Lepton flavour violating decays in the Grimus – Neufeld model

(based on 2206.00661 and 2211.14384)

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Overview: Grimus-Neufeld model, neutrino masses and LFV

- Grimus–Neufeld model [GN '89] = 2HDM + 1 sterile neutrino
  - can include neutrino masses and mixings with seesaw + radiative
  - can give LFV processes
- We look at GNM's specific scenario:
  - sterile Majorana mass is small
    $\Rightarrow$enhanced LFV decay rates
    $\Rightarrow$Approximate Z_2 symmetry in Yukawa sector
    $\Rightarrow$Makes it similar to other popular models: scotogenic, scoto-seesaw.
- Larger LFV decay rates make restrictions from experiments possible on scalar+neutrino sector.
  $\Rightarrow$ We put constrains on scalar+ neutrino sector from LFV

GN

- The Lagrangian:

$$\mathscr{L} = \mathscr{L}_{2HDM} - \frac{1}{2} MNN - Y_i^{(1)} \ell_i \varepsilon H_1 N - Y_i^{(2)} \ell_i \varepsilon H_2 N + h.c., \quad \ell_i = \begin{pmatrix} \nu_i \\ \ell_i^- \end{pmatrix}$$

where ℓ – lepton doublet, N–sterile neutrino, $j = e, \mu, \tau$; H_1 and H_2 in the Higgs basis ($\langle H_1 \rangle = \frac{1}{\sqrt{2}} v$, $\langle H_2 \rangle = 0$).

- We say that if

$$y^2 \equiv \sum_i \left| Y_i^{(1)} \right|^2 \ll 1$$

we have approximate Z_2 symmetry in Yukawa sector:

$$\text{SM particles} \to +\text{SM particles}, \quad N, H_2 \to -N, -H_2$$

- y – Z_2 small symmetry breaking parameter
$\Rightarrow$ tiny seesaw scale (next slide)

tiny $Y_i^{(1)}$ and tiny seesaw

$$\mathscr{L} = \mathscr{L}_{2HDM} - \frac{1}{2} M N N - Y_i^{(1)} \ell_i \varepsilon H_1 N - Y_i^{(2)} \ell_i \varepsilon H_2 N + h.c.$$

- First two additional terms (when $\langle H_1 \rangle \to \frac{1}{\sqrt{2}} v$) lead to two non-vanishing neutrino masses at tree-level (labeled $m_3 < m_4$):

$$m_3 = \frac{y^2 v^2}{2 m_4}, m_4 - m_3 = M, y^2 \equiv \sum_i \left| Y_i^{(1)} \right|^2 .$$

  - scale of active neutrinos $m_3 = O(0.01\text{eV})$.
  - seesaw: $m_4 \gg m_3 \Rightarrow m_4 \approx M$
  - we assume $y < O\left(10^{-7}\right) \Rightarrow m_4 < 10\text{GeV}$ – a tiny seesaw scale.
- The last term in $\mathscr{L}$ induces radiative neutrino mass generation and LFV.

$Y_i^{(2)}$: radiative mass and LFV

$$\mathscr{L} = \mathscr{L}_{2HDM} - \frac{1}{2} MNN - Y_i^{(1)} \ell_i \varepsilon H_1 N - Y_i^{(2)} \ell_i \varepsilon H_2 N + h.c., \quad Y_i^{(1)} \ll 1$$

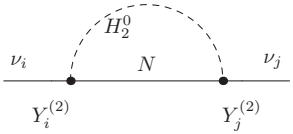


Figure: Radiative mass generation gives m_2^{pole}.

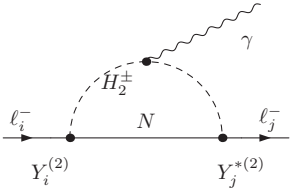


Figure: LFV decay $\ell_i \to \ell_j \gamma$
$A_{\text{penguin}} \propto \left(Y^{(2)}\right)^2 \frac{1}{m_{H^\pm}^2}$

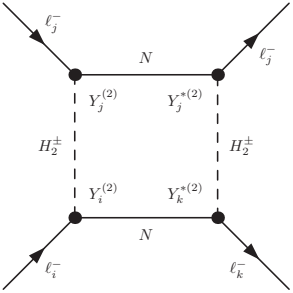
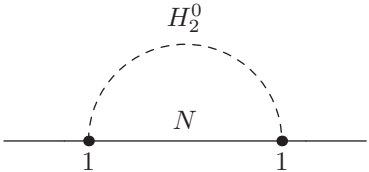


Figure: LFV decay $\ell_i \to \ell_j \ell_j \ell_k$,
$A_{\text{box}} \propto \left(Y^{(2)}\right)^4 \frac{1}{m_{H^\pm}^2}$

Light neutrino mass matrix

- Define at external momentum $p^2 = 0$, in the Higgs basis:

$\Lambda =$



$$\overset{\text{if } Z_2}{=} \quad \frac{m_4}{32\pi^2} \ln \frac{m_H^2}{m_A^2} \sim \lambda_5 m_4 \,,$$

- We have a 3×3 mass matrix for light neutrinos of rank 2 in flavor basis:

$$m_{ij} = \underbrace{Y_i^{(1)} Y_j^{(1)} \frac{v^2}{2m_4}}_{\text{Seesaw}} + \underbrace{Y_i^{(2)} Y_j^{(2)} \Lambda}_{\text{Radiative}} + O\left(Y^{(1)} \times \text{loop}\right)$$

- We diagonalize

$$U^* m U^\dagger = \text{diag}\left(0, m_2^{\text{pole}}, m_3^{\text{pole}}\right), \; m_2^{\text{pole}} < m_3^{\text{pole}} .$$

If normal ordering assumed, $m_2^{\text{pole}} = \sqrt{\Delta m_{21}^2}$, $m_3^{\text{pole}} = \sqrt{\Delta m_{31}^2}$ and $U^\dagger \approx U_{\text{PMNS}}$.

parametrization of $Y_i^{(2)}$

- Yukawa couplings in Flavour basis (parameters, not fixed by neutrino data are in red, $i = e, \mu, \tau$):

$$Y_i^{(1)} = i e^{-i\phi(\Lambda)} \sqrt{\frac{2 m_3^{\text{pole}} m_4}{v^2 z(r, \omega_{22})}} \left(0, R_{22}, \frac{m_3^{\text{pole}}}{m_2^{\text{pole}}} R_{32}\right)_j U_{ji}\,,,$$

$$Y_i^{(2)} = \text{sign}(\Lambda) \sqrt{\frac{m_2^{\text{pole}}}{|\Lambda| \cdot z(r, \omega_{22})}} \left(0, R_{22}, \frac{m_3^{\text{pole}}}{m_2^{\text{pole}}} R_{32}\right)_j U_{ji}\,, \quad U = \begin{cases} U_{PMNS}^\dagger & \text{for NO} \\ O_{IO} U_{PMNS}^\dagger & \text{for IO} \end{cases},$$

$$R = \begin{pmatrix} \cos r\, e^{i\omega_{22}} & -\sin r\, e^{-i\omega_{32}(r,\omega_{22})} e^{i\phi(\Lambda)} \\ \sin r\, e^{i\omega_{32}(r,\omega_{22})} & \cos r\, e^{-i\omega_{22}} e^{i\phi(\Lambda)} \end{pmatrix}, \begin{pmatrix} \nu_{\text{radiative}} \\ \nu_{\text{seesaw}} \end{pmatrix} = R \begin{pmatrix} \nu_2 \\ \nu_3 \end{pmatrix}$$

- r is a mixing angle between seesaw and radiative, ω_{22}– a free phase.
- Note: there exist single parameter point (r, ω_{22}) for each flavour i, that $Y_i^{(2)} = 0$.
- Recall that for $m_4 < v \Rightarrow \Lambda \sim m_4$, so $Y_i^{(1)} \sim \sqrt{m_4}\,, Y^{(2)} \sim \frac{1}{\sqrt{m_4}} \Rightarrow Y^{(2)} \sim \frac{1}{Y^{(1)}}$.

Bounds from LFV

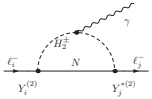


Figure: $A_{\text{penguin}} \propto \frac{f(r,\omega_{22})}{\Lambda m_{H^\pm}^2}$

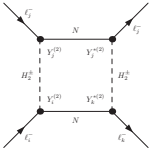


Figure: $A_{\text{box}} \propto \frac{g(r,\omega_{22})}{\Lambda^2 m_{H^\pm}^2}$

- Upper bounds on $Y^{(2)}$ come from:
  - $\text{Br}\left(\ell_i \to \ell_j \gamma\right)$ (lower bound on **"photon factor"** Λm_H^2)
  - $\text{Br}\left(\ell_i \to \ell_j \ell_k \ell_k\right)$,$\text{Br}\left(\ell_i \to \ell_k \ell_k \ell_k\right)$ (lower bound on **"box factor"** $\Lambda^2 m_H^2$*)

    *If boxes dominate over penguins
  - Perturbative unitarity (from $\sum_i \left|Y_i^{(2)}\right|^2 < 8\pi$ we get lower bound on Λ)

Photon dominance

- Photon dominance: $A_{\text{penguin}} \gg A_{\text{box}}$ then:

$$\mathrm{Br}(l_i \to 3 l_j) \approx \left[-\frac{5 \cdot \alpha}{18\pi} + \frac{\alpha}{3\pi}\left(-\frac{11}{4} + \ln \frac{m_i^2}{m_j^2} \right) \right] \cdot \mathrm{Br}(l_i \to l_j \gamma),$$

  2-body decay experiments are more constraining (put bounds on $\Lambda m_{H^\pm}^2$) in this regime.

- We have:

$$A_{\text{box}}/A_{\text{penguin}} \sim \frac{\Lambda m_{H^\pm}^2}{\Lambda^2 m_{H^\pm}^2} = \frac{1}{\Lambda} = m_{H^\pm}^2 \frac{(\Lambda m_{H^\pm}^2)}{(\Lambda m_{H^\pm}^2)^2}$$

- For $\Lambda m_{H^\pm}^2 = \text{const}$, $A_{\text{box}}/A_{\text{penguin}} \sim m_{H^\pm}^2 \Rightarrow$ increase $m_{H^\pm}$, can deviate from photon dominance
  - Then 3-body decays can become more constraining (put bounds on $\Lambda^2 m_{H^\pm}^2$) .

Photon dominance for low $m_{H^\pm}$: constraints from $\ell_i \to \ell_j \gamma$

- $\mu \to e\gamma$ is most constraining in *almost* all of the parameter space (give **"typical"** constraint on $\Lambda m_{H^\pm}^2$).
- $\tau \to e\gamma$ and $\tau \to \mu\gamma$ become important in a tiny parameter space around $Y_e^{(2)} = 0$ and $Y_\mu^{(2)} = 0$ (called **"special"** regions, give **"absolute"** constraint on $\Lambda m_{H^\pm}^2$)

Constraints from $\mu \to e\gamma$

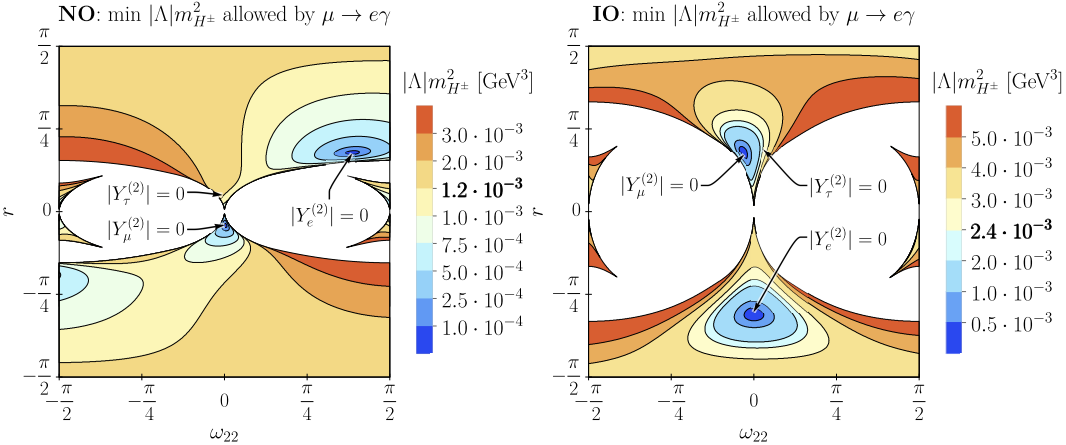


Figure: Lower bound on $\Lambda m^2_{H^\pm}$ from $\mu \to e\gamma$ as a function of r and ω_{22} for NO and IO. White regions are dissalowed by neutrino sector. The $\mu \to e\gamma$ vanishes at two points in $r - \omega$ plane, indicated in the plots by $\left|Y^{(2)}_{e,\mu}\right| = 0$. Typical lower bound: $|\Lambda|\, m^2_{H^\pm} \gtrsim 10^{-4}\text{GeV}^3$.

Very lowest $\Lambda m^2_{H^\pm}$ from τ decays

- The special solutions of $Y^{(2)}_{e,\mu} = 0$ makes $\mu \to e\gamma$ vanish, but τ decays have non-zero prediction at those points:
  - τ decays give bounds $\Lambda m^2_{H^\pm}$ on , when $Y^{(2)}_{e,\mu} = 0$

Process and parameter point	NO, $\lvert\Lambda\rvert\, m^2_{H^\pm}$ [GeV^3]	IO, $\lvert\Lambda\rvert\, m^2_{H^\pm}$ [GeV^3]
$\tau \to e\gamma$ at $Y^{(2)}_{\mu} = 0$	$1.9 \cdot 10^{-6}$	$4.0 \cdot 10^{-6}$
$\tau \to \mu\gamma$ at $Y^{(2)}_{e} = 0$	$1.3 \cdot 10^{-5}$	$7.6 \cdot 10^{-6}$

Table: Lower bound on $\Lambda m^2_{H^\pm}$ in special points.

- **Absolute** bound (when whole model is excluded) is the lower of the two.

Final results

strongest constraint	Absolute lower bound		
	parameter	Normal (Inverted) ordering	$m_{H^\pm}/\text{TeV}$
$\tau \to e\gamma$	$\lvert\Lambda\rvert\, m_{H^\pm}^2/\left(10^{-6}\text{GeV}^3\right)$	1.9(4.0)	$< 1.2(0.4)$
$\tau \to 3e$	$\Lambda^2 m_{H^\pm}^2/\left(10^{-18}\text{GeV}^4\right)$	2.6(98.7)	$1.2\,(0.4) \div 3.4\,(4.7)$
pert. unitarity	$\lvert\Lambda\rvert/\left(10^{-12}\text{GeV}\right)$	0.5(2.1)	$> 3.2(4.7)$
	Typical lower bound		
	Parameter	Normal (Inverted) ordering	$m_{H^\pm}/\text{TeV}$
$\mu \to e\gamma$	$\lvert\Lambda\rvert\, m_{H^\pm}^2/\left(10^{-5}\text{GeV}^3\right)$	8.8 (4.9)	$< 1.5(1.7)$
$\mu \to 3e$	$\Lambda^2 m_{H^\pm}^2/\left(10^{-15}\text{GeV}^4\right)$	3.4 (0.85)	$> 1.5(1.7)$
pert. unitarity	didn't reach for $m_{H^\pm} < 5$ TeV.		

- Typical (what you whould typically get from random scan) is at least an order of magnitude stronger than the absolute bound.

Connection to scalar sector

- For easy interpretation, let us assume Z_2 in scalar sector, and $m_{H^\pm} \approx m_A \approx m_H$, then:

$$|\Lambda|\, m_{H^\pm}^2 = \frac{m_{H^\pm}^2 m_4}{32\pi^2} \ln \frac{m_H^2}{m_A^2} \approx |\lambda_5|\, m_4 \cdot \frac{v^2}{32\pi^2}$$

$$\Lambda^2 m_{H^\pm}^2 \approx \lambda_5^2 \frac{m_4^2}{m_{H^\pm}^2} \cdot \frac{v^4}{(32\pi^2)^2}$$

which roughly leads to:

$$\text{absolute for NO(IO):} \begin{cases} |\lambda_5| \gtrsim 1\,(2) \cdot 10^{-2}\, \frac{\text{keV}}{m_4} & m_{H^\pm} < 1.2(0.4)\,\text{TeV} \\ |\lambda_5| \gtrsim 1\,(5) \cdot 10^{-2}\, \frac{m_{H^\pm}/\text{TeV}}{m_4/\text{keV}} & m_{H^\pm} > 1.2(0.4)\,\text{TeV} \end{cases}$$

$$\text{typical:} \begin{cases} |\lambda_5| \gtrsim \frac{\text{keV}}{m_4} & m_{H^\pm} < 1.5(1.7)\,\text{TeV} \\ |\lambda_5| \gtrsim \frac{m_{H^\pm}/\text{TeV}}{m_4/\text{keV}} & m_{H^\pm} > 1.5(1.7)\,\text{TeV} \end{cases}$$

- LFV relates neutrinos, with Peccei-Quinn symmetry breaking operator in the scalar sector $\mathscr{L} \ni \frac{1}{2}\lambda_5 \left(H_2^\dagger H_1\right)^2$

Conclusions

- Small λ_5 (Peccei-Quinn symmetry breaking), and small m_4 (or, equivalently, small Z_2 symmetry breaking coupling, $Y^{(1)}$) can give signatures in LFV decays in GNM.
  - Signatures for large m_4 are not likely (give way weaker constraints)
- Results directly apply to scoto-seesaw model and give qualitative behaviour for scotogenic model too (both of them have exact Z_2, but more sterile neutrinos)
- There is at least an order of magnitude difference between completely excluded ("absolute") and most likely (or "typically") excluded values.

Thank you!