Is it possible to recover the correct theory of gravity from cosmology?

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Introduction

The most significant discovery in cosmology in the last 20–30 years:

$$d_L(z, \text{observ}) > d_L(z, \text{Canon. Cosmol. Model})$$

for supernovae Ia.
Interpretation:
$a(t) \propto t^p$, $p > 1$ — acceleration.
Is the interpretation correct?
I. The large scale inhomogeneous Lemaître–Tolman spacetime (Andrzej Krasiński, Warszawa):
in a toy model one finds the effect without global acceleration.

II. Small scale inhomogeneous spacetime — the backreaction problem.
The real Universe: $g_{\mu\nu}$, $T_{\mu\nu}$ and EFE ($G = c = 1$)

$$G_{\mu\nu}(g) + \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu}.$$

Now average $g_{\mu\nu}$ and $T\mu\nu$ to R–W spacetime:
$g_{\mu\nu} \to g^{(0)}_{\mu\nu}$, $T_{\mu\nu} \to T^{(0)}_{\mu\nu}$ independently $\Rightarrow$

$$G_{\mu\nu}(g^{(0)}) + \Lambda g^{(0)}_{\mu\nu} = 8\pi(T^{(0)}_{\mu\nu} + \tau_{\mu\nu}),$$

$\tau_{\mu\nu}$ — the average of the contributions to EFE arising from the departures of g and T from $g^{(0)}$ and $T^{(0)}$ $\Rightarrow$
$\tau_{\mu\nu}$ — the *backreaction effects* of the matter inhomogeneities on $g^{(0)}$ $\Leftrightarrow$ *effective stress–energy tensor* of the inhomogeneities.

In cosmology we *NEVER* use an exact solution $g_{\mu\nu}$ for the exact $T_{\mu\nu}$. We *assume* that an R–W metric $g'^{(0)}_{\mu\nu}$ is generated by $T^{(0)}_{\mu\nu}$ via *averaged* EFE

$$G_{\mu\nu}(g'^{(0)}) + \Lambda g'^{(0)}_{\mu\nu} = 8\pi T^{(0)}_{\mu\nu}.$$

In practice $g'^{(0)}_{\mu\nu} \Rightarrow a'(t) = t^{1/2}(radiationera), t^{2/3}$ (galactic era).
If $\tau_{\mu\nu} \neq 0 \Rightarrow g'^{(0)}_{\mu\nu} \neq g^{(0)}_{\mu\nu}$.
The backreaction problem: is

$$\tau_{\mu\nu} \approx 0?$$

Green and Wald (2006–2014): $\tau_{\mu\nu}$ *is* ≈ 0.
T Buchert, M Carfora, G F R Ellis, E W Kolb, M A H MacCallum, J J Ostrowski, S Räsänen, B F Roukema, L Andersson, A A Coley and D L Wiltshire, arXiv 1505.07800v1,
,, *Is there proof that backreaction of inhomogeneities is irrelevant in cosmology?*" — the backreaction may be significant.
S R Green and R M Wald, arXiv 1506.06452v1,
,,*Comments on backreaction*" — $\tau_{\mu\nu} \approx 0$, a harsh response.
Conclusion:
The polemics is sharp, the problem is open.

Assume: the spacetime is (almost) R–W and the acceleration is real. How to account for it?

1. ΛCDM model.

It well fits all observational data (Planck satellite 2013) $\Rightarrow$ *phenomenologically* is fully satisfactory and *CONCEPTUALLY* is NOT satisfactory:

— it is extremely hard to calculate Λ from the first principles (Weinberg 1989),

— „fine tuning" problem: $\rho_\Lambda \approx 3\rho_{DM}$ — a coincidence.

2. Dark energy („quintessence"):

dynamical classical field with $p_X \approx -\rho_X < 0$. Most fantastic concepts (Chaplygin gas).

3. A distinct theory of gravity $\Rightarrow$ alternative dynamics for $a(t)$.

Two questions:

— is it possible to reject GR?

— is it worth doing?

Multitude of gravity theories

Physics of gravitation is exceptional:
in all other branches of physics there is one theory or at most a couple of competing theories,
for gravitation there is infinite number of existing or potential theories and GR is just a point in a continuous space of conceivable theories.
All the ,,alternative gravity theories" are merely various modifications, generalizations and complications of Einstein's GR, they would never arise without it. Why is it so?

Experimental cause:
the number of experiments and observations that are accessible to us is small and their variety is very small.
We cannot produce gravit. fields and gravit. effects in such a number and variety as we can in electromagnetism. We cannot create strong gravit. waves or black holes. We do not rule over gravitation as we do over electromagnetism.
Apparently: gravitation is a macroscopic phenomenon related to large masses. It is unclear if it has a microscopic quantum foundation.
In consequence:
many gravity theories fit sufficiently well to the scarce set of empirical data. Modifications of GR go in all possible directions $\Rightarrow$ classes of alternative theories are disparate $\Rightarrow$ no complete classification, no experts.

Theories most interesting for cosmologists: *metric nonlinear gravity* (NLG) theories — differ from GR only in equations of motion: gravity is described by one unifying tensor field $g_{\mu\nu}$ which in the initial formulation of the theories is interpreted as a spacetime metric with a Lagrangian

$$L = f(g_{\alpha\beta}, R_{\alpha\beta\mu\nu}),$$

where f is any smooth scalar function.
Most investigated: *restricted NLG theories*:

$$L = f(R), \qquad L = R \qquad \mathrm{GR}.$$

Called ,,curvature quintessence scenario".
Appeared very early (Hermann Weyl circa 1919). Various motivations: avoidance of singularities, quantization of gravity, string theory. All the expectations failed.

The second question:
why to employ the alternative theories instead of assuming that the cosmic acceleration is driven in GR by a kind of self–interacting scalar (vector or tensor) matter field?
Answer:
one must then introduce ad hoc some *classical* field unknown to laboratory physics. All known species of matter exist as quanta of quantized matter fields and are described by the Standard Particle Model.
Dark energy is a classical field:
— it does not fit the Standard Model,
— it contradicts the tenet of modern physics: all matter fields are quantized.
Gravitational field is classical and need not be quantized, is independent of the Standard Model.
Replacement of GR by an NLG theory is more conservative than the concept of dark energy.

The leading idea:
if the first real trouble for GR has appeared in cosmology, then the cosmological data should be crucial for recognizing the correct theory in the space of alternative theories.
If the correct theory is a specific $L = f(g_{\alpha\beta}, R_{\alpha\beta\mu\nu})$ rather than $L = R$ (GR), then the question: *why this* f? — is postponed to some future.
Now one seeks for some f fitting all available cosmological data.
GR: $L = R \Rightarrow$ EFE are of *second* order,
NLG: field equations for $g_{\mu\nu}$ are of *fourth* order.
In cosmology NLG theories are applied only to the spatially flat R–W spacetime:

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2),$$

$\Rightarrow$ field equations reduce to one quasi–linear third order ODE for $a(t)$ — quasi–Friedmannian eq. (QFE).
Solar system tests are purely auxiliary and actually are ambiguous.

In vacuum:
GR: Friedmann eq. $\dot{a}^2 = 0$ — only Minkowski spacetime,
NLG theory: nontrivial $a(t)$ solutions.
NLG cosmology:
at present epoch the nonlinear effects in QFE dominate over matter contributions and the vacuum solutions $a(t)$ account for the acceleration.
Only approximate solutions are known (QFE very complicated).
The typical research program is:
seek for $L = f(g_{\alpha\beta}, R_{\alpha\beta\mu\nu})$ such that it admits a vacuum solution $a(t)$ yielding the sequence
inflation $\rightarrow$ deceleration $\rightarrow$ acceleration.

Example (Carroll et al. 2004):

$$L = R^2 + R + \frac{1}{R}.$$

then assume:

- in the early universe the R^2 term is dominant giving rise to some kind of inflation (Starobinsky 1980),
- in the intermediate curvature period (radiation and galactic eras) $1s < t < 10^{10}$ years the R term dominates and the standard cosmological model is valid,
- at present and in future the decreasing curvature makes the $1/R$ term dominant what results in an accelerated expansion.

This kind of argument is explicitly or implicitly present in most works on NLG theories.

Results.
Multitude of Lagrangians $L = f(R)$ fitting observ. data:
rational functions (polynomial/polynomial), irrational functions, $\exp(cR)$, a combination of 2 confluent hypergeometric fcts., etc.
Conjecture (Nojiri & Odintsov): ANY cosmological evolution may be realized by some $f(R)$. Also

$$L = R + \frac{1}{R_{\alpha\beta}R^{\alpha\beta}} + \frac{1}{R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}}$$

fits the Supernovae Ia data (the accelerated expansion).

This line of research — very suggestive — is misleading. All these outcomes are only for R–W metric. R–W spacetime is 'flexible'—is versatile in that it contains an *arbitrary* function $a(t)$.
The QFE for $a(t)$ has solutions for ANY $f \Longrightarrow$ R–W metric is a *universal* solution for all NLG theories.
Yet Minkowski ($\mathcal{M}_4$), de Sitter (dS) and anti–de Sitter (AdS) spacetimes are not universal.
Is it all meaningful?

Most researchers seem to be unaware of the wealth and variety of possibilities.

The Lagrangian of a generic NLG theory,
$L = f(R, R_{\alpha\beta}R^{\alpha\beta}, R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}, \ldots)$, depends on all 14 invariants of Riemann tensor.

In the simplest case of $L = f(R)$ one may invoke a corollary of the Cantor's theorem in set theory:
the cardinality number of the set of smooth functions $f(R)$ is equal to continuum.

In other words: the set of all NLG theories is the continuum.

If one believes that the accelerated cosmic expansion requires dynamics different from that in GR, then how to select the correct Lagrangian?

Criticism of the cosmological search of the correct theory

My main message:
applying cosmological observations is the most unreliable way for recovering the true theory of gravitation.

I.

In GR $L = R$ and Einstein field equations are of 2nd order.
The field equations of NLG theories are of 4th order ('higher derivative gravity').
Dynamics is completely different from GR! $g_{\mu\nu}$ has in general 8 d.o.f. (GR: 2 d.o.f.) $\Longrightarrow$ the space of solutions is larger than that of GR, also in cosmology. It may contain astonishing solutions.

II. I emphasize:

the standard cosmological model in GR has the following generic features:

- if cosmic matter satisfies the strong energy condition (SEC) then Hawking–Penrose singularity theorem implies that our universe contains a singularity,
- in R–W spacetime the singularity must be in the past (the Big Bang) and $a(t)$ monotonically grows from zero independently of the particular particle content and other properties of the cosmic matter (eq. of state).

If the SEC does not hold: e. g. for a self–interacting scalar field Φ in GR: $G_{\mu\nu} = 8\pi G T_{\mu\nu}(\Phi) \Longrightarrow$

- there are oscillatory solutions for $t \to \infty$,
- there may be no past singularity.

In general a condition equivalent to SEC is not satisfied in NLG theories. If in a given NLG theory the answer to the questions:
i) is past singularity universal?
ii) are oscillatory solutions excluded?
is 'NO', then
all the successes of the Standard Cosmological Model are lost and whole cosmology (not only in R–W spacetime) must be constructed anew.
Up to now: only special solutions are known and the phase space analysis is inconclusive.

III.

The fundamental problem of NLG cosmology:

is it possible to uniquely and effectively reconstruct the underlying $L = f(R)$ or $f(R_{\alpha\beta\mu\nu})$ from an exact form of $a(t)$ (taken from observations) treated as a solution of the corresponding QFE?

Response:

it is practically impossible.

Why the failure?

R–W spacetime: *we never use exact solutions for realistic matter* (*all particles of the Standard Particle Model and their clumpy localizations*).

In standard cosmology we assume: radiation era ends at $t \approx t_{\rm rec}$ and $p = \frac{1}{3}\rho$ is *suddenly* replaced by $p = 0$. There is an exact solution for *radiation + nonrelativistic matter* — it is in form $t = t(a)$ and cannot be inverted.

The standard Friedmann cosmology with *approximate* solutions:

- $a \propto \sqrt{t}$ in radiation era,
- $a \propto t^{2/3}$ in galactic era

is a good fit to the real universe prior to the acceleration era $\Longrightarrow$ slight modifications $a(t) + \delta a(t)$ also well fit the data and are *approximate* solutions to some QF eq. $\Longrightarrow$ give rise to a gravity theory different from GR.

R–W spacetime is the most unsuitable one for recovering the L. It is doubly deceptive in this search:
i) it is an approximation to the real world since the universe is not perfectly homogeneous and isotropic;
ii) the solutions that are always used, $a \propto \sqrt{t}$ and $t^{2/3}$, are NOT exact ones.
Both the R–W spacetime and the solutions form a math. model being an *approximation* to the physical world.
Conclusion:
It is impossible to uniquely recover the correct gravity L from approximate (observational) cosmological solutions.
Final conclusion:
Cosmology is not a way leading to the true theory of gravity.