Calculation of multi-scale, multi-loop integrals with SecDec 3

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Project in collaboration with
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Matter to the deepest, Ustroń, Sep 14th, 2015

http://secdec.hepforge.org/

Precise theoretical predictions in the LHC era

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- Numerical methods are in general easier to automate, multiple scales do not pose a problem
  $\rightarrow$ highly interesting in light of the current need for predictions involving massive particles!

Numerical evaluation of Feynman integrals

Many people are/have been working on PURELY numerical methods, e.g. Anastasiou/Beerli/Kunszt et al., Becker/Reuschle/Weinzierl et al., Binoth/Heinrich et al., Boughezal/Melnikov/Petriello et al., Czakon et al., Freitas et al., Kurihara et al., Nagy/Soper et al., Passarino et al., ...

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  - Speed / accuracy

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- Problems beyond the one-loop level mainly are
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  - Numerical convergence in the presence of integrable singularities (e.g. thresholds) (solved with SecDec 2)
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- Problems beyond the one-loop level mainly are
  - Extraction of IR and UV singularities (solved with SecDec 1)
  - Numerical convergence in the presence of integrable singularities (e.g. thresholds) (solved with SecDec 2)
  - Speed / accuracy (further improved in SecDec 3)

Public codes using the sector decomposition method

Idea and method of sector decomposition pioneered by
Hepp '66, Denner & Roth '96, Binoth & Heinrich '00

Public codes:

- sector_decomposition (uses GiNaC) Bogner & Weinzierl '07
  supplemented with `CSectors` Gluza, Kajda, Riemann, Yundin '10
  for construction of integrand in terms of Feynman parameters
- FIESTA* (uses Mathematica, C) A.V. Smirnov, V.A. Smirnov, Tentyukov '08 '09, A.V. Smirnov '13
- SecDec* (uses Mathematica, Fortran/C++)
  Carter & Heinrich '10; SB, Carter, Heinrich '12; SB & Heinrich '13; SB, Heinrich, Jones, Kerner, Schlenk, Zirke '15

*Multi-scale integrals not limited to the Euclidean region
SB, J. Carter & G. Heinrich '12; A.V. Smirnov '13

The program SecDec can deal with...

Feynman graph	or	parametric function

SecDec is a tool to numerically compute

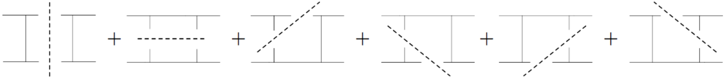
- General **Feynman** integrals for **arbitrary** kinematics and with numerators
- Integrals **matching** a Feynman integral **structure**
- More general **parametric** functions

Feynman loop integrals

- Scalar multi-loop integral in Feynman parametrization

$$G=\frac{(-1)^{N_\nu}}{\prod_{j=1}^{N}\Gamma(\nu_j)}\Gamma(N_\nu-LD/2)\int\limits_0^\infty\prod_{j=1}^{N}\mathrm{d}x_j\;x_j^{\nu_j-1}\,\delta(1-\sum_{l=1}^{N}x_l)\frac{\mathcal{U}^{N_\nu-(L+1)D/2}(\vec{x})}{\mathcal{F}^{N_\nu-LD/2}(\vec{x},s_{ij})}$$

  with $N_\nu=\sum_{j=1}^{N}\nu_j$ in D dimensions with L loops, N propagators to power ν_j
- Feynman integrals with (contracted) numerators of rank R
- $\mathcal{U}$ and $\mathcal{F}$ can be constructed via **topological cuts** or by specifying the individual propagators in momentum space



$$\mathcal{F}=-s_{12}\,x_1x_3-s_{23}\,x_2x_4\;-\;p_1^2\,x_1x_2\;-\;p_2^2\,x_2x_3\;-\;p_3^2\,x_3x_4\;-\;p_4^2\,x_4x_1\,.$$

Modified Feynman loop integrals

More general **user-defined polynomial integrals** matching the Feynman loop integral structure

$$G_{user} = P(\varepsilon) \int_0^1 \prod_{j=1}^{N} \mathrm{d}x_j \; {x_j}^{a_j(\varepsilon)} \; \mathcal{N}(\vec{x}, s_{ij}, \varepsilon) \, \mathcal{U}^{\mathrm{expoU}(\varepsilon)}(\vec{x}, s_{ij}) \; \mathcal{F}^{\mathrm{expoF}(\varepsilon)}(\vec{x}, s_{ij})$$

with a prefactor P and a numerator function $\mathcal{N}$ and exponents a_j

- $\mathcal{U}$ and $\mathcal{F}$ can have negative exponents, also $a_j < 0$ allowed
- integrals without δ-constraint
- user has more responsibility when computing an integral in the physical region

Multi-dimensional parameter integrals

A general parametric function can be

- a phase space integral where IR divergences are regulated dimensionally, e.g.

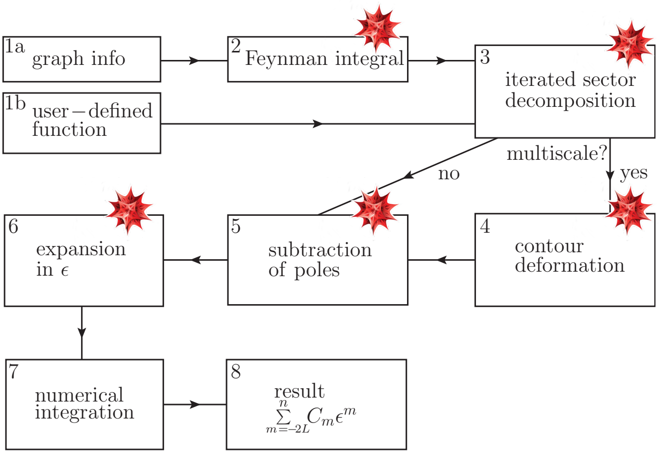
$$\int \mathrm{d}\Phi^D |\mathrm{ME}|^2 \propto \int \mathrm{d}s_{13}\,\mathrm{d}s_{23}\, s_{13}^{-1-\varepsilon}\, \frac{\mathcal{F}(s_{13},s_{23})}{s_{13}+s_{23}}$$
$$\to \int_0^1 \mathrm{d}x\,\mathrm{d}y\, x^{-1-\varepsilon}\, \frac{\mathcal{F}(x,y)}{x+y}$$

- functions of the type of hypergeometric functions, e.g.

$$ {}_3F_2(a_1,\ldots,a_3;b_1,b_2;\beta) \propto$$
$$\iint_0^1 \mathrm{d}x\mathrm{d}y\; x^{a_1-1}(1-x)^{b_1-a_1-1}y^{a_2-1}(1-y)^{b_2-a_2-1}(1-\beta xy)^{-a_3}$$

- NEW in SecDec 3: additional ε-dependent functions $g(\varepsilon,\vec{x})$ can be included (no iterated sector decomposition applied)

Operational sequence of the SecDec 3 program



Numerical integration:

CUBA library Hahn et al. '04 '11 or BASES Kawabata '95

Summary of new features in SecDec version 3

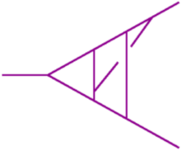
SB, Heinrich, Jones, Kerner, Schlenk, Zirke '15

- Two additional **decomposition algorithms** based on computational geometry (avoid infinite recursion)
- Numerators can be given in terms of **inverse propagators**
- **Linear** propagators can be treated
- ε-dependent symbolic functions allowed in parametric integrals
- Restructured **user input** helps interfacing with reduction programs
- 2 new **integrators** included CQuad, NIntegrate
- Usage of batch systems facilitated, scans over parameter ranges accelerated
- Internal **structure** largely **rewritten**

SecDec is ready for large-scale applications!

Download SecDec 3

http://secdec.hepforge.org/



SecDec

Sophia Borowka, Gudrun Heinrich, Stephen Jones, Matthias Kerner, Johannes Schlenk, Tom Zirke

A program to evaluate dimensionally regulated parameter integrals numerically

home download program user manual faq changelog

NEW: Version 3.0 of the program can be downloaded as SecDec-3.0.7.tar.gz.

User interface in SecDec 3

SecDec needs 3 input files:

- param.input: minimal info needed to run SecDec

```
graph=Box2L
epsord=0
```

- kinem.input: contains point name and numerical values for kinematics

```
p1 -3 -2
p2 1 4
```

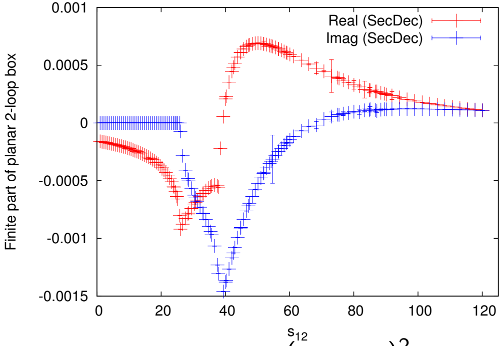
- math.m: graph definition, enhanced flexibility

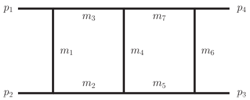
```
momlist={k1,k2};
proplist={ k1^2, (k1+p2)^2,
           (k1-p1)^2, (k1-k2)^2,
           (k2+p2)^2, (k2-p1)^2,
           (k2+p2+p3)^2, (k1+p3)^2 };
powerlist={1,1,1,1,1,1,1,-1};
ExternalMomenta={p1,p2,p3,p4};
externallegs=4;
prefactor=Gamma[1+eps]^2;
KinematicInvariants = {s,t};
Masses={};
ScalarProductRules = {
  SP[p1,p1]->0,
  SP[p2,p2]->0,
  SP[p3,p3]->0,
  SP[p4,p4]->0,
  SP[p1,p2]->s/2,
  SP[p2,p3]->t/2,
  SP[p1,p3]->-s/2-t/2
};
Dim=4-2*eps;
```

All-massive planar 7-propagator 2L box

- 13 independent mass scales, full numerical approach
  $\Rightarrow$ Many scales are not a bottleneck

Finite part





$s_{12} = (p_1 + p_2)^2$

$m_1^2 = 2, \ m_2^2 = 6,$
$m_3^2 = 7, \ m_4^2 = 8,$
$m_5^2 = 9, m_6^2 = 10,$
$m_7^2 = 12, \ p_1^2 = 1,$
$p_2^2 = 3, \quad p_3^2 = 4,$
$p_4^2 = 5, s_{23} = -0.25$

- timings: 10-80 secs (SecDec 2),
  rel. accuracy 10^{-3}, abs. accuracy: 10^{-8}

$s_{12} + s_{23} + s_{13} = (\sum_{i=1}^4 p_i)^2$

SB Jun '14

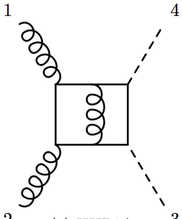
Higgs-boson pair production in gluon fusion

- Interesting for measurement of Higgs-boson self-coupling
- LO (1-loop) known Glover, van der Bij '88
- NLO in $m_t \to \infty$ limit Plehn, Spira, Zerwas '96; Dawson, Dittmaier, Spira '98
- NLO with $m_t \to \infty$ but supplemented with $1/m_t$ expansion Grigo, Hoff, Melnikov, Steinhauser '13
- NNLO in $m_t \to \infty$ limit De Florian, Mazzitelli '13
- NNLO $m_t \to \infty$ with all matching coefficients Grigo, Melnikov, Steinhauser '14
- NNLO $m_t \to \infty$ + NNLL threshold resummation De Florian, Mazzitelli '15
- Full mass dependence in real radiation part + matching to parton shower Frederix, Hirschi, Mattelaer, Maltoni, Torrielli, Vryonidou, Zaro '14; Maltoni, Vryonidou, Zaro '14
- Full top-mass dependence at NLO missing so far!

gg → HH - Two-loop integrals

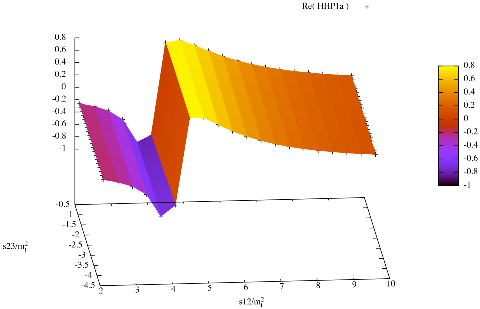
SB, Heinrich, Greiner, Jones, Kerner, Luisoni, Mastrolia,
Schlenk, Schubert, Stoyanov, Di Vita, Zirke

- Requires computation of unknown two-loop integrals
- 4 independent scales: s_{12}, s_{23}, m_H, m_t
- numerical evaluation with SecDec



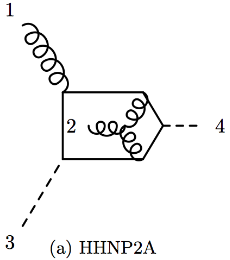
(a) HHP1A

$m_H = 125$ GeV
$m_t = 173$ GeV



Plot by Gudrun Heinrich

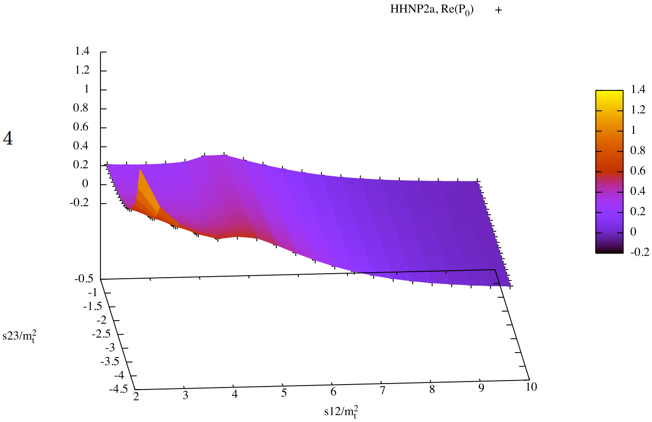
gg → HH - Two-loop integral examples



(a) HHNP2A

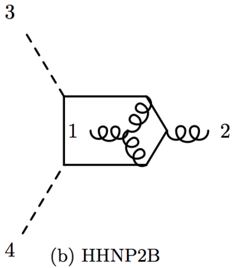
$$I = \frac{P_{-1}}{\varepsilon} + P_0$$

$m_H = 125$ GeV
$m_t = 173$ GeV



Plot by Gudrun Heinrich

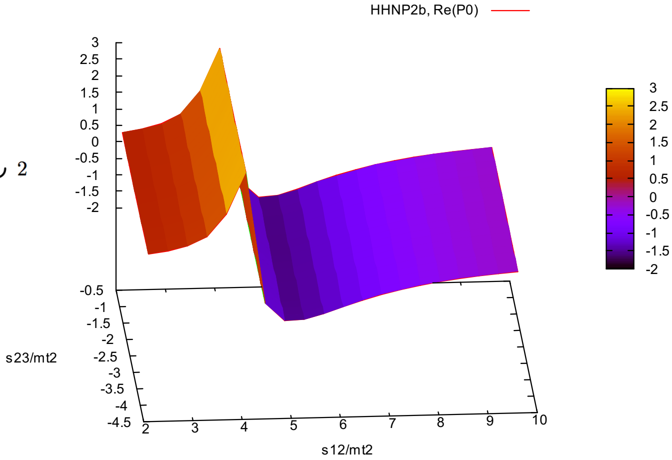
$gg \to HH$ - Two-loop integral examples



(b) HHNP2B

$$I = \frac{P_{-1}}{\varepsilon} + P_0$$

$$m_H = 125\,\text{GeV}$$
$$m_t = 173\,\text{GeV}$$



Plot by Gudrun Heinrich

Summary and Outlook

Summary

- SecDec allows for computation of diverse integrals contributing to scattering amplitudes
- SecDec 3: new decomposition strategies, improved user interface, negative propagator powers and linear propagators allowed, integrators added, efficiency increased
- New features in SecDec 3 prepare for large(r) scale applications

Outlook

- More speed improvements, complex masses
- Interface to other programs, e.g. FIRE, LiteRed, Reduze, Loopedia (see Viktor Papara's talk)
- Application to phenomenologically relevant processes

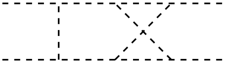
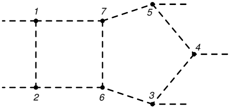
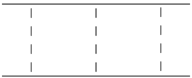
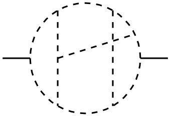
Backup

Install SecDec 3

- **Install**:
  tar xzvf SecDec-3.0.7.tar.gz
  cd SecDec-3.0.7
  make
  (make check)

- **Prerequisites**:
  Mathematica (version 7 or above), Perl, Fortran and/or C++ compiler, NORMALIZ Bruns, Ichim, Roemer, Soeger for usage of geometric decomposition strategies

Comparison of decomposition strategies

Diagram	Strategy X	Strategy G1	Strategy G2
	282 sectors 1 s	266 sectors 8 s	166 sectors 4 s
	368 sectors 1 s	360 sectors 9 s	235 sectors 5 s
	548 sectors 3 s	506 sectors 15 s	304 sectors 4 s
	infinite recursion	72 sectors 5 s	76 sectors 1 s
	27336 sectrs 5510 s	32063 sectrs 11856 s	27137 sectrs 443 s