The flavour problem and family symmetry beyond the Standard Model

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Outline

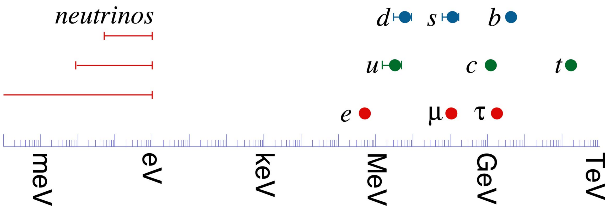
The problems in the Standard Model

Parameters in the lepton sector

Good agreement with the experimental data, but many parameters:
6 masses of leptons, 3 mixing angles, 1 or 3 CP phases.

Some open questions- **FLAVOUR PROBLEM**

- Existence of exactly 3 families in the Standard Model
- The hierarchy of the fermions' masses



The problems in the Standard Model

- Explanation of **PMNS** and **CKM** parameters
- What is the neutrino mass?
- Why the neutrino mass is so small?
- Have we got normal (**NO**) or inverted ordering (**IO**) in the neutrino´s mass spectrum?
- What is the nature of neutrino (Dirac or Majorana)?
- Is CP symmetry violated in the lepton sector at all?

Symmetry realization

BASIC IDEA

$$G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y \longrightarrow G_{SM} \times G_{flavour}$$

$G_{flavour}$:

- is not a gauge symmetry
- can be continuous or finite
- its imposition on the Lagrangian can reduce the number of parameters in the SM.

Two types of methods:

1. "**bottom-up**": $EXP \longrightarrow \mathbf{U_{PMNS}} \longrightarrow u_i \longrightarrow G_i \longrightarrow \mathbf{G_{Flavour}}$
2. "**top-down**": $\mathbf{G_{Flavour}} \longrightarrow G_i \longrightarrow u_i \longrightarrow \mathbf{U_{PMNS}}$

Studies within SM

U_{TBM}

Before year 2012 experimental data were in agreement with the so called "**T**ri**B**i**M**aximal" (**TBM**) mixing matrix:

$$U_{TBM} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} 0.816 & 0.577 & 0.0 \\ -0.408 & 0.577 & 0.707 \\ 0.408 & -0.577 & 0.707 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}.$$

$$\theta_{13} = 0^\circ, \quad \theta_{23} \cong 45^\circ, \quad \theta_{12} \cong 35^\circ.$$

- third column - maximal mixing between ν_μ and ν_τ:

$$|U_{\mu 3}| = |U_{\tau 3}| = 1/\sqrt{2},$$

- second column - equal mixing between ν_e, ν_μ and ν_τ.

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- In 2004 hypothesis linking U_{TBM} matrix with discrete symmetry group: A_4.
- Within almost a decade lots of other proposals: $U(1)^3 \times Z_2^3 \rtimes S_3$, $Z_2^3 \rtimes S_3$, S_4, T', $\Delta(27)$, $\Delta(54)$.

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[1] P. F. Harrison, D. H. Perkins, W. G. Scott: *Tri-bimaximal mixing and the neutrino oscillation data*

Studies within SM

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U_{TBM}

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$$U_{TBM} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} 0.819 & 0.551 & 0.158 \\ -0.512 & 0.581 & 0.632 \\ 0.256 & -0.599 & 0.758 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}.$$

$$\theta_{13} = 8.5^\circ \pm 0.2^\circ, \quad \theta_{23} = 45^\circ \pm 3^\circ, \quad \theta_{12} = 34^\circ \pm 1^\circ.$$

- In 2004 hypothesis linking U_{TBM} matrix with discrete symmetry group: A_4.
- Within almost a decade lots of other proposals: $U(1)^3 \times Z_2^3 \rtimes S_3$, $Z_2^3 \rtimes S_3$, S_4, T', $\Delta(27)$, $\Delta(54)$.
- 2012 - new oscillation data excluding TBM pattern.

Building model based on the SM- attempts after 2012[2]

To our knowledge all non-abelian subgroups of *SU(3)* up to the order **511** have been regarded as the possible family symmetries.

The results are following:

1. No group can accommodate simultaneously all three columns of the PMNS matrix,
2. Many groups can accommodate first or second column of the mixing matrix,
3. Only one group can accommodate the third column ($\Delta 150$) of mixing matrix giving the following results:

$$sin^2 2\theta_{13} = 0,11 \; (0,098 \pm 0,013 \; in \; \textbf{EXP}),$$

$$sin^2 2\theta_{23} = 0,94 \; (> 0,92 \; in \; \textbf{EXP}).$$

[2] C.S.Lam: *Symmetry of Leptonic Mass Matrices*

Studies within the SM-summary and conclusions

In this approach masses can be fitted only by taking appropriate values of the parameters.
For all studied groups there are:

- **3 real parameters** in the case of charged leptons mass matrix ,
- **4 complex parameters** for neutrino mass matrix (Majorana neutrino).

The biggest disadvantages of the model

In that approach the imposed family symmetry is broken into two subgroups (for charged leptons and neutrino mass matrices).
Obtaining the masses requires fitting of the parameters.

Our model

The Suggestion

Let us extend the scalar sector by one additional Higgs doublet.
Maybe relation between 2 doublets
(through the 2-dimensional representation of family symmetry)
will not force us to break the imposed symmetry.

MOTIVATION

- The addition of the nonabelian symmetry to the Standard Model has been widely studied
- These models didn't give any reasonable results
- We had to break the family symmetry in order to get non-trivial mixing matrix
- We want to explain not only parameters in the mixing parameters, but also the hierarchy problem

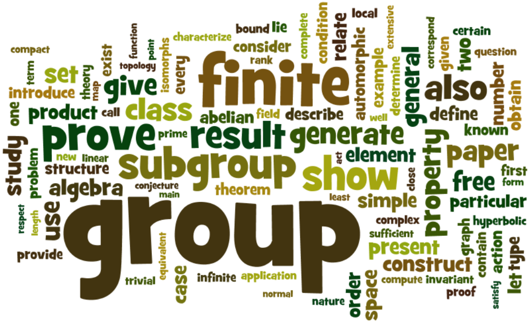
Step by Step

THE MODEL VERIFICATION

One has to:

- find the groups **G** which fulfil the requirements of our model,
- impose flavour symmetry **G** on the Yukawa Lagrangian,
- calculate the Yukawa matrices,
- create mass matrices and mixing matrix,
- in the case of the agreement with the experiment :
  - studies of the symmetry restrictions on the Higgs part of Lagrangian,
  - verification of the Higgs Lagrangian with respect to its physical properties.

What are the groups G?



1. *Discrete* : the number of elements in the group is finite,
2. *Non-abelian*,
3. *Subgroup* of **U(3)** (particularly of **SU(3)**),
4. Possesses **3** and **2**-dimensional irreducible representations (the order divisible by 6),

The requirements for the flavour group

Which groups can fulfil the requirements of our model?

- We want to have the following transformation law for Higgs doublets:

$$\begin{bmatrix} \Phi_1' \\ \Phi_2' \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \Phi_1 \\ \Phi_2 \end{bmatrix},$$

where

Φ_1, Φ_2-- the Higgs doublets before transformation,

Φ_1', Φ_2'-- the Higgs doublets after transformation,

$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$--2-dimensional representation of $\mathbf{G}$.

2-DIMENSIONAL REPRESENTATION IS NEEDED!

Imposing the symmetry

Since the Yukawa couplings are parameters within SM we have to impose family symmetry which can possibly explain the mixing parameters and mass hierarchy.

Yukawa Lagrangian in our model:

$$\mathcal{L} = - \sum_{\alpha,\beta=e,\mu,\tau,\, j=1,2} [\Gamma'^{l}_{\alpha\beta j} \bar{L}'_{\alpha L} \Phi_j l'_{\beta r} + \Gamma'^{\nu}_{\alpha\beta j} \bar{L}'_{\alpha L} \tilde{\Phi}_j \nu'_{\beta r}] + H.c..$$

Each term can be rewritten in more compact way:

$$\mathcal{L} = a^\dagger (\Gamma_j \Phi_j) b + H.c.,$$

where:

$$a = \begin{bmatrix} L_e \\ L_\mu \\ L_\tau \end{bmatrix}, b = \begin{bmatrix} f_e \\ f_\mu \\ f_\tau \end{bmatrix}, \Phi_j = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ v_j + H_j \end{bmatrix}, \tilde{\Phi}_j = i\sigma_2 \Phi^*$$

Γ_j- an arbitrary 3-dimensional matrix.

The invariance equation

At this point we impose **the family symmetry G**:

$$a \longrightarrow D_a a, \quad b \longrightarrow D_b b, \quad \Phi \longrightarrow D_\Phi \Phi$$

$$\mathcal{L} = a^\dagger(\Gamma_j \Phi_j) b + H.c. \longrightarrow \mathcal{L}' = a^\dagger (D_a)^\dagger (\Gamma_j (D_\Phi)_{jk} \Phi_k) D_b b + H.c.$$

where:

D_a and D_b $\longrightarrow$ 3 dimensional irreducible representations of G,
D_Φ $\longrightarrow$ 2 dimensional irreducible representation of G.

The invariance equation

Requiring $\mathcal{L} = \mathcal{L}'$ one gets the following condition:

$$(D_a(f))^\dagger (\Gamma_j (D_\Phi)_{jk}(f)) D_b(f) = \Gamma_k \quad \forall f \in G.$$

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[3] O.Ludl : *Systematic analysis of finite symmetry groups and their application to the lepton sector*

Solution to the invariance equation

The invariance equation can be rewritten in the form:

$$(D_a)^\dagger(\Gamma_j(D_\Phi)_{jk})D_b = \Gamma_k,$$

$$[D_\Phi]_{jk}[D_a]^\dagger_{\alpha\gamma}[\Gamma_j]_{\gamma\delta}[D_b]_{\delta\beta} = [\Gamma_k]_{\alpha\beta},$$

$$[D_\Phi]^T_{kj}[D_a]^\dagger_{\alpha\gamma}[D_b]^T_{\beta\delta}[\Gamma_j]_{\gamma\delta} = [\Gamma_k]_{\alpha\beta},$$

which in the more simplified version looks like the eigenproblem :

$$\mathbf{N}\Gamma = \Gamma,$$

$$\mathbf{N}=[D_\Phi]^T \otimes [D_a]^\dagger \otimes [D_b]^T$$

$$DIM(N) = 2 \times 3 \times 3 = 18$$

Solution to the invariance equation and its interpretation

1. Construction of the matrix **N** for all generators
2. Finding the eigensubspace for all generators
3. Finding the common eigensubspace
4. Establishing the base vector for common eigensubspace

Solution to the equation

The base vector of the common eigensubspace of all matrices **N** constitutes the solution to the invariance equation.

INTERPREATION OF THE SOLUTION

It can be proven that matrices Γ_j are the Clebsch-Gordan coefficients for the following decomposition:

$$D_a \otimes D_b^* = \oplus_\lambda D_\lambda$$

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[4] O.Ludl :*Systematic analysis of finite symmetry groups and their application to the lepton sector*

Construction of the mass matrices and mixing matrix

In order to get the solution for mass matrices we must look for the following Clebsch-Gordan coefficients:

$D_{L_L} \otimes D^*_{l_R} = \otimes_\lambda D_\lambda$ -here we look for the 2-dimensional representation

$D_{L_L} \otimes D^*_{\upsilon_R} = \otimes_\lambda D_\lambda$ -here we look for the 2^*-dimensional representation

$$M^\nu_{\alpha\beta} = \frac{1}{\sqrt{2}} \sum_i \upsilon_i (\Gamma^\nu_i)_{\alpha\beta},$$

$$M^l_{\alpha\beta} = \frac{1}{\sqrt{2}} \sum_i \upsilon_i (\Gamma^l_i)_{\alpha\beta}.$$

DIAGONALIZATION

$$V^{l\dagger}_L M^l V^l_R = M^l_{diagonal}$$

$$V^{\nu\dagger}_L M^\nu V^\nu_R = M^\nu_{diagonal}$$

$$U_{PMNS} = V^{l+}_L V^\nu_L$$

Preliminary results

In order to make calculations faster and easier we have used **GAP**-program for discrete algebra computation.

We have used two libraries:

1. Small Group Library,
2. Repsn.

Searching for the flavour group

- All non-abelian discrete subgroups of SU(3) and U(3) up to the order **1024** were considered (49 499 125 314 groups),
- Only **62 groups** fulfill the requirements of our model,
- Among them **17 groups** are the subgroups of SU(3).

Preliminary results

All found groups were classified according to [5].

[[i,j]]	The Group Description (in *GAP*)	The Group classification	Is subgroup of SU(3)?
[[24,12]]	S_4	$\Delta(24) = \Delta(6 \times 2^2)$	✓
[[48,30]]	$A_4 \rtimes C_4$	$S_4(2)$	
[[54,8]]	$((C_3 \times C_3) \rtimes C_3) \rtimes C_2$	$\Delta(54) = \Delta(6 \times 3^2)$	✓
[[96,64]]	$((C_4 \times C_4) \rtimes C_3) \rtimes C_2$	$\Delta(96) = \Delta(6 \times 4^2)$	✓
[[96,65]]	$A_4 \rtimes C_8$	$S_4(3)$	
[[108,11]]	$((C_3 \times C_3) \rtimes C_3) \rtimes C_4$	$\Delta(6 \times 3^2, 2)$	
[[150,5]]	$((C_5 \times C_5) \rtimes C_3) \rtimes C_2$	$\Delta(150) = \Delta(6 \times 5^2)$	✓
[[162,10]]	$(((C_3 \times C_3) \rtimes C_3 \times C_3) \rtimes C_3) \rtimes C_2$		
[[162,12]]	$((C_9 \times C_3) \rtimes C_3) \rtimes C_2$		
[[162,14]]	$((C_9 \times C_3) \rtimes C_3) \rtimes C_2$	$D(9,1,1;2,1,1)$	✓
[[162,44]]	$((C_9 \times C_3) \rtimes C_3) \rtimes C_2$	$\Delta'(6 \times 3^2, 2, 1)$	
[[192,182]]	$((C_4 \times C_4) \rtimes C_3) \rtimes C_4$	$\Delta(6 \times 4^2, 2)$	
[[192,186]]	$A_4 \rtimes C_{16}$	$S_4(4)$	
[[216,17]]	$((C_3 \times C_3) \rtimes C_3) \rtimes C_8$	$\Delta(6 \times 3^2, 4)$	

[5] P.O.Ludl ``On the finite subgroups of U(3) of order smaller than 512''

The Higgs part of Lagrangian

Obviously, we want whole Lagrangian to be invariant under transformation of the flavour group G.

The general Higgs potential can be presented as:

$$V(\Phi) = \mu^2 B_{ij}(\Phi_i^\dagger \Phi_j) + \lambda C_{ijkl}(\Phi_i^\dagger \Phi_j)(\Phi_k^\dagger \Phi_l)$$

What are the restrictions imposed by the symmetry group G?

Can we create the Higgs Lagrangian with 2 different $VEVS$?

1. From I Schur Lemma : $B = \lambda \mathbf{I}$, the coefficients C_{ijkl} depend on the group.
2. **YES**, we can: $V(\Phi_1, \Phi_2) = \lambda_1(\Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2 - f_1)^2$.

The minimum is for: $\Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2 = f_1$:

$$\Phi_1 = v_1 = \sqrt{f_1^1} e^{i\beta_1^1}, \quad \Phi_2 = v_2 = \sqrt{f_1^2} e^{i\beta_1^2}, \quad f_1^1 + f_1^2 = f_1.$$

Conclusions

- **The masses** and **mixing parameters** have to be explained,
- **The tribimaximal ansatz** must be replaced,
- It is necessary to go **beyond the SM** in order to deal with the flavour problem,
- **Extension of the scalar sector** may provide the answer,
- The further analysis of Higgs Lagrangian in the context of flavour symmetry must be carried out

THANK YOU FOR YOUR ATTENTION