Magnetic moment $(g-2)_\mu$, EWSM and SUSY

Dominik Stöckinger

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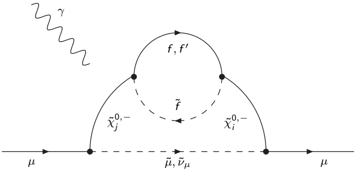


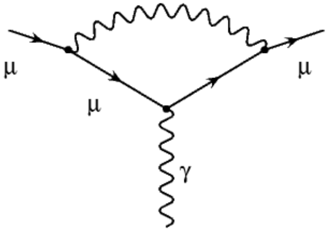
Ustron conference "Matter at the Deepest", September 2015

Muon $g-2$ at Brookhaven $\longrightarrow$ Fermilab

$$a_\mu^{\text{exp}} = (11\,659\,208.9 \quad (6.3)_{\text{tot}}) \times 10^{-10}$$
$$a_\mu^{\text{FNAL}} = (?????????? \quad (1.6)_{\text{tot}}) \times 10^{-10}$$

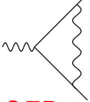
3–4 σ deviation from a_μ^{SM} $\Rightarrow$ improve SM and BSM theory!

Muon $(g-2)$ in SM

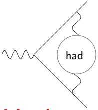


$$a_\mu = \frac{\alpha(0)}{2\pi} + \ldots$$

[in units 10^{-10}]

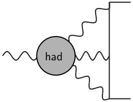


QED: $11\,658\,471.8 \quad (0.0)$

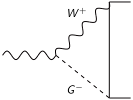


Had vp: $682.5 \;\; (4.2)$

→ D. Nomura

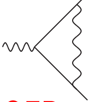


Had lbl: $10.5 \;\; (2.6)$



Weak: $15.36 \;\; (0.10)$

→ next section



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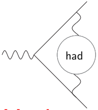

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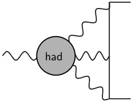
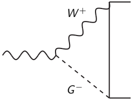


Had lbl: $10.5 \;\; (2.6)$



Weak: $15.36 \;\; (0.10)$

→ next section

SM prediction too low by $\approx (30 \pm 8) \times 10^{-10}$!

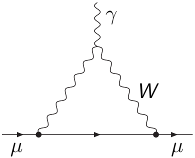
Outline

1. SM weak contributions
   - "Final" result for weak contributions (1, 2, 3-loop)
2. SUSY
   - SUSY: Structure of result
   - SUSY: Precision prediction
   - Large a_μ^{SUSY} with TeV-scale masses
3. Conclusions

Outline

1. SM weak contributions
   - "Final" result for weak contributions (1, 2, 3-loop)

Re-evaluation of a_μ(weak) [Gnendiger, DS, Stöckinger-Kim '13]



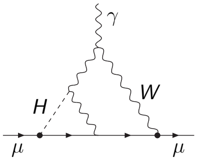
$$a_\mu^{\mathrm{EW}(1)} = \frac{G_F}{\sqrt{2}} \frac{m_\mu^2}{8\pi^2} \left[\frac{5}{3} + \frac{1}{3}(1-4s_W^2)^2 \right]$$
$$= (19.480 \pm 0.001_{M_W}) \times 10^{-10}$$

Decide parametrization: G_F instead of $\alpha(0)/M_W^2$

M_W predicted by SM: $M_W = 80.363 \pm 0.013$ GeV (don't use $M_W^{\mathrm{exp}} = 80.385 \pm 0.015$ GeV)

Parametrization at n-loops: $G_F \alpha^{(n-1)}(0)$ $\rightsquigarrow$ consistent with [Czarnecki, Marciano, Vainshtein '03]

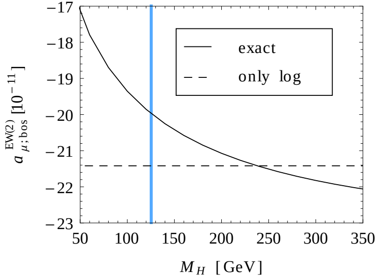
2-loop bosonic contributions



large M_H-limit: [Czarnecki, Krause, Marciano '95];

full: [Heinemeyer, DS, Weiglein '04] but G_F^2-parametrization

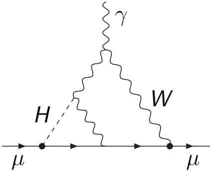
Recalculation with $G_F\alpha(0)$ parametrization:

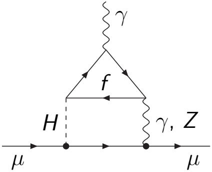


$$a_{\mu;\mathrm{bos}}^{\mathrm{EW}(2)} = (-1.997 \pm 0.003) \times 10^{-10}$$

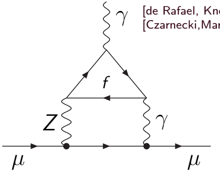
Error: essentially from M_H

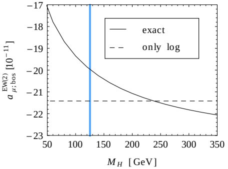
Re-evaluation of a_μ(weak) [Gnendiger, DS, Stöckinger-Kim '13]

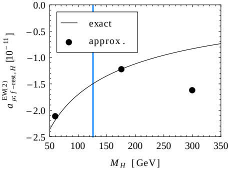


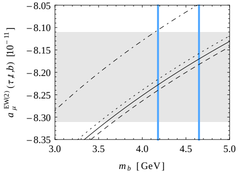


[de Rafael, Knecht, et al '95–'02]
[Czarnecki,Marciano,Vainshtein '03]





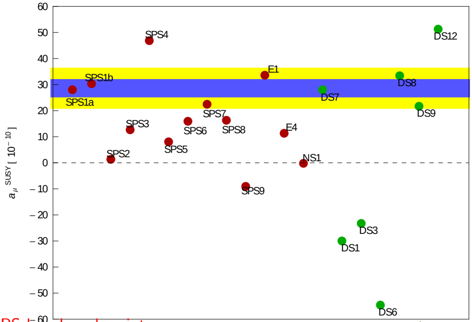




- exact evaluation of M_H-dependent parts
- consistent parametrization of 1-, 2-, 3-loop $\propto G_F\alpha^{n-1}$
- 3-loop logs $G_F\alpha^2\log^2(M_W/m_\mu)$ from [Czarnecki,Marciano,Vainshtein '03]
- final result: $(15.36 \pm 0.10) \times 10^{-10}$

Outline

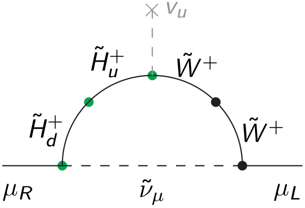
2. SUSY
   - SUSY: Structure of result
   - SUSY: Precision prediction
   - Large a_μ^{SUSY} with TeV-scale masses



SPS benchmark points
[v.Weitershausen,Schäfer, Stöckinger-Kim,DS '10]

LHC Inverse Problem ($300 fb^{-1}$)
can't be distinguished at LHC
[Sfitter: Adam, Kneur, Lafaye, Plehn, Rauch, Zerwas '10]

Main parameter dependence of a_μ^{SUSY}



Two Higgs(ino) sector parameters:

$\tan\beta = \frac{v_u}{v_d}$, $\mu = H_u - H_d$ transition

Diagram contains:

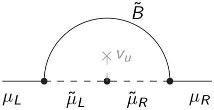
$\propto y_\mu \, v_u \, \mu$

Likewise for the muon mass!

Main parameter dependence of a_μ^{SUSY}

Two Higgs(ino) sector parameters:

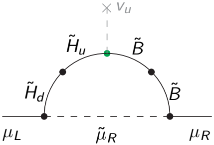
$$\tan\beta = \frac{v_u}{v_d}, \quad \mu = H_u - H_d \text{ transition}$$



$\propto \mu$ for $\mu \to \infty$

Diagram contains:

$$\propto y_\mu \, v_u \;\, \mu$$

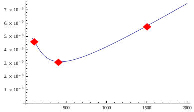


$\propto$ other sign!

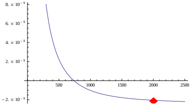
Likewise for the muon mass!

Further diagrams also $\propto y_\mu \, v_u \;\, \mu$ but different additional parameters

small/large μ



small/large $M_{\tilde{\mu}_L}$



[Fargnoli,Gnendiger,Passehr,DS,Stöckinger-Kim '13]

The general structure

[Marchetti,Mertens,Nierste,DS '08; Bach,Park,DS,Stöckinger-Kim'15]

$$a_\mu^{\mathrm{SUSY}} = \frac{y_\mu v_u \; a_\mu^{\mathsf{red}}}{m_\mu^{\mathsf{pole}}} + \ldots$$

$$m_\mu^{\mathsf{pole}} = y_\mu v_d + \underbrace{y_\mu v_u \Delta_\mu^{\mathsf{red}}}_{\text{often neglected}} + \ldots$$

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Insert Yukawa, neglecting higher orders:

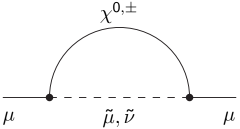
$$a_\mu^{\mathrm{SUSY}} \approx \tan\beta \; a_\mu^{\mathsf{red}}$$

$$\approx 12 \times 10^{-10} \tan\beta \, \mathrm{sign}(\mu) \left(\frac{100\mathrm{GeV}}{M_{\mathsf{SUSY,\ universal}}} \right)^2$$

Status of SUSY prediction: uncertainty 3×10^{-10} [DS '06]

1-Loop

$\propto \tan\beta$



complete

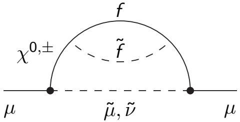
[Fayet '80],...

[Kosower et al '83],[Yuan et al '84],...

[Lopez et al '94],[Moroi '96]

2-Loop (SUSY 1L)

e.g. $\propto \log \frac{M_{\rm SUSY}}{m_\mu}$



QED,$\tan^2\beta$

$f, \tilde{f}$-loops

[Degrassi,Giudice '98]

[Marchetti, Mertens, Nierste, DS '08]

[Schäfer, Stöckinger-Kim,

v. Weitershausen, DS '10]

[Fargnoli,Gnendiger,Passehr,DS,

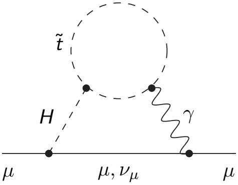
Stöckinger-Kim '13]

(remaining: 65000 diagrams computed,

1 class of counterterms missing)

2-Loop (SM 1L)

e.g. $\propto \tan\beta\, \mu\, m_t$



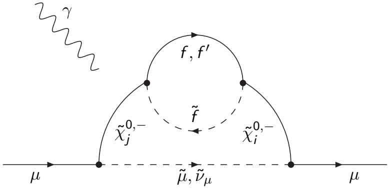
complete

[Chen,Geng'01][Arhib,Baek '02]

[Heinemeyer,DS,Weiglein '03]

[Heinemeyer,DS,Weiglein '04]

New: contributions with $f\tilde{f}$ loops [Fargnoli, Gnendiger, Passehr, DS, Stöckinger-Kim '13]



Motivation:

- maximum complexity: 5 heavy + 2 light scales
- remaining class with dependence on squarks
- contains large logs, $\Delta\rho$

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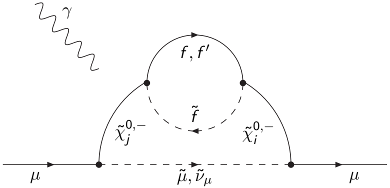

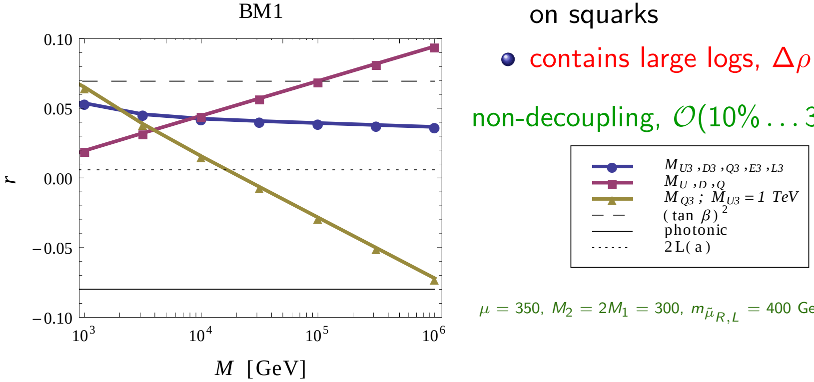

Motivation:

- maximum complexity: 5 heavy + 2 light scales
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- contains large logs, $\Delta\rho$

non-decoupling, $\mathcal{O}(10\% \ldots 30\%)$

$\mu = 350$, $M_2 = 2M_1 = 300$, $m_{\tilde{\mu}_{R,L}} = 400$ GeV, $\tan\beta = 40$

Contributions involving $\Delta\rho$

One-loop ambiguity

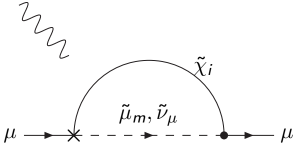
$$\left.\begin{array}{lll} a_\mu^{\rm 1L} & = \alpha(0)\ldots & = 29.4 \\ a_\mu^{\rm 1L} & = \alpha(M_Z)\ldots & = 31.6 \\ a_\mu^{\rm 1L} & = \alpha(G_F)\ldots & = 30.5 \end{array}\right\}$$

differ by $\Delta\alpha$, $\Delta\rho$: 2L$f\tilde{f}$-terms

Fixed by full $2Lf\tilde{f}$ calculation

(for SPS1a, unit: 10^{-10})

Contributions involving $\Delta\rho$



$$= a_\mu^{1\mathrm{L}} \times \left(\ldots + \frac{\delta\left(e^2/s_W^2\right)}{e^2/s_W^2} \right)$$

$$= a_\mu^{1\mathrm{L}} \times \left(\Delta\alpha - \frac{c_W^2}{s_W^2}\Delta\rho + \ldots \right)_{f,\tilde{f}\text{-loops}}$$

One-loop ambiguity

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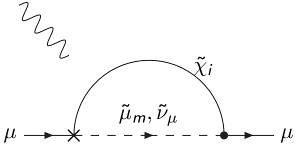

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One-loop ambiguity

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differ by $\Delta\alpha$, $\Delta\rho$: 2L$f\tilde{f}$-terms

Fixed by full $2Lf\tilde{f}$ calculation

$$a_\mu^{1\mathrm{L}+2\mathrm{L}f\tilde{f}} = 32.2$$

(for SPS1a, unit: 10^{-10})

Radiative muon mass

[Bach,Park,DS,Stöckinger-Kim'15]

$$a_\mu^{\rm SUSY} = \frac{y_\mu v_u \ a_\mu^{\rm red}}{m_\mu^{\rm pole}} + \ldots$$

$$m_\mu^{\rm pole} = y_\mu v_d + \underbrace{y_\mu v_u \Delta_\mu^{\rm red}}_{\text{now important}} + \ldots$$

Now take $v_d \to 0$, $\tan\beta \to \infty$ (no divergence of physical quantities)

Radiative muon mass

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Muon and all down-type masses arise from loops!
Yukawas large but not too large (see later)

Radiative muon mass

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$$a_\mu^{\rm SUSY} \to \frac{a_\mu^{\rm red}}{\Delta_\mu^{\rm red}}$$

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Now take $v_d \to 0$, $\tan\beta \to \infty$ (no divergence of physical quantities)

$$a_\mu^{\rm SUSY} \approx \frac{12 \times 10^{-10} \;\; \mathrm{sign}(\mu) \left(\frac{100\mathrm{GeV}}{M_{\text{SUSY, universal}}}\right)^2}{-0.0018 \; \mathrm{sign}(\mu)}$$

sign wrong! But note: all masses were set equal!

Large a_μ in MSSM for $\tan\beta \to \infty$, different masses

[Bach,Park,DS,Stöckinger-Kim, '15]

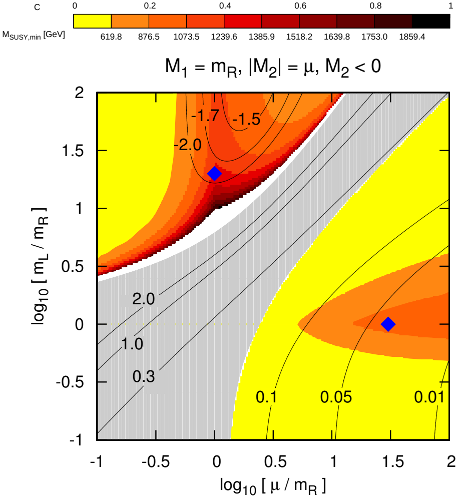
Generally: $a_\mu^{\rm SUSY} \to \frac{a_\mu^{\rm red}}{\Delta_\mu^{\rm red}}$

coloured: a_μ positive

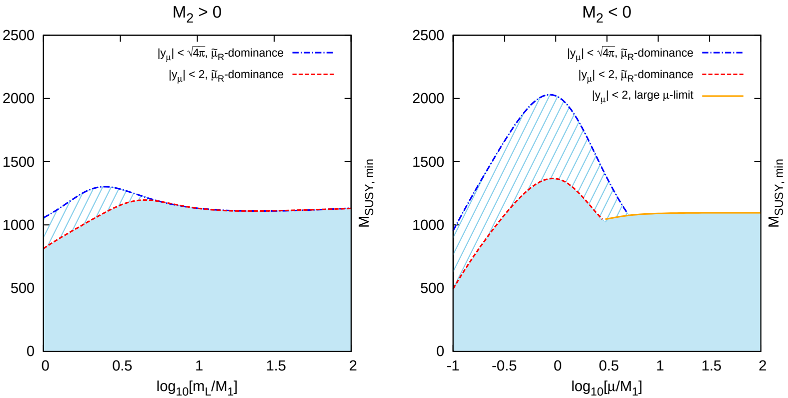
Sample TeV-scale masses:

μ	M_1	M_2	m_L	m_R	$a_\mu/10^{-9}$
15	1	-1	1	1	3.01
1.3	1.3	-1.3	26	1.3	2.90

Experimental constraints ok: B-physics, Higgs-physics [Dobrescu,Fox; Altmannshofer,Straub '11], vacuum stability [Bach,Park,DS,Stöckinger-Kim, '15]



The "largest" possible SUSY masses (:= a_μ explained for $\tan\beta \to \infty$)

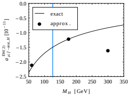


Outline

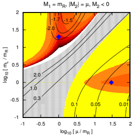
3 Conclusions

Summary: a_μ as a probe of the SM and beyond

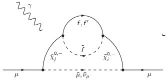
- Recent progress on all aspects of $a_\mu^{\rm SM}$
  - $a_\mu^{\rm Exp} - a_\mu^{\rm SM} \approx (30 \pm 8) \times 10^{-10}$

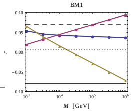


- $a_\mu^{\rm N.P.,SUSY}$ very model-dependent, typically $\mathcal{O}(\pm 1 \ldots 50) \times 10^{-10}$
  - TeV-scale SUSY for $\tan\beta \to \infty$ can explain a_μ
  - many scenarios with light sparticles, too
  - but e.g. CMSSM, MRSSM cannot any more!



- SUSY precision predictions available
  - large logs, reduce α-ambiguity
  - still, theory error too large





New measurements within next 5 years $\Rightarrow$ Promising!!