Bound muon decay and its role in New Physics searches

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Matter To The Deepest

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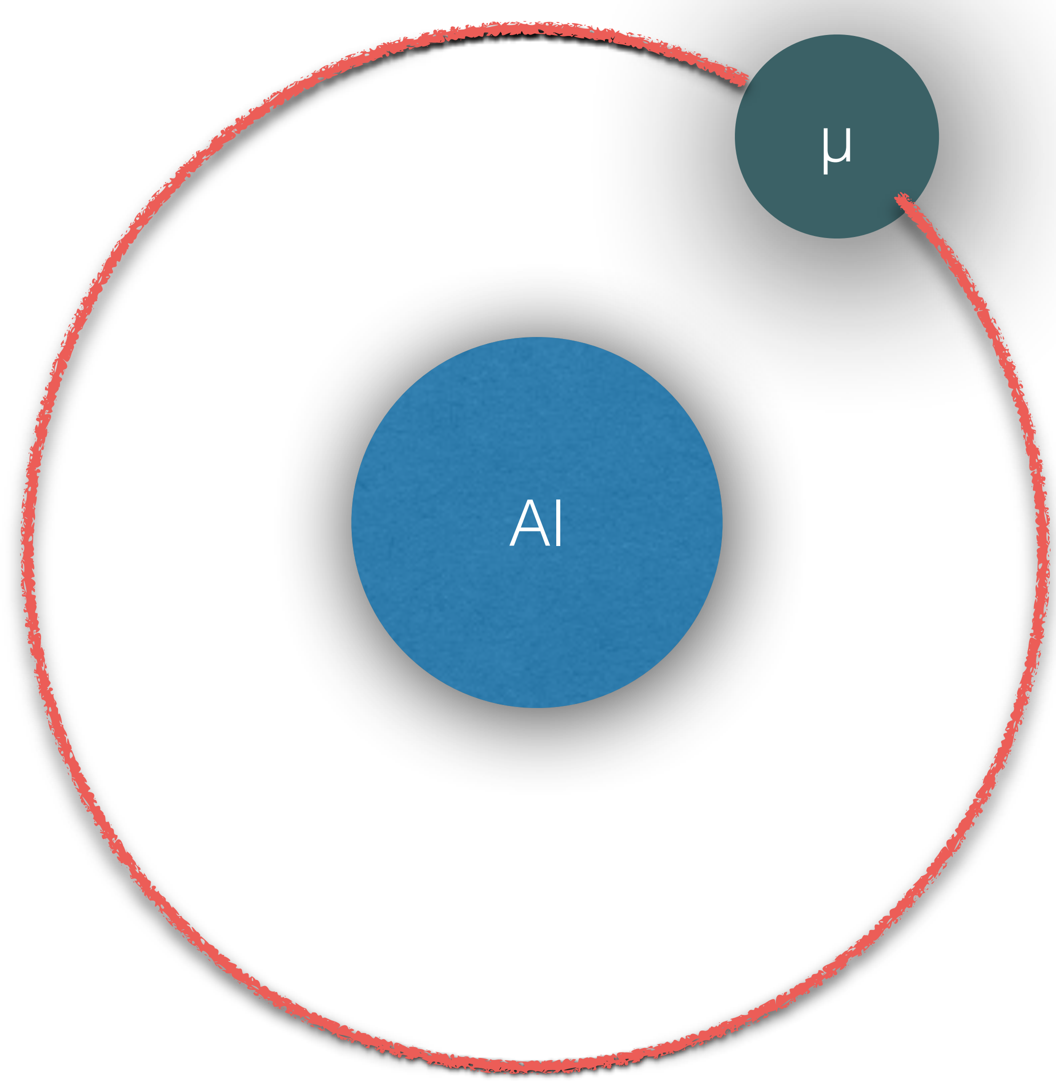
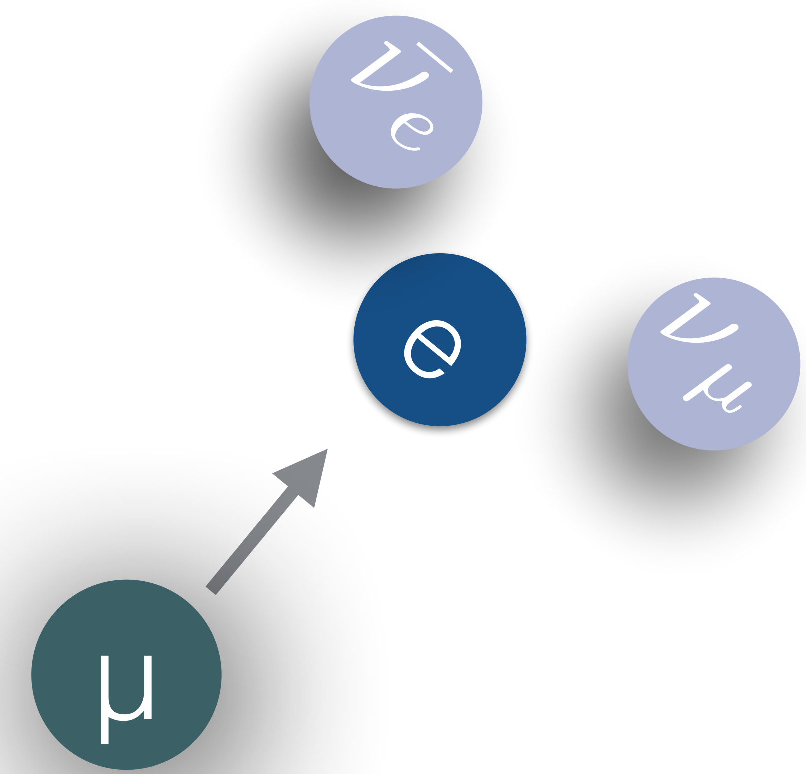
Outline

- Decay in orbit spectrum:
  - central region
  - endpoint region
- Summary



Free muon decay

- Well known SM process
- Source of New Physics constraints
- NLO corrections calculated in 1950s
- NNLO corrections are also known
- Only lepton flavour conserving decay modes have been observed
- Anomalous magnetic moment may indicate a need for a NP contributions

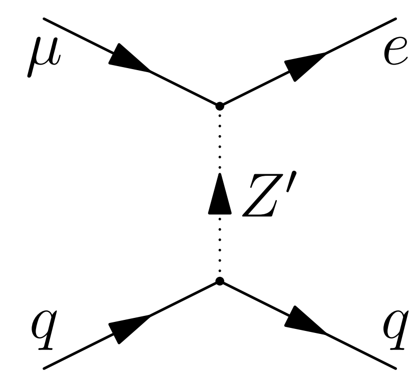


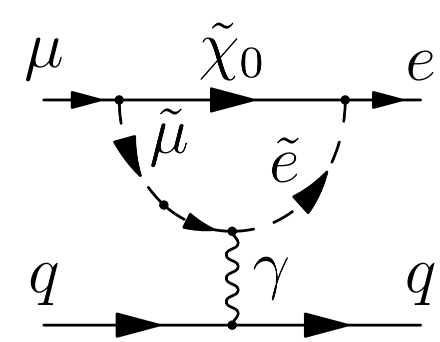
Off—diagonal dipole moments

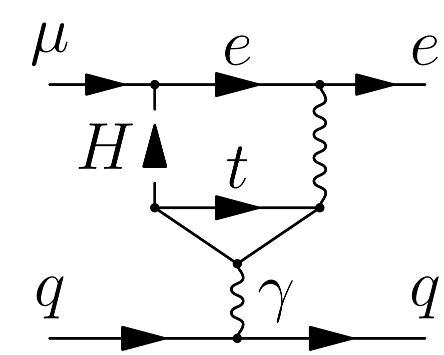
- Similar type of operators may contribute to g-2 and Charged Lepton Flavour Violation (CLFV)

- CLFV is suppressed in SM

- Three interesting CLFV processes
  - $\mu \to e\gamma$
  - muon electron conversion
  - $\mu \to eee$



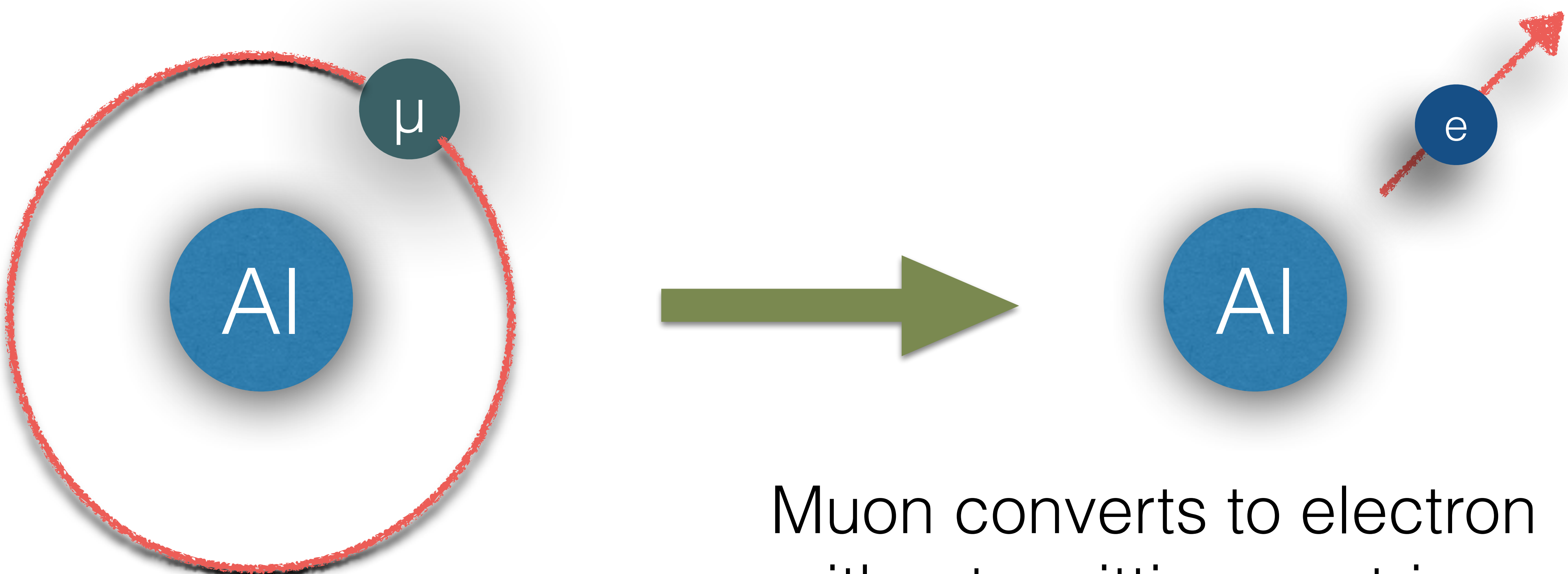




Three processes with bound muons

Proces		SM rate	Why important?
Conversion	$(\mu^- N) \rightarrow N + e^-$	Negligible	Observation indicates New Physics
Decay in Orbit DIO	$(\mu^- N) \rightarrow N + e^- + \overline{\nu_e} + \nu_\mu$	Approximately equal to free muon decay rate	Background to conversion
Capture	$(\mu^- N) \rightarrow N' + \nu_\mu$	Depends on Z	Normalization factor for conversion

Muon electron conversion



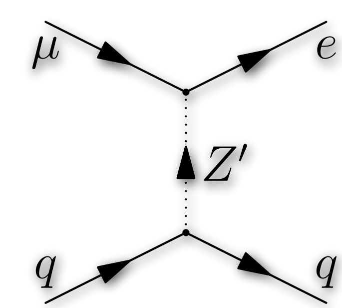
Muon converts to electron
without emitting neutrinos
Lepton family number not conserved

Muon electron conversion

- Clean experimental signature — mono-energetic electron

- Current limit on the ratio R of the conversion to the capture $R < 7 \times 10^{-13}$
- Planned experiments expect to improve R by ~4 orders of magnitude, equivalent to probing New Physics scale up to 10 000 TeV!

- Conversion can probe larger class of operators than $\mu \to e\gamma$

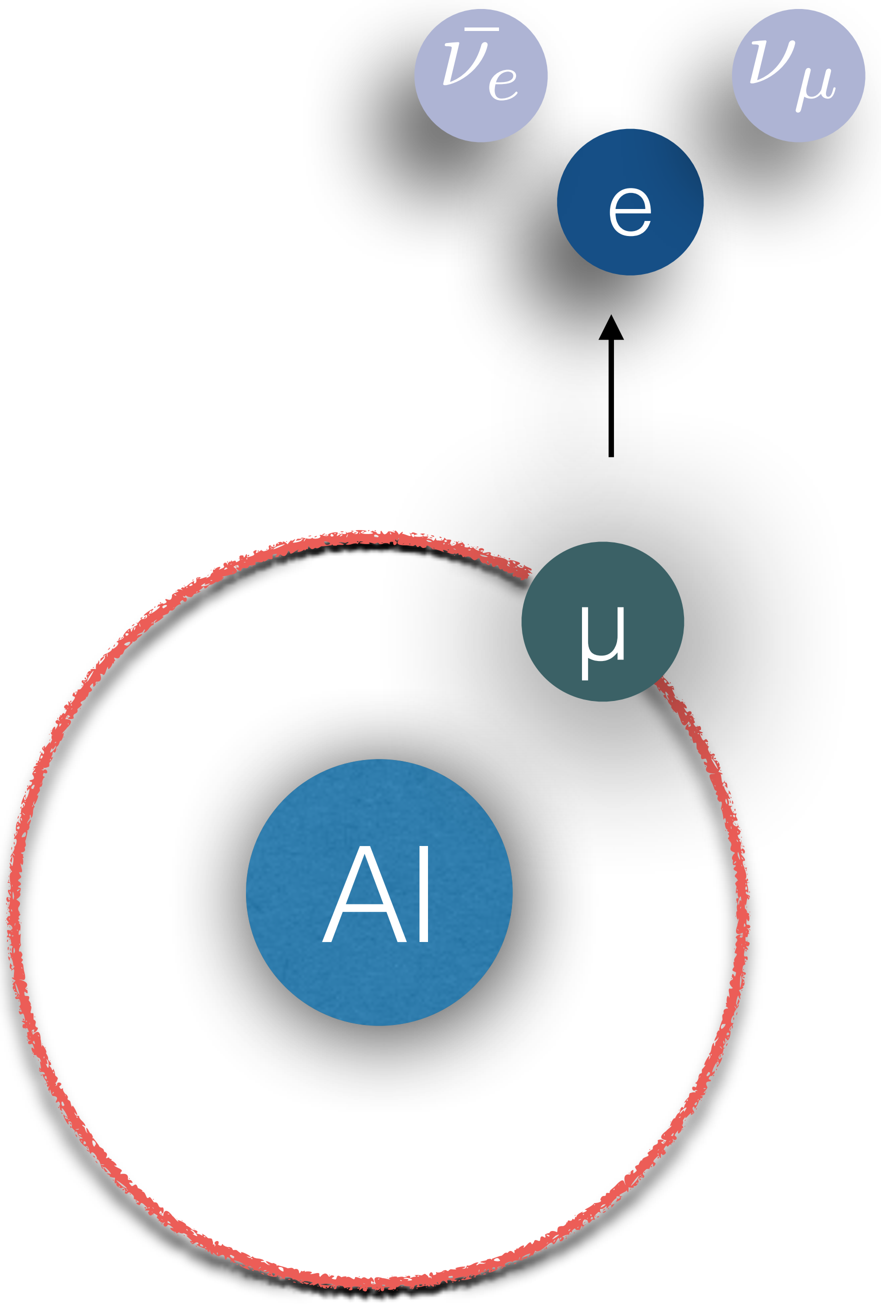


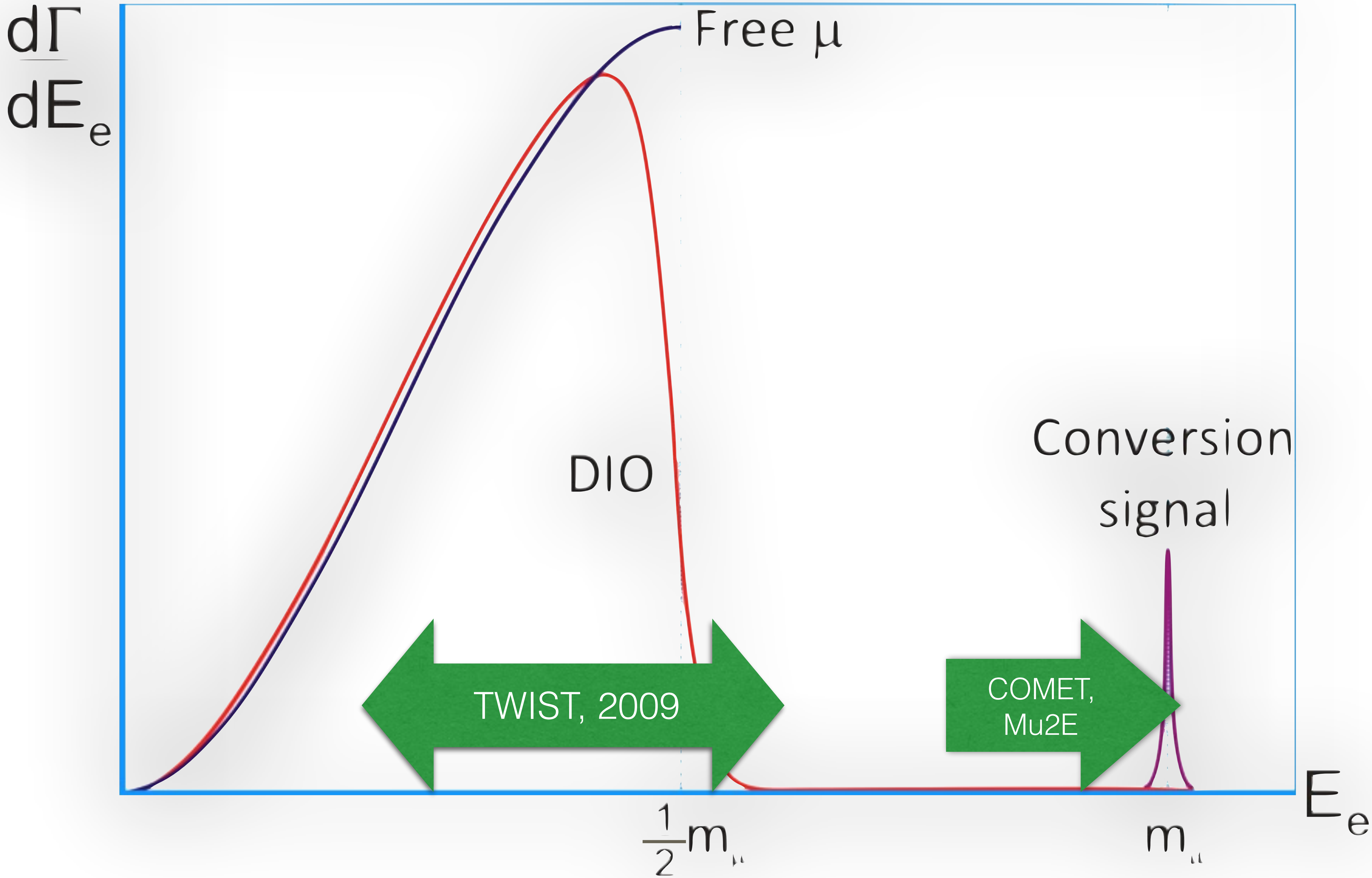
Bound muon decay

- Muon DIO: standard muon decay into an electron and two neutrinos, with the muon and a nucleus forming bound state

- For a free muon, energy and momentum conservation restricts electron spectrum to $E_e < \frac{m_\mu}{2}$

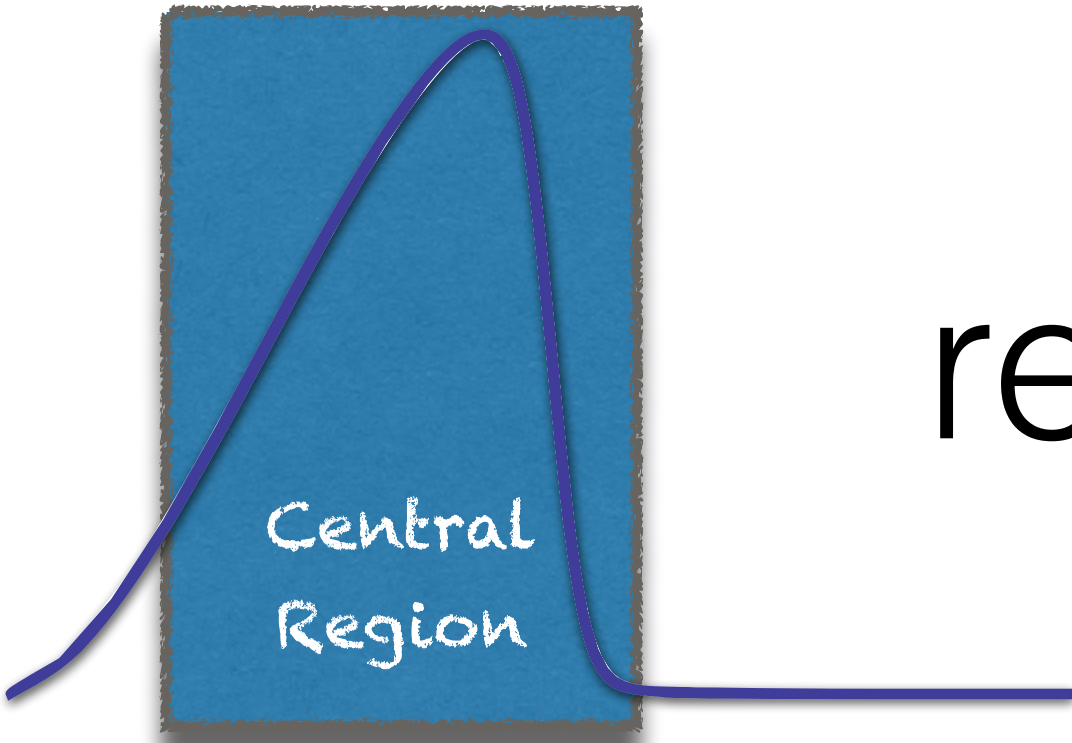
- For DIO, momentum can be exchanged between the nucleus and both the muon and the electron





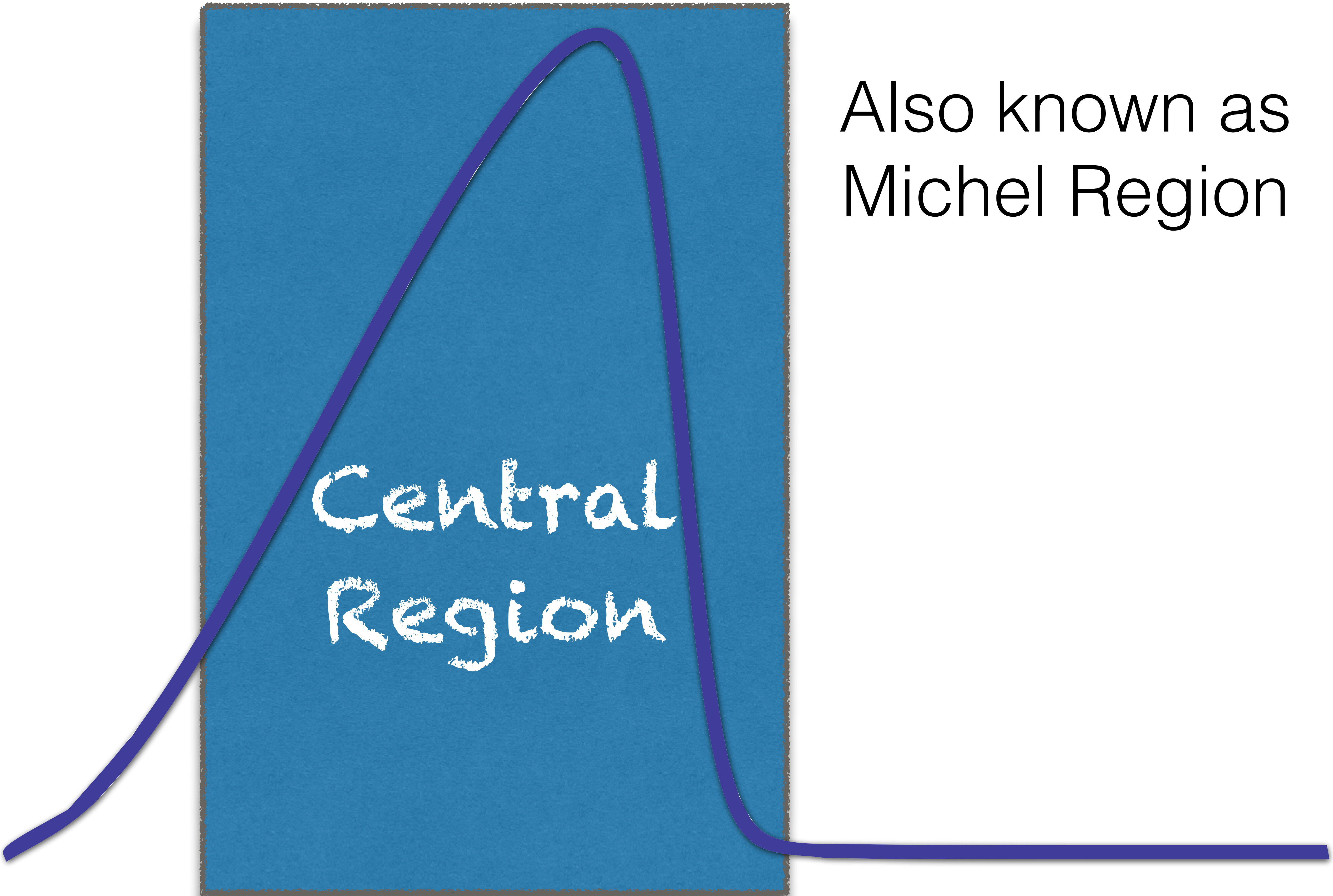
DIO Spectrum

Two regions



Most important effect:	
muon motion in an atom	**exchange of a hard photon**
Corrections:	
final state interaction	**finite size of the nucleus**
	recoil effects

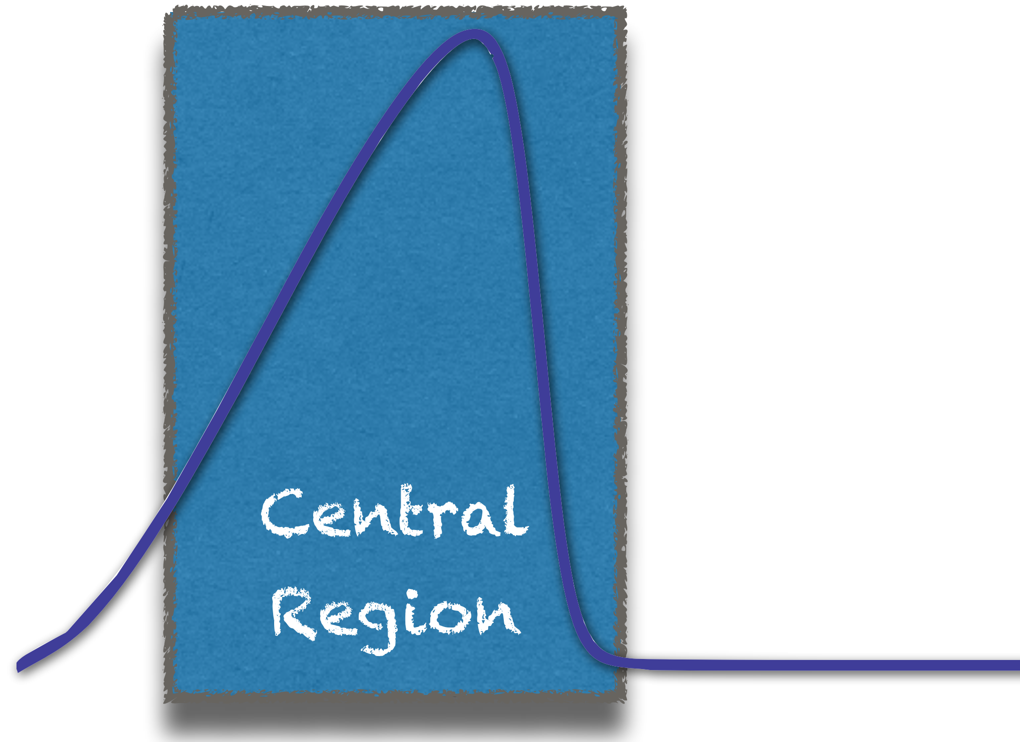
Radiative corrections!



Also known as
Michel Region

Central region

$$m_\mu Z\alpha \ll E_e \lesssim \frac{m_\mu}{2}$$



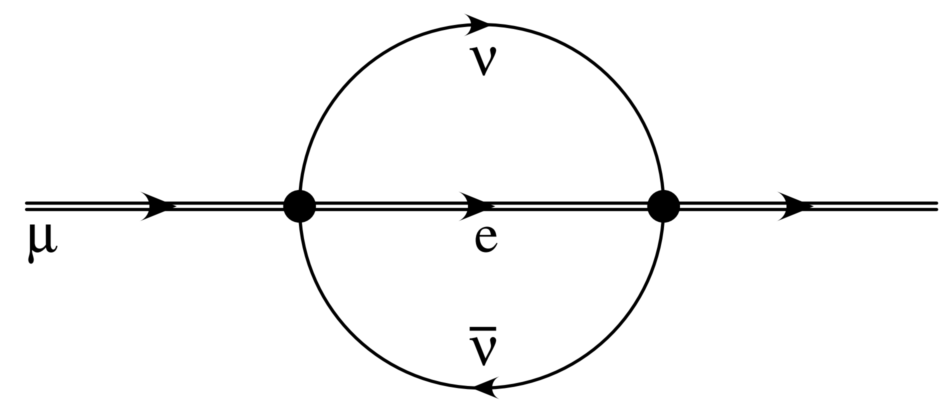
- Typical momentum transfer between nucleus and muon is of the order of $m_\mu Z\alpha$
- Requires resummation
- Dominant effect — muon motion in the initial state
- Similar problems — decays of heavy quarks in mesons

QED shape function

- Charged particle in the external field is almost on-shell

$$\frac{1}{(p_e+\pi)^2} \approx \frac{1}{p_e^2 + 2p_e \cdot \pi} \to \delta(p_e^2 + 2p_e \cdot \pi)$$

$$\pi_\mu = i\partial_\mu - eA_\mu$$



- We are interested only in the leading corrections

QED shape function

- Shape function is defined as an expectation value:

$$S(\lambda) = \int \mathrm{d}^3x\psi^\star(x)\delta(\lambda - n\cdot\pi)\psi(x)$$

Momentum distribution

Final state interaction, required by gauge invariance

- We work in light-cone gauge

$$n \cdot A = 0$$

- Normalization: $\int_{-\infty}^{\infty} d\lambda S(\lambda) = 1$

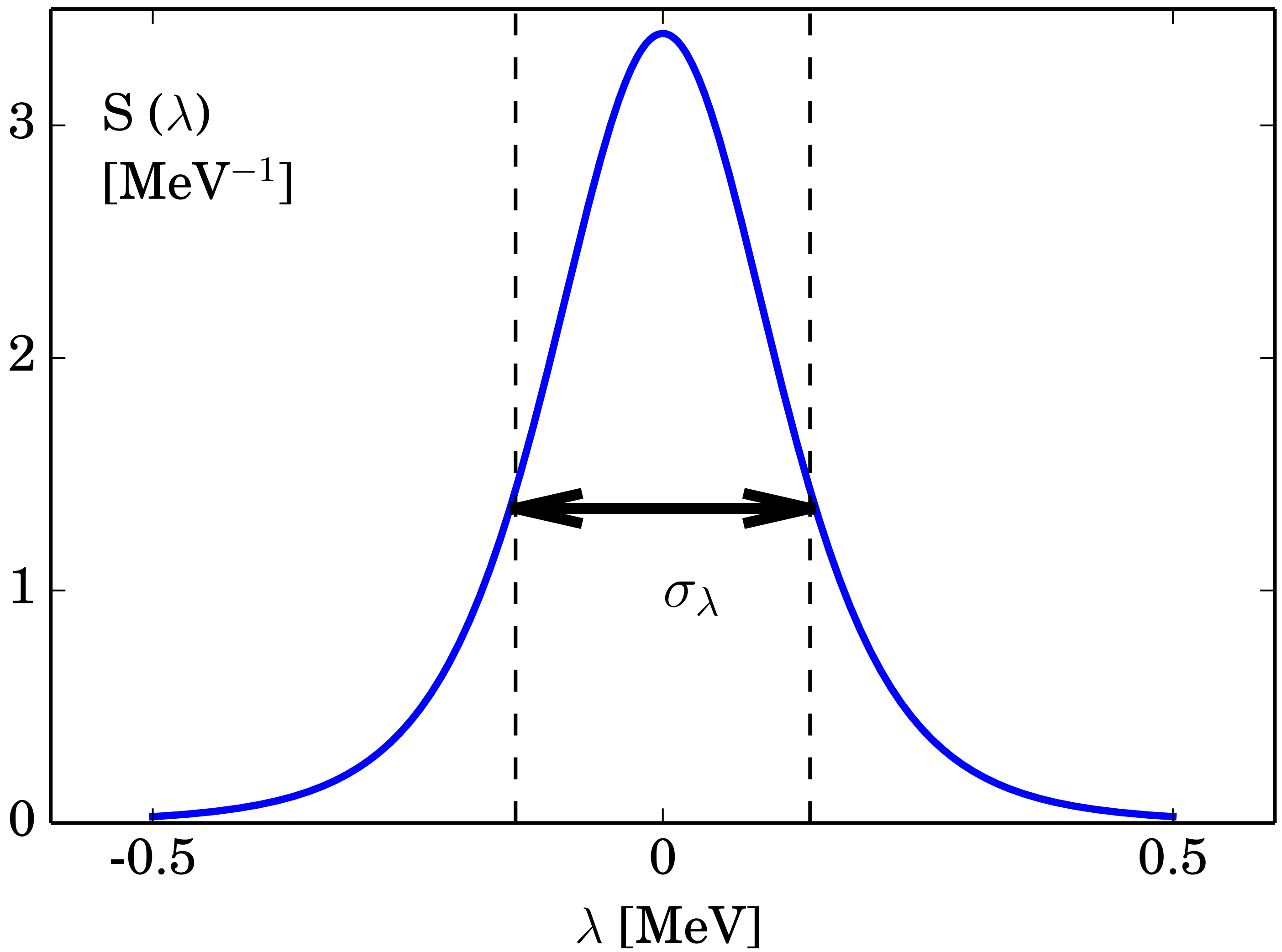
Power counting

- $\lambda \sim \frac{p_e^2}{2E_e} \sim m_\mu Z\alpha$ (muon momentum in an atom)
- Shape function behaves as $S(\lambda) \sim \frac{1}{Z\alpha}$
- First moment is zero in the leading order

$$\int d\lambda \lambda S(\lambda) \sim (Z\alpha)^2$$

- Second moment $\int d\lambda \lambda^2 S(\lambda) = \frac{1}{3}(m_\mu Z\alpha)^2$

QED shape function



A. Czarnecki, R.S.

Phys.Rev. D92 (2015), 053004

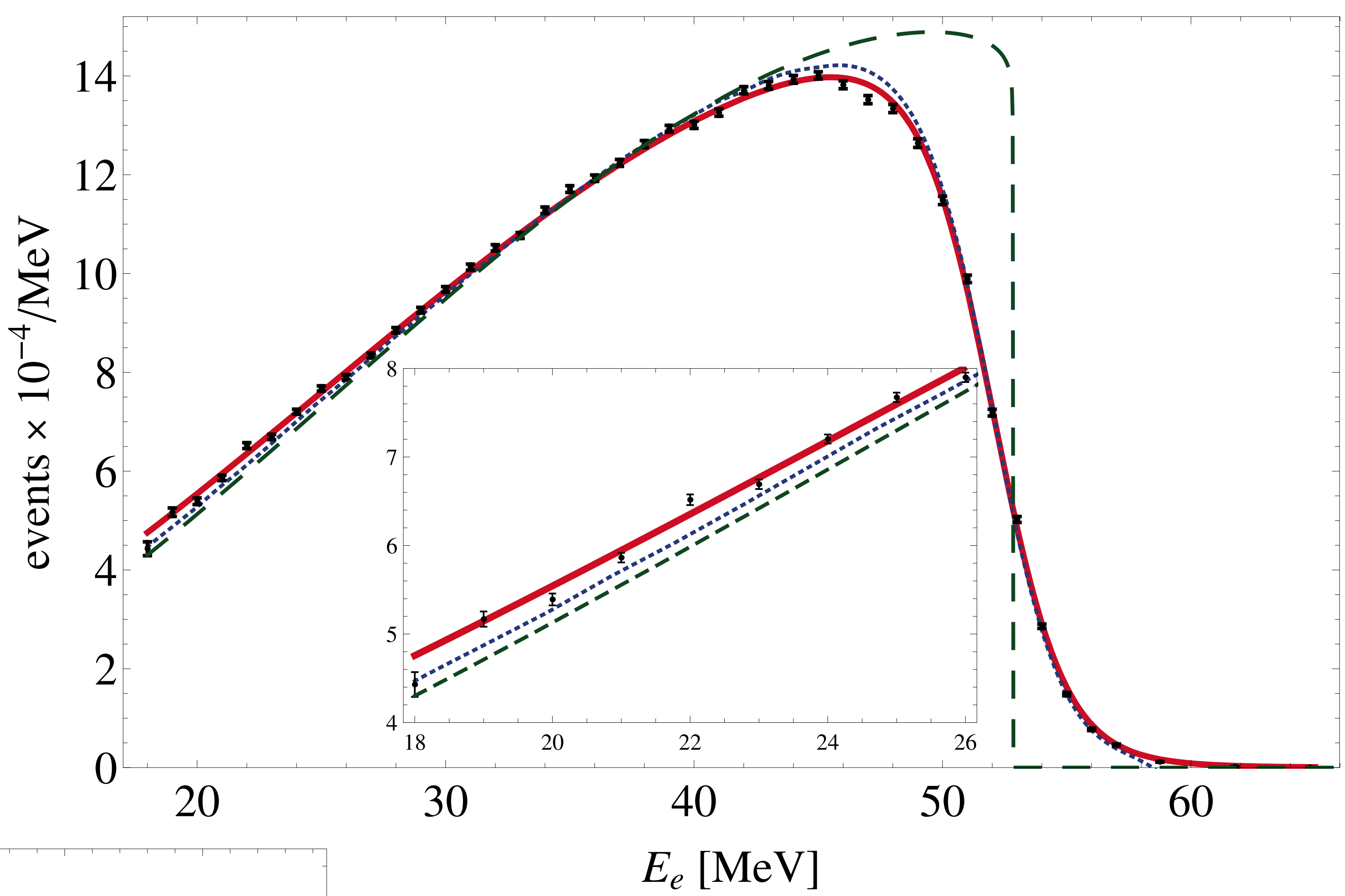
Spectrum can be calculated using factorization formula

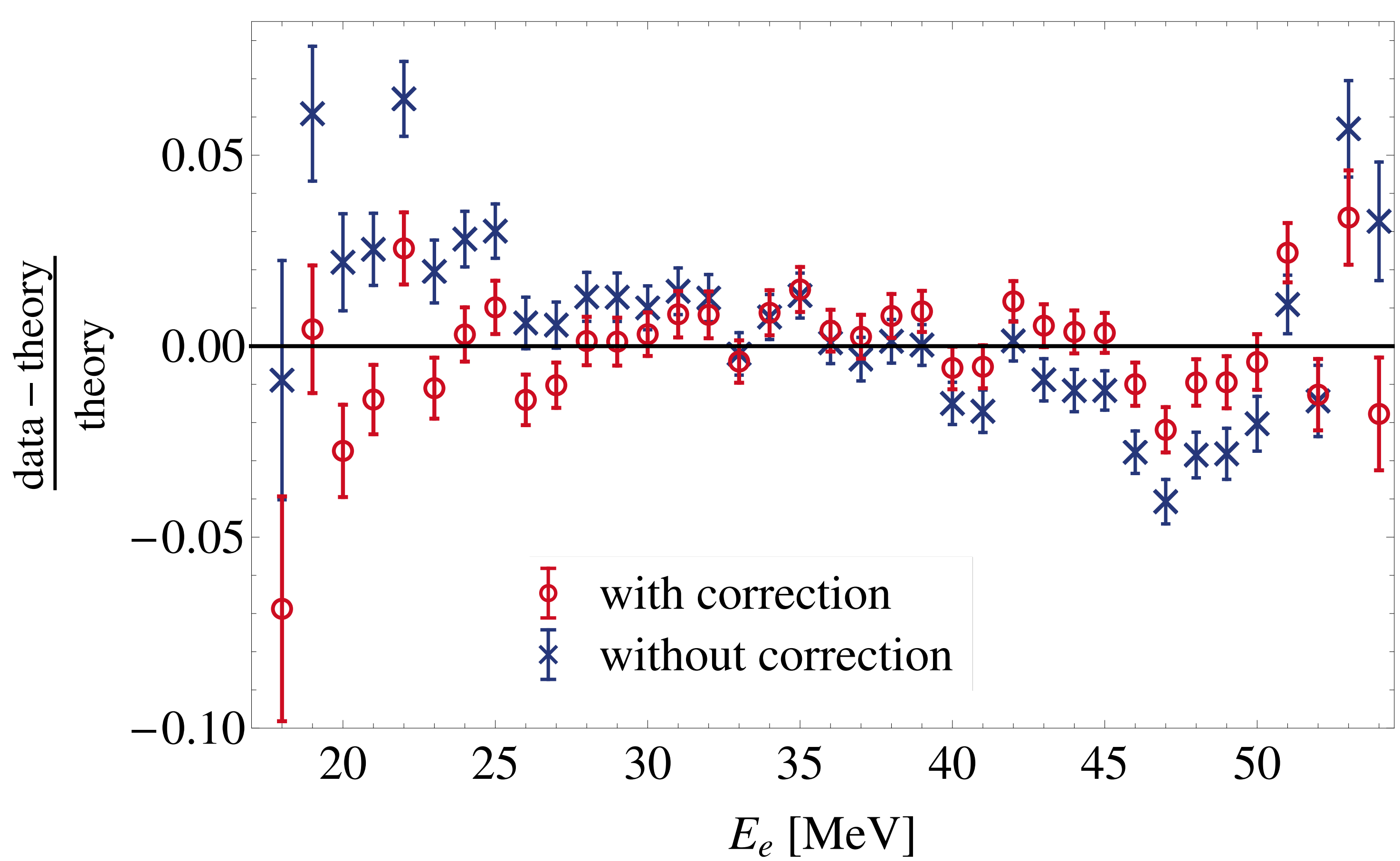
$$\frac{d\Gamma_{DIO}}{dE_e} = \frac{d\Gamma_{Free}}{dE_e} * S$$

Leading corrections

and their relation
to the experimental data

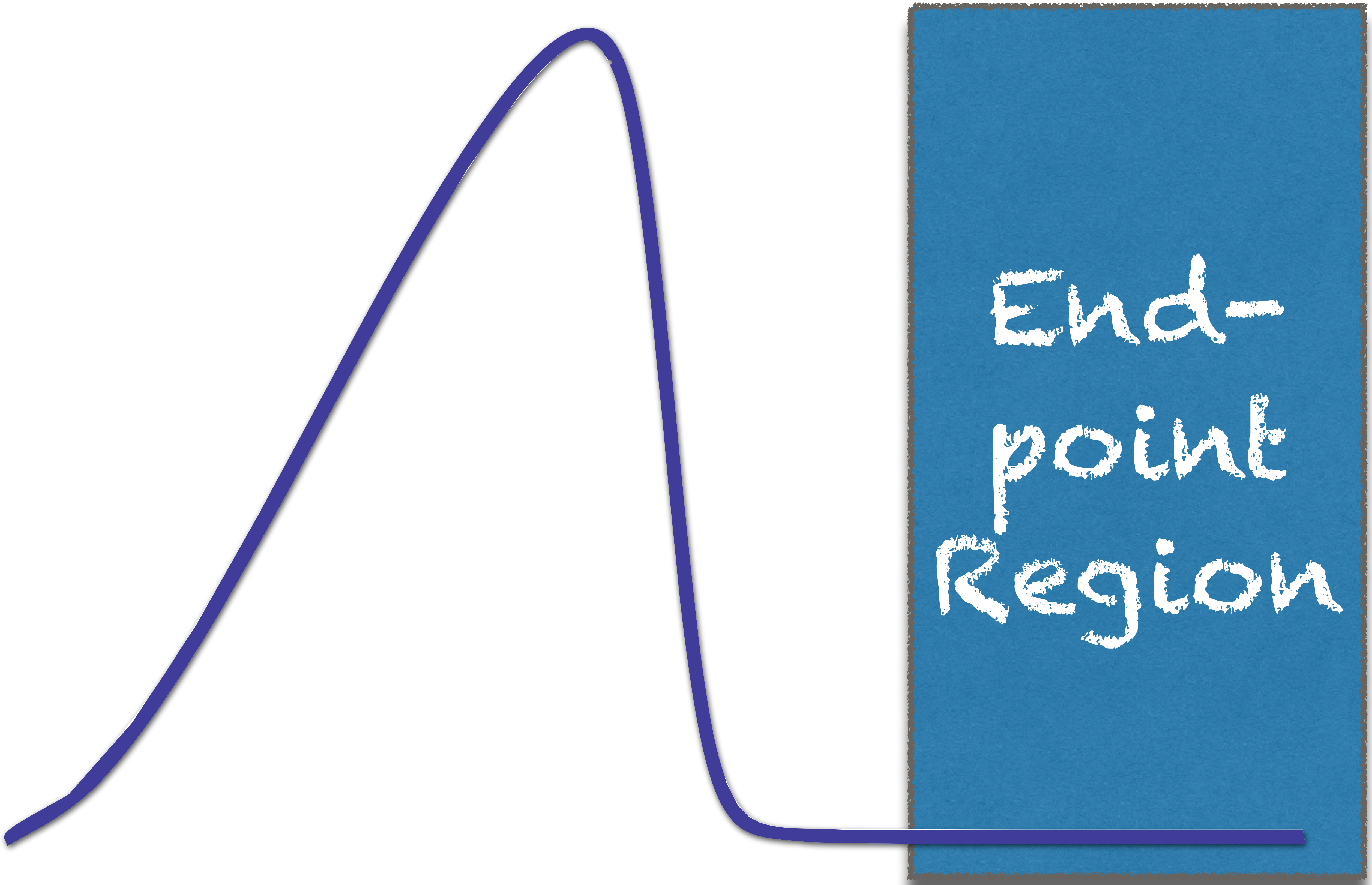
TWIST 2009





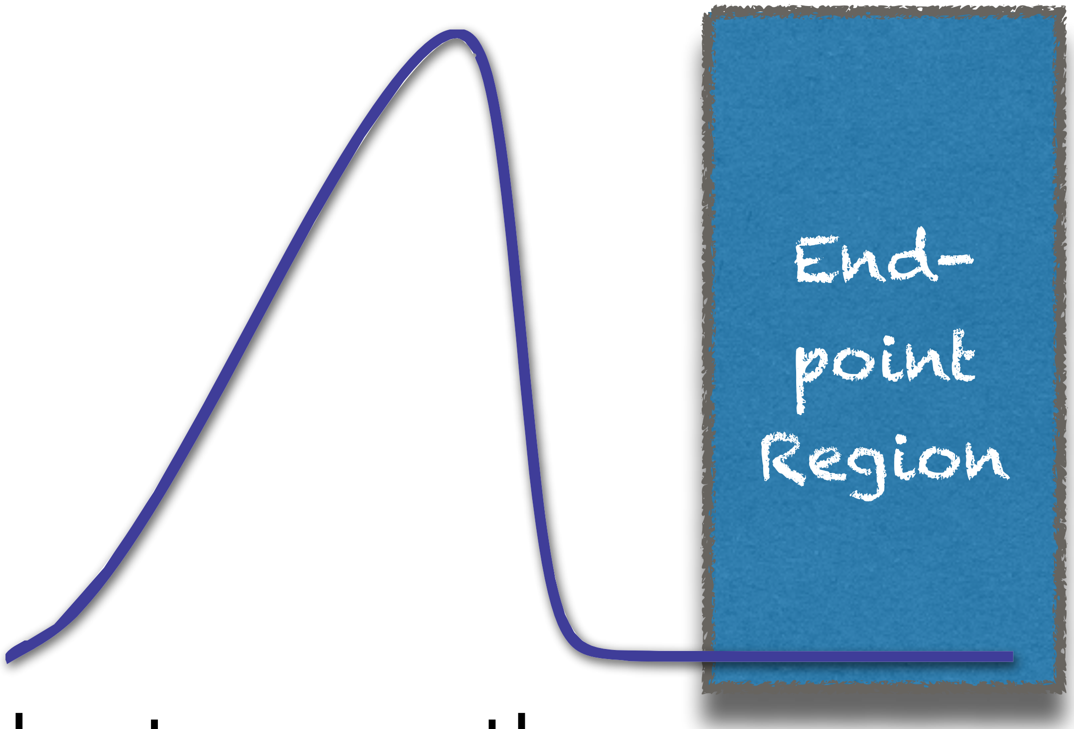
A. Czarnecki, M. Dowling,
X. Garcia i Tormo , W. Marciano, R.S.

Phys. Rev. D90 (2014), 093002


End-
point
Region

Endpoint region

$$E_e \sim m_\mu$$



- Typical momentum transfer between the nucleus and the muon is of the order of the muon mass
- Both wave functions and propagators can be expanded in powers of $Z\alpha$

$$\frac{m_\mu}{\Gamma_{Free}} \frac{d\Gamma}{dE_e} \approx \frac{1024}{5\pi} (Z\alpha)^5 \left(\frac{\Delta}{m_\mu}\right)^5 \qquad \Delta = E_{max} - E_e$$

Endpoint energy

$$E_{max} = m_\mu + E_b + E_{rec}$$

$$E_b \approx -m_\mu \frac{(Z\alpha)^2}{2}$$

Binding energy

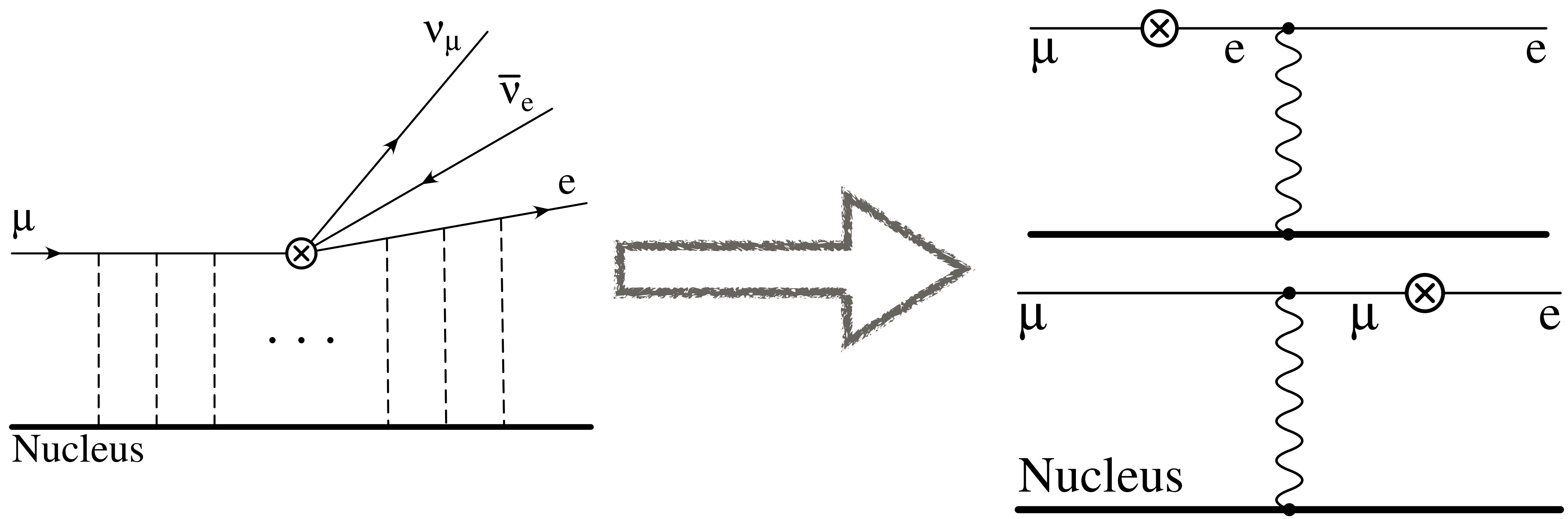
$$E_{rec} \approx -\frac{m_\mu^2}{2m_N}$$

Recoil energy

(kinetic energy of the nucleus)

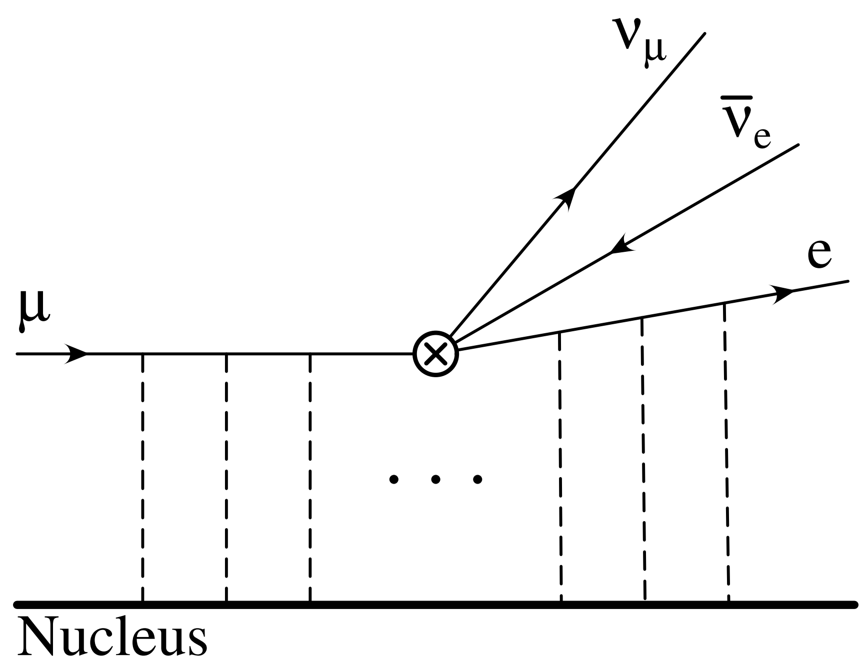
Both corrections decrease the endpoint energy

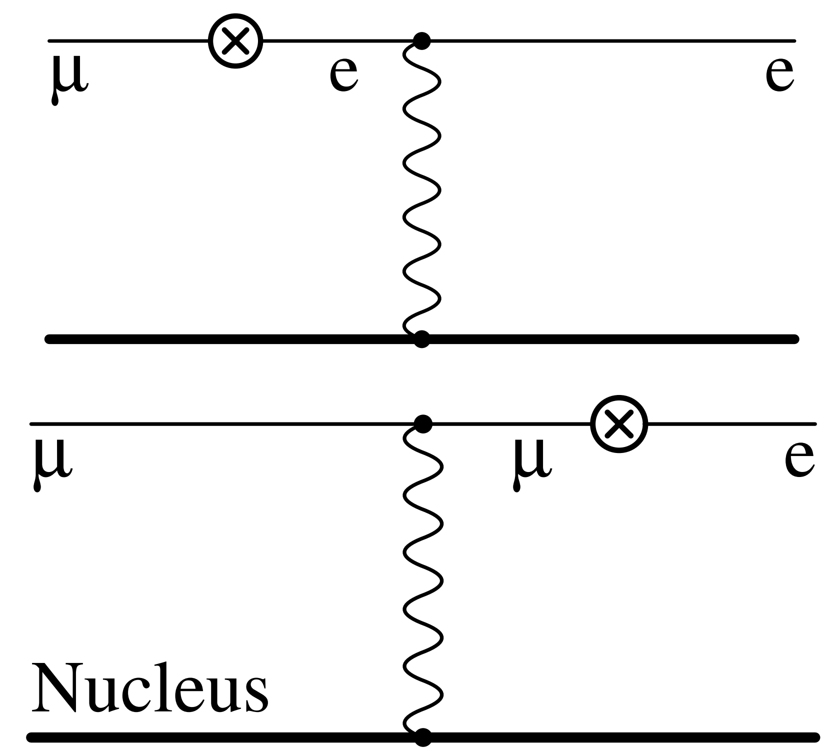
Endpoint expansion



$$\int \frac{\mathrm{d}^3\nu}{\nu_0}\,\frac{\mathrm{d}^3\overline{\nu}_0}{\overline{\nu}_0}\,\delta\left(\Delta-\nu_0-\overline{\nu}_0\right)\ldots\psi\ldots\overline{\psi}\sim\Delta^5$$

Endpoint expansion

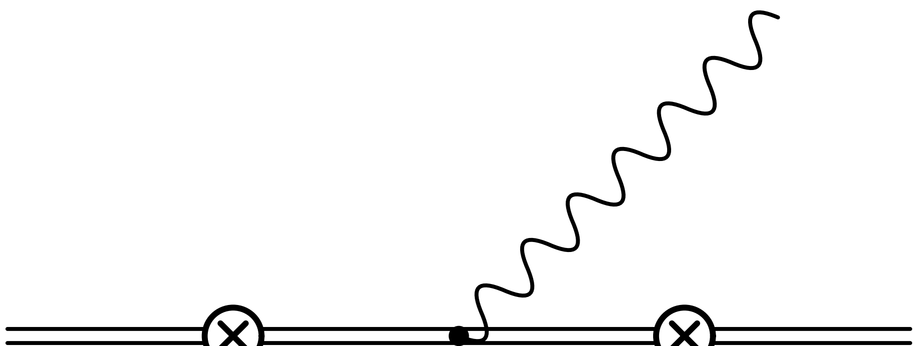


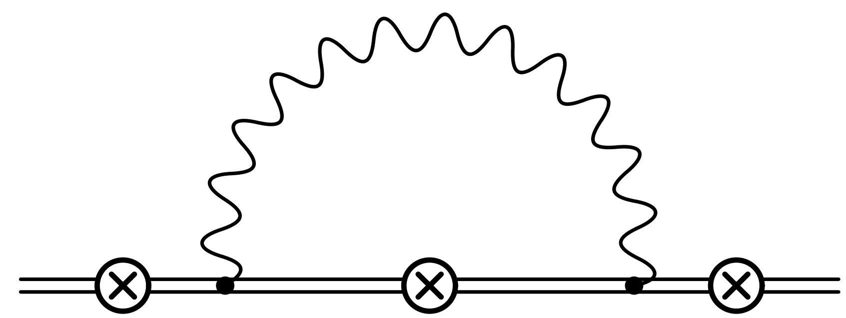


$$V\left(\vec{k}^2\right) = -\frac{Z\alpha}{2\pi^2\vec{k}^2}$$

$$\mathcal{A} \sim \psi(0) \times V(m_\mu^2) \sim (Z\alpha)^{\frac{3}{2}} \times Z\alpha = (Z\alpha)^{\frac{5}{2}}$$

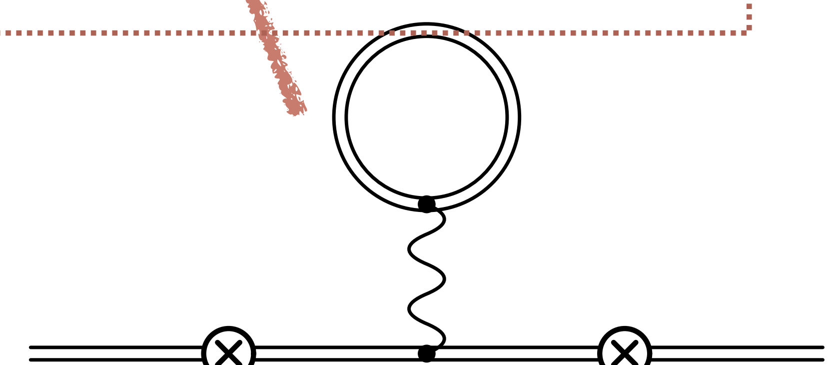
Radiative corrections





$$\frac{1}{\Gamma_{Free}}\frac{d\Gamma}{dE_e} = \Delta^5 \frac{1024}{5\pi m_\mu^6}(Z\alpha)^5 \left(\frac{\Delta}{m_\mu}\right)^{\frac{\alpha}{\pi}\delta_S} (1 + \frac{\alpha}{\pi}\delta_{VP} + \frac{\alpha}{\pi}\delta_H)$$

$$\delta_S \approx 10.0, \quad \delta_H \approx -15.6, \quad \delta_{VP} \approx 6.1$$



A. Czarnecki, R.S.

arXiv:1505.05237

Higher order terms

- Expansion parameter is $\pi Z\alpha$, again very similar to the calculations of photoelectric effect
- Higher order terms were calculated numerically; they give -21% correction for a point-like nucleus
- Finite-size nucleus corrections suppress the higher order terms
- Also higher orders in Δ may be required for precise determination of experimental background

Summary

- Searches for rare decays require accurate predictions for the SM background
- TWIST measurement of the DIO spectrum is sensitive to radiative corrections
- Muon DIO spectrum:
  - We have radiative corrections in regions relevant for experiment
  - Ultimate goal is a correction to the spectrum in the whole energy range