High–precision luminosity at e^+e^- colliders: theory status and challenges

Guido Montagna

Dipartimento di Fisica, Università di Pavia & INFN, Sezione di Pavia
guido.montagna@pv.infn.it

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Ustron, Matter to the Deepest

Based on work with C.M. Carloni Calame, O. Nicrosini, F. Piccinini *et al.*

Luminosity at e^+e^- colliders: Bhabha scattering

- Luminosity $\mathcal{L}$: machine parameter underlying any cross section measurement

$$\boxed{\sigma = \frac{N}{\mathcal{L}}}$$

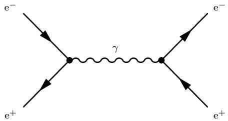
- At e^+e^- colliders, $\mathcal{L}$ can be precisely determined using an appropriate reference process

$$\boxed{\mathcal{L} = \frac{N_{\rm obs}}{\sigma_{\rm theory}}}$$

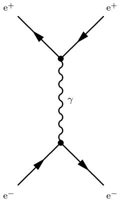
$N_{\rm obs}$: small exp. error

$\sigma_{\rm theory}$: precise theory input

- Best reference process: **QED Bhabha scattering**



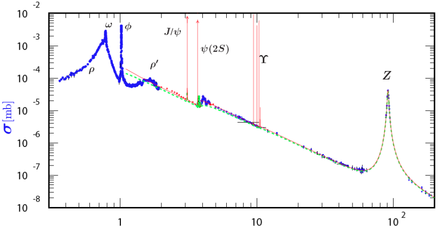
LEP: **small–angle Bhabha**
TLEP/ILC/CEPC



Flavor factories: **large–angle Bhabha**

The quest for precision

Flavor factories



- Luminosity measured with $0.1 \div 1\%$ precision
- Measurement of $\sigma_{\rm had} \longrightarrow g-2$ and $\Delta\alpha_{\rm had}(q^2)$

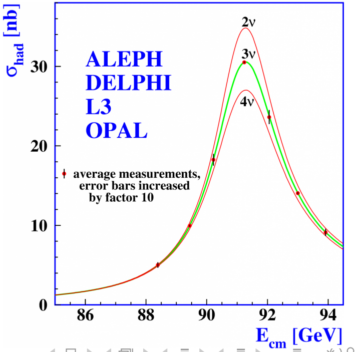
$a_\mu^{\rm exp.} - a_\mu^{\rm th.} \sim 3-4\sigma$ $\qquad a_\mu \doteq (g-2)_\mu/2$

$M_W^{\rm exp.} - M_W^{\rm SM} \sim 2\sigma$ $\qquad M_W$: W mass

LEP

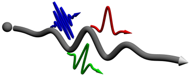
- Luminosity measured with sub–per mille precision
- Measurement of $\sigma^0_{\rm had} \longrightarrow$ number of neutrinos

$N_\nu^{\rm exp.} - 3 \sim 2\sigma$ (theory dominated)



Luminosity and radiative corrections

- **Precision luminosity** $\longrightarrow$ **precision calculations**, including QED radiative corrections



- QED corrections enhanced by large collinear logarithms $L = \ln(Q^2/m_e^2)$

LO	α^0		
NLO	αL	α	
NNLO	$\frac{1}{2}\alpha^2 L^2$	$\frac{1}{2}\alpha^2 L$	$\frac{1}{2}\alpha^2$
h.o.	$\sum_{n=3}^{\infty} \frac{\alpha^n}{n!} L^n$	$\sum_{n=3}^{\infty} \frac{\alpha^n}{n!} L^{n-1}$	$\cdots$

$L = \log(s/m_e^2) \simeq 15$ — Large–angle Bhabha at flavor factories

$L = \log(|t|/m_e^2) \simeq 17$ — Small–angle Bhabha at LEP and TLEP–Z

$L = \log(|t|/m_e^2) \simeq 20$ — Small–angle Bhabha at TLEP above $t\bar{t}$ threshold

- **Monte Carlo generators** needed for
  - realistic simulations
  - data–theory comparison under complex event selection criteria

Monte Carlo ingredients

- Fixed–order: complete NLO corrections
- QED resummation: collinear Structure Functions, Parton Shower, exclusive exponentiation (YFS)
- Matching: NLO $\otimes$ resummation $\longrightarrow$ partial inclusion of $\mathcal{O}(\alpha^2 L)$ photonic corrections at NNLO
- Vacuum polarization
- $Z-$exchange diagrams (high energies)

set up	a.	b.	c.	d.
$\delta_{\rm NLO}$	-11.61	-14.72	-16.03	-19.57
$\delta_{\rm NLO}^{\rm non\text{-}log}$	-0.34	-0.56	-0.34	-0.56
$\delta_{\rm HO}$	0.39	0.82	0.73	1.44
$\delta_{\alpha^2 L}$	0.04	0.08	0.05	0.10
$\delta_{\rm VP}$	1.76	2.49	4.81	6.41

Size of radiative corrections (in per cent) to the Bhabha cross section at meson factories from BabaYaga@NLO. Bare e^+/e^-

a. / b. $\sqrt{s} \simeq 1$ GeV, $E_{\rm min} = 0.8\, E_{\rm beam}$, $\xi_{\rm max} = 10^\circ$, $20^\circ < \theta_\pm < 160^\circ / \, 55^\circ < \theta_\pm < 125^\circ$
c. / d. $\sqrt{s} = 10$ GeV, $E_{\rm min} = 0.8\, E_{\rm beam}$, $\xi_{\rm max} = 10^\circ$, $20^\circ < \theta_\pm < 160^\circ / \, 55^\circ < \theta_\pm < 125^\circ$

Luminosity at flavor factories: generators

Luminosity measured with $0.1 \div 1\%$ precision using **large–angle Bhabha** (and $e^+e^- \to \gamma\gamma$) as reference process, **simulated with two independent generators**

Generator	Processes	Theory	Accuracy
BabaYaga 3.5	$e^+e^-, \gamma\gamma, \mu^+\mu^-$	QED Parton Shower	$\sim 0.5\%$
BabaYaga@NLO	$e^+e^-, \gamma\gamma, \mu^+\mu^-$	$\mathcal{O}(\alpha)$ + QED PS	$\sim 0.1\%$
BHWIDE	e^+e^-	$\mathcal{O}(\alpha)$ YFS	$\sim 0.1\%$
MCGPJ	$e^+e^-, \gamma\gamma, \mu^+\mu^-$	$\mathcal{O}(\alpha)$ + coll. SF	$\sim 0.2\%$

Reference MC – Babayaga@NLO

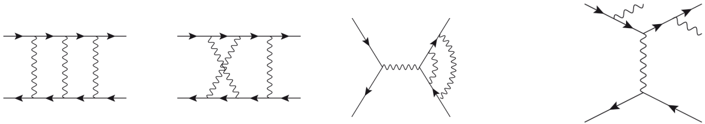
- BabaYaga 3.5/BabaYaga@NLO `http://www2.pv.infn.it/~hepcomplex/babayaga.html`
  Used by BaBar, Belle, BESIII, CLEO, KEDR and KLOE.
  Carloni Calame *et al.*, 2000 / 2006
- BHWIDE `http://placzek.web.cern.ch/placzek/bhwide/`
  Used by BaBar, BESIII, KEDR, KLOE and SND.
  Jadach, Placzek and Ward, 1997
- MCGPJ `http://cmd.inp.nsk.su/~sibid/`
  Used by CMD, Belle and SND.
  Arbuzov *et al.*, 2005 / Eidelman *et al.*, 2011

Sources of uncertainty and Bhabha at NNLO in QED

- **Sources of uncertainty**:
  - Technical precision: bugs, approximations in numerical algorithms ...
  - Theoretical precision: vacuum polarization (parametric, driven by $\sigma_{\rm had}$) and incomplete NNLO corrections

- **NNLO QED corrections to Bhabha available $\longrightarrow$ benchmark for MC accuracy**
  - Photonic corrections (dominant contribution)

Penin, 2005 / 2006
Becher and Melnikov, 2007



  - Electron loop corrections

Bonciani *et al.*, 2004 / 2005
Actis *et al.*, 2007

  - Heavy fermion and hadronic loops

Becher and Melnikov, 2007 / Bonciani *et al.*, 2008
Actis *et al.*, 2008 / Kühn and Uccirati, 2009

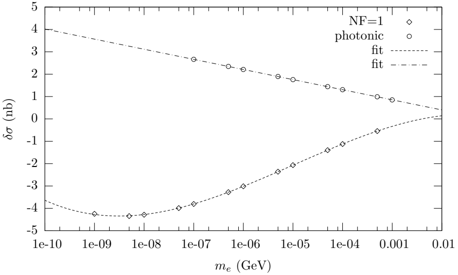
  - Soft+Virtual corrections to hard bremsstrahlung

Jadach, Ward *et al.*, 1996, 2001
Actis *et al.*, 2010

Comparison to NNLO: accuracy of BabaYaga@NLO

□ NNLO Photonic (Penin)

Carloni Calame *et al.*, 2006



- ▷ $\delta\sigma \doteq \sigma^{\rm NNLO}_{\rm Penin} - \sigma^{\rm NNLO}_{\rm BabaYaga@NLO}$
  $\delta\sigma \le 0.2$ ‰ $\sigma_{\rm LO}$
- ▷ $\delta\sigma/\sigma_{\rm LO} \propto \alpha^2 L$ and infrared–safe

□ Leptonic and hadronic pairs

Carloni Calame *et al.*, 2011

	$\sqrt{s}$		$\sigma_{\rm BY}$(nb)	$S_{e^+e^-}$ [‰]	$S_{\rm lep}$ [‰]	$S_{\rm had}$ [‰]	$S_{\rm tot}$ [‰]
KLOE	1.020	NNLO		-3.935(5)	-4.472(5)	1.02(4)	-3.45(4)
		BabaYaga	455.71	-3.445(2)	-4.001(2)	0.876(5)	-3.126(5)
BES	3.650	NNLO		-1.469(9)	-1.913(9)	–1.3(1)	-3.2(1)
		BabaYaga	116.41	-1.521(4)	-1.971(4)	-1.071(4)	-3.042(5)
BaBar	10.56	NNLO		-1.48(2)	-2.17(2)	-1.69(8)	-3.86(8)
		BabaYaga	5.195	-1.40(1)	-2.09(1)	-1.49(1)	-3.58(2)
Belle	10.58	NNLO		-4.93(2)	-6.84(2)	-4.1(1)	-10.9(1)
		BabaYaga	5.501	-4.42(1)	-6.38(1)	-3.86(1)	-10.24(2)

- ▷ BabaYaga@NLO accuracy (well) below 1‰

Luminosity at flavor factories: total theoretical uncertainty

Updated from: Actis *et al.*, EPJ C66 (2010) 585
arXiv:0912.0749

Source of unc. (%)	1–2 GeV	BESIII	BaBar/Belle
	Vacuum Polarization [1]		
$\|\delta_{\rm VP}\|$ [Jegerlehner]	—	0.01	0.03
$\|\delta_{\rm VP}\|$ [HMNT]	0.02	0.01	0.02
	NNLO		
$\|\delta^{\alpha^2}_{\rm photonic}\|$ [2]	0.02	0.02	0.02
$\|\delta^{\alpha^2}_{\rm pairs}\|$ [3]	0.03	0.02	$0.03 \div 0.07$
$\|\delta^{\alpha^2}_{\rm SV,H}\|$ [4]	0.05 / 0.03	0.05 / 0.03	0.05 / 0.03
$\|\delta^{\alpha^2}_{\rm HH}\|$	—	—	—
$\|\delta_{\rm total}\|$ **quadrature**	$\mathbf{0.07/0.05}$	$\mathbf{0.06/0.04}$	$\sim \mathbf{0.07 \div 0.09}$

- ▷ **Comparable to luminosity theoretical uncertainty at LEP**
- ▷ In proximity of ψ/Υ's resonances, accuracy deteriorates: $\mathcal{L}$ affected by $\sigma_{\rm had}$ uncertainty!

[1] From $\Delta\alpha_{\rm had}(q^2) \pm \delta_{\rm had}$, $\delta_{\rm had}$ returned by VP parameterization.

[2] Carloni Calame *et al.*, 2006: BabaYaga@NLO vs. NNLO photonic by Penin

[3] Carloni Calame *et al.*, 2011: BabaYaga@NLO vs. NNLO (leptonic and hadronic) pairs by DESY Zeuthen – Katowice

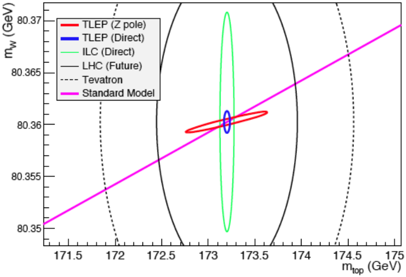
[4] Estimated from LEP studies by Jadach, Ward *et al.*
Conservative, WG Report / Less conservative, Jadach *et al.* 1999, 2001

TLEP and luminosity

The TLEP Design Study Working Group, M. Bicer *et al.*
JHEP 1401 (2014) 164, `arXiV:1308.6176`

TLEP: e^+e^- circular collider at c.m. energies from 90 to 350 GeV for SM precision tests after the Higgs discovery

- ▷ $\sqrt{s} \simeq 90$ GeV: Z pole (Tera$Z$)
- ▷ $\sqrt{s} \simeq 160$ GeV: WW threshold (Oku$W$)
- ▷ $\sqrt{s} \simeq 240$ GeV: ZH production threshold
- ▷ $\sqrt{s} \simeq 350$ GeV: $\mathrm{t\bar{t}}$ threshold (MegaTop)

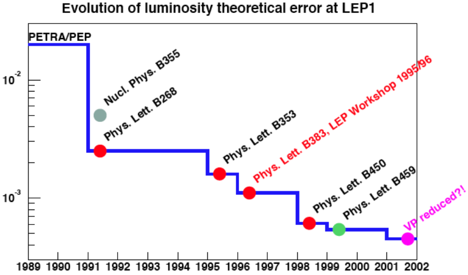


To the extent that the aforementioned issues are properly addressed and solved, there should be no significant difficulty to achieve luminosity measurements with an experimental precision similar to that obtained at LEP, typically a few times 10^{-4}. At the Z peak it would be of interest to achieve even better precision, e.g., for the measurement of the invisible width hence the number of light neutrinos, which will require a more precise construction of the luminometers. The main limitation on the luminosity measurement, however, would presently come from the theoretical calculation of the low angle Bhabha cross section. Clearly, progress in this aspect would pay great dividends.

LEP experience will be the benchmark for future theoretical work, **with accuracy at 10^{-4} level**

Luminosity at LEP

Jadach, `arXiv:hep-ph/0306083`



- ▷ $\mathcal{L}$ measured with $\simeq \mathbf{0.3 \div 0.4‰}$ **exp. accuracy** using **small–angle Bhabha** $(3^\circ \div 6^\circ\ /\ 1^\circ \div 3^\circ)$
- ▷ 1990 – 2000: theoretical uncertainty reduced from **1% to** $\simeq \mathbf{0.5 \div 0.6‰}$

Reference MC – BHLUMI $\mathcal{O}(\alpha)$ YFS. Used by all four LEP collaborations.

Jadach, Ward *et al.*, 1989 / 1992 / 1997

Other generators/codes used in the assessment of the th. uncertainty at LEP

- ☐ OLDBIS+LUMLOG. $\mathcal{O}(\alpha)$ + LL approx. — Jadach *et al.*, 1990
- ☐ BHAGEN95. $\mathcal{O}(\alpha)$ + coll. SF — Caffo, Czyz and Remiddi, 1995 / 1997
- ☐ SABSPV. $\mathcal{O}(\alpha)$ + coll. SF — Cacciari *et al.*, 1995 / Montagna *et al.*, 1996
- ☐ NNLBHA. Semi–analytical to $\mathcal{O}(\alpha^2 L)$ accuracy. — Arbuzov *et al.*, 1995

Luminosity at LEP: total theoretical uncertainty

Adapted from Jadach, `ArXiv:hep-ph/0306083`
Excluding technical precision

Type of correction/uncert.	Ref. [1]	Refs. [2,3]	Ref. [4]	Ref. [5]
Missing photonic $\mathcal{O}(\alpha^2 L)$	0.15%	0.10% [5]	0.027% [6]	0.027%
Missing photonic $\mathcal{O}(\alpha^3 L^3)$	0.008%	0.015% [7]	0.015%	0.015%
Vacuum polarization	0.04%	0.04% [8]	0.04%	0.04%
Light pairs	0.03%	0.03%	0.03%	0.01% [9]
$Z-$exchange	0.015%	0.015% [10]	0.015%	0.015%
Total	**0.16%**	**0.11%**	**0.061%**	**0.054%**

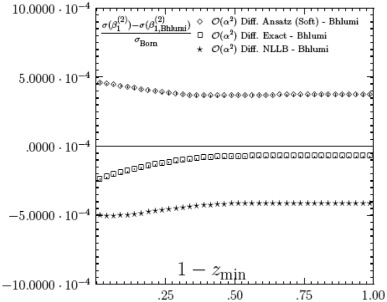
[1] **Pre–LEP2 Workshop**: Jadach *et al.*, Phys. Lett. B353 (1995) 362
[2] **LEP2 Workshop**: Jadach, Nicrosini *et al.*, `hep-ph/9602393`
[3] **LEP2 Workshop**: Arbuzov *et al.*, Phys. Lett. B383 (1996) 238 [`hep-ph/9605239`]
[4] Ward, Jadach *et al.*, Phys. Lett. B450 (1999) 262 [`hep-ph/9811245`]
[5] Montagna *et al.*, Nucl. Phys. B547 (1999) 39 [`hep-ph/9811436`]
Phys. Lett. B459 (1999) 649 [`hep-ph/9905235`]

5 From comparisons of independent codes differing in $\mathcal{O}(\alpha^2 L)$ contents
6 New analysis of $\mathcal{O}(\alpha^2)$ sub–leading contributions in BHLUMI using NNLO calculations
7 Estimate of missing LL contributions in BHLUMI
8 Induced by hadronic contribution to $\Delta\alpha$
9 Exact calculation of NNLO leptonic pairs
10 Uncertainty in QED corrections to $\gamma(t) - Z(s)$ interference

TLEP: what next?

LEP theoretical uncertainty can be reduced **by a factor of** $2 \div 3$ with

- **Improvements in BHLUMI.xx**
  - Inclusion of missing subleading $\mathcal{O}(\alpha^2)$ photonic + pairs corrections
  - New $\Delta\alpha_{\rm had}$ parameterizations
- **New complete NNLO generators**
  - + QED resummation beyond $\mathcal{O}(\alpha^2)$
  - + $Z-$exchange
- New tests of physical (weak corrections) + technical precision



SABH at TLEP / $3^\circ \div 6^\circ$

Energy	TLEP $-Z$	TLEP WW	TLEP ZH	TLEP $t\bar{t}$
$Z-$exchange LO	$+0.064\%$	-0.062%	-0.044%	-0.030%
$\Delta\alpha$	$+5.17\%$	$+6.27\%$	$+7.14\%$	$+7.99\%$
$\delta\Delta\alpha_{\rm had}$	$\pm\mathbf{0.021\%}$	$\pm\mathbf{0.027\%}$	$\pm\mathbf{0.030\%}$	$\pm\mathbf{0.032\%}$

▷ **Theoretical accuracy limited by $\Delta\alpha_{\rm had}$ uncertainty!**

Summary

- Precision tests of the SM at e^+e^- colliders require **high–precision luminosity measurements**

- High–precision luminosity measurements rely on **precision calculation of the Bhabha process** encoded into MC generators

- The accuracy of the theoretical predictions (LEP and flavor factories) is at the **sub–per mille level and robust** (Bhabha at NNLO in QED)

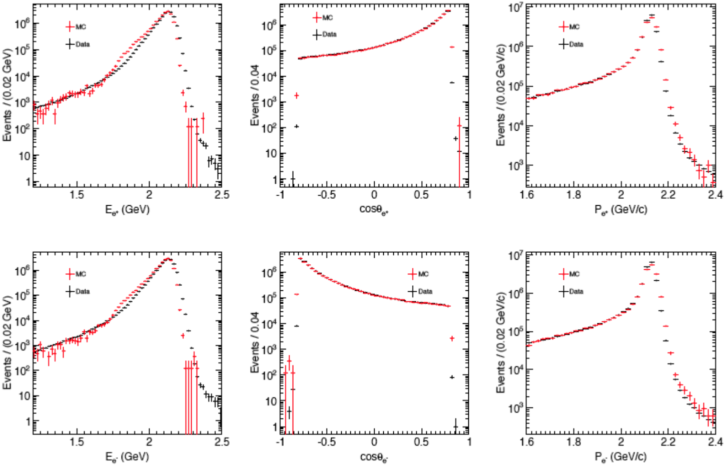
- For next–generation experiments (TLEP/ILC/CEPC), the LEP theoretical uncertainty can be reduced by **a factor of** $2 \div 3$ with
  - improvements in existing reference codes (BHLUMI)
  - new NNLO + h.o. generators (e.g. BabaYaga@NNLO)
  - new tests of physical + technical precision

- For the challenging 10^{-4} precision, all the theoretical ingredients are at hand but the **accuracy is presently limited by the** $\Delta\alpha_{\rm had}$ **uncertainty**
  $\hookrightarrow$ **new** $\sigma_{\rm had}$ **data needed**

Extra

Luminosity measurement using BabaYaga

BESIII Coll., `ArXiv:1503.03408`
$\mathcal{L}$ to 1% precision using Bhabha events
MC = BabaYaga 3.5

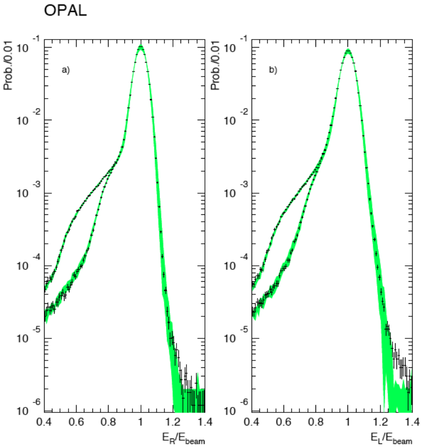


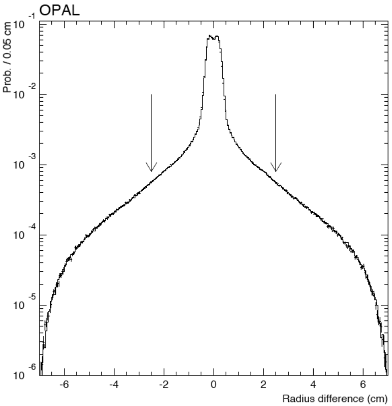
▷ New BESIII measurement of $e^+e^- \to \pi^+\pi^-$ cross section based on $\mathcal{L}$ measurement with total 5‰ uncertainty (BabaYaga@NLO)

BESIII Coll., `ArXiv:1507.08188`

Luminosity measurement using BHLUMI

Opal Coll., `ArXiv:hep-ex/9910066`
$\mathcal{L}$ with 3.4×10^{-4} uncertainty
MC: BHLUMI 4.04





OPAL data vs. BHLUMI predictions for the energy distribution of the small–angle Bhabha events in the right and left calorimeter (left plot) and the acoplanarity distribution (right plot).