49. Zjazd Fizyków Polskich,
Katowice, 10 września 2025

S. Sarkar, A. Orthey, RA,
arXiv:2312.04405

S. Sarkar, C. Datta,
S. Halder, RA,
PRL 2025

# Bell nonlocality and its applications in certification of quantum states and measurements


Remigiusz Augusiak

*Center for Theoretical Physics, Polish Academy of Sciences*



Funded by
the European Union

# Quantum technologies

- New quantum technologies

Quantum computers



Quantum key distribution



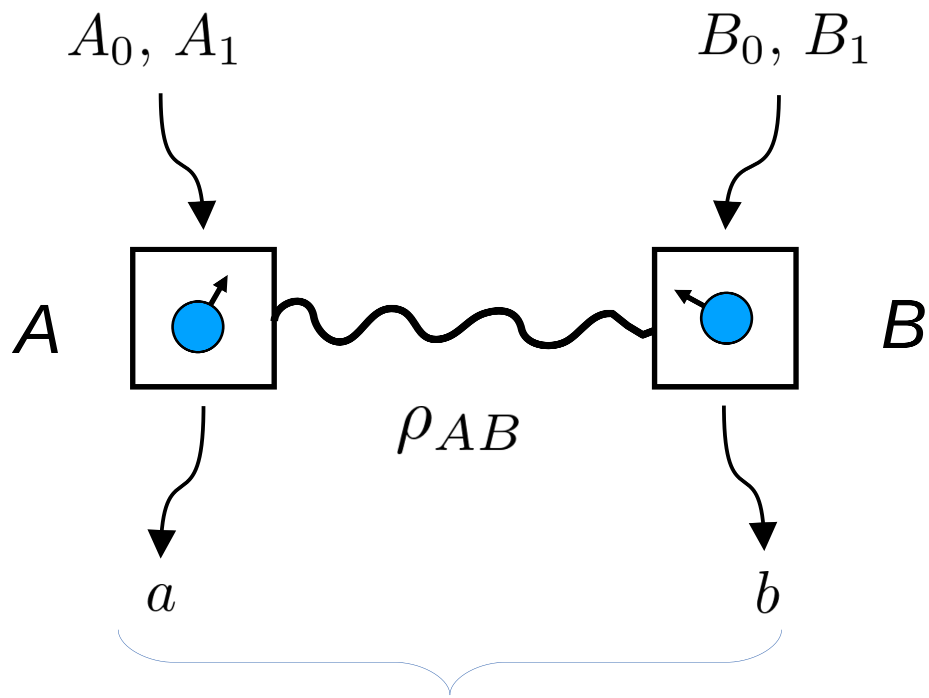
Quantum random number generators

- **Question:** *How to certify that these devices work according to their specification and operate on a given state/perform given measurements*

Device-indendent certification (self-testing)

# Bell nonlocality

- **Bell scenario:** two parties performing measurements on their local systems



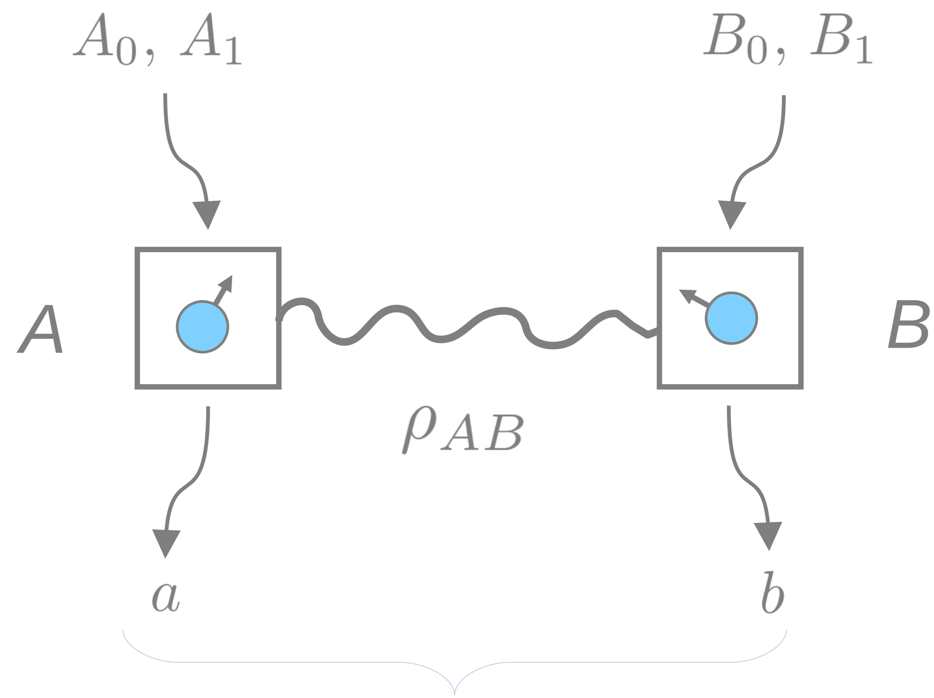

$$A_x^2 = B_y^2 = \mathbb{1}$$

$$a, b = \pm 1$$

$$\langle A_x B_y \rangle = \mathrm{tr}(\rho_{AB} A_x \otimes B_y)$$

# Bell nonlocality

- **Bell scenario:** two parties performing measurements on their local systems




$$A_x^2 = B_y^2 = \mathbb{1}$$

$$a, b = \pm 1$$

$$\langle A_x B_y \rangle = \mathrm{tr}(\rho_{AB} A_x \otimes B_y)$$

- **Bell inequalities:**

$$I_{\mathrm{CHSH}} := \langle A_0 B_0 \rangle + \langle A_0 B_1 \rangle + \langle A_1 B_0 \rangle - \langle A_1 B_1 \rangle \leq 2$$

LHV models/ classical correlations

$$I_{\mathrm{CHSH}} > 2$$

Bell nonlocality

# Bell nonlocality

- Non-locality is a resource for device-independent certification

  - Security of quantum key distribution
    [Ekert, PRL (1991); A. Acín *et al.*, PRL (2007)]

  - Certification of true randomness
    [Pironio *et al.*, Nature (2010); Colbeck, Renner, Nat. Phys. (2012)]

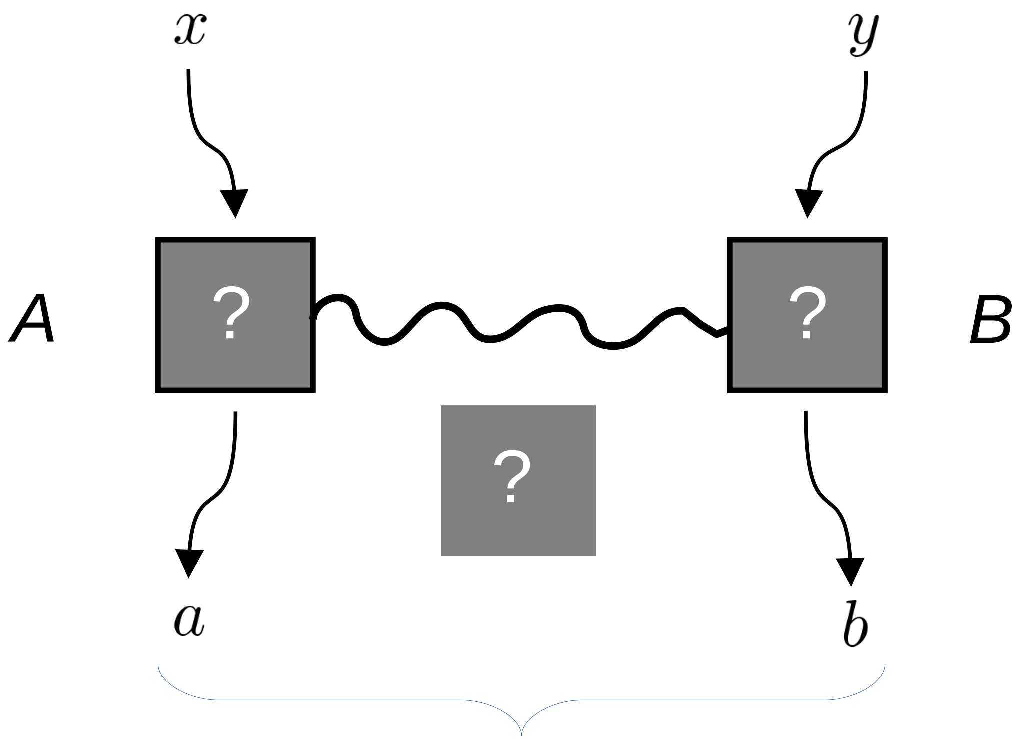
  - Device-independent entanglement certification
    [J.-D. Bancal *et al.*, PRL (2011)]

  - Certification of system's dimension
    [N. Brunner *et al.*, PRL (2008)]

  - Self-testing
    [Mayers, Yao, QIC (2004)]

# DI independent certification

- The idea of device-independent certification

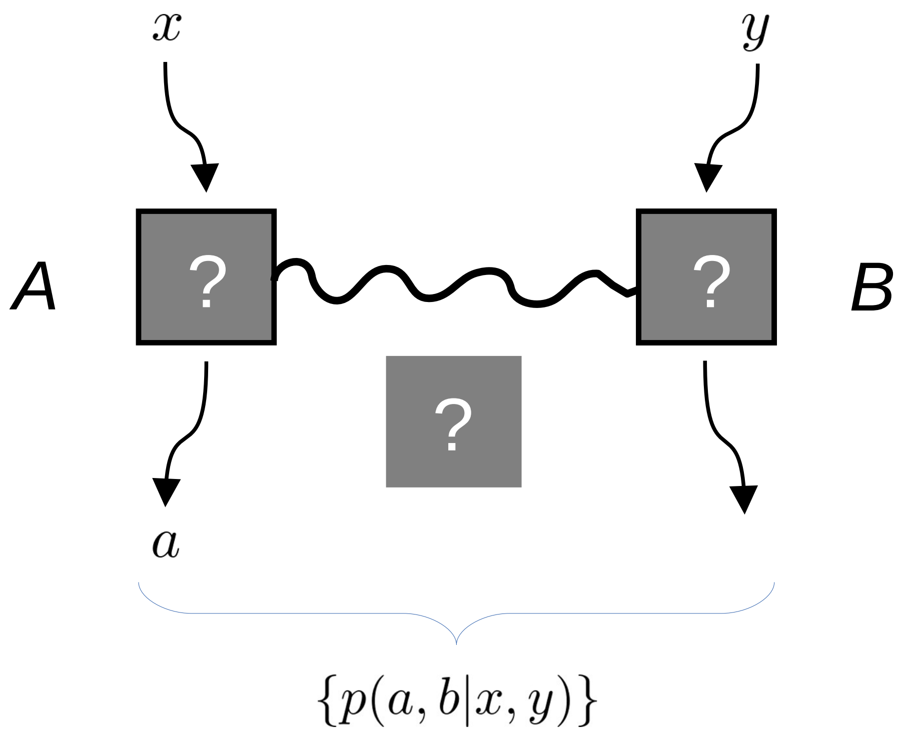



$$\{p(a, b|x, y)\}$$

$$\langle A_x B_y \rangle$$

# DI independent certification

- The idea of device-independent certification



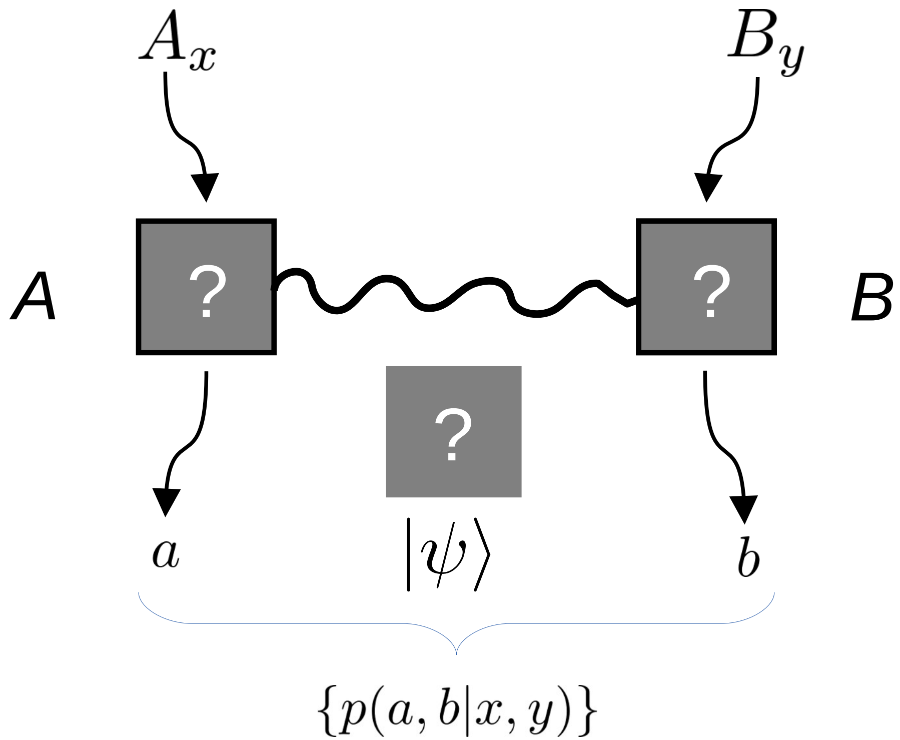

- Given $\{p(a,b|x,y)\}$
- a violation of a Bell inequality

$$\sum_{a,b,x,y} \alpha_{a,b,x,y}\, p(a,b|x,y) = \beta > \beta_C$$

→ Deduce properties of the state and measurements

# Self-testing

- Self-testing




Reference experiment we want to certify

- **Self-testing:**

$$\exists_{U_A, U_B} \qquad (U_A \otimes U_B)|\psi\rangle = |\psi'\rangle \otimes |\text{aux}\rangle \qquad \{|\psi'\rangle, A'_x, B'_y\}$$

$$U_A\, A_x\, U_A^\dagger = A'_x \otimes \mathbb{1}$$

$$U_B\, B_y\, U_B^\dagger = B'_y \otimes \mathbb{1} \qquad |\psi\rangle \sim |\psi'\rangle \quad A_i \sim A'_i$$

Etc.

# Self-testing

[Tsirelson 93; S. Popescu, D. Rohrlich, 92 Mayers, Yao, 2003]

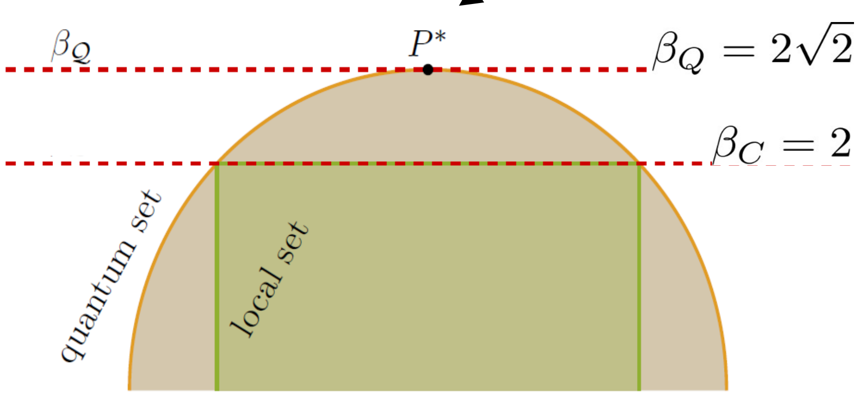
▶ **Example**: *Self-testing from violation of the CHSH Bell inequality*

$$I_{\mathrm{CHSH}} := \langle A_0 B_0 \rangle + \langle A_0 B_1 \rangle + \langle A_1 B_0 \rangle - \langle A_1 B_1 \rangle = 2\sqrt{2}$$

(maximal quantum value)

$$\begin{cases} |\psi_{AB}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \otimes |\mathrm{aux}\rangle \\ A_0 = X \otimes \mathbb{1} \qquad B_0 = \frac{1}{\sqrt{2}}(X + Z) \otimes \mathbb{1} \\ A_1 = Z \otimes \mathbb{1} \qquad B_1 = \frac{1}{\sqrt{2}}(X - Z) \otimes \mathbb{1} \end{cases}$$

(only pure states are self-testable)




$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

# Self-testing entangled states

nature COMMUNICATIONS

ARTICLE

Received 13 Dec 2016 | Accepted 31 Mar 2017 | Published 26 May 2017 DOI: 10.1038/ncomms15485 OPEN

All pure bipartite entangled states can be self-tested

Andrea Coladangelo[1], Koon Tong Goh[2] & Valerio Scarani[2,3]

Letter | Published: 13 February 2023

**Quantum networks self-test all entangled states**

Ivan Šupić, Joseph Bowles, Marc-Olivier Renou, Antonio Acín & Matty J. Hoban

*Nature Physics* **19**, 670–675 (2023) | Cite this article

arXiv:2412.13266 [pdf, other] quant-ph

All pure multipartite entangled states of qubits can be self-tested up to complex conjugation

**Authors:** Maria Balanzó-Juandó, Andrea Coladangelo, Remigiusz Augusiak, Antonio Acín, Ivan Šupić

*How to self-test a mixed state?*

# Self-testing quantum measurements

- Some particular classes of measurements in various scenarios

Article | Open access | Published: 01 August 2024

## All real projective measurements can be self-tested

Ranyiliu Chen, Laura Mančinska & Jurij Volčič

Nature Physics **20**, 1642–1647 (2024) | Cite this article



PHYSICAL REVIEW LETTERS **121**, 250507 (2018)

## Self-Testing Entangled Measurements in Quantum Networks

Marc Olivier Renou,[1,*] Jędrzej Kaniewski,[2,3] and Nicolas Brunner[1]

## Self-Testing in Prepare-and-Measure Scenarios and a Robust Version of Wigner's Theorem

Miguel Navascués, Károly F. Pál, Tamás Vértesi, and Mateus Araújo

Phys. Rev. Lett. **131**, 250802 (2023) - Published 21 December, 2023

*How to self-test a arbitrary measurements?*

# Certification of measurements and states in quantum networks

S. Sarkar, A. C. Orthey, R.A.

arXiv:2312.04405

S. Sarkar, C. Datta, S. Halder, R.A.

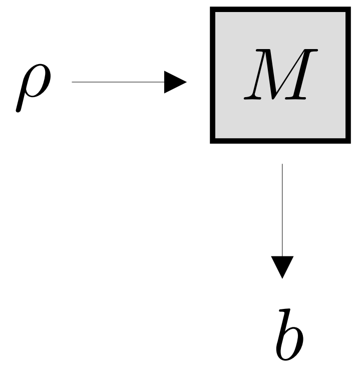
Phys. Rev. Lett. (2025)

# Quantum measurements

- Quantum measurements

$$M = \{R_b\}_b \quad \text{s.t.} \quad R_b \in \mathcal{B}(\mathcal{H})$$

$$R_b \geq 0 \qquad \sum_b R_b = \mathbb{1}$$




$$p(b) = \mathrm{Tr}[R_b \rho]$$

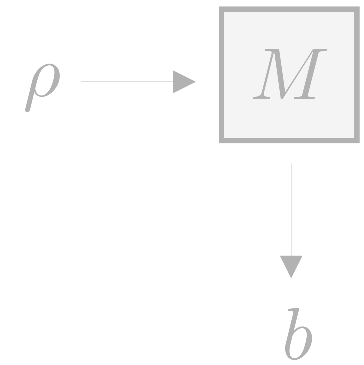
- projective measurements
- POVM's (generalized)

# Quantum measurements

- Quantum measurements

$$M = \{R_b\}_b \quad \text{s.t.} \quad R_b \in \mathcal{B}(\mathcal{H})$$

$$R_b \geq 0 \qquad \sum_b R_b = \mathbb{1}$$

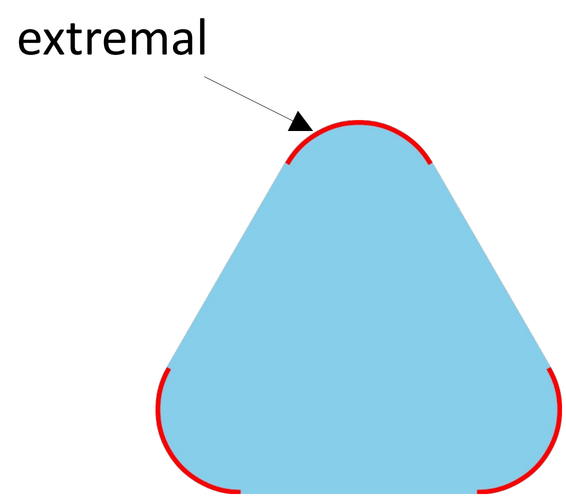



$$p(b) = \mathrm{Tr}[R_b \rho]$$

- projective measurements
- POVM's (generalized)

- Extremal quantum measurements

  - Quantum measurements form a convex set
  - $R_b \neq p M_b^1 + (1-p) M_b^2$
  - projective measurement
  - extremal POVM's




[Wikipedia]

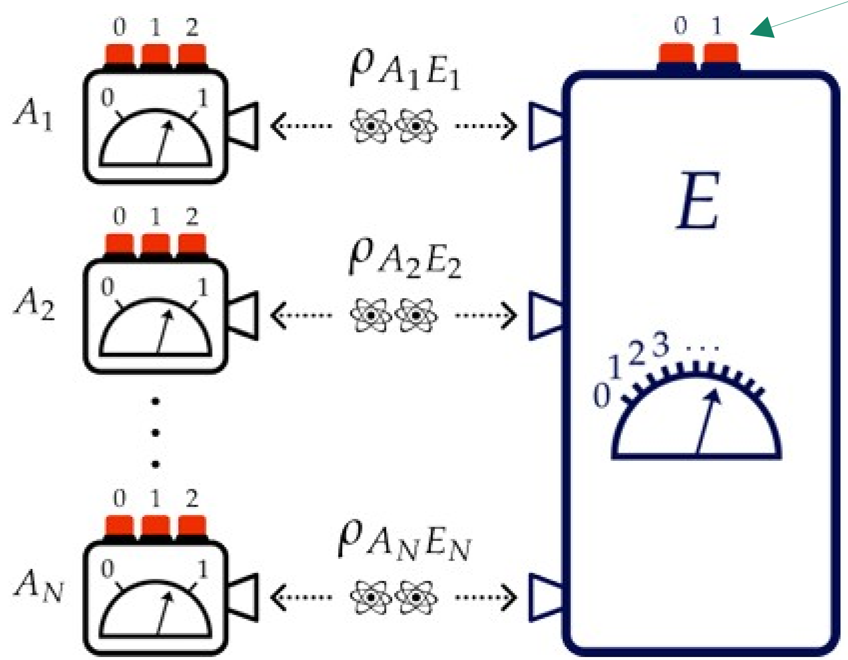
[D'Ariano, Lo Presti, Perinotti, J. Phys. A (2005)]

# Scenario

Star network

three two-outcome measurements

- 1st Eve's meas. to certify the network (GHZ basis)




- 2nd Eve's measeurment to be certified $\{R_b\}_b$

Correlations

$$\{p(a_1,\ldots,a_N,l|x_1,\ldots,x_N,z)\}$$

**Objective**: *Characterize Eve's second measurement*

# Results

- Certification of the sources

$$p(a_1,\ldots,a_N,l|x_1,\ldots,x_N,0)$$

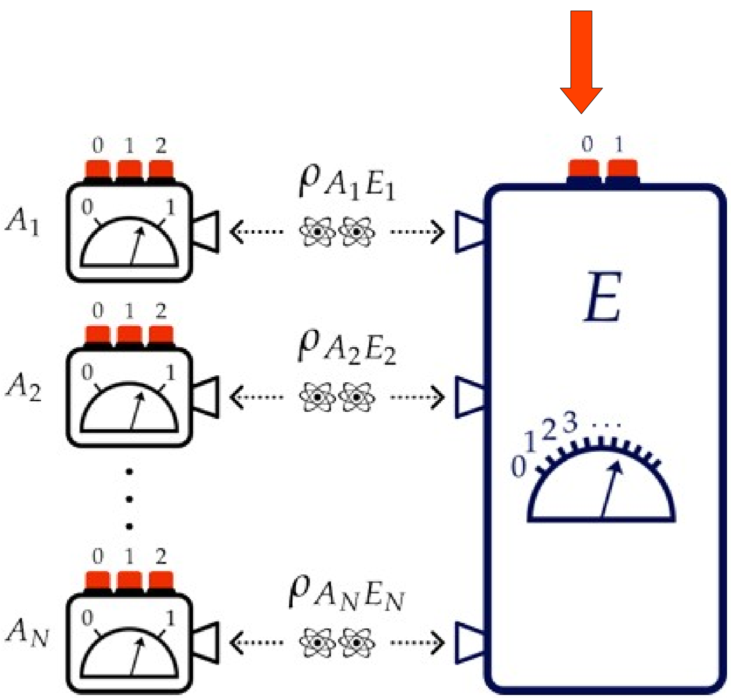
$$\mathcal{I}_l=(-1)^{l_1}\left[(N-1)\left\langle\tilde{A}_{1,1}\prod_{i=2}^{N}A_{i,1}\right\rangle+\sum_{i=2}^{N}(-1)^{l_i}\left\langle\tilde{A}_{1,0}A_{i,0}\right\rangle-(-1)^{l_1}\sum_{i=2}^{N}(-1)^{l_i}\left\langle A_{1,2}A_{i,2}\prod_{\substack{j=2\\ j\neq i}}^{N}A_{j,1}\right\rangle\right]\leqslant\beta_C$$

Maximal violation for any $l$

First Eve's measurement

$$\forall i \qquad \rho_{A_iE_i}\sim|\phi_+\rangle\langle\phi_+|$$

$$|\phi_+\rangle=\frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)$$



# Results

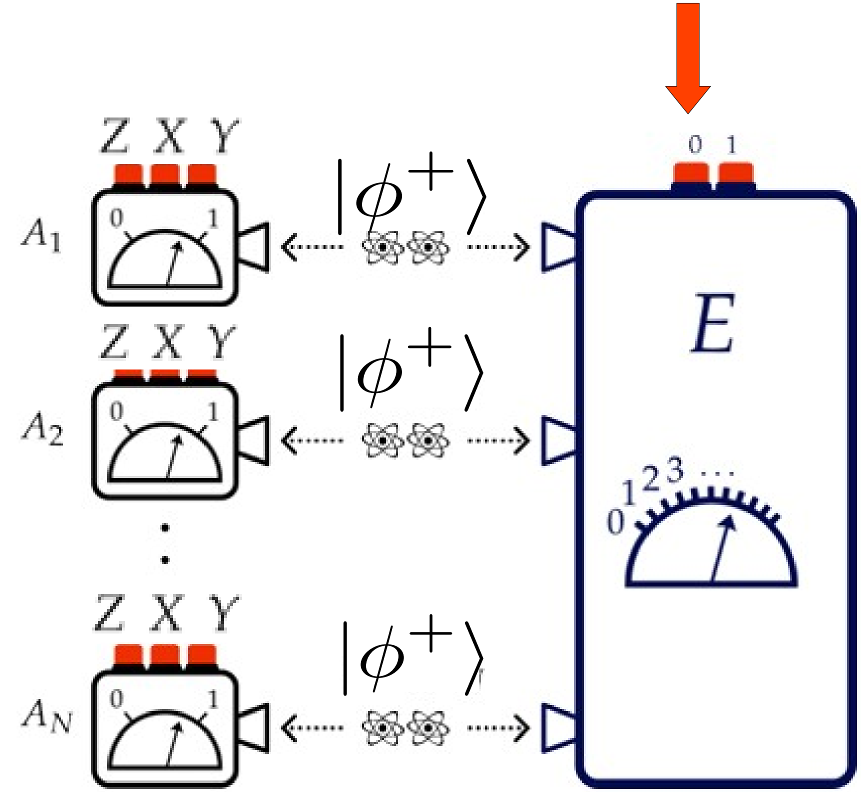
- Certification of external parties' measurements

$$A_{i,0} \sim X \qquad A_{i,1} \sim Z$$

and $\qquad (i = 1, \ldots, N)$

$$A_{i,2} \sim Y \qquad A_{i,2} \sim -Y$$

First Eve's measurement




Tomographically complete set of measurements

⬍

DI tomography of the second Eve's measurement

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad Y = \begin{pmatrix} 0 & -\mathbb{i} \\ \mathbb{i} & 0 \end{pmatrix}$$

# Results

- Certification of the given composite measurement

$$\{R_b\}_b$$

- projective measurements
- extremal POVM's

$$\mathcal{H} = (\mathbb{C}^d)^{\otimes N}$$

- We want to show that

$$R_b \sim R'_b$$

$$U_E \, R_b \, U_E^\dagger = R'_b \otimes \mathbb{1}_{E''}$$

For some given known extremal measurement

$$\{R'_b\}_b$$

$$\mathcal{H} = (\mathbb{C}^2)^{\otimes N}$$

# Results

- Swap of the measurement

$$\mathrm{Tr}[A_i \otimes R_b |\phi^+\rangle\langle\phi^+|] = \mathrm{Tr}\left[A_i R_b^T\right]$$

Pauli matrices

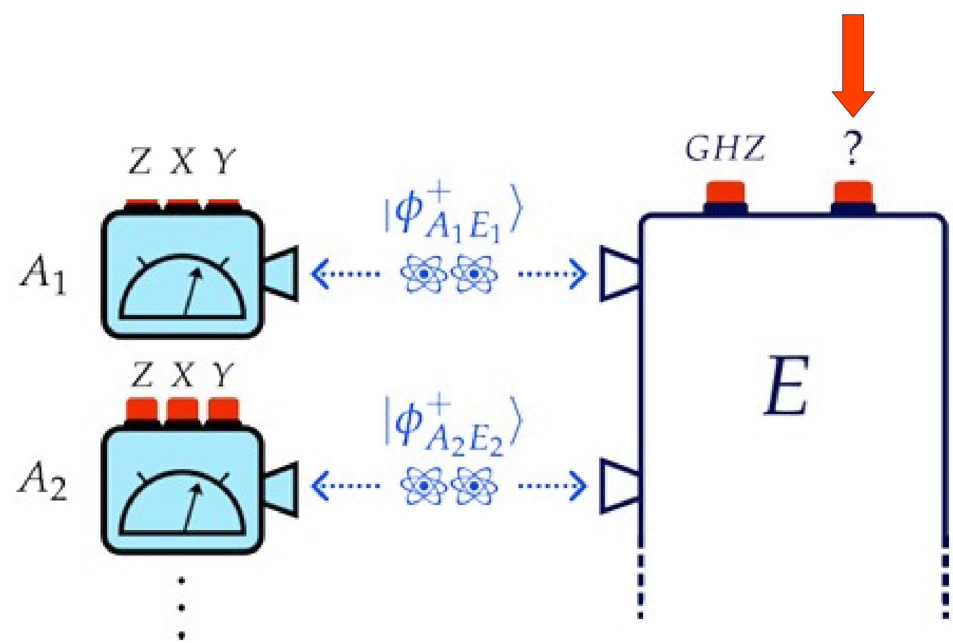
$$\{R_b\}$$

Second Eve's measurement

Local quantum tomography of the measurement

$$R_b = (R_b')^* \otimes \mathbb{1} \quad \text{or} \quad R_b = R_b' \otimes \mathbb{1}$$

$$A_{i,2} \sim Y \qquad\qquad A_{i,2} \sim -Y$$




- Generalization to any extremal measurement on $\mathbb{C}^D \ (D \leq 2^N)$

# Cartification of arbitrary measurements

- What about arbitrary nonextremal measurement

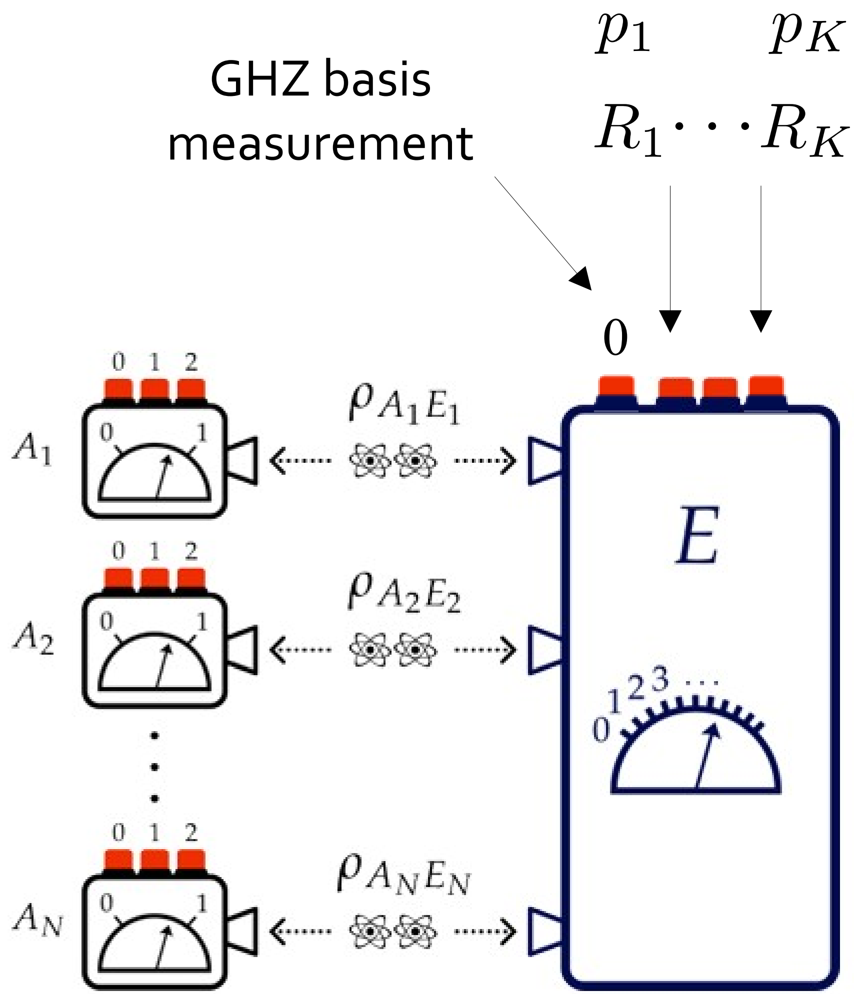
$$M = \{M_b\}$$ nonextremal

$$\Downarrow$$

$$M = \sum_{i=1}^{K} p_i R_i$$

$$R_i = \{R_b^{(i)}\}$$ extremal

$$M_b = \sum_i p_i R_b^{(i)}$$



# Certification of quantum states

- Certified preparation ofquantum states states (with post-selection)
  - pure states

$$\{M_b\} \quad \text{s.t.} \quad M_0 = |\psi\rangle\langle\psi|$$

  - mixed states

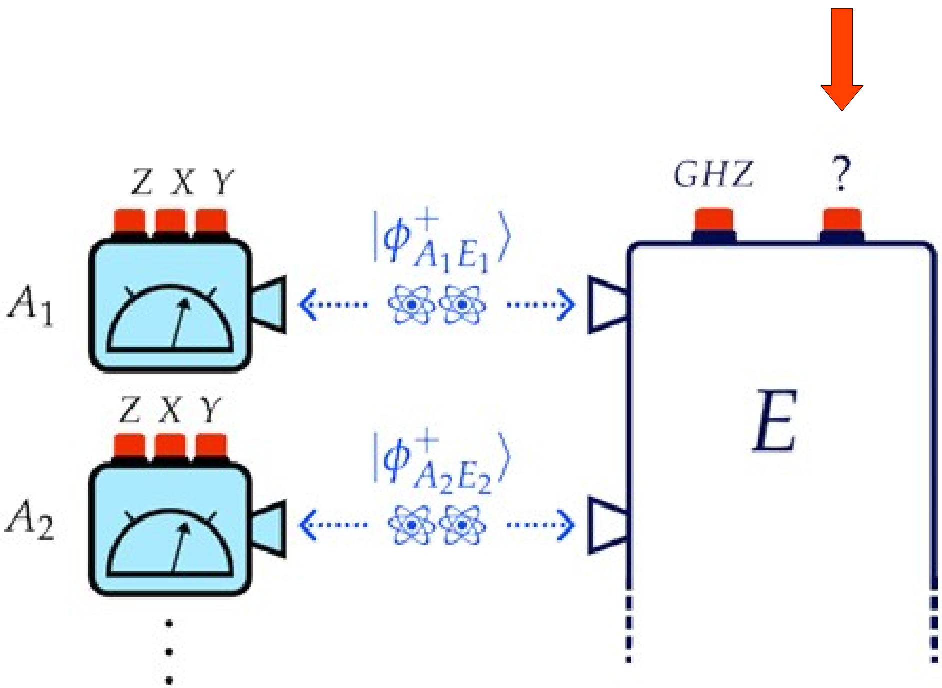
$$\rho = \sum_k p_k |\psi_k\rangle\langle\psi_k| \quad \text{on} \quad \mathbb{C}^d$$

$$\{M_k, N_b\} \quad \text{s.t.} \quad M_k = p_k |\psi_k\rangle\langle\psi_k|$$

$3d$ – outcome extremal POVM

Impossible in the standard Bell scenario!

Second Eve's measurement



# Summary and outlook

- A universal scheme for certification of quantum measurements (projective and extremal)
- The schemes allows for (indirect) certification of mixed states

- Explore how robust are the schemes to noises and experimental imperfections

$$I_l \geq \beta_Q - \epsilon \quad (\epsilon > 0) \quad \stackrel{?}{\Longrightarrow} \quad \|\Lambda(R_l) - M_l\| \leq f(\epsilon)$$

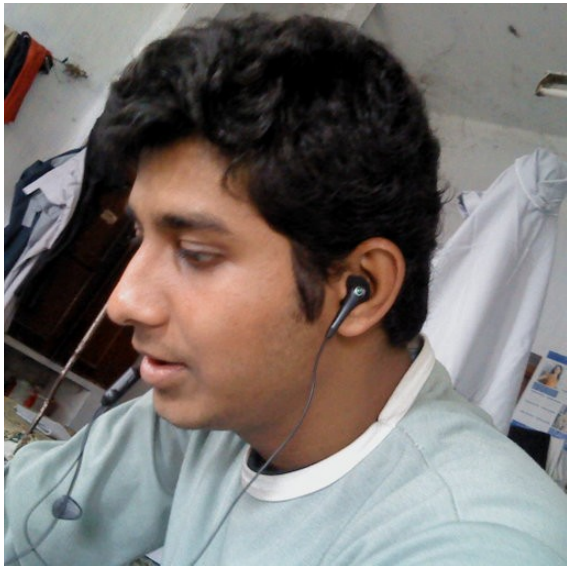
- Extension to arbitrary quantum maps → certification of q. computations
- Make the schemes more practical (lower number of measurements, outcomes etc.)
- Relax the assumption that the sources are independent

# The team

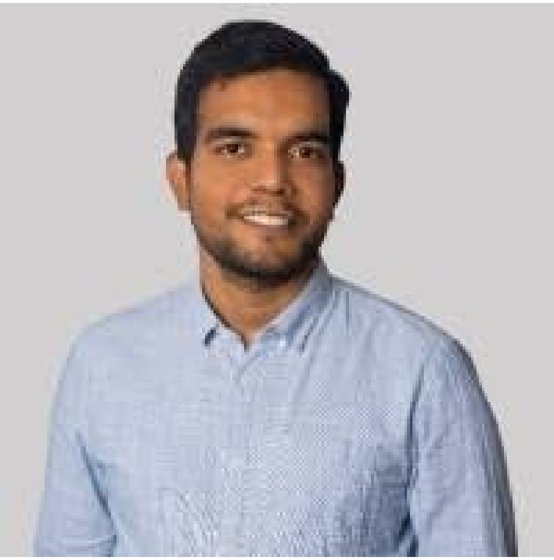
Shubhayan Sarkar

Alexandre Orthey

Saronath Halder



Chandan Datta



Gautam Sharma

# Call for Group Leader of the New Quantum Modelling Group


## at the Center for Theoretical Physics of the Polish Academy of Sciences in Warsaw

**within the ERA Chair project EUCENTRAL financed by the European Union**


**The call will open in 2026**

This project is funded by the European Union under Horizon Europe (project No. 101186579)

EU Central
Modelling Center for Quantum Technologies

Funded by the European Union

cft

HR EXCELLENCE IN RESEARCH