

# The neutron star crust-core transition environment in terms of the symmetry energy.

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# A very concise characteristic of a neutron star

- Typical masses of  $\sim 2M_{\odot}$   
( $M_{\odot} = 2 \times 10^{30}$  kg is the solar mass)
- Typical radii  $R \sim 10 - 14$  km.
- Typical compactness parameter  $R_S/R \equiv 2GM/c^2R \sim 0.4$ ,  $R_S$  is the Schwarzschild radius
- Central density - exceeds several times the density of atomic nuclei ( $2.6 \times 10^{14}$  gcm $^{-3}$ ), depending on the composition and the equation of state

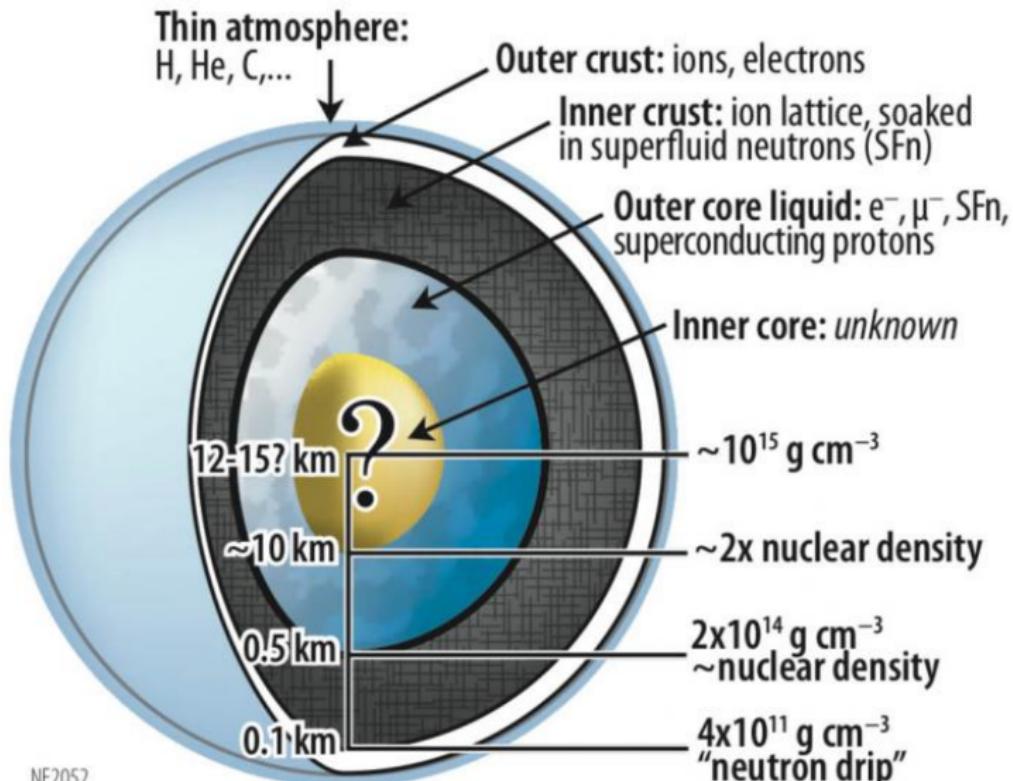
The ultimate degree of compactness - the most compact stars ever observed

Comparing the observed neutron stars' properties with theoretical predictions can provide information on the obscure properties of dense nuclear matter in their interiors.

## Structure of a neutron star - stratification of matter inside a neutron star model

- atmosphere - thin plasma layer where the spectrum of thermal electromagnetic neutron star radiation is formed
- crust
  - outer crust -  $10^6$  gcm $^{-3} < \rho < 4 \times 10^{11}$  gcm $^{-3}$
  - inner crust -  $4 \times 10^{11}$  gcm $^{-3} < \rho < \rho_t$
- core
  - outer core - homogeneous n, p, e, ( $\mu$ ), charge neutral matter ( $Y_e = Y_p$ ) in  $\beta$ -equilibrium
  - inner core

# Internal structure of a neutron star

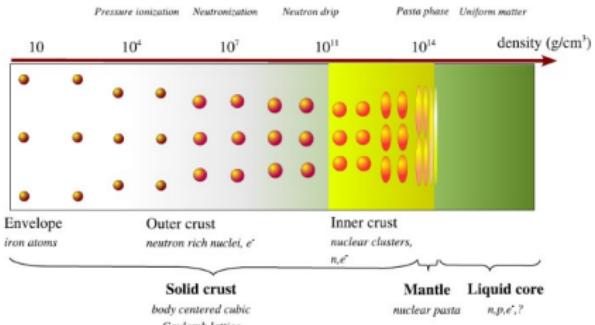


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Credit: K.C. Gendreau et al. (2012), SPIE, 8443, 13

# Neutron star model - equation of state (EoS) an essential ingredient

- outer crust - electrons with Coulomb lattice corrections (BPS, HS)
- inner crust - neutron-rich nuclei in a Coulomb lattice, electrons and free neutrons (BBP), pasta phase
- core EoS - different approaches, e.g., phenomenological models, microscopic models BHF, DBHF



(N. Chamel and P. Haensel. Liv. Rev. Rel. 11, 10, 2008)

## The model

Relativistic mean field models are written in terms of parameters fitted to reproduce properties of bulk nuclear matter and finite nuclei.

The non-linear Walecka model with  $\sigma$  and  $\omega$  self-interaction and different types of mesonic cross-terms

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\text{int}}, \quad \mathcal{L}_{\text{int}} = \bar{\psi} \left( g_\sigma \sigma - (g_\omega \omega_\mu + \frac{1}{2} g_\rho \vec{\tau} \cdot \vec{\rho}_\mu) \gamma^\mu \right) \psi + \mathcal{L}_{\text{NL}}.$$

## The model

$\mathcal{L}_{\text{NL}}$  includes self and cross-meson interaction terms added to remodel the high-density limit of the EoS and the symmetry energy

$$\begin{aligned}\mathcal{L}_{\text{NL}} = & -\frac{A}{3}\sigma^3 - \frac{B}{4}\sigma^4 + \frac{C}{4}(g_\omega^2 \omega_\mu \omega^\mu)^2 + g_\sigma g_\omega^2 \sigma(\omega_\mu \omega^\mu)(\alpha_1 + \frac{1}{2}\alpha'_1 g_\sigma \sigma) \\ & + g_\sigma \sigma g_\rho^2 (\vec{\rho}_\mu \vec{\rho}^\mu)(\alpha_2 + \frac{1}{2}\alpha'_2 g_\sigma \sigma) + \frac{1}{2}\alpha'_3 (g_\omega g_\rho)^2 (\omega_\mu \omega^\mu)(\vec{\rho}_\mu \vec{\rho}^\mu).\end{aligned}$$

EoS - in terms of binding energy - general form

$$E(n_b, \delta) = \frac{\varepsilon(n_b, \delta)}{n_b} - M, \quad n_b = n_n + n_p, \quad \delta = \frac{n_n - n_p}{n_b} = 1 - 2Y_p, \quad Y_p = \frac{n_p}{n_b}$$

$$E(n_b, \delta) = E_0(n_b) + E_{\text{sym},2}(n_b)\delta^2 + E_{\text{sym},4}(n_b)\delta^4 + \dots$$

The symmetric nuclear matter

$$E_0(n_b) = E_0(n_0) + \frac{K_0}{2!} \left( \frac{n_b - n_0}{3n_0} \right)^2 + \frac{J_0}{3!} \left( \frac{n_b - n_0}{3n_0} \right)^3 + \frac{I_0}{4!} \left( \frac{n_b - n_0}{3n_0} \right)^4 + \dots$$

# Properties of nuclear matter

The asymmetric nuclear matter

$$\begin{aligned} E_{\text{sym},2}(n_b) &= E_{\text{sym},2}(n_0) + L_{\text{sym},2} \left( \frac{n_b - n_0}{3n_0} \right) + \frac{K_{\text{sym},2}}{2!} \left( \frac{n_b - n_0}{3n_0} \right)^2 \\ &\quad + \frac{J_{\text{sym},2}}{3!} \left( \frac{n_b - n_0}{3n_0} \right)^3 + \frac{I_{\text{sym},2}}{4!} \left( \frac{n_b - n_0}{3n_0} \right)^4 + \dots \end{aligned}$$

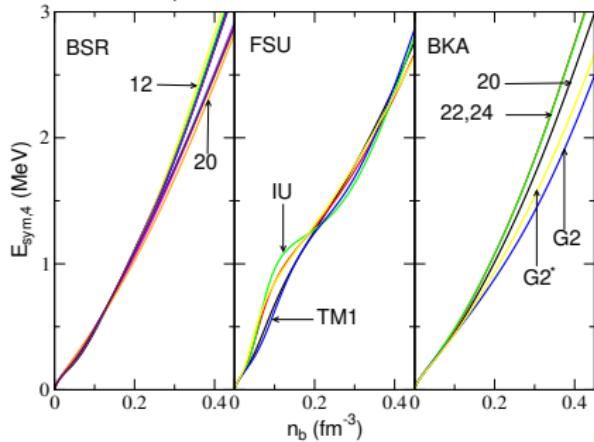
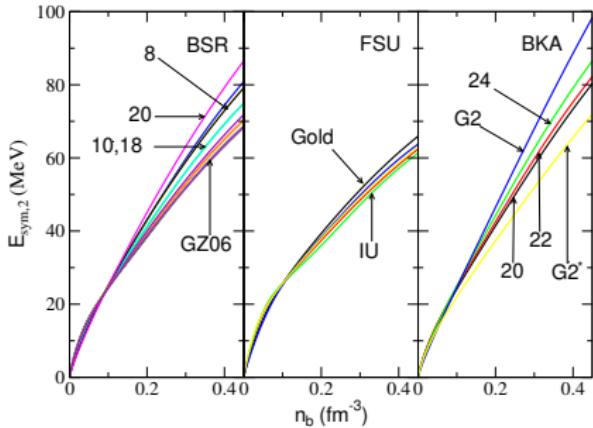
$$\begin{aligned} E_{\text{sym},4}(n_b) &= E_{\text{sym},4}(n_0) + L_{\text{sym},4} \left( \frac{n_b - n_0}{3n_0} \right) + \frac{K_{\text{sym},4}}{2!} \left( \frac{n_b - n_0}{3n_0} \right)^2 \\ &\quad + \frac{J_{\text{sym},4}}{3!} \left( \frac{n_b - n_0}{3n_0} \right)^3 + \frac{I_{\text{sym},4}}{4!} \left( \frac{n_b - n_0}{3n_0} \right)^4 + \dots \end{aligned}$$

Characteristics of models regarding the nonlinear meson couplings.

Model	$\sigma - \omega^2$	$\sigma^2 - \omega^2$	$\sigma - \rho^2$	$\sigma^2 - \rho^2$	$\rho^2 - \omega^2$	$Q_\rho$
BSR	+	+	+	+	+	
FSUGZ03	+	+	+	+	+	$m_\rho^2 + g_\sigma \sigma g_\rho^2 (2\alpha_2 + \alpha'_2 g_\sigma \sigma)$
FSUGZ06	+	+	+	+	+	$+ \alpha'_3 (g_\omega g_\rho)^2 \omega^2$
BKA	+	+	+	-	-	$m_\rho^2 + 2\alpha_2 g_\sigma g_\rho^2 \sigma$
G2*	+	+	+	-	-	$m_\rho^2 + 2\alpha_2 g_\sigma g_\rho^2 \sigma$
FSUGold	-	-	-	-	+	
FSUGold4	-	-	-	-	+	$m_\rho^2 + \alpha'_3 (g_\omega g_\rho)^2 \omega^2$
IU-FSU	-	-	-	-	+	

# The symmetry energy

$Q_\rho$  modifies the symmetry energy  $E_{\text{sym},2}(n_b) = \frac{k_F^2}{6E_F} + \frac{g_\rho^2 n_b}{8Q_\rho}$ ,  $E_F = (k_F^2 + M_{\text{eff}}^2)^{1/2}$



## Constraints

- symmetric nuclear matter
  - incompressibility  $K_0 = 230 \pm 20$  MeV
  - skewness parameter  $-800 < J_0 < 400$  MeV
- symmetry energy
  - symmetry energy parameter  $E_{\text{sym}}(n_0) = 31.7 \pm 3.2$  MeV
  - symmetry energy slope parameter  $L_{\text{sym}} = 58.7 \pm 28.1$  MeV
  - $-400 \text{ MeV} < K_{\text{sym}} < 100 \text{ MeV}$
  - $-200 \text{ MeV} < J_{\text{sym}} < 800 \text{ MeV}$

Neutron star maximum mass above  $2 M_\odot$  (PSR J0348+0432, PSR J1614-2230)

# Location of the inner edge of a neutron star crust

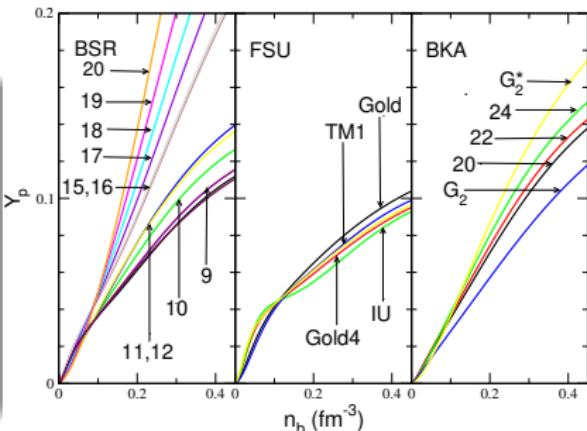
## The equilibrium proton fraction

The chemical equilibrium for reactions:

$n \rightarrow p + e^- + \bar{\nu}_e$  and  $p + e^- \rightarrow n + \nu_e$   
requires  $\mu_e = \mu_n - \mu_p = 2 \frac{\partial E}{\partial \delta}$ ,

the chemical potential  $\mu_i = \frac{\partial E_i}{\partial Y_i}$ ,  $i=(n, p, e)$

$$\begin{aligned} \hbar c (3\pi^2 n_b)^{1/3} Y_p^{1/3} &= 4(1 - 2Y_p) E_{\text{sym},2}(n_b) \\ &+ 8(1 - 2Y_p)^3 E_{\text{sym},4}(n_b) \end{aligned}$$



## Stability condition of the n, p, e matter against the growth of small density fluctuations

$$-\left(\frac{\partial P}{\partial v}\right)_\mu > 0, \quad -\left(\frac{\partial \mu_{\text{asym}}}{\partial q_c}\right)_v > 0,$$

The total pressure of the system  $P = P_N + P_e$ ,  $v$  - volume per baryon number,  $n_b = 1/v$ ,  
 $q_c$  - charge per baryon number,  $\mu_{\text{asym}} = \mu_n - \mu_p$ .

Estimation of the transition density  $n_t$  - from the first inequality.

## Transition density and the corresponding pressure

$$\begin{aligned} P(n_t) = & n_t^2 \left. \frac{dE_0(n_b)}{dn_b} \right|_{n_t} + n_t^2 (1 - 2Y_t)^2 \left( \left. \frac{dE_{\text{sym},2}(n_b)}{dn_b} \right|_{n_t} + (1 - 2Y_t)^2 \left. \frac{dE_{\text{sym},4}(n_b)}{dn_b} \right|_{n_t} \right) + \\ & + n_t Y_t (1 - 2Y_t) (E_{\text{sym},2}(n_t) + 2E_{\text{sym},4}(n_t)(1 - 2Y_t)^2). \end{aligned}$$

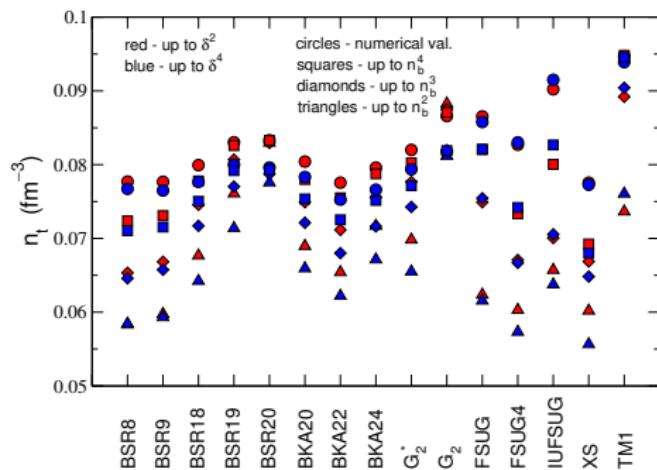
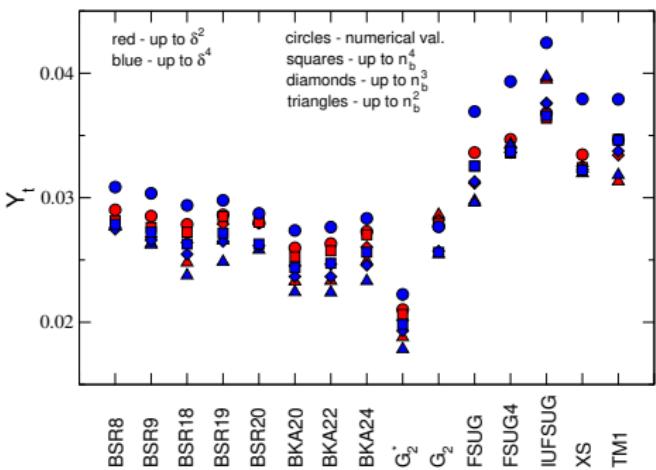
The transition pressure in terms of parameters characterizing properties of asymmetric nuclear matter:

$$\begin{aligned} P(n_t) \approx & \frac{n_t^2(n_t - n_0)}{9n_0^2} (K_0 + K_{\text{sym},2}\delta_t^2 + K_{\text{sym},4}\delta_t^4) + \\ & + L_{\text{sym},2} \left( \frac{n_t(n_t - n_0)Y_t\delta_t}{3n_0} + \frac{n_t^2\delta_t^2}{3n_0} \right) \\ & + n_t Y_t \delta_t (E_{\text{sym},2}(n_0) + 2E_{\text{sym},4}(n_0)\delta_t^2) + \\ & + L_{\text{sym},4} \left( \frac{2n_t(n_t - n_0)Y_t\delta_t^3}{3n_0} + \frac{n_t^2\delta_t^4}{3n_0} \right) \end{aligned}$$

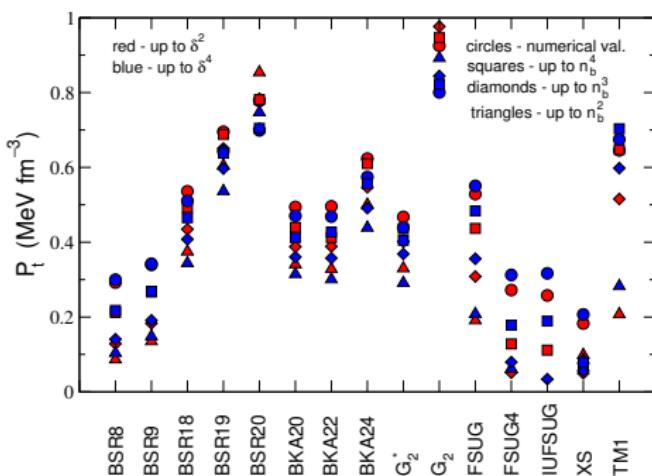
Pure neutron matter:  $\delta_t = 1, Y_t = 0$

$$\begin{aligned} P(n_t) \approx & \left( \frac{n_t}{3n_0} \right)^2 (n_t - n_0) (K_0 + K_{\text{sym},2} + K_{\text{sym},4}) + \\ & + \left( \frac{n_t}{3n_0} \right) n_t (L_{\text{sym},2} + L_{\text{sym},4}) \end{aligned}$$

# Effects of higher-order isospin and density terms of the symmetry energy on the core-crust transition density, pressure, and equilibrium proton fraction



# Effects of higher-order isospin and density terms of the symmetry energy on the core-crust transition density, pressure, and equilibrium proton fraction

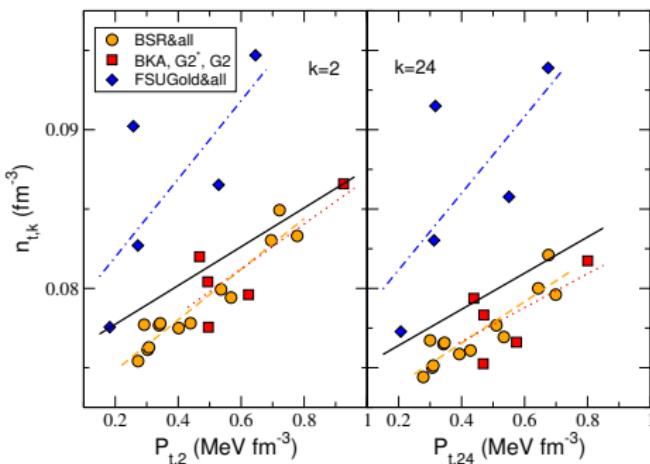


Inclusion of the fourth-order symmetry energy term:

- increases the value of  $Y_t$ , especially for models with  $\omega - \rho$  coupling
- shifts the inner-crust-core transition to lower densities for models with  $\sigma - \rho$  coupling and to higher densities when the  $\omega - \rho$  coupling is present
- lowers the corresponding transition pressure for  $\sigma - \rho$  and increases for  $\omega - \rho$  coupling

Functions representing the symmetry energy's density dependence affect the obtained results.

# Relations between the characteristics of the nuclear matter EoS and a neutron star properties related to its core edge location



- Correlation between the transition density  $n_t$  and the symmetry energy slope  $L_{\text{sym}}$  and the curvature  $K_{\text{sym}}$ 
  - The parabolic approximation -  $n_t$  is anti-correlated with  $K_{\text{sym}}$ , while the anti-correlation with  $L_{\text{sym}}$  is practically non-existent
  - Inclusion of the fourth-order term - both variables  $K_{\text{sym}}$  and  $L_{\text{sym}}$  are equally anti-correlated with  $n_t$  - the individual role of  $L_{\text{sym}}$  in the analysis of the variability of the transition density  $n_t$  increases
- Correlation between transition density  $n_t$  and selected factors - construction of the hierarchy of regression models  $n_t$  vs.  $(P_t)$ ,  $(P_t, L_{\text{sym}})$ ,  $(P_t, L_{\text{sym}}, K_{\text{sym}})$ . The fit of the regression function to the sample points increased significantly.

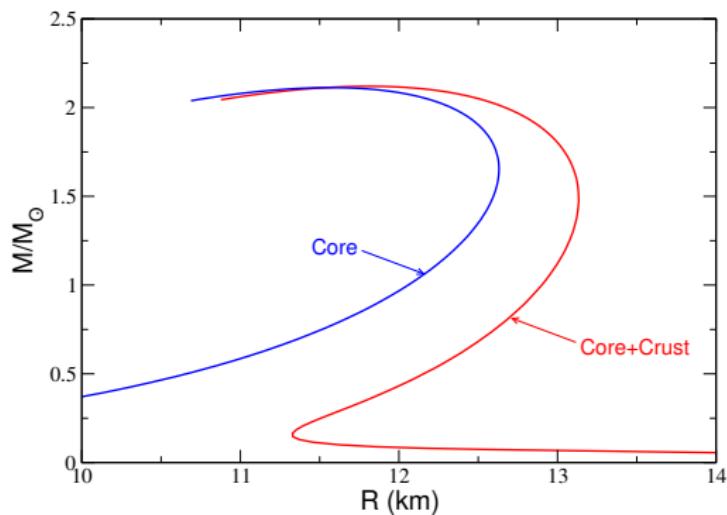
## Effect of the crust on the mass-radius relation

The mass-radius relation obtained for the core model with  $\omega - \rho$  coupling (TM1).

This coupling significantly reduces the symmetry energy slope  $L_{\text{sym}}$  value, starting from 110.79 MeV.

The inner crust is BBP EoS, and the outer crust is BPS EoS.

- changes the radius of a neutron star, the value of the change depends on the mass and reaches the most significant value for stars with low and intermediate masses
- does not affect the value of the maximum mass



# Conclusions

- Symmetry energy decisively influences the environmental conditions at the crust-core boundary
- The essential factors that alter the form of the function representing the symmetry energy are the approximation used - **parabolic vs. the fourth-order**, details of the models considered - RMF models - **the presence of nonlinear couplings between mesons**
- Imprint of the density and pressure in crustal observables
  - The crust thickness
  - Fraction of the neutron star mass contained in the crust

$$M_{cr} \sim 8\pi R_c^3 P_t (R_c/R_S - 1)$$

- Sensitivity of the crustal moment of inertia on the transition pressure

$$I_c \sim \frac{16\pi}{3} \frac{R_c^6}{R_S} P_t,$$

$R_c$  - the core radius,  $R_S$  - the Schwarzschild radius

- There are clear correlations between transition density  $n_t$  and selected factors ( $P_t, L_{sym}, K_{sym}$ ) - dependence on the adopted form of symmetry energy.