NLO for hybrid k_T -factorization

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Collinear factorization in QCD

Automated calculations up to NLO since over decade.

$$d\sigma^{LO} = \int \frac{dx_{in}}{x_{in}} \frac{d\overline{x}_{\overline{in}}}{\overline{x}_{\overline{in}}} f_{in}(x_{in}) f_{\overline{in}}(\overline{x}_{\overline{in}}) dB(x_{in}, \overline{x}_{\overline{in}}) \tag{1}$$

initial states:

$$\begin{aligned} k^{\mu}_{in} &= x_{in} P^{\mu} \\ k^{\mu}_{\overline{in}} &= \overline{x}_{\overline{in}} \overline{P}^{\mu} \end{aligned} \tag{2}$$

Collinear factorization in QCD at NLO

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initial states:

$$k_{in}^{\mu} = x_{in} P^{\mu}$$
$$k_{\overline{in}}^{\mu} = \overline{x}_{\overline{in}} \overline{P}^{\mu}$$

$$d\sigma^{NLO} = \int \frac{dx_{in}}{x_{in}} \frac{d\overline{x}_{\overline{in}}}{\overline{x}_{\overline{in}}} \left\{ f_{in}(x_{in}) f_{\overline{in}}(\overline{x}_{\overline{in}}) \left[dV(x_{in}, \overline{x}_{\overline{in}}) + dR(x_{in}, \overline{x}_{\overline{in}}) \right] \right\} \quad (3)$$

Not finite

Collinear factorization in QCD at NLO

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$$\begin{aligned} d\sigma^{NLO} \quad = \quad & \int \frac{dx_{in}}{x_{in}}\frac{d\overline{x}_{\overline{in}}}{\overline{x}_{\overline{in}}} \Bigg\{ f_{in}(x_{in}) f_{\overline{in}}(\overline{x}_{\overline{in}}) \left[dV(x_{in}, \overline{x}_{\overline{in}}) + dR(x_{in}, \overline{x}_{\overline{in}}) \right]_{\text{cancelling}} \\ + \quad & \Bigg[f_{in}(x_{in}) \frac{-\alpha_s}{2\pi\epsilon} \int_{\overline{x}_{\overline{in}}}^{1} d\overline{z} \mathcal{P}_{\overline{in}}(\overline{z}) f_{\overline{in}}(\overline{x}_{\overline{in}}/\overline{z}) \\ & f_{\overline{in}}(\overline{x}_{\overline{in}}) \frac{-\alpha_s}{2\pi\epsilon} \int_{x_{in}}^{1} dz \mathcal{P}_{in}(z) f_{in}(x_{in}/z) \Bigg] dB(x_{in}, \overline{x}_{\overline{in}}) \Bigg\} \end{aligned}$$

Not finite at all

Collinear factorization in QCD at NLO

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$$\begin{aligned}
d\sigma^{NLO} \;=\; & \int \frac{dx_{in}}{x_{in}} \frac{d\overline{x}_{\overline{in}}}{\overline{x}_{\overline{in}}} \Big\{ f_{in}(x_{in}) f_{\overline{in}}(\overline{x}_{\overline{in}}) \left[dV(x_{in}, \overline{x}_{\overline{in}}) + dR(x_{in}, \overline{x}_{\overline{in}}) \right]_{\text{cancelling}} \\
+ & \Big[f_{in}(x_{in}) \frac{-\alpha_s}{2\pi\epsilon} \int_{\overline{x}_{\overline{in}}}^{1} d\overline{z} \mathcal{P}_{\overline{in}}(\overline{z}) f_{\overline{in}}(\overline{x}_{\overline{in}}/\overline{z}) \\
& + f_{\overline{in}}(\overline{x}_{\overline{in}}) \frac{-\alpha_s}{2\pi\epsilon} \int_{x_{in}}^{1} dz \mathcal{P}_{in}(z) f_{in}(x_{in}/z) \Big] dB(x_{in}, \overline{x}_{\overline{in}}) \\
+ & \left[f_{in}^{(1)}(x_{in}) f_{\overline{in}}(\overline{x}_{\overline{in}}) + f_{in}(x_{in}) f_{\overline{in}}^{(1)}(\overline{x}_{\overline{in}}) \right] \frac{\alpha_s}{2\pi} dB(x_{in}, \overline{x}_{\overline{in}}) \Big\}
\end{aligned}$$

Finite at all

$$f_{\overline{in}}^{(1)}(\overline{x}_{\overline{in}}) - \frac{1}{\epsilon} \int_{\overline{x}_{\overline{in}}}^{1} d\overline{z} \mathcal{P}_{\overline{in}}(\overline{z}) f_{\overline{in}}(\overline{x}_{\overline{in}}/\overline{z}) = finite$$

$$f_{in}^{(1)}(x_{in}) - \frac{1}{\epsilon} \int_{x_{in}}^{1} dz \mathcal{P}_{in}(z) f_{in}(x_{in}/z) = finite$$

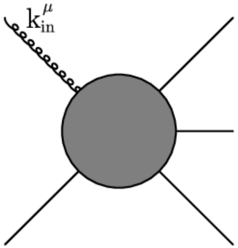
Objective

Hybrid k$_T$ factorization in QCD

Establish the same within hybrid k$_T$-factorization, for which the LO cross section formula is:

$$d\sigma^{LO} = \int \frac{dx_{in}}{x_{in}} \frac{d^2k_T}{\pi} \frac{d\bar{x}_{\overline{in}}}{\bar{x}_{\overline{in}}} F_{in}(x_{in}, k_T) f_{\overline{in}}(\bar{x}_{\overline{in}}) dB^*(x_{in}, k_T, \bar{x}_{\overline{in}}) \tag{4}$$

- The amplitudes inside $B^*(x_{in}, k_T, \bar{x}_{\overline{in}})$ depend explicitly on k_T.
- They involve a space-like initial-state gluon with momentum $k_{in}^{\mu} = x_{in}P^{\mu} + k_T^{\mu}$



- Such amplitudes need care to be well-defined, to be gauge invariant
- We apply the auxiliary-parton method, and our objective is within this constraint

Auxiliary parton method

We put our interest on process with one space-like gluon. $\omega(p_1) = g(p_1)/q(P_1)$

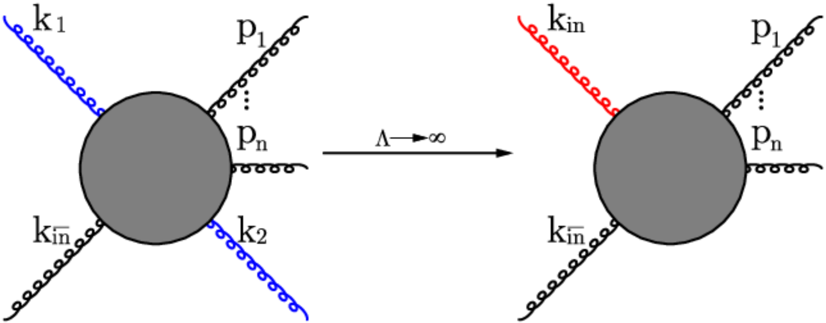
$$g^*(k_{in})\omega_{\overline{in}}(k_{\overline{in}}) \to \omega_1(p_1)\omega_2(p_2)\cdots\omega_n(p_n).$$

This process is obtained via named auxiliary parton method from process

$$q(k_1(\Lambda))\omega_{\overline{in}}(k_{\overline{in}}) \to q(k_2(\Lambda))\omega_1(p_1)\omega_2(p_2)\cdots\omega_n(p_n)$$

with light-like momenta parametrized with Λ

$$k_1^\mu = \Lambda P^\mu,\ k_2^\mu = p_\Lambda^\mu = (\Lambda - x_{in})P^\mu - k_T^\mu + \frac{|k_T|^2}{2P^\mu \cdot \overline{P}^\mu(\Lambda - x_{in})}\overline{P}^\mu.$$



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Their difference is

$$k_1^\mu - k_2^\mu = k_{in}^\mu + O(\Lambda^{-1}) = x_{in}P^\mu + k_T^\mu + O(\Lambda^{-1})$$

Taking $\Lambda \to \infty$ one will obtain the matrix element with space-like gluon

$$\frac{x_{in}^2|k_T|^2}{g_s^2 C_{aux}\Lambda^2}|\overline{M}^{aux}|^2(\Lambda P, k_{\overline{in}}; p_\Lambda, \{p_i\}_{i=1}^n) \xrightarrow{\Lambda\to\infty} |\overline{M}^*|^2(k_{in}, k_{\overline{in}}; \{p_i\}_{i=1}^n) \quad (5)$$

As auxiliary partons we can choose quarks as well as gluons. Then

$$C_{aux-q} = \frac{N_c^2 - 1}{N_c},\ C_{aux-g} = 2N_c.$$

The NLO contributions - schematically

$$d\sigma^{NLO} = \int \frac{dx_{in}}{x_{in}} \frac{d^2k_T}{\pi} \frac{d\overline{x}_{\overline{in}}}{\overline{x}_{\overline{in}}} \{F_{in}(x_{in}, |k_T|) f(\overline{x}_{\overline{in}}) \left[dV^*(x_{in}, k_T, \overline{x}_{\overline{in}}) + dR^*(x_{in}, k_T, \overline{x}_{\overline{in}})\right] + \left[F_{in}^{(1)}(x_{in}, |k_T|) f(\overline{x}_{\overline{in}}) + F_{in}(x_{in}, |k_T|) f^{(1)}(\overline{x}_{\overline{in}})\right] dB^*(x_{in}, k_T, \overline{x}_{\overline{in}})\} \tag{6}$$

Virtual contributions

$$dV^* = dV^{*fam} + dV^{*unf}$$

- Familiar contribution conserve smooth on-shell $k_T \to 0$
- Unfamiliar contribution $dV^{*unf} = a_\epsilon N_c Re(\mathcal{V}_{aux}) dB^*$

$$a_\epsilon = \frac{\alpha_s}{2\pi} \frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}; \quad \epsilon = \frac{4 - dim}{2}$$

$$\mathcal{V}_{aux} = \left(\frac{\mu^2}{|k_T|^2}\right)^\epsilon \left[\frac{2}{\epsilon} \ln\frac{\Lambda}{x_{in}} - i\pi + \overline{\mathcal{V}}_{aux}\right] + \mathcal{O}(\epsilon) + \mathcal{O}(\Lambda^{-1})$$

$$\overline{\mathcal{V}}_{aux-q} = \frac{1}{\epsilon}\frac{13}{6} + \frac{\pi^2}{3} + \frac{80}{18} + \frac{1}{N_c^2}\left[\frac{1}{\epsilon^2} + \frac{3}{2}\frac{1}{\epsilon} + 4\right] - \frac{n_f}{N_c}\left[\frac{2}{3}\frac{1}{\epsilon} + \frac{10}{9}\right]$$

$$\overline{\mathcal{V}}_{aux-g} = -\frac{1}{\epsilon^2} + \frac{\pi^2}{3}$$

Details in E. Blanco, A. Giachino, A. v. Hameren, P. Kotko: One-loop gauge invariant amplitudes with a space-like gluon.

Real radiation

Real contribution we defined as

$$dR^{*fam}(k_{in}, k_{\overline{in}}; \{p_i\}_{i=1}^{n+1}) = \frac{a_\epsilon \mu^{2\epsilon}}{\pi_\epsilon} \frac{1}{|k_T|^2} d\Sigma^*_{n+1}(k_{in}, k_{\overline{in}}; \{p_i\}_{i=1}^{n+1}) J_R(\{p_i\}_{i=1}^{n+1}) \tag{7}$$

$$a_\epsilon = \frac{\alpha_s}{2\pi} \frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}; \quad \pi_\epsilon = \frac{\pi^{1-\epsilon}}{\Gamma(1-\epsilon)}$$

- One parton more in a final state (compared to Born)
- One collinear pair and / or one soft parton
- The singularities look the same as if the initial-state gluon were on-shell

- Independent of the type of auxiliary partons
- No $ln\Lambda$

Did we miss something?

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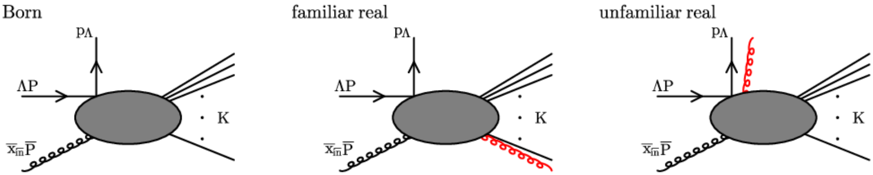
$$dR^* = dR^{*fam} + dR^{*unf}$$

Came from phase space where the radiative gluon take part in consumption of the Λ

- depends of type of auxiliary partons
- violates the smooth on-shell limit and smooth large Λ limit

Unfamiliar real contribution



In the unfamiliar case the radiative gluon participates in the consumption of Λ $\qquad k_T = q_T + r_T$

$$\frac{x_{in}^2|q_T + r_T|^2}{g_s^2 C_{aux}\Lambda^2}|\overline{M}^{aux}|^2((\Lambda + x_{in})P, k_{\overline{in}}; x_r\Lambda P + r_T + \overline{x}_r\overline{P}, x_q\Lambda P + q_T + \overline{x}_q\overline{P}, \{p_i\}_{i=1}^n)$$

$$\xrightarrow{\Lambda\to\infty} \mathcal{Q}_{aux}(x_q, q_T, x_r, r_T)|\overline{M}^*|^2(x_{in}P - q_T - r_T, k_{\overline{in}}; \{p_i\}_{i=1}^n)$$

The phase space also factorizes, we can perform analytical integration, the result is:

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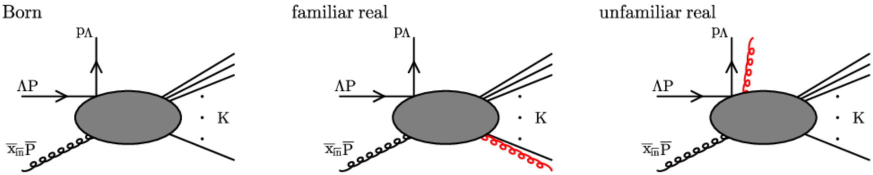

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$$\xrightarrow{\Lambda\to\infty} \mathcal{Q}_{aux}(x_q, q_T, x_r, r_T)|\overline{M}^*|^2(x_{in}P - q_T - r_T, k_{\overline{in}}; \{p_i\}_{i=1}^n)$$

The phase space also factorizes, we can perform analytical integration, the result is:

$$dR^{*unf}(k_{in}, k_{\overline{in}}; \{p_i\}_{i=1}^{n+1}) = \left\{a_\epsilon N_c\left(\frac{\mu^2}{|k_T|^2}\right)^\epsilon\left[-\frac{2}{\epsilon}\ln\frac{2P\cdot\overline{P}\Lambda}{|k_T|^2} + \overline{R}_{aux}\right] + \mathcal{O}(\epsilon, \Lambda^{-1})\right\} dB^*(k_{in}, k_{\overline{in}}; \{p_i\}_{i=1}^n)$$

Unfamiliar contributions - completed

Collection of virtual and real unfamiliar contribution brings

$$\Delta_{unf} dB^* = dR^{*unf} + dV^{*unf}$$

general unfamiliar contribution is given by

$$\Delta_{unf} = \frac{a_\epsilon N_c}{\epsilon}\left(\frac{\mu^2}{|k_T|^2}\right)^\epsilon \left[\mathcal{J}_{aux} + \mathcal{J}_{univ} + \mathcal{J}_{univ} - 2ln\frac{2P\cdot\overline{P}x_{in}}{|k_T|^2}\right]$$

where

$$\mathcal{J}_{univ} = \frac{11}{6} - \frac{n_f}{3N_c} - \frac{\mathcal{K}}{N_c}(-\epsilon) \qquad \text{with} \qquad \mathcal{K} = N_c\left(\frac{67}{18} - \frac{\pi^2}{6}\right) - \frac{5n_f}{9}$$

$$\mathcal{J}_{aux-g} = \frac{11}{6} + \frac{n_f}{3N_c^3} + \frac{n_f}{6N_c^3}(-\epsilon), \qquad \mathcal{J}_{aux-q} = \frac{3}{2} - \frac{1}{2}(-\epsilon)$$

- No $ln\Lambda$
- Target impact factor corrections as in Ciafaloni, Colferai 1999.
- Other terms also known in literature (Regge trajectory, renormalization of the coupling constant)

Familiar real collinear singularities

The dR^{*fam} has a singularity when a radiative gluon becomes collinear to $\overline{P}$ which leads to divergence $\Delta_{\overline{coll}}$ with splitting as $\dfrac{1}{z(1-z)} - 2 + z(1-z)$ included.

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Tree-level matrix elements with a space-like gluon still have a singularity when a radiative gluon becomes collinear to P.

$$|\overline{M}^*|^2\left(x_{in}P + k_T, k_{\overline{in}}; r, \{p_i\}_{i=1}^n\right) \xrightarrow{r\to x_r P} \frac{2N_C}{P\cdot r}\frac{x_{in}^2}{x_r(x_{in}-x_r)^2}|\overline{M}^*|^2\left((x_{in}-x_r)P + k_T, k_{\overline{in}}; \{p_i\}_{i=1}^n\right) \quad (8)$$

Similar to usual collinear gluon splitting with only the $\dfrac{1}{z(1-z)}$ part.

This leads to a non-cancelling divergence similar to the collinear case given by

$$\Delta^*_{coll}(x_{in}, k_T) = -\frac{\alpha_\epsilon}{\epsilon}\int_{x_{in}}^1 dz\left[\frac{2N_C}{[1-z]_+} + \frac{2N_C}{z} + \gamma_g\delta(1-z)\right]F\left(\frac{x_{in}}{z}, k_T\right) \quad (9)$$

Summary

General NLO formula

$$d\sigma^{NLO} = \int \frac{dx_{in}}{x_{in}} \frac{d^2k_T}{\pi} \frac{d\overline{x}_{\overline{in}}}{\overline{x}_{\overline{in}}} \Big\{ F_{in}(x_{in}, k_T) f_{\overline{in}}(\overline{x}_{\overline{in}}) \left[dR^*(x_{in}, k_T, \overline{x}_{\overline{in}}) + dV^*(x_{in}, k_T, \overline{x}_{\overline{in}}) \right]_{cancelling}$$
$$+ \left[F^{NLO}_{in}(x_{in}, k_T) + F_{in}(x_{in}, k_T)\Delta_{unf}(x_{in}, k_T) + \Delta^*_{coll}(x_{in}, k_T) \right] f_{\overline{in}}(\overline{x}_{\overline{in}}) dB^*(x_{in}, k_T, \overline{x}_{\overline{in}})$$
$$\left[f^{NLO}\overline{in}(\overline{x}_{\overline{in}}) + \Delta_{\overline{coll}} \right] F_{in}(x_{in}, k_T) dB^*(x_{in}, k_T, \overline{x}_{\overline{in}}) \Big\} \tag{10}$$

The collinear divergences Δ^*_{coll} and $\Delta_{\overline{coll}}$
$f^{NLO}\overline{in}(\overline{x}_{\overline{in}}) + \Delta_{\overline{coll}} \rightarrow$ finite as in collinear factorization
$F^{NLO}_{in}(x_{in}, k_T) + F_{in}(x_{in}, k_T)\Delta_{unf}(x_{in}, k_T) + \Delta^*_{coll}(x_{in}, k_T) \rightarrow$ finite ? still necessity for scheme for regularization

Details in A. v. Hameren, L. Motyka, G. Ziarko: Hybrid kT-factorization and impact factors at NLO. J. High Energ. Phys. 2022, 103 (2022). https://doi.org/10.1007/JHEP11(2022)103 [SPRINGER]

Thank you for listening!

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