Constraints on the Higgs sector from radiative mass generation of neutrinos

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the 312-νSM and its parameters

needed cancellations from m_{ν_2}

restrictions from m_{ν_3}

The 312-νSM

Standard Model (SM) + one fermionic singlet + two Higgs doublets

- is not a new idea: W. Grimus and H. Neufeld, Nucl. Phys. B **325** (1989) 18.

The 312-νSM has parameters additionally to the "original" SM

- the singlet Majorana mass term M_R
- the parameters due to the second Higgs doublet
  - the Yukawa couplings to the second Higgs doublet

    $(Y_E^{(2)})_{jk}$ lepton doublets and charged lepton singlets ℓ_{Rj}

    $(Y_N^{(2)})_k$ lepton doublets and the neutral fermionic singlet N_R
  - the additional parameters in the Higgs sector

    H. E. Haber and D. O'Neil, Phys. Rev. D **83** (2011) 055017 [arXiv:1011.6188 [hep-ph]].

    * $m^2_{H_2}$, $m^2_{H_3}$, $m^2_{H^\pm}$ masses of the additional Higgs bosons
    * θ_{12}, θ_{13} mixing angles between the neutral Higgs fields
    * Z_2, Z_3, Z_7 parameters of the Higgs potential,

      not fixed by tree level mass relations

The 312-νSM: Tree level predictions

- three additional scalars: two neutral and one charged
  - but they are not our primary concern ...
- the mixing gives a $(3+1) \times (3+1)$ symmetric mass matrix

$$M_\nu = \begin{pmatrix} M_L & M_D^\top \\ M_D & M_R \end{pmatrix} \qquad \text{with} \qquad \begin{array}{rcl} M_L &=& 0_{3\times3} \\ M_D &=& (m_{Ne}, m_{N\mu}, m_{N\tau}) \end{array}$$

- diagonalized $U_{(\nu)} M_\nu U_{(\nu)}^\top = \mathrm{diag}(m_1, m_2, m_3, m_4)$ by the unitary matrix

$$U_{(\nu)} = \begin{pmatrix} U_{1e} & U_{1\mu} & U_{1\tau} & 0 \\ U_{2e} & U_{2\mu} & U_{2\tau} & 0 \\ \frac{is(M_D^*)_e}{m_3} & \frac{is(M_D^*)_\mu}{m_3} & \frac{is(M_D^*)_\tau}{m_3} & -is \\ \frac{c(M_D^*)_e}{m_4} & \frac{c(M_D^*)_\mu}{m_4} & \frac{c(M_D^*)_\tau}{m_4} & c \end{pmatrix} \qquad \text{where} \qquad \begin{array}{rcl} c^2 &=& \frac{m_4}{m_4+m_3} \\ s^2 &=& \frac{m_3}{m_4+m_3} \end{array}$$

- only two of the four neutral fermions have a non-zero mass: m_3 and m_4
  - but m_2 can be generated at one-loop level

The 312-νSM: Tree level parameters and relations

- use the second Higgs coupling $(Y_N^{(2)})_k$ to distinguish the tree-level massless "neutrinos" $\zeta_{1,2}$

  with $U_{1k}(Y_N^{(2)})_k = 0$ and $U_{2k}(Y_N^{(2)})_k = d\sqrt{2}$

  $\Rightarrow$ U_{1k} is defined to be orthogonal to both Yukawa couplings

  – then U_{2k} has to be orthogonal to $(M_D^*)_k$ and to U_{1k}

- with mixing matrix R of the neutral Higgses, defined by the angles $s_{1j} := \sin\theta_{1j}$ and $c_{1j} := \cos\theta_{1j}$,

$$R = \begin{pmatrix} c_{12}c_{13} & -s_{12} & -c_{12}s_{13} \\ s_{12}c_{13} & c_{12} & -s_{12}s_{13} \\ s_{13} & 0 & c_{13} \end{pmatrix} = \begin{pmatrix} q_{11} & \mathrm{Re}[q_{12}] & \mathrm{Im}[q_{12}] \\ q_{21} & \mathrm{Re}[q_{22}] & \mathrm{Im}[q_{22}] \\ q_{31} & \mathrm{Re}[q_{32}] & \mathrm{Im}[q_{32}] \end{pmatrix}$$

  we get an effective coupling for $h_n^0\bar{\zeta}_\alpha\zeta_\beta$

$$y_{\alpha\beta\,n} = \frac{1}{2}\sum_{k=1}^{3}[q_{n1}\frac{1}{v}(M_D)_k + q_{n2}^*\frac{1}{\sqrt{2}}(Y_N^{(2)})_k](U_{\alpha k}U_{\beta N} + U_{\beta k}U_{\alpha N})$$

  – with the properties

$$y_{1\beta\,n} = y_{22\,n} = 0 \qquad y_{23\,n} = -is\,d\,q_{n2}^* \qquad y_{24\,n} = c\,d\,q_{n2}^*$$

One-loop predictions

- loop contribution from the neutral Higgses to the mass correction

$$\delta m_\alpha^{\text{loop}} = \sum_{n=1}^{3}\sum_{\gamma=3}^{4} \text{Re}[(y_n)_{\alpha\alpha}]\frac{2m_\gamma}{m_{H_n}^2}\text{Re}[(y_n)_{\gamma\gamma}]A_0(m_\gamma^2) + m_\gamma B_0(p^2, m_{H_n}^2, m_\gamma^2)\text{Re}[(y_n)_{\alpha\gamma}(y_n)_{\gamma\alpha}]$$

  – giving $\delta m_1^{\text{loop}} = 0$ and with $p^2 = m_2^2 = 0$

$$\delta^{\text{loop}} m_2 = d^2 \sum_{n=1}^{3} \frac{\text{Re}[q_{n2}^2]}{m_4 + m_3}[m_4^2 B_0(0, m_{H_n}^2, m_4^2) - m_3^2 B_0(0, m_{H_n}^2, m_3^2)]$$

- using the measured values

$$\Delta m_{\text{sol}}^2 \sim 7.5 \times 10^{-23}\,\text{GeV}^2 \quad \text{and} \quad \Delta m_{\text{atm}}^2 \sim 2.5 \times 10^{-21}\,\text{GeV}^2$$

  – we estimate $m_2^{\text{phys}} \sim 10^{-11}$ GeV and $m_3 \sim m_3^{\text{phys}} \sim 5 \times 10^{-11}$ GeV

  – we take $m_{H_3} \geq m_{H_2} > m_{H_1} = 125$ GeV and $m_4 > 85$ GeV

  ⇒ we can safely ignore m_3 in $\delta^{\text{loop}} m_2$

    * this allows a further analytic simplification

One-loop predictions: $m_2^{\text{phys}} = m_2^{\text{bare}} + \delta m_2 = 0 + \delta^{\text{loop}} m_2$

- expressing all masses as ratios to the Higgs mass $m_{H_1} = 125\,\text{GeV}$

$$m_{H_2} = r_2 \times m_{H_1} \qquad m_{H_3} = r_3 \times m_{H_1} \quad \text{and} \quad m_4 = R_4 \times m_{H_1}$$

  and using the simple form for $B_0(0,a,b) = 1 - \frac{a \ln[a] - b \ln[b]}{a-b}$ we get

$$\delta^{\text{loop}} m_2 = -m_{H_1} \times d^2 R_4 \sum_{n=1}^{3} \text{Re}[q_{n2}^2] \frac{R_4^2 \ln[R_4^2] - r_n^2 \ln[r_n^2]}{R_4^2 - r_n^2}$$

  or putting in the values for q_{n2}^2:

$$\delta^{\text{loop}} m_2 = -m_{H_1} \times d^2 R_4 \left[(s_{12}^2 - c_{12}^2 s_{13}^2) \frac{R_4^2 \ln[R_4^2]}{R_4^2 - 1} + (c_{12}^2 - s_{12}^2 s_{13}^2) \frac{R_4^2 \ln[R_4^2] - r_2^2 \ln[r_2^2]}{R_4^2 - r_2^2} + (0 - c_{13}^2) \frac{R_4^2 \ln[R_4^2] - r_3^2 \ln[r_3^2]}{R_4^2 - r_3^2} \right]$$

- this function is linear in s_{1k}^2

  $\Rightarrow$ the extremal values are given by the CP conserving limit

$$s_{12} \cdot c_{12} \cdot s_{13} \cdot c_{13} = 0$$

- each selection of the parameters $m_{H_2}^2$, $m_{H_3}^2$, θ_{12}, θ_{13}, and m_4
  - gives an allowed (very narrow) range of d
    - but certain values of the angles have to be excluded, as they produce an exact cancellation of the contributions from the different Higgses

One-loop predictions: from $\delta^{\text{loop}} m_2$

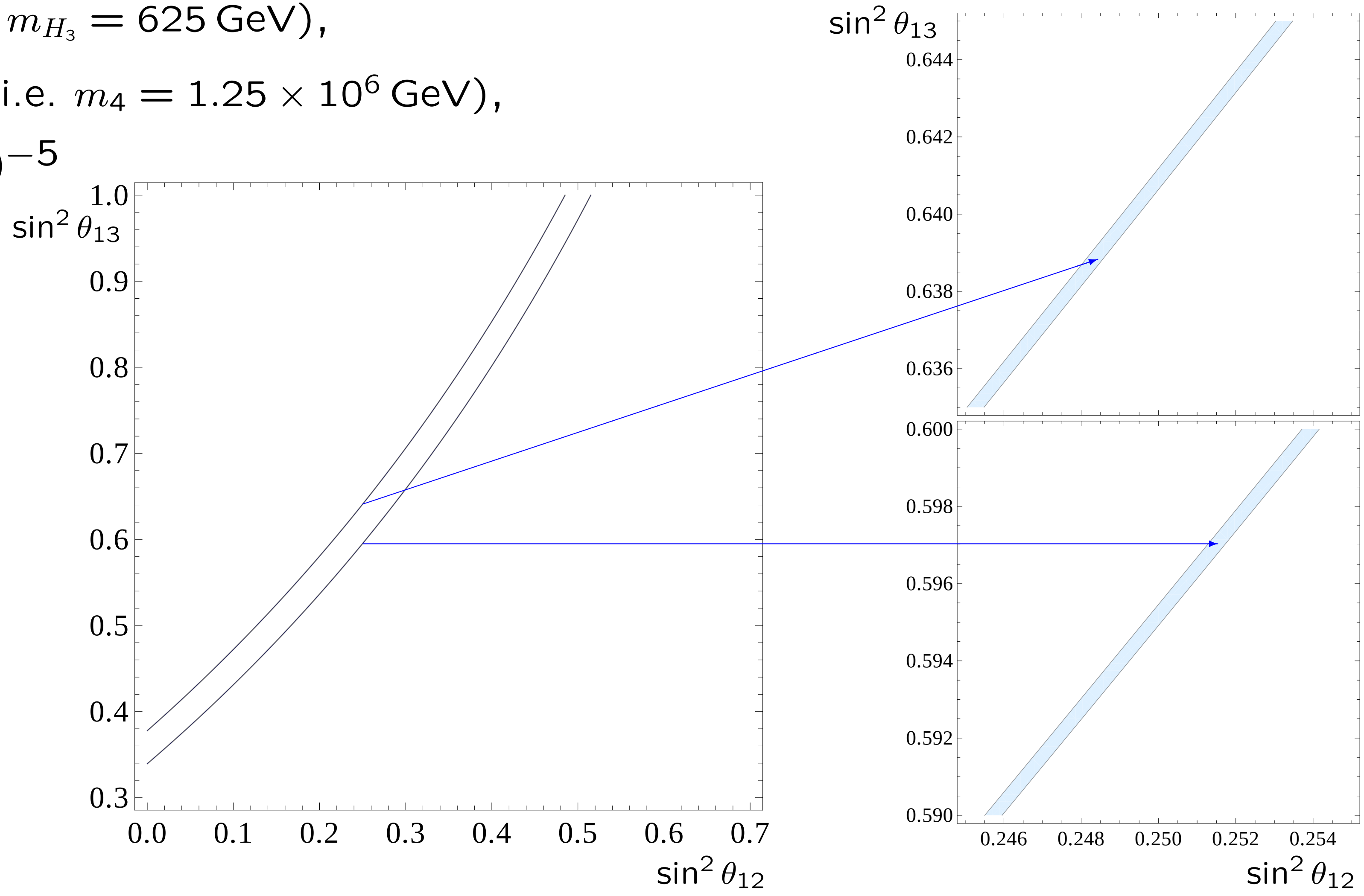
- plotting the allowed ranges of angles $\sin^2\theta_{12}$ and $\sin^2\theta_{13}$ for

  $r_2 = 4$ (i.e. $m_{H_2} = 500\,\text{GeV}$),

  $r_3 = 5$ (i.e. $m_{H_3} = 625\,\text{GeV}$),

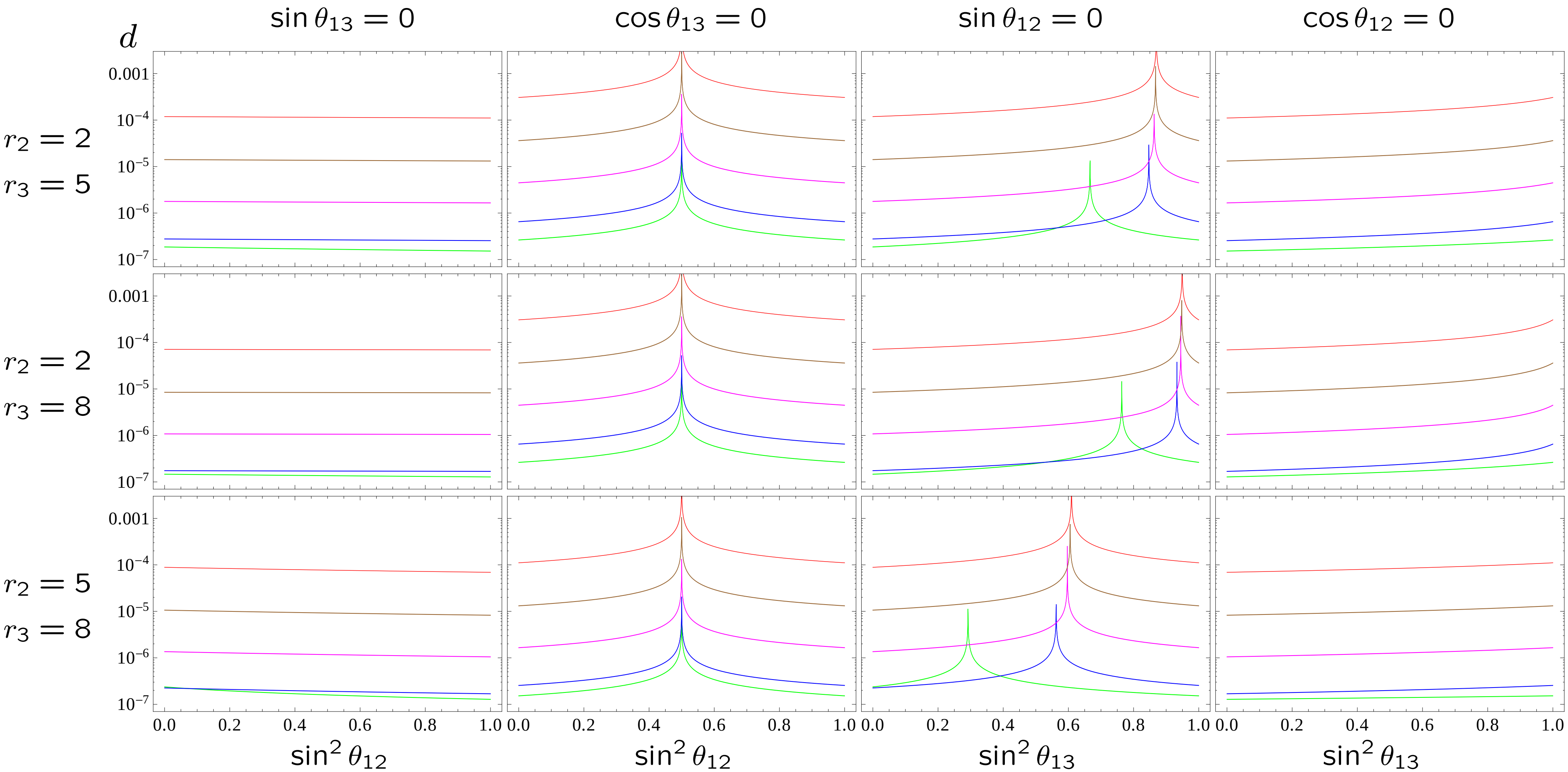
  $R_4 = 10^4$ (i.e. $m_4 = 1.25 \times 10^6\,\text{GeV}$),

  and $d = 10^{-5}$



One-loop predictions: from $\delta^{\text{loop}} m_2$ – CP conserving cases

- plotting $d(\theta)$ for given values of r_2, r_3, and $R_4 = \{1.5, 150, 1.5 \times 10^4, 1.5 \times 10^6, 1.5 \times 10^8\}$



One-loop predictions: using $\delta^{\text{loop}} m_3$

How can we do that?

- when renormalizing the Lagrangian expressed in the mass eigenstates
  - one gets a counter term $\delta^{\text{ct}} m$ for each non vanishing mass m
    - we have $m_3 > 0$ already at tree level ...

"Trick" of Grimus and Lavoura

W. Grimus and L. Lavoura, JHEP **0011** (2000) 042 [arXiv:hep-ph/0008179].

- renormalize the Lagrangian expressed in interaction eigenstates
  - the counter term for the mass matrix

$$\delta^{\text{ct}} M_\nu = \begin{pmatrix} \delta^{\text{ct}} M_L & (\delta^{\text{ct}} M_D)^\top \\ \delta^{\text{ct}} M_D & \delta^{\text{ct}} M_R \end{pmatrix} \qquad \text{has} \qquad \delta^{\text{ct}} M_L = 0_{3\times 3}$$

    - since $M_L^{\text{tree}} = 0_{3\times 3}$
  - the counter term $\delta^{\text{ct}} (M_D)_k = \frac{1}{\sqrt{2}} [(\delta^{\text{ct}} v)(Y_N^{(2)})_k + v(\delta^{\text{ct}} Y_N^{(2)})_k]$
    - is fixed by the vacuum and the Higgs coupling
  - $\delta^{\text{ct}} M_R$ is "fixed" by the not measured heavy singlet ... and ignored

One-loop predictions: using $\delta^{\text{loop}} m_3$

constructing $\delta^{\text{loop}} M_L$

- inverting the seesaw relation $M_\nu = U^\dagger_{(\nu)} \text{diag}(0,0,m_3,m_4) U^*_{(\nu)}$
  - we get $(M_L)_{jk} = (U_{(\nu)})^*_{aj} m_a (U_{(\nu)})^*_{ak}$
    * where we have to use

$$m_D^2 := M_D M_D^\dagger \qquad m_{3,4} = \frac{1}{2}\left[\mp M_R + \sqrt{M_R^2 + 4 m_D^2}\right]$$

and

$$U_{(\nu)} = \begin{pmatrix} U_{1e} & U_{1\mu} & U_{1\tau} & 0 \\ U_{2e} & U_{2\mu} & U_{2\tau} & 0 \\ \frac{is(M_D^*)_e}{m_3} & \frac{is(M_D^*)_\mu}{m_3} & \frac{is(M_D^*)_\tau}{m_3} & -is \\ \frac{c(M_D^*)_e}{m_4} & \frac{c(M_D^*)_\mu}{m_4} & \frac{c(M_D^*)_\tau}{m_4} & c \end{pmatrix} \quad \text{where} \quad \begin{array}{rcl} 2sc & = & 2m_D/\sqrt{M_R^2 + 4m_D^2} \\ 0 & = & U_{1k}(M_D)_k = U_{2k}(M_D)_k \\ 0 & = & U_{1k}(Y_N^{(2)})_k \end{array}$$

- renormalized parameters of the interaction Lagrangian are enough
  - to calculate $(\delta^{\text{loop}} M_L)_{jk} = (U_{(\nu)})^*_{aj}\, \delta^{\text{loop}} m_a (U_{(\nu)})^*_{ak}$
  - to get the predicted masses, diagonalize $\begin{pmatrix} \delta^{\text{loop}} M_L & (M_D)^\top \\ M_D & M_R \end{pmatrix}$
    * as the contributions from $\delta^{\text{loop}} M_D$ and $\delta^{\text{loop}} M_R$ are subleading [G-L]

[G-L] W. Grimus and L. Lavoura, JHEP **0011** (2000) 042 [arXiv:hep-ph/0008179].

One-loop predictions: using $\delta^{\text{loop}} m_3$

$\delta^{\text{loop}} m_a$ has in principle four contributions

- distinguished by the bosons in the loop
  - Z-boson, neutral Higgses H_n^0, W-boson, charged Higgs $H^\pm$
    - where the Goldstone bosons are included with their respective gauge bosons
    - for simplicity we pick Feynman gauge for the gauge boson propagators
- $\delta_Z^{\text{loop}} m_3$ has a contribution from $m_4 \times$ Loopfunction $\gg m_3$

  $\Rightarrow$ we need to cancel this contribution by $\delta_{H_n^0}^{\text{loop}} m_3$
    - as $\delta_W^{\text{loop}} m_3 < \delta_Z^{\text{loop}} m_3$ and $\delta_{H^\pm}^{\text{loop}} m_3 < \delta_{H_n^0}^{\text{loop}} m_3$
    - we get the charged contributions from the attempt to extend [G-L]
- we require for the cancellation

$$\delta^{\text{loop}} m_3 \;\leq\; 2\, m_3^{\text{phys}}$$

  - together with the restrictions from $\delta^{\text{loop}} m_2$
    - we will again use the four CP conserving cases

One-loop predictions: using $\delta^{\text{loop}} m_3$

Cancellations are always possible

- but restrict the additional parameters:
  - the scalar product of the two Yukawa couplings:

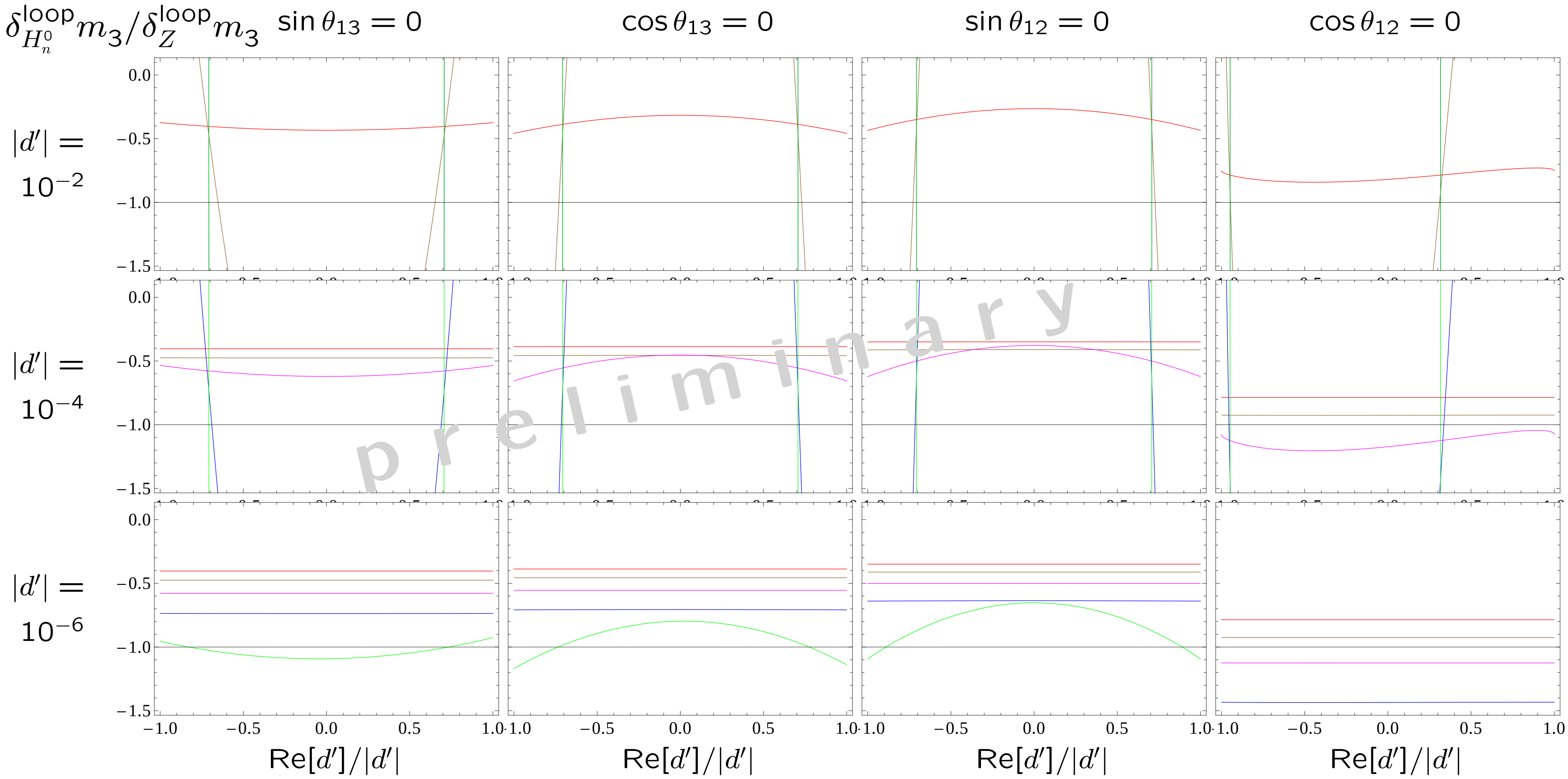
$$d' := \frac{1}{\sqrt{2}} U_{3k} (Y_N^{(2)})_k = -\frac{is}{m_3\sqrt{2}} (M_D^*)_k (Y_N^{(2)})_k = -\frac{isv}{2m_3} Y_N^{(1)\dagger} Y_N^{(2)}$$

  - the phase of d' cannot not be absorbed
    - it is a physical parameter
  - but both, modulus and phase are restricted to allow cancellations

- $\delta^{\text{loop}}_{H^\pm} m_3$ can have a contribution of the order of m_3
  - needed for a full numerical analysis

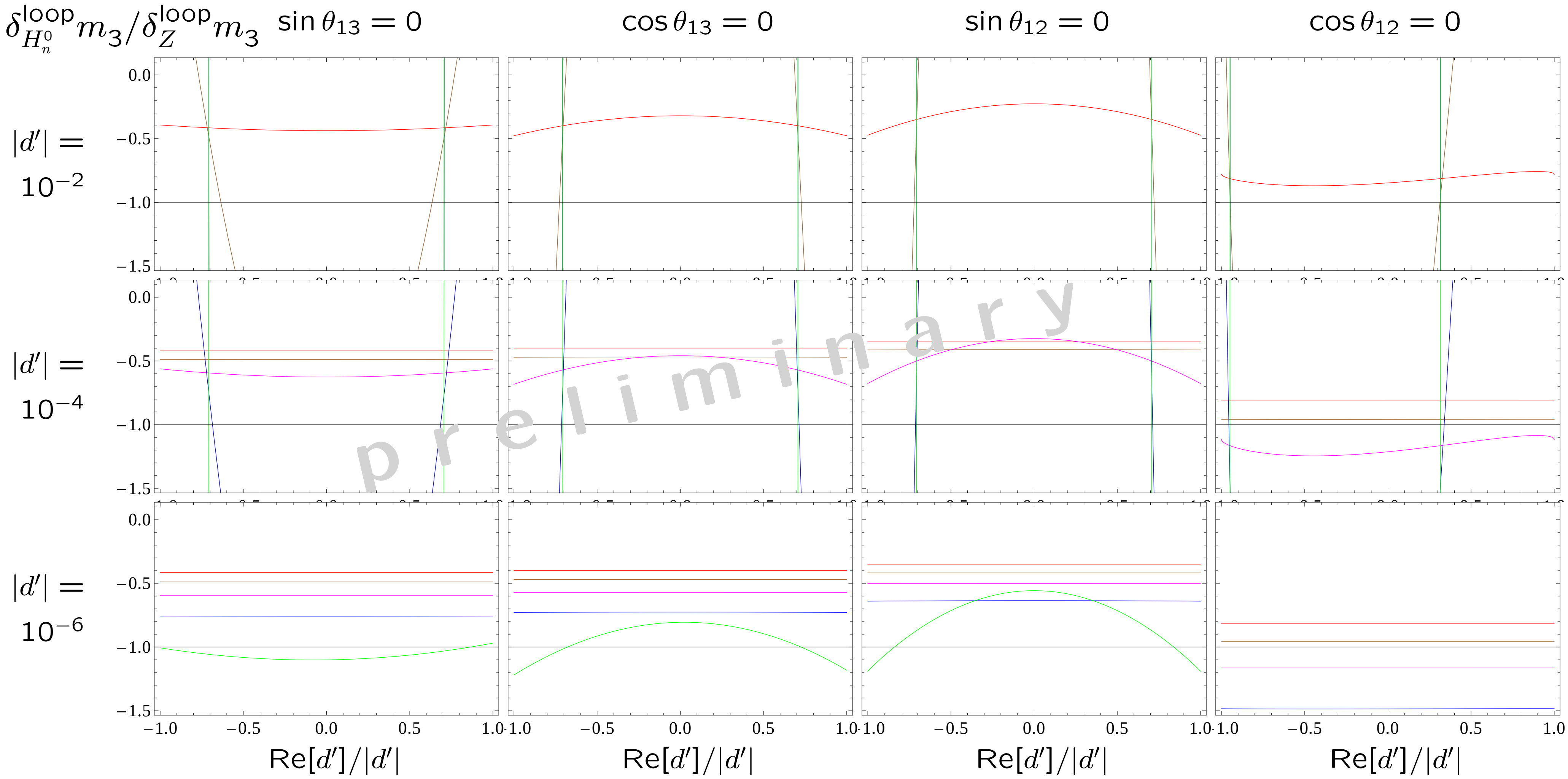
One-loop predictions: using $\delta^{\text{loop}} m_3$ – CP conserving cases

- plotting $\delta^{\text{loop}}_{H_n^0} m_3 / \delta^{\text{loop}}_{Z} m_3$ for given values of $r_2 = 2$, $r_3 = 5$, and $R_4 = \{1.5, 150, 1.5 \times 10^4, 1.5 \times 10^6, 1.5 \times 10^8\}$



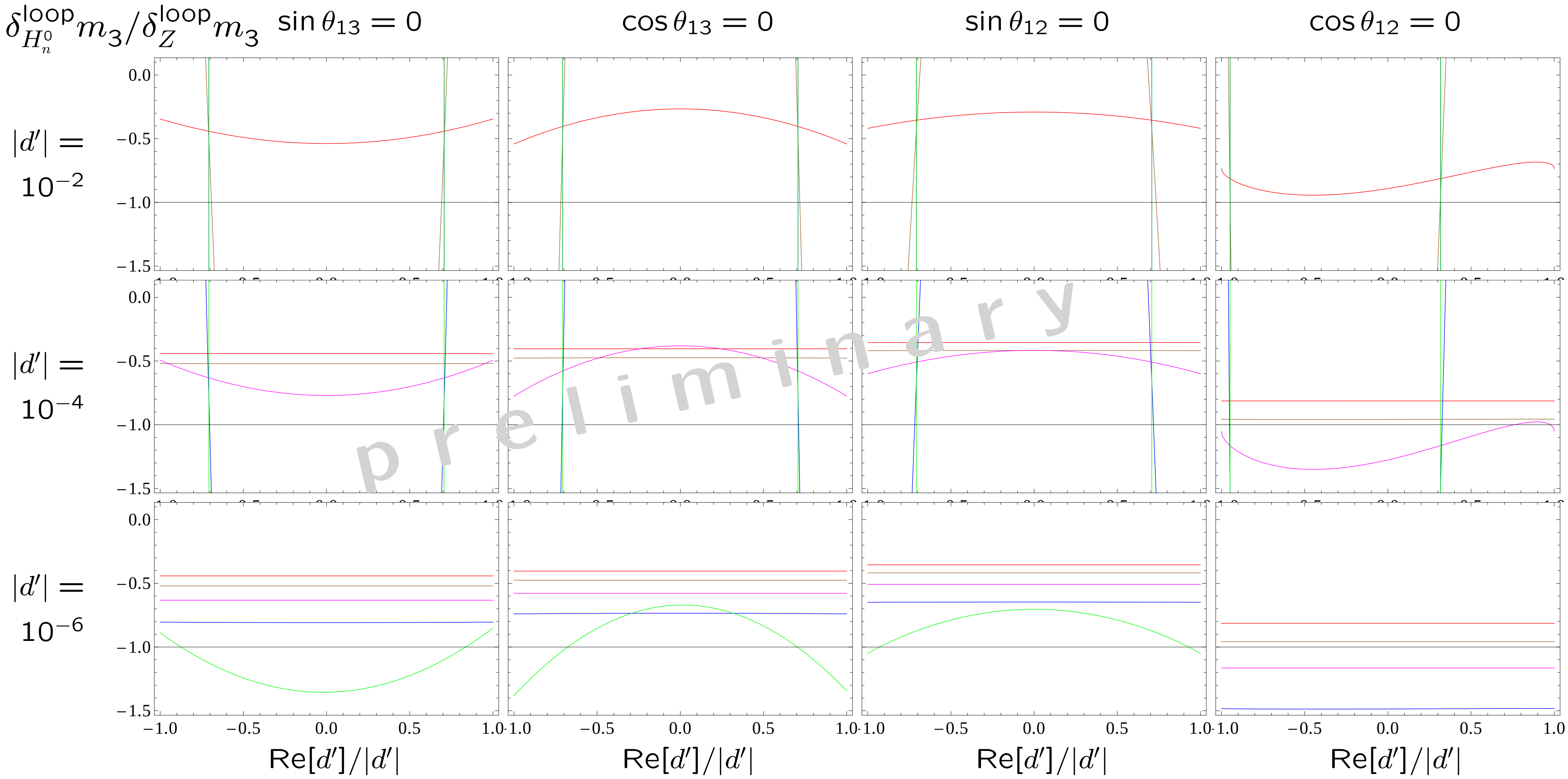
One-loop predictions: using $\delta^{\text{loop}} m_3$ – CP conserving cases

- plotting $\delta^{\text{loop}}_{H^0_n} m_3 / \delta^{\text{loop}}_{Z} m_3$ for given values of $r_2 = 2$, $r_3 = 8$, and $R_4 = \{1.5, 150, 1.5 \times 10^4, 1.5 \times 10^6, 1.5 \times 10^8\}$



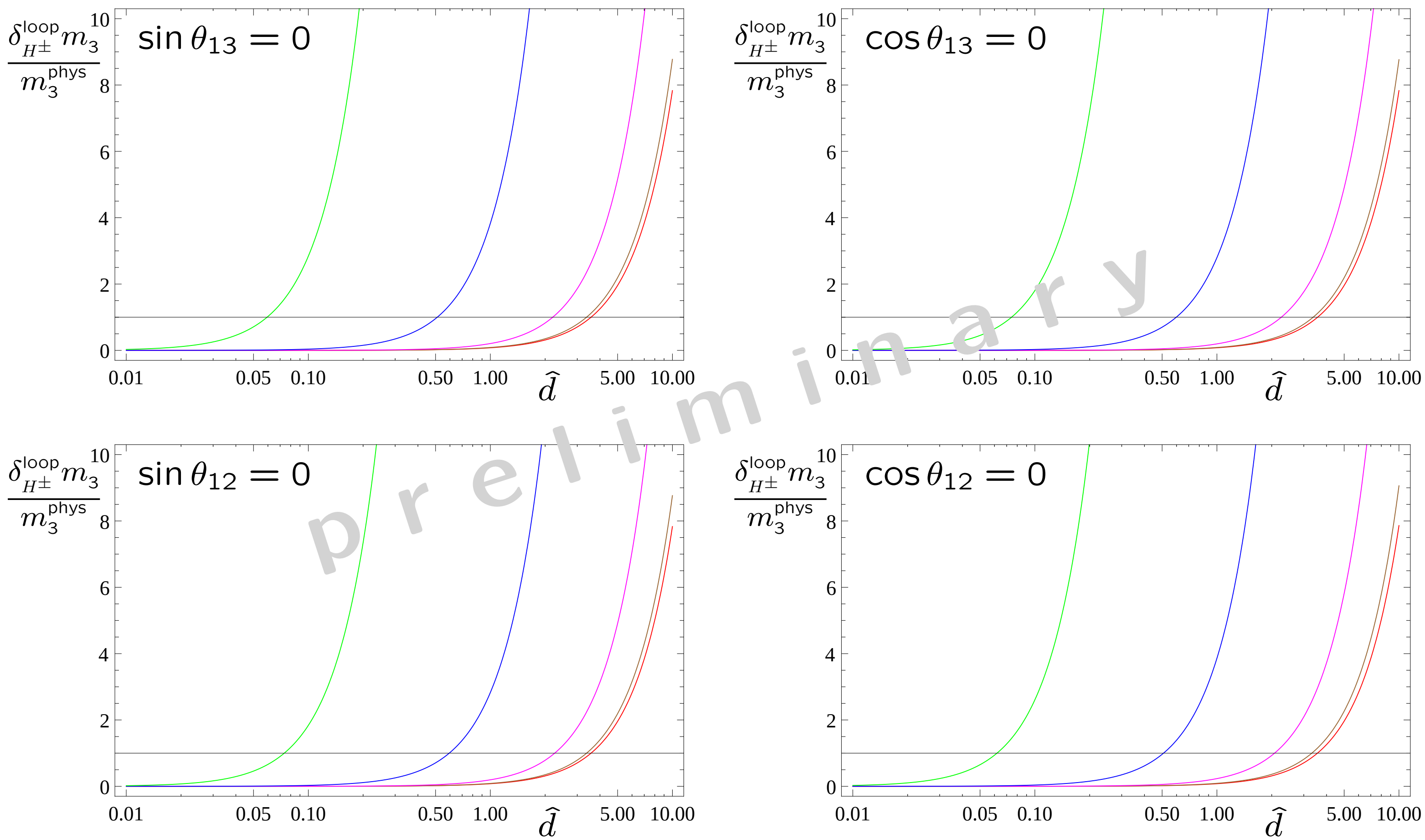
One-loop predictions: using $\delta^{\text{loop}} m_3$ – CP conserving cases

- plotting $\delta^{\text{loop}}_{H^0_n} m_3 / \delta^{\text{loop}}_Z m_3$ for given values of $r_2 = 5$, $r_3 = 8$, and $R_4 = \{1.5, 150, 1.5 \times 10^4, 1.5 \times 10^6, 1.5 \times 10^8\}$



One-loop predictions: using $\delta^{\text{loop}} m_3$ – CP conserving cases

- plotting $\delta^{\text{loop}}_{H^\pm} m_3 / m_3^{\text{phys}}$ for given values of $m_{H^\pm} = 200\,\text{GeV}$ and $R_4 = \{1.5, 150, 1.5 \times 10^4, 1.5 \times 10^6, 1.5 \times 10^8\}$ depending on the diagonal element $\hat{d} = (Y_E^{(2)})_{ee} = (Y_E^{(2)})_{\mu\mu} = (Y_E^{(2)})_{\tau\tau}$



The 312-νSM: Conclusions

Our model

- basically fixes the Yukawa couplings of the neutral singlet
- and restricts the other parameters somewhat

Thank you

for discussion

and comments

and of course for the conference! ☺

Backup

One-loop predictions: from $\delta^{\mathrm{loop}} m_2$ – CP conserving cases

- plotting $d(\theta)$ for given values of r_2, r_3, and $R_4 = \{1.5, 150, 1.5 \times 10^4, 1.5 \times 10^6, 1.5 \times 10^8\}$