

THE STRUCTURE OF THE REAL LINE IN QUANTUM MECHANICS AND COSMOLOGY

Paweł Klimasara

(in collaboration with Torsten Asselmeyer-Maluga*, Krzysztof Bielas and Jerzy Król)



University of Silesia in Katowice, Poland
Institute of Physics, Department of Astrophysics and Cosmology



*German Aerospace Center (DLR), Berlin, Germany

17th September 2015

The plan of the presentation

- 1) FOUNDATIONS OF MATHEMATICS
- 2) FORCING IN QUANTUM MECHANICS
- 3) FORCING IN COSMOLOGY
- 4) COSMIC INFLATION DRIVEN
BY EXOTIC SMOOTHNESS
- 5) DISCUSSION AND PERSPECTIVES

Foundations of mathematics

ZF(C) = ZERMELO-FRAENKEL SET THEORY
(WITH THE AXIOM OF CHOICE)

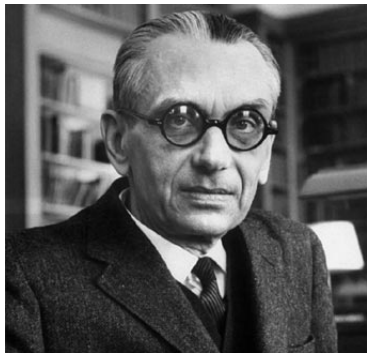
Foundation of (almost) all of mathematics

IT IS WIDELY ACCEPTED PARADIGM THAT MOST OF
THE MATHEMATICS REQUIRED BY THEORIES OF
PHYSICS IS BASED ON THE FORMAL THEORY
OF SETS, E.G. ZFC

⇒ THE REAL LINE \mathbb{R} LIES IN THE
FRAMEWORK OF SET THEORY

AXIOMATIZATION AS THE FINAL STEP OF
FORMALIZATION OF A THEORY:

NO PLACE FOR AMBIGUITY?



KURT GÖDEL
(1906 - 1978)

MODEL OF A THEORY

A mathematical universe
(a class) within which the
theorems of this theory are
fulfilled.

THE ZFC SET THEORY
HAS INFINITELY MANY
NON-ISOMORPHIC MODELS
- THERE IS NO UNIQUE
UNIVERSE FOR
MATHEMATICS

What differs non-isomorphic models of ZFC?

DIFFERENT VIEW ON PROPERTIES "AS SEEN" FROM
INSIDE AND OUTSIDE OF A MODEL: THE IDEA OF
ABSOLUTNESS OF A FORMULA

THE SET OF REAL NUMBERS IS NOT ABSOLUTE

Some formal object can be a real number in one model,
while in another model it is not.

For some models of ZFC:

$\mathcal{M}_1, \mathcal{M}_2$ - non-isomorphic models of ZFC

$$\mathbb{R}_{\mathcal{M}_1}, \mathbb{R}_{\mathcal{M}_2} \subset \mathbb{R} \quad \mathbb{R}_{\mathcal{M}_1} \neq \mathbb{R}_{\mathcal{M}_2}$$

DIFFERENT
MODELS OF
ZFC



DIFFERENT
SETS OF
REALS

Preliminary assumptions

PERHAPS, DURING THE EVOLUTION OF THE UNIVERSE, THERE WERE SOME CHANGES OF THE MODEL OF ZFC (AND SO THE REAL LINE AS WELL)?

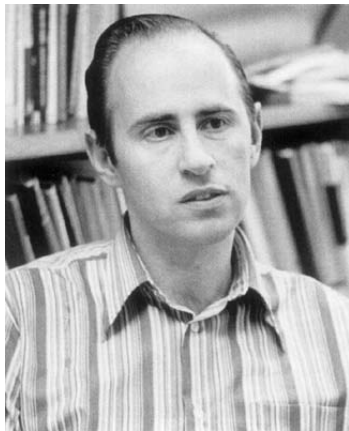
WE PROPOSE THE MODEL WHERE IT ACTUALLY HAPPENED (DURING THE INFLATION ERA)

MATHEMATICAL LOGIC PROVIDES FORMAL TOOLS TO DESCRIBE SUCH CHANGE

FORCING - A FORMAL
METHOD OF EXTENDING
GIVEN MODEL OF ZF(C)

THE 1964 FIELDS MEDAL

Using his original method,
Paul Cohen showed that
Continuum Hypothesis ($2^{\aleph_0} = \aleph_1$)
is independent of the axioms
of ZFC.



PAUL COHEN
(1934 - 2007)

FORCING CAN ADD SOME "MISSING" REALS TO A MODEL

$\mathcal{M}[G]$ - *generic* extension by special *measure* forcing of a model \mathcal{M}

RAREFACTION OF THE REAL LINE

$\mathbb{R}_{\mathcal{M}}$ is merely a meager subset of $\mathbb{R}_{\mathcal{M}[G]}$
(it is in precise sense of negligible size)

$$\mathcal{M} \xrightarrow{\text{forcing}} \mathcal{M}[G] \quad \mathbb{R}_{\mathcal{M}} \subset \mathbb{R}_{\mathcal{M}[G]}$$

A **MEAGER SET** = COUNTABLE SUM
OF NOWHERE-DENSE SETS

(\star) It has inner (Lebesgue) measure 0
and full outer measure.

LEMMA

Every Lebesgue measurable subset of a set that fulfills (\star) has measure 0.

Forcing in quantum mechanics

PAUL A. BENIOFF (1976)

NEITHER ANY MODEL OF
ZFC NOR ITS GENERIC
EXTENSION BY FORCING
ARE SUFFICIENT FOR
GRASPING THE
MATHEMATICS OF QM
ALONG WITH ITS
STATISTICAL PREDICTIONS



⇒ Particular position of QM formalism in mathematics

Historical remarks

WILLIAM BOOS (1996) \diamond ROBERT A. VAN WESEP (2006)

If there are "semiclassical states" realizing LHV program, then they are generic ultrafilters (formal objects specific to forcing constructions).

JERZY KRÓL (2004)

"DYNAMICAL NETWORK OF MODELS"
(adding result of a measurement by forcing)

Micro to macroscale shift

- ⇒ QM is defined w.r.t. the reals $\mathbb{R}_{\mathcal{M}}$ of some model \mathcal{M} of ZFC.
- ⇒ Then, the model varies and new reals are added (by forcing).

IN MACROSCALE (GR) WE USE
THE FULL REAL LINE \mathbb{R}
FOR A DESCRIPTION
OF SPACE AND TIME

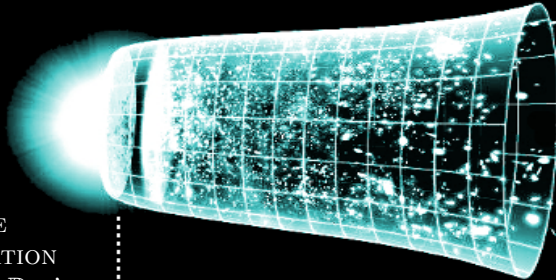
(★) - Suppose that the real numbers, which parametrize space, come from the quantum realm via continuous measurement (\sim position observable).

JERZY KRÓL \diamond P. K. (2015)

UNDER (★) THERE IS ALWAYS A NONTRIVIAL FORCING ON THE MEASURE ALGEBRA ON \mathbb{R}^3 , WHEN PASSING FROM THE QUANTUM REGIME TO THE CLASSICAL WORLD DESCRIBED BY GR

Forcing in cosmology

The general assumptions



SPACETIME
PARAMETRIZATION
VIA CHANGING $\mathbb{R}_{\mathcal{M}}$ 'S



THE FULL REAL LINE \mathbb{R} PARAMETRIZATION

★ The growth of inherent density of reals \leftrightarrow spacetime inflation

The cosmological constant problem

The zero-point energy of quantum field corresponding to a particle of mass m :

$$\frac{E}{V} = \int \frac{d^3k}{(2\pi)^3} \frac{\sqrt{\mathbf{k}^2 + m^2}}{2}$$

IN PARTICLE PHYSICS SUCH CONTRIBUTIONS TO Ω_Λ APPEAR TO BE TOO LARGE BY MANY ORDERS OF MAGNITUDE.

The cosmological constant problem

⇒ Let us evaluate this integral under the assumption that the zero-modes of quantum fields lie in the model \mathcal{M} (the internal space coordinates are parameterized by the real numbers $\mathbb{R}_{\mathcal{M}}$).

ZERO-MODES ARE DESCRIBED AS IF THE SPACE IS SPANNED ON THE INTERNAL $\mathbb{R}_{\mathcal{M}}^3$. HOWEVER, THE DEGREES OF FREEDOM OF GR REQUIRE THE FULL REAL LINE \mathbb{R} FOR THE PARAMETRIZATION OF SPACE COORDINATES

The cosmological constant problem

In this case the integral is evaluated over the non-measurable subset $\mathbb{R}_{\mathcal{M}}^3 \subset \mathbb{R}^3$.

- ⇒ IN GENERAL SUCH LEBESGUE INTEGRAL DOES NOT EXIST.
- ⇒ WE LOOK FOR CONTRIBUTIONS OVER ALL MEASURABLE SUBSETS OF $\mathbb{R}_{\mathcal{M}}^3$.

SINCE $\mathbb{R}_{\mathcal{M}}^3$ IS MEAGER, ALL SUCH SUBSETS HAVE MEASURE 0 AND THEIR CONTRIBUTIONS TO Ω_{Λ} VANISH.

Cosmic inflation driven by exotic smoothness

Obtaining non-zero CC

SO WHY THE VALUE OF DENSITY $\Omega_\Lambda \sim 10^{-27} \frac{\text{kg}}{\text{m}^3}$
IS NON-ZERO (AND SO SMALL)?

★ (TORSTEN ASSELMAYER-MALUGA, JERZY KRÓL; 2014)
ONE CAN MAKE USE OF THE EXOTIC SMOOTH GEOMETRY ON \mathbb{R}^4

EXOTIC \mathbb{R}^4



(CODIMENSION-1)
FOLIATION OF \mathbb{R}^4



EVOLUTION
W.R.T.
THE FOLIATION

The transition:

WILDLY
EMBEDDED
3-SPHERE



HOMOLOGY
3-SPHERE θ

generates the realistic inflation in the simple model

$$S^3 \times_{\theta} \mathbb{R} \subset \mathbb{R}^4.$$

THE MORSE FUNCTION OF THE
HANDLE-DECOMPOSITION OF $S^3 \times_{\theta} \mathbb{R}$ GENERATES
THE SHAPE OF THE POTENTIAL $V(\phi)$

\Rightarrow IT IS REALISTIC IF WE CHOOSE THE
EXOTIC SMOOTH \mathbb{R}^4

(there are infinitely many *-continuum-* different exotic
 \mathbb{R}^4 's and $S^3 \times_{\theta} \mathbb{R}$'s)

Discussion and perspectives

Summary

- THERE IS NO SINGLE MODEL OF ZFC THAT CAN BE USED FOR THE CORRECT REPRESENTATION OF MATHEMATICS OF QM AND ITS STATISTICAL PREDICTIONS. ONE NEEDS TO REFER TO SOME DYNAMICS BETWEEN MODELS.



- THE STRUCTURE OF THE REAL LINE (E.G. SET-THEORETIC FORCING) ENCODES THE FORMAL SHIFT FROM QUANTUM TO CLASSICAL POSITION OBSERVABLE.



- OBTAINING FULL REAL LINE FROM QM VIA CONTINUOUS MEASUREMENTS MUST INVOLVE NONTRIVIAL FORCING.

Summary

- LOCALIZATION OF QUANTUM REALM WITHIN A ZFC MODEL LEADS TO VANISHING OF ZERO-MODES OF QUANTUM FIELDS.



- IF WE CHOOSE AN EXOTIC SMOOTH \mathbb{R}^4 STRUCTURE FOR A SPACETIME, THEN ONE CAN OBTAIN THE SHAPE OF INFLATION POTENTIAL $V(\phi)$ AND A REALISTIC COSMOLOGICAL CONSTANT.

PLANS FOR FUTURE WORK:

- ★ Direct description of the cosmic inflation in such forcing-based model of the universe. In particular, we want to give a forcing mechanism for emerging exotic \mathbb{R}^4 in the early universe (hence the inflation).
- ★ Attempt to find the impact of meager (\sim rarefied) real line on the various distortions of CMB
- ★ Description of quantum entanglement in the forcing language



Thank You for a Patient Hearing