gaillard, Lee, 1974]
[Altarelli, Maiani, 1974]

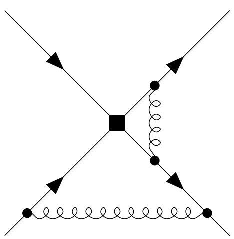


[Grinstein *et al.*, 1990]

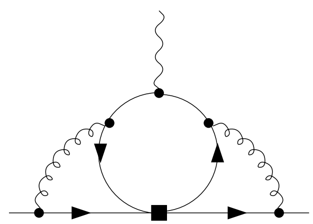


[Shifman *et al.*, 1978]
[Grigjanis *et al.*, 1988]

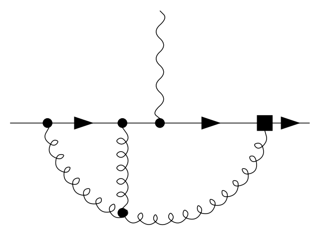
NLO



[Altarelli *et al.*, 1981]
[Buras, Weisz, 1990]

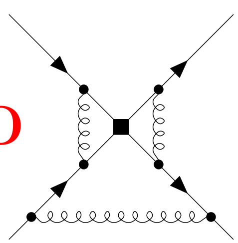


[Chetyrkin, MM, Münz, 1997]

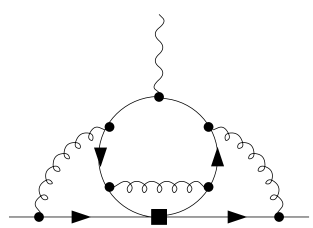


[MM, Münz, 1995]

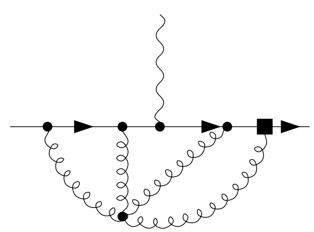
NNLO



[Gorbahn, Haisch, 2004]



[Czakon, Haisch, MM, 2006]



[Gorbahn, Haisch, MM, 2005]

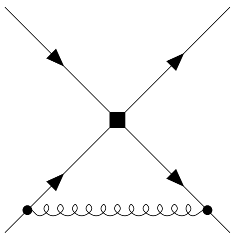
Resummation of large logarithms $\left(\alpha_s \ln \frac{\mu_0^2}{\mu_b^2}\right)^n$ in the $b \to s\gamma$ amplitude.

RGE for the Wilson coefficients: $$\mu \frac{d}{d\mu} C_j(\mu) = C_i(\mu)\gamma_{ij}(\mu)$$ Evolution from $\mu_0 \sim M_W, m_t$ down to $\mu_b \sim m_b$.

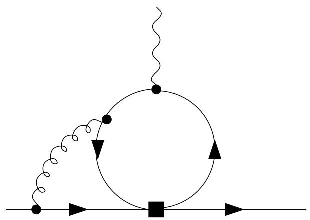
The anomalous dimension matrix γ_{ij} is found from the effective theory renormalization constants, e.g.:

Z_{22} Z_{27} Z_{87}

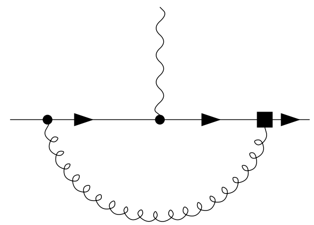
LO



[Gaillard, Lee, 1974]
[Altarelli, Maiani, 1974]

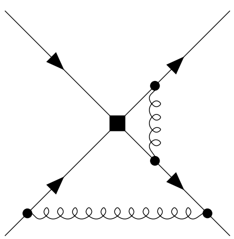


[Grinstein *et al.*, 1990]

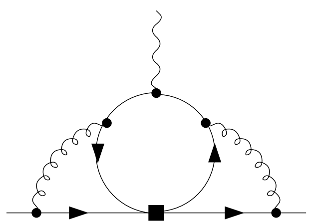


[Shifman *et al.*, 1978]
[Grigjanis *et al.*, 1988]

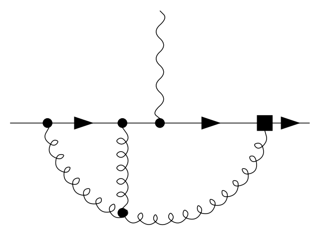
NLO



[Altarelli *et al.*, 1981]
[Buras, Weisz, 1990]

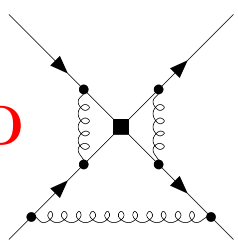


[Chetyrkin, MM, Münz, 1997]

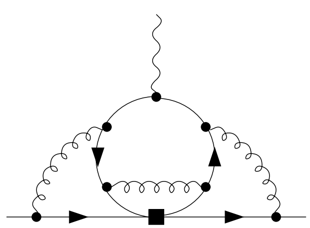


[MM, Münz, 1995]

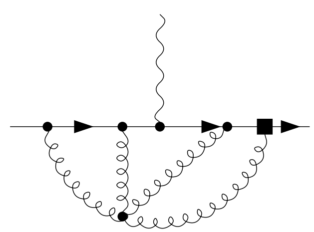
NNLO



[Gorbahn, Haisch, 2004]



[Czakon, Haisch, MM, 2006]
$\sim 2 \times 10^4$ diagrams,
-4% effect on $\mathcal{B}_{s\gamma}^{\rm SM}$



[Gorbahn, Haisch, MM, 2005]

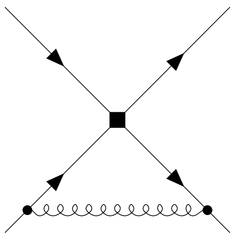
Resummation of large logarithms $\left(\alpha_s \ln \frac{\mu_0^2}{\mu_b^2}\right)^n$ in the $b \to s\gamma$ amplitude.

RGE for the Wilson coefficients: $$\mu \frac{d}{d\mu} C_j(\mu) = C_i(\mu) \gamma_{ij}(\mu)$$ Evolution from $\mu_0 \sim M_W, m_t$ down to $\mu_b \sim m_b$.

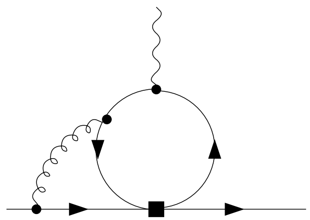
The anomalous dimension matrix γ_{ij} is found from the effective theory renormalization constants, e.g.:

Z_{22} | Z_{27} | Z_{87}

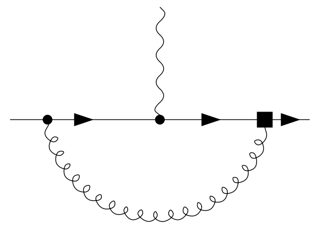
LO



[Gaillard, Lee, 1974]
[Altarelli, Maiani, 1974]

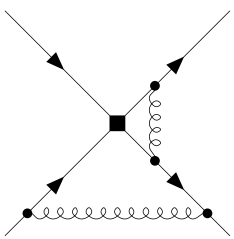


[Grinstein *et al.*, 1990]

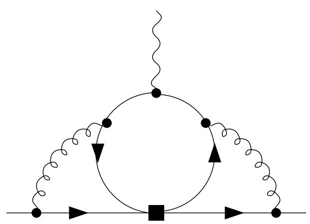


[Shifman *et al.*, 1978]
[Grigjanis *et al.*, 1988]

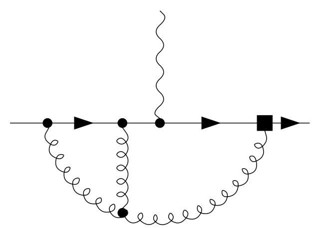
NLO



[Altarelli *et al.*, 1981]
[Buras, Weisz, 1990]

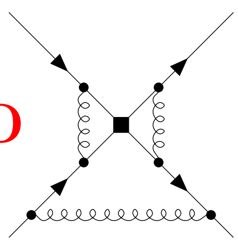


[Chetyrkin, MM, Münz, 1997]

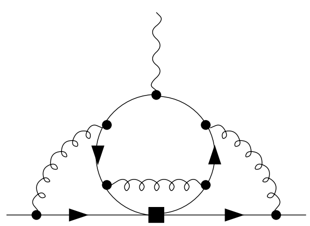


[MM, Münz, 1995]

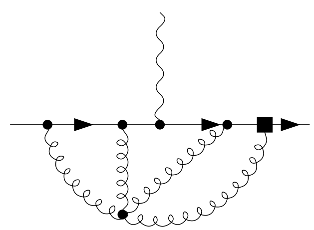
NNLO



[Gorbahn, Haisch, 2004]



[Czakon, Haisch, MM, 2006]
$\sim 2 \times 10^4$ diagrams,
-4% effect on $\mathcal{B}_{s\gamma}^{\rm SM}$



[Gorbahn, Haisch, MM, 2005]

Since 2006, all the Wilson coefficients $C_1(\mu_b), \ldots, C_8(\mu_b)$ are known at the NNLO.