Recent Advances in Phenomenology with EFT

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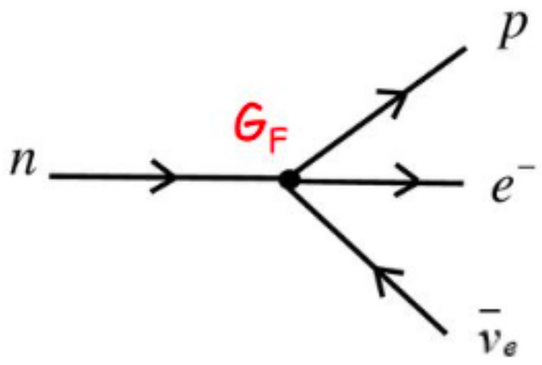
Overview

- A brief intro to Effective Field Theory: SMEFT
- Bottom-Up: construction of BSMEFT scenarios
  - Left-Right Symmetric Model as an effective theory
- Top-down: construction of effective action
- Universal one-loop effective action up to d_8
- Summary

Part I : A brief intro to Effective Field Theory

Lesson from the Past: Fermi's Description for Beta Decay

In 1933, Fermi introduced a four-fermion vertex to explain the theory of beta-decay



Coupling strength is given by Fermi Constant,

$$G_F = 1.166 \times 10^{-5} \text{GeV}^{-2}$$

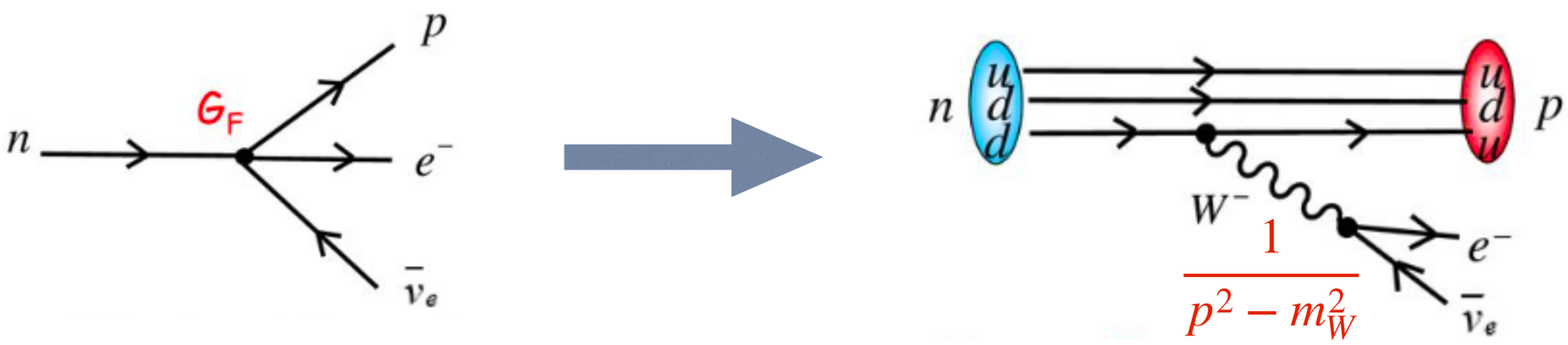
The problem:

Cross-section, $\sigma \propto G_F^2 s$, grows with increasing energy

Fermi's theory breaks down at high energies.

Lesson from the Past: Fermi's Description for Beta Decay

The solution:



Introducing a massive propagator remedies the problem at high energy

The lesson:

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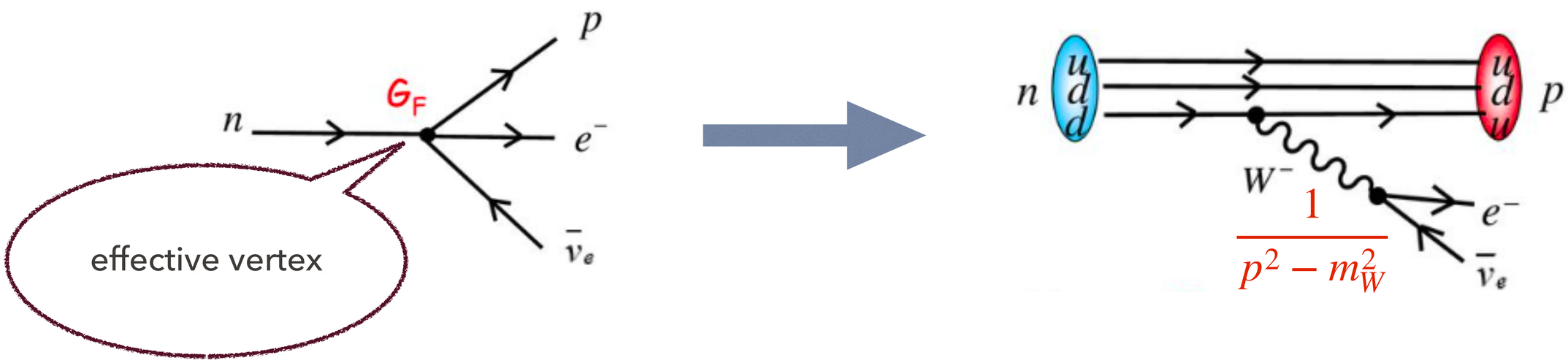

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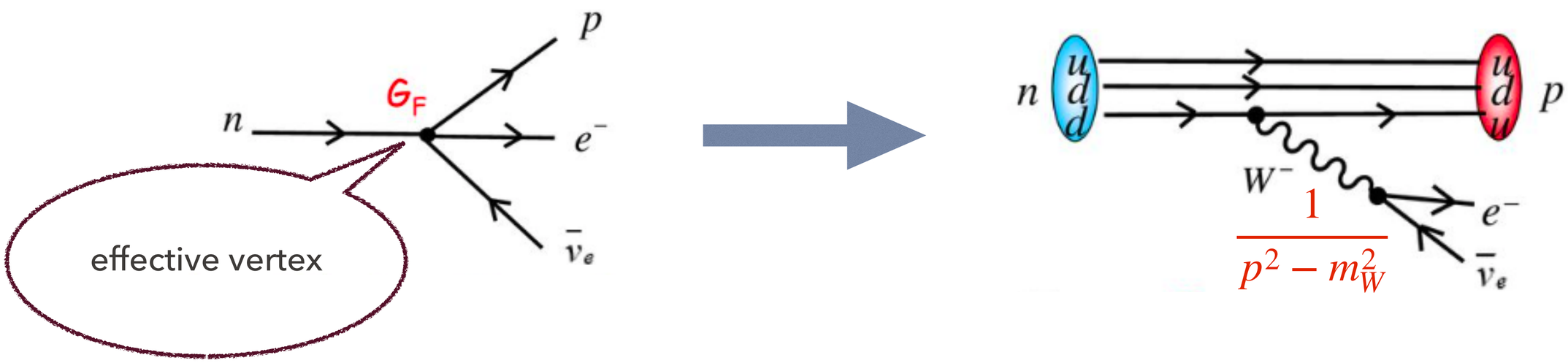


Introducing a massive propagator remedies the problem at high energy

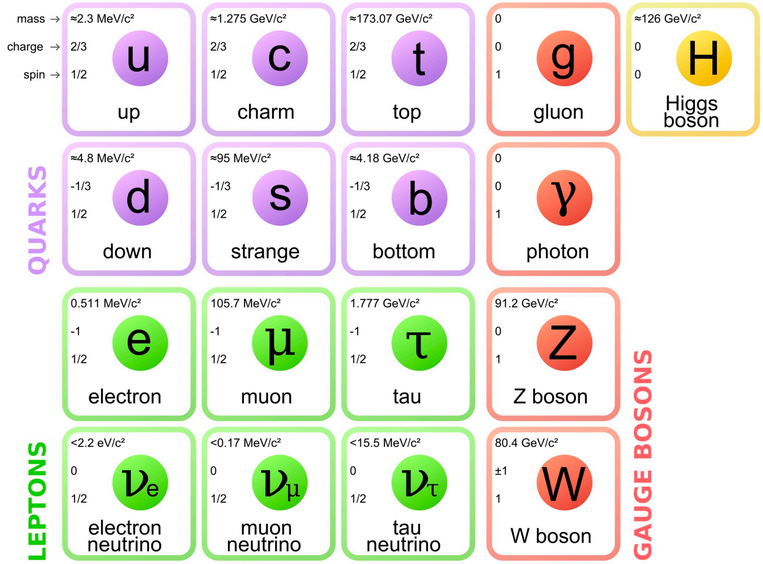
The lesson:

Utility of Effective Field Theory : Parameterising lack of information in terms of higher mass dimension operators

Georgi ARNPS 1993 43:209-52

Manohar arXiv:1804.05863

The Standard Model Effective Field Theory



The Standard Model (SM) of Elementary Particles

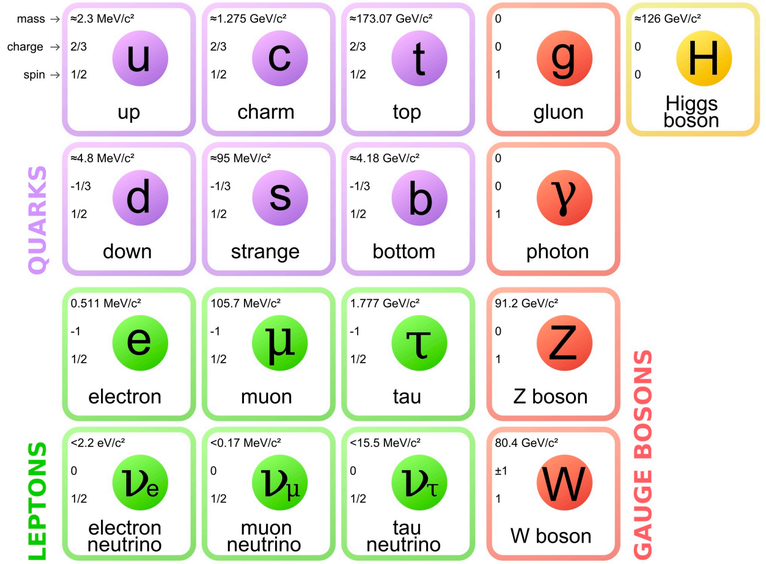
Shortcomings of the SM

- How to explain the small but non-zero masses for neutrinos?
- Why there is more matter than anti-matter?
- What is the composition of dark matter and dark energy?

....and more

No direct hint for new physics yet!!

The Standard Model Effective Field Theory



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In the meantime: keep on making more precise predictions for observables, accommodating for SM deviations

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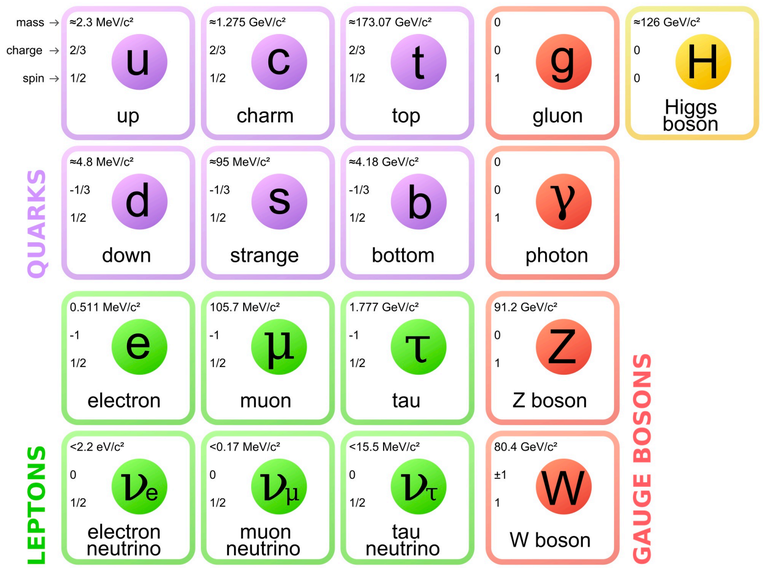

The Standard Model (SM) of Elementary Particles

$$\text{SM} + \text{EFT} = \text{SMEFT}$$

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_{n=5}^{\infty} \sum_{i=1}^{N} \frac{1}{\Lambda^{(n-4)}} \mathcal{C}_i \, \mathcal{O}_i$$

SMEFT Lagrangian

High energy scale

Wilson Coefficients

Effective Operators

Mass of the lightest "heavy field" beyond the SM Lagrangian

Built with the SM fields and respect SM gauge symmetry

Some more examples

- The HEFT theory, a non-linear Higgs potential is studied.
- The LEFT theory, an EFT to describe the SM below the electroweak scale

Which road to take: Bottom-Up or Top-Down?

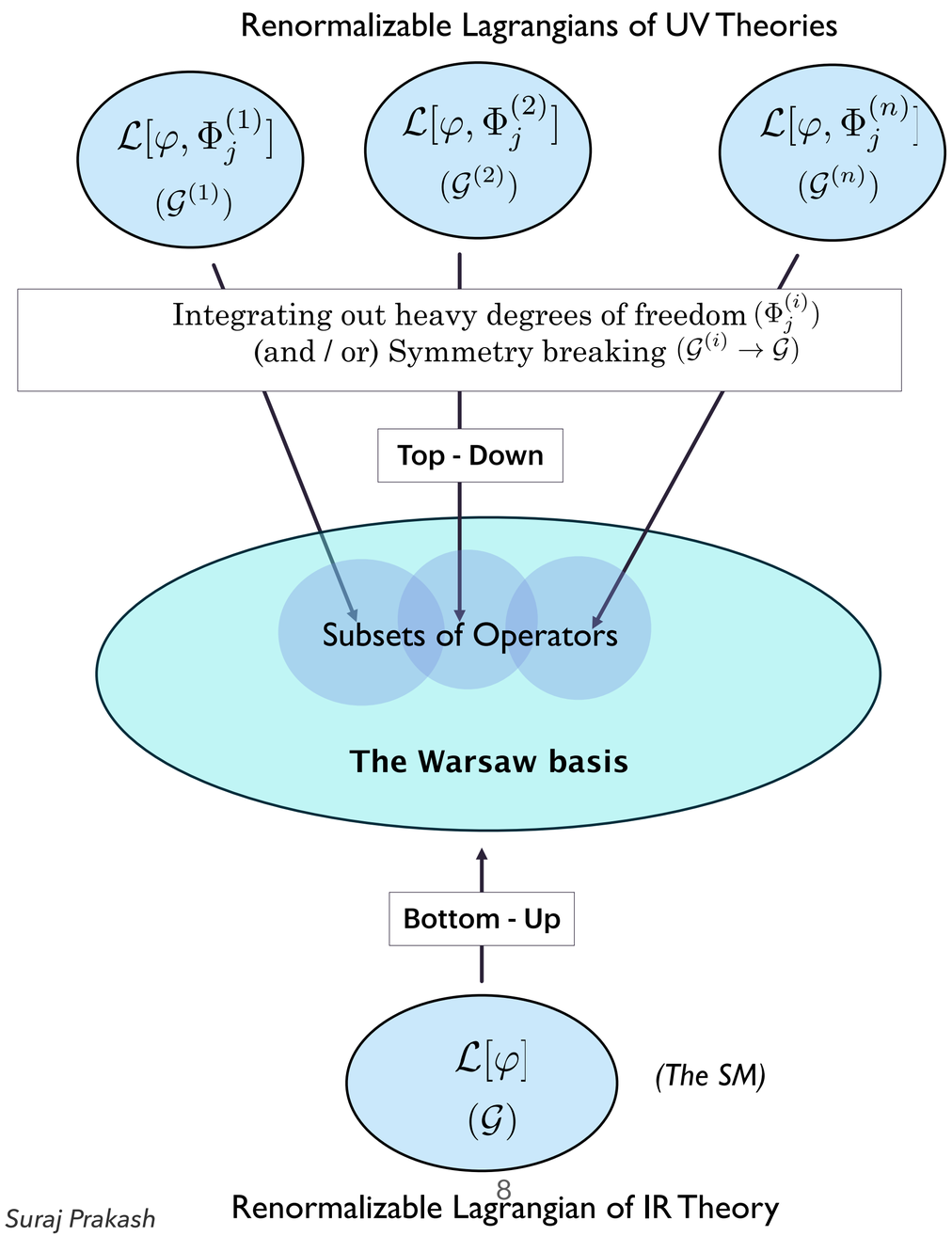

Image courtesy: Suraj Prakash

Part II : Bottom-Up: construction of BSMEFT scenarios

Symmetry guided approach for operator construction

Question: How to construct a complete and independent set of operators without any idea about the full Lagrangian?

Need information only about fields and how they transform under given symmetry

Hilbert Series :

$$\mathcal{H}[\phi] = \prod_{j=1}^{n} \int_{\mathcal{G}_j} \underbrace{d\mu_j}_{\text{Haar Measure}} \quad \underbrace{PE[\varphi, R]}_{\text{Plethystic Exponential}} \quad ,$$

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Group Invariant Polynomial

depends only the symmetry group involved

depends on how states transform under the groups

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A robust technique to construct operator bases for LEFT, SMEFT or extensions of SMEFT

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What are the phenomenological implications of such scenarios?

Left-Right Symmetric Model as an effective theory

The internal symmetry group: $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$

Particle content:

Field	$SU(3)_C$	$SU(2)_L$	$SU(2)_R$	$U(1)_{B-L}$	Spin	Lorentz Group
Φ	1	2	2	0	0	Scalar
Δ_L	1	3	1	2	0	Scalar
Δ_R	1	1	3	2	0	Scalar
Q_L	3	2	1	1/3	1/2	Spinor
Q_R	3	1	2	1/3	1/2	Spinor
L_L	1	2	1	-1	1/2	Spinor
L_R	1	1	2	-1	1/2	Spinor
G_μ^A	8	1	1	0	1	Vector
$W_{\mu,L}^I$	1	3	1	0	1	Vector
$W_{\mu,R}^I$	1	1	3	0	1	Vector
B_μ	1	1	1	0	1	Vector

From Hilbert series we can construct dimension-6 structures $(\phi^2 X^2)$:

$$\mathcal{O}_{\Delta W}^{RrW_LW_L} : \mathrm{Tr}\left[\Delta_R^\dagger \Delta_R W_{L\mu\nu} W_L^{\mu\nu}\right] , \qquad \mathcal{O}_{\Delta W}^{RrW_RW_R} : \mathrm{Tr}\left[\Delta_R^\dagger \Delta_R W_{R\mu\nu} W_R^{\mu\nu}\right] ,$$
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Gauge kinetic terms get modified in the presence of $\phi^2 X^2$ operators :

$$\mathcal{L}_{\text{gauge,kin}}^{(4)+(6)} = -\begin{pmatrix}\partial_\mu W_{L\nu}^- & \partial_\mu W_{R\nu}^-\end{pmatrix}\begin{pmatrix}1-\frac{2\Theta_{W_{LL}}}{\Lambda^2} & 0\\ 0 & 1-\frac{2\Theta_{W_{RR}}}{\Lambda^2}\end{pmatrix}\begin{pmatrix}\partial^\mu W_L^{+\nu}\\ \partial^\mu W_R^{+\nu}\end{pmatrix}$$
$$-\frac{1}{2}\begin{pmatrix}\partial_\mu W_{3L\nu}\\ \partial_\mu W_{3R\nu}\\ \partial_\mu B_\nu\end{pmatrix}^T\begin{pmatrix}1-\frac{2\Theta_{3L3L}}{\Lambda^2} & 0 & 0\\ 0 & 1-\frac{2\Theta_{3R3R}}{\Lambda^2} & -\frac{2\Theta_{3RB}}{\Lambda^2}\\ 0 & -\frac{2\Theta_{3RB}}{\Lambda^2} & 1-\frac{2\Theta_{BB}}{\Lambda^2}\end{pmatrix}\begin{pmatrix}\partial^\mu W_{3L}^\nu\\ \partial^\mu W_{3R}^\nu\\ \partial^\mu B^\nu\end{pmatrix}.$$

Here, the parameters are given as

$$\Theta_{W_{LL}} = v_R^2 \mathcal{C}_{\Delta W}^{RrW_LW_L}, \qquad \Theta_{3R3R} = v_R^2\left(\mathcal{C}_{\Delta W}^{RrW_RW_R} - \mathcal{C}_{\Delta W}^{RW_RrW_R}\right),$$
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Gauge field redefinition :

$$W_L^{\pm\mu} \to \left(1+\frac{\Theta_{W_{LL}}}{\Lambda^2}\right) W_L^{\pm\mu}, \qquad W_{3R}^\mu \to \left(1+\frac{\Theta_{3R3R}}{\Lambda^2}\right) W_{3R}^\mu + \frac{\Theta_{3RB}}{\Lambda^2} B^\mu$$
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$$W_{3L}^{\mu} \to \left(1+\frac{\Theta_{3L3L}}{\Lambda^2}\right) W_{3L}^{\mu}.$$

Modifications in neutral-current and charge-current interactions

$$\mathcal{L}_{\nu Q}^{\rm CC} \supset \frac{g^2}{2\mathcal{M}_{W_{1,2}}^2} \bar{e}_L \gamma^\mu \nu_L \left(\epsilon^{1,2}_{(e\nu)_L*(ud)_L} \bar{u}_L \gamma^\mu d_L + \epsilon^{1,2}_{(e\nu)_L*(ud)_R} \bar{u}_R \gamma^\mu d_R\right) + \text{h.c.}$$

Here, $\epsilon^{1,2}_{(e\nu)_L*(ud)_L} = \epsilon^{1,2}_{(e\nu)_L} \cdot \epsilon^{1,2}_{(ud)_L}$ and $\epsilon^{1,2}_{(e\nu)_L*(ud)_R} = \epsilon^{1,2}_{(e\nu)_L} \cdot \epsilon^{1,2}_{(ud)_R}$ with

$$\epsilon^1_{(e\nu)_L}, \epsilon^1_{(ud)_L} = \cos\xi \left(1+\frac{\Theta_{W_{LL}}}{\Lambda^2}\right), \qquad \epsilon^2_{(e\nu)_L}, \epsilon^2_{(ud)_L} = \sin\xi \left(1+\frac{\Theta_{W_{LL}}}{\Lambda^2}\right)$$

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$$\mathcal{L}_{\nu Q}^{\rm NC} \supset \frac{g^2}{4\cos^2\theta_W \mathcal{M}_{Z_{1,2}}^2} \bar{\nu}_L \gamma^\mu \nu_L$$

$$\times \left(\zeta^{1,2}_{\nu_L*u_L} \bar{u}_L \gamma^\mu u_L + \zeta^{1,2}_{\nu_L*u_R} \bar{u}_R \gamma^\mu u_R + \zeta^{1,2}_{\nu_L*d_L} \bar{d}_L \gamma^\mu d_L + \zeta^{1,2}_{\nu_L*d_R} \bar{d}_R \gamma^\mu d_R\right)$$

Here, $\zeta^{1,2}_{\nu_L*f} = \zeta^{1,2}_{\nu_L} \cdot \zeta^{1,2}_f$, with $f \in \{u_L, u_R, d_L, d_R\}$ and

$$\zeta^1_{f_L} = \left[a_L^f \cos\theta_2 + b_L^f \sin\theta_2\right], \qquad f = \nu, e, u, d,$$

$$\zeta^1_{f_R} = \left[a_R^f \cos\theta_2 + b_R^f \sin\theta_2\right], \qquad f = e, u, d,$$

$$\zeta^2_{f_L} = \left[a_L^f \sin\theta_2 - b_L^f \cos\theta_2\right], \qquad f = \nu, e, u, d,$$

$$\zeta^2_{f_R} = \left[a_R^f \sin\theta_2 - b_R^f \cos\theta_2\right], \qquad f = e, u, d.$$

Produces correction in low-energy observables like weak mixing angle, Fermi constant and ρ parameter etc.

Part III : Top-down: construction of effective action

Top-Down EFT: Integrating out heavy fields

Consider Φ to be a heavy scalar that we wish to integrate out

$$e^{iS_{\text{eff}}[\phi](M)} = \int D\Phi \, e^{iS[\phi,\Phi](M)}$$

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Expand Φ around its classical minima,

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Action can be expanded around the minima , $$S[\phi, \Phi_c + \eta] = S[\phi, \Phi_c] + \left.\frac{\delta S[\phi, \Phi]}{\delta \Phi}\right|_{\Phi=\Phi_c} \eta + \frac{1}{2} \left.\frac{\delta^2 S[\phi, \Phi]}{\delta^2 \Phi}\right|_{\Phi=\Phi_c} \eta^2 + \mathcal{O}(\eta^3)$$

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$$e^{iS_{\text{eff}}[\phi]} = \int D\eta\, e^{iS[\phi,\Phi_c+\eta]} \approx e^{S[\Phi_c]}\left[\det\left(-\frac{\delta^2 S}{\delta\Phi^2}\bigg|_{\Phi_c}\right)\right]^{-1/2}$$

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Tree-level piece

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Tree-level piece

One-loop piece

Top-Down EFT: Integrating out heavy fields

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Dependent only on light fields

Gaillard M.K. Nucl.Phys. B268 (1986) 669-692

Cheyette O. Nucl. Phys. B 297 (1988) 183

Henning et. al. JHEP01(2016)023

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How to obtain tree-level Lagrangian, $$D_\mu \frac{\partial}{\partial(D_\mu\Phi)}\mathcal{L}(\phi,\Phi) = \frac{\partial}{\partial\Phi}\mathcal{L}(\phi,\Phi)$$ *Euler - Lagrange equation*

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How to obtain tree-level Lagrangian, $$D_\mu \frac{\partial}{\partial(D_\mu\Phi)}\mathcal{L}(\phi,\Phi) = \frac{\partial}{\partial\Phi}\mathcal{L}(\phi,\Phi) \quad \Rightarrow \Phi_c[\phi]$$

Computation of one-loop effective Lagrangian

Let's start with a UV Lagrangian, $\mathscr{L}\left[\phi, \Phi\right] \supset \Phi^\dagger (P^2 - m^2 - U(\phi))\, \Phi + (\Phi^\dagger B(\phi) + \text{h.c.}) + \mathscr{O}(\Phi^3)$

$$S_{\text{eff,1-loop}} \approx i c_s \,\text{Tr}\log\left(-\frac{\delta^2 S}{\delta \Phi^2}\bigg|_{\Phi_c} \right) = i c_s \,\text{Tr}\log\left(-P^2 + m^2 + U(\phi) \right)$$

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Inserting a set of momentum and spatial states,

$$\int d^d x\, \mathscr{L}_{\text{eff}}[\phi] = i c_s \int d^d x \int \frac{d^d q}{(2\pi)^d} < q\,|\,x > \text{tr}\log(-P^2 + m^2 + U) < x\,|\,q >$$

$$= i c_s \int d^d x \int \frac{d^d q}{(2\pi)^d}\, e^{iq.x}\, \text{tr}\log(-P^2 + m^2 + U)\, e^{-iq.x}$$

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$$\begin{aligned}
\mathcal{L}^{(dim-6)}_{1-loop}[\phi,\Phi_c] =& \frac{c}{(4\pi)^2}\,\text{tr}\Bigg\{ m^2\left(1+\log\frac{\mu^2}{m^2}\right) U + m^0\left[\frac{1}{12}\left(1+\log\frac{\mu^2}{m^2}\right) G'^2_{\mu\nu} + \frac{1}{2}\log\frac{\mu^2}{m^2} U^2\right] \\
&+ \frac{1}{m^2}\Bigg[-\frac{1}{60}\left(P_\mu G'_{\mu\nu}\right)^2 - \frac{1}{90} G'_{\mu\nu} G'_{\nu\sigma} G'_{\sigma\mu} - \frac{1}{12}\left(P_\mu U\right)^2 - \frac{1}{6} U^3 \\
&- \frac{1}{12} U G'_{\mu\nu} G'_{\mu\nu}\Bigg] + \frac{1}{m^4}\Bigg[\frac{1}{24} U^4 + \frac{1}{12} U\left(P_\mu U\right)^2 + \frac{1}{120}\left(P^2 U\right)^2 + \frac{1}{24}\left(U^2 G'_{\mu\nu} G'_{\mu\nu}\right) \\
&- \frac{1}{120}\left[(P_\mu U),(P_\nu U)\right] G'_{\mu\nu} - \frac{1}{120}\left[U[U, G'_{\mu\nu}]\right] G'_{\mu\nu}\Bigg] + \frac{1}{m^6}\Bigg[-\frac{1}{60} U^5 - \frac{1}{20} U^2 \left(P_\mu U\right)^2 \\
&- \frac{1}{30}\left(U P_\mu U\right)^2\Bigg] + \frac{1}{m^8}\left[\frac{1}{120} U^6\right]\Bigg\}.
\end{aligned}$$

Henning et. al. JHEP01(2016)023

Drozd et. al. JHEP03(2016)180

Computation of one-loop effective Lagrangian

Let's start with a UV Lagrangian, $\mathscr{L}\left[\phi,\Phi\right] \supset \Phi^\dagger(P^2 - m^2 - U(\phi))\,\Phi + (\Phi^\dagger\, B(\phi) + \text{h.c.}) + \mathscr{O}(\Phi^3)$

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Inserting

1. Role of dimension-eight operators in an EFT for 2HDM.
Phys.Rev.D 106(2022) 5, 055012.
Sally Dawson, Duarte Fontes, Samuel Homiller, Matthew Sullivan

2. On the impact of dimension-eight SMEFT operators on Higgs measurements.
JHEP 02 (2019) 123
Chris Hays, Adam Martin, Veronica Sanz, Jack Setford

To go beyond d_6, we need a simplified and easy-to-implement approach for matching!!

Universal One-loop Effective Action upto D8

$$
\begin{aligned}
\mathcal{L}_{\rm eff}^{d\leq 8} =& \frac{c_s}{(4\pi)^2} M^4 \left[-\frac{1}{2}\left(\ln\left[\frac{M^2}{\mu^2}\right] - \frac{3}{2}\right)\right] + \frac{c_s}{(4\pi)^2}\,{\rm tr}\left\{ M^2 \left[-\left(\ln\left[\frac{M^2}{\mu^2}\right] - 1\right) U\right]\right. \\
&+ M^0\, \frac{1}{2}\left[-\ln\left[\frac{M^2}{\mu^2}\right] U^2 - \frac{1}{6}\ln\left[\frac{M^2}{\mu^2}\right] (G_{\mu\nu})^2\right] \\
&+ \frac{1}{M^2}\frac{1}{6}\left[-U^3 - \frac{1}{2}(P_\mu U)^2 - \frac{1}{2}U\,(G_{\mu\nu})^2 - \frac{1}{10}(J_\nu)^2 + \frac{1}{15}\,G_{\mu\nu}\,G_{\nu\rho}\,G_{\rho\mu}\right] \\
&+ \frac{1}{M^4}\frac{1}{24}\left[U^4 - U^2(P^2U) + \frac{4}{5}U^2(G_{\mu\nu})^2 + \frac{1}{5}(U\,G_{\mu\nu})^2 + \frac{1}{5}(P^2U)^2\right. \\
&\quad - \frac{2}{5}U\,(P_\mu U)\,J_\mu + \frac{2}{5}U(J_\mu)^2 - \frac{2}{15}(P^2U)(G_{\rho\sigma})^2 + \frac{1}{35}(P_\nu J_\mu)^2 \\
&\quad - \frac{4}{15}U\,G_{\mu\nu}G_{\nu\rho}G_{\rho\mu} - \frac{8}{15}(P_\mu P_\nu U)\,G_{\rho\mu}G_{\rho\nu} + \frac{16}{105}G_{\mu\nu}J_\mu J_\nu \\
&\quad + \frac{1}{420}(G_{\mu\nu}G_{\rho\sigma})^2 + \frac{17}{210}(G_{\mu\nu})^2(G_{\rho\sigma})^2 + \frac{2}{35}(G_{\mu\nu}G_{\nu\rho})^2 \\
&\quad \left. + \frac{1}{105}G_{\mu\nu}G_{\nu\rho}G_{\rho\sigma}G_{\sigma\mu} + \frac{16}{105}(P_\mu J_\nu)G_{\nu\sigma}G_{\sigma\mu}\right] \\
&+ \frac{1}{M^6}\frac{1}{60}\left[-U^5 + 2\,U^3(P^2U) + U^2(P_\mu U)^2 - \frac{2}{3}U^2G_{\mu\nu}U\,G_{\mu\nu} - U^3(G_{\mu\nu})^2\right. \\
&\quad + \frac{1}{3}U^2(P_\mu U)J_\mu - \frac{1}{3}U\,(P_\mu U)(P_\nu U)\,G_{\mu\nu} - \frac{1}{3}U^2J_\mu(P_\mu U) \\
&\quad - \frac{1}{3}U\,G_{\mu\nu}(P_\mu U)(P_\nu U) - U\,(P^2U)^2 - \frac{2}{3}(P^2U)(P_\nu U)^2 - \frac{1}{7}((P_\mu U)G_{\mu\alpha})^2 \\
&\quad + \frac{2}{7}U^2G_{\mu\nu}G_{\nu\alpha}G_{\alpha\mu} + \frac{8}{21}U\,G_{\mu\nu}U\,G_{\nu\alpha}G_{\alpha\mu} - \frac{4}{7}U^2(J_\mu)^2 - \frac{3}{7}(U\,J_\mu)^2 \\
&\quad + \frac{4}{7}U\,(P^2U)(G_{\mu\nu})^2 + \frac{4}{7}(P^2U)U(G_{\mu\nu})^2 - \frac{2}{7}U\,(P_\mu U)J_\nu G_{\mu\nu} \\
&\quad - \frac{2}{7}(P_\mu U)U\,G_{\mu\nu}J_\nu - \frac{4}{7}U\,(P_\mu U)G_{\mu\nu}J_\nu - \frac{4}{7}(P_\mu U)U\,J_\nu G_{\mu\nu} \\
&\quad + \frac{4}{21}U\,G_{\mu\nu}(P^2U)G_{\mu\nu} + \frac{11}{21}(P_\alpha U)^2(G_{\mu\nu})^2 - \frac{10}{21}(P_\mu U)J_\nu U\,G_{\mu\nu} \\
&\quad - \frac{10}{21}(P_\mu U)G_{\mu\nu}U\,J_\nu - \frac{2}{21}(P_\mu U)(P_\nu U)G_{\mu\alpha}G_{\alpha\nu} + \frac{10}{21}(P_\nu U)(P_\mu U)G_{\mu\alpha}G_{\alpha\nu} \\
&\quad - \frac{1}{7}(G_{\alpha\mu}(P_\mu U))^2 - \frac{1}{42}((P_\alpha U)G_{\mu\nu})^2 - \frac{1}{14}(P_\mu P^2U)^2 - \frac{4}{21}(P^2U)(P_\mu U)J_\mu \\
&\quad \left. + \frac{4}{21}(P_\mu U)(P^2U)J_\mu + \frac{2}{21}(P_\mu U)(P_\nu U)(P_\mu J_\nu) - \frac{2}{21}(P_\nu U)(P_\mu U)(P_\mu J_\nu)\right] \\
&+ \frac{1}{M^8}\frac{1}{120}\left[U^6 - 3\,U^4(P^2U) - 2\,U^3(P_\nu U)^2 + \frac{12}{7}U^2(P_\mu P_\nu U)(P_\nu P_\mu U)\right. \\
&\quad + \frac{26}{7}(P_\mu P_\nu U)U\,(P_\mu U)(P_\nu U) + \frac{26}{7}(P_\mu P_\nu U)(P_\mu U)(P_\nu U)U + \frac{9}{7}(P_\mu U)^2(P_\nu U)^2 \\
&\quad + \frac{9}{7}U\,(P_\mu P_\nu U)U\,(P_\nu P_\mu U) + \frac{17}{14}((P_\mu U)(P_\nu U))^2 + \frac{8}{7}U^3G_{\mu\nu}U\,G_{\mu\nu} \\
&\quad + \frac{5}{7}U^4(G_{\mu\nu})^2 + \frac{18}{7}G_{\mu\nu}(P_\mu U)U^2(P_\nu U) + \frac{9}{14}(U^2G_{\mu\nu})^2 \\
&\quad + \frac{18}{7}G_{\mu\nu}U\,(P_\mu U)(P_\nu U)U + \frac{18}{7}(P_\mu P_\nu U)(P_\mu U)U\,(P_\nu U) \\
&\quad + \left(\frac{8}{7}G_{\mu\nu}U\,(P_\mu U)U\,(P_\nu U) + \frac{26}{7}G_{\mu\nu}(P_\mu U)U\,(P_\nu U)U\right) \\
&\quad \left. + \left(\frac{24}{7}G_{\mu\nu}(P_\mu U)(P_\nu U)U^2 - \frac{2}{7}G_{\mu\nu}U^2(P_\mu U)(P_\nu U)\right)\right] \\
&+ \frac{1}{M^{10}}\frac{1}{210}\left[-U^7 - 5\,U^4(P_\nu U)^2 - 8\,U^3(P_\mu U)U(P_\mu U) - \frac{9}{2}(U^2(P_\mu U))^2\right] \\
&\left. + \frac{1}{M^{12}}\frac{1}{336}\left[U^8\right]\right\}.
\end{aligned}
$$

$$U_{ij} = \frac{\delta^2\mathcal{L}_{UV}}{\delta\Phi_i\delta\Phi_j}$$

$$G_{\mu\nu} = [P_\mu, P_\nu],$$

$$J_\mu = P_\nu G_{\nu\mu} = [P_\nu, [P_\nu, P_\mu]]$$

UB, J Chakrabortty, S Rahaman, K Ramkumar
e-Print: 2306.09103 [hep-ph].

Universal One-loop Effective Action upto D8

$$\mathcal{L}_{\rm eff}^{d\leq 8} = \frac{c_s}{(4\pi)^2} M^4 \left[-\frac{1}{2}\left(\ln\left[\frac{M^2}{\mu^2}\right] - \frac{3}{2}\right)\right] + \frac{c_s}{(4\pi)^2}\mathrm{tr}\left\{ M^2 \left[-\left(\ln\left[\frac{M^2}{\mu^2}\right] - 1\right) U\right]\right.$$

$$+ M^0 \frac{1}{2}\left[-\ln\left[\frac{M^2}{\mu^2}\right] U^2 - \frac{1}{6}\ln\left[\frac{M^2}{\mu^2}\right] (G_{\mu\nu})^2\right]$$

$$+ \frac{1}{M^2}\frac{1}{6}\left[-U^3 - \frac{1}{2}(P_\mu U$$

$$+ \frac{1}{M^4}\frac{1}{24}\left[U^4 - U^2(P^2U$$

$$- \frac{2}{5}U\,(P_\mu U)$$

$$- \frac{4}{15}U\,G_{\mu\nu}G$$

$$+ \frac{1}{420}(G_{\mu\nu}G$$

$$+ \frac{1}{105}G_{\mu\nu}G_\nu$$

$$+ \frac{1}{M^6}\frac{1}{60}\left[-U^5 + 2U^3(P^2U) +$$

$$+ \frac{1}{3}U^2(P_\mu U)J_\mu - \frac{1}{3}U$$

$$- \frac{1}{3}U\,G_{\mu\nu}(P_\mu U)(P_\nu U$$

$$+ \frac{2}{7}U^2 G_{\mu\nu}G_{\nu\alpha}G_{\alpha\mu} +$$

$$+ \frac{4}{7}U\,(P^2U)(G_{\mu\nu})^2 + \frac{4}{7}(P^2U)U(G_{\mu\nu})^2 - \frac{2}{7}U\,(P_\mu U)J_\nu G_{\mu\nu}$$

$$- \frac{2}{7}(P_\mu U)U\,G_{\mu\nu}J_\nu - \frac{4}{7}U\,(P_\mu U)G_{\mu\nu}J_\nu - \frac{4}{7}(P_\mu U)U\,J_\nu G_{\mu\nu}$$

$$+ \frac{4}{21}U\,G_{\mu\nu}(P^2U)G_{\mu\nu} + \frac{11}{21}(P_\alpha U)^2(G_{\mu\nu})^2 - \frac{10}{21}(P_\mu U)J_\nu U\,G_{\mu\nu}$$

$$- \frac{10}{21}(P_\mu U)G_{\mu\nu}U\,J_\nu - \frac{2}{21}(P_\mu U)(P_\nu U)G_{\mu\alpha}G_{\alpha\nu} + \frac{10}{21}(P_\nu U)(P_\mu U)G_{\mu\alpha}G_{\alpha\nu}$$

$$- \frac{1}{7}(G_{\alpha\mu}(P_\mu U))^2 - \frac{1}{42}((P_\alpha U)G_{\mu\nu})^2 - \frac{1}{14}(P_\mu P^2U)^2 - \frac{4}{21}(P^2U)(P_\mu U)J_\mu$$

$$+ \frac{4}{21}(P_\mu U)(P^2U)J_\mu + \frac{2}{21}(P_\mu U)(P_\nu U)(P_\mu J_\nu) - \frac{2}{21}(P_\nu U)(P_\mu U)(P_\mu J_\nu)\Big]$$

$$+ \frac{1}{M^8}\frac{1}{120}\Big[U^6 - 3U^4(P^2U) - 2U^3(P_\nu U)^2 + \frac{12}{7}U^2(P_\mu P_\nu U)(P_\nu P_\mu U)$$

$$U)U + \frac{9}{7}(P_\mu U)^2(P_\nu U)^2$$

$$)^2 + \frac{8}{7}U^3 G_{\mu\nu}U\,G_{\mu\nu}$$

$$U^2 G_{\mu\nu})^2$$

$$)U\,(P_\nu U)$$

$$(P_\nu U)U\Big)$$

$$P_\nu U)\Big)\Big]$$

$$\frac{9}{2}(U^2(P_\mu U))^2\Big]$$

Some nice features of this formulation!

1. It's universal, i.e., does not depend on the specific form of the UV theory as well as IR DoFs

2. Equally applicable for LEFT, SMEFT or any other effective theory at any scale

3. Can be easily implemented in matching tools like CoDEx, or any other to get the WCs

$$G_{\mu\nu} = [P_\mu, P_\nu],$$

$$J_\mu = P_\nu G_{\nu\mu} = [P_\nu, [P_\nu, P_\mu]]$$

UB, J Chakrabortty, S Rahaman, K Ramkumar
e-Print: 2306.09103 [hep-ph].

Summary

- EFT provides a useful tool for parameterizing the effects of new physics without any idea of its origin
- SMEFT: contains effective operators made of SM fields and respects SM symmetry
- Bottom-Up approach and BSM theories as low-energy effective theory
- Top-Down approach and matching using functional methods
- Universal one-loop effective action at dim-8

Thanks for your attention!

Backup slides

Heat Kernel: A Brief Review

Consider the part of the UV Lagrangian that's bi-linear in Φ, $\mathcal{L}^{\Phi} = \Phi^{\dagger}(D^2 + U + M^2)\Phi = \Phi^{\dagger}(\Delta)\Phi,$

The Heat-Kernel for operator Δ can be written as, $K(t, x, y, \Delta) = < y | e^{-t\Delta} | x >$

The Heat-Kernel satisfies the heat equation, $(\partial_t + \Delta)\, K(t, x, y, \Delta) = 0$

with initial condition, $K(0, x, y, \Delta) = \delta(x - y)$

A.A.Bel'kov et al., hep-ph/9506237

I.G.Avramidi, Nuc.Phys.B, 355(1991)

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The HK for the operator Δ, $K(t, x, y, \Delta) = K_0(t, x, y)\, H(t, x, y, \Delta).$

The HK for the free operator $\Delta_0 = \partial_\mu \partial^\mu + M^2$

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Contains the information about the interaction, U

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$$(4\pi t)^{-d/2}\, \mathrm{Exp}\left[\frac{(x-y)^2}{4t} - tM^2\right]$$

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$$(4\pi t)^{-d/2}\, \mathrm{Exp}\left[\frac{(x-y)^2}{4t} - tM^2\right]$$

$$\sum_k \frac{(-t)^k}{k!} b_k(x, y)$$

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$$\sum_k \frac{(-t)^k}{k!} \underbrace{b_k(x, y)}_{\text{Heat-Kernel coefficients}}$$

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One-loop effective action and Heat-Kernel coefficients

Consider the part of the UV Lagrangian that's bi-linear in Φ, $\mathcal{L}^{\Phi} = \Phi^{\dagger}(D^2 + U + M^2)\Phi = \Phi^{\dagger}(\Delta)\Phi,$

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The Heat-Kernel satisfies the heat equation, $(\partial_t + \Delta)\, K(t, x, y, \Delta) = 0$

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The HK for the operator Δ, $K(t, x, y, \Delta) = K_0(t, x, y)\ H(t, x, y, \Delta).$

- How to obtain one-loop effective action in terms of Heat-Kernel coefficients?

$$\mathscr{L}_{\text{eff, 1-loop}} = c_s\, \mathrm{tr} \log(-P^2 + U + M^2) = c_s\, \mathrm{tr} \int_0^{\infty} \frac{dt}{t} e^{-t\Delta} = c_s\, \mathrm{tr} \int_0^{\infty} \frac{dt}{t} K(t, x, x, \Delta)$$

Use:

$$\ln \lambda = -\int_0^{\infty} \frac{dt}{t} e^{-t\lambda}$$

$$= c_s \int_0^{\infty} \frac{dt}{t} (4\pi t)^{-d/2} e^{-tM^2} \sum_k \frac{(-t)^k}{k!}\, \mathrm{tr}[b_k] = \frac{c_s}{(4\pi)^{d/2}} \sum_{k=0}^{\infty} M^{d-2k} \frac{(-1)^k}{k!} \Gamma[k - d/2]\, \underbrace{\mathrm{tr}[b_k]}_{\text{Heat-Kernel coefficients}}$$

One-loop effective action and Heat-Kernel coefficients

✤ How to obtain one-loop effective action in terms of Heat-Kernel coefficients?

$$\mathscr{L}_{\text{eff, 1-loop}} = \frac{c_s}{(4\pi)^{d/2}} \sum_{k=0}^{\infty} M^{d-2k} \frac{(-1)^k}{k!} \Gamma[k - d/2]\, \text{tr}[b_k]$$

Operators of the form $\mathcal{O}(D^r U^s)$ appear in the HKC $b_n(x,x)$ where $n = r/2 + s$.

Start with the initial condition, $b_0(x,x) = I$

Use recursive relation, $\mathcal{O}(D^r U^s) \equiv [b_{r/2+s}][[U^s]] = \sum_{k=0}^{n=r/2+s} \frac{n!\,(n-1)!}{k!(2n-k)!} \{k\, D^{2(n-k)} \{U b_{k-1}[[U^{s-1}]]\} - T_{2(n-k)}\, b_k[[U^s]]\}_{z=0}$

$$\Rightarrow\ T_{\mu_1\mu_2\ldots\mu_m} = D_{\mu_1} T_{\mu_2\ldots\mu_m} + R_{\mu_2\ldots\mu_m,\mu_1},$$

in the above equation, $R_{\mu_2\ldots\mu_m,\mu_1} = [D_{\mu_2}\ldots D_{\mu_m}, D_{\mu_1}]$,

$$\Rightarrow\ R_{\mu_2\mu_3\ldots\mu_m,\mu_1} = G_{\mu_2\mu_1} D_{\mu_3}\ldots D_{\mu_m} + D_{\mu_2} D_{\mu_3\ldots\mu_m,\mu_1},$$