How well could we calculate luminosity at FCCee?

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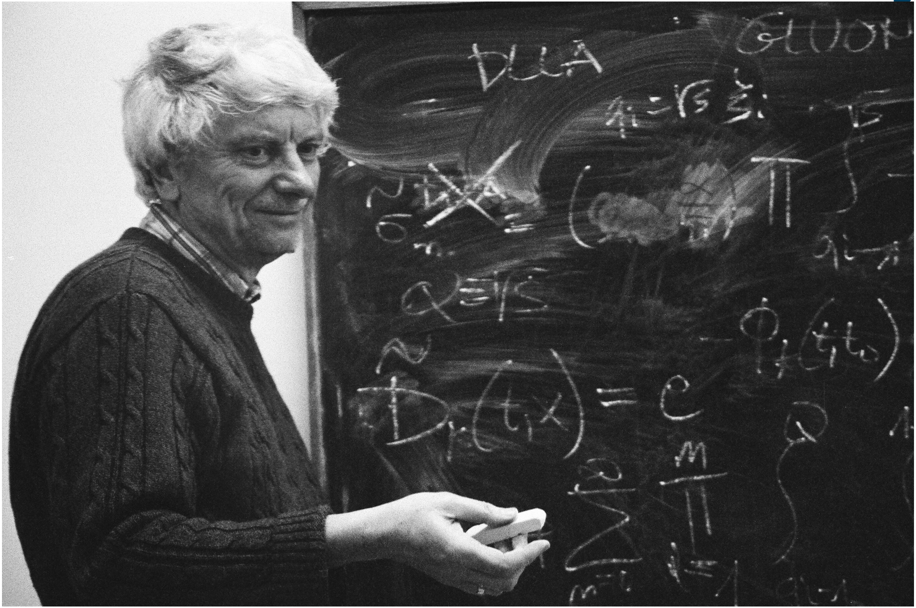
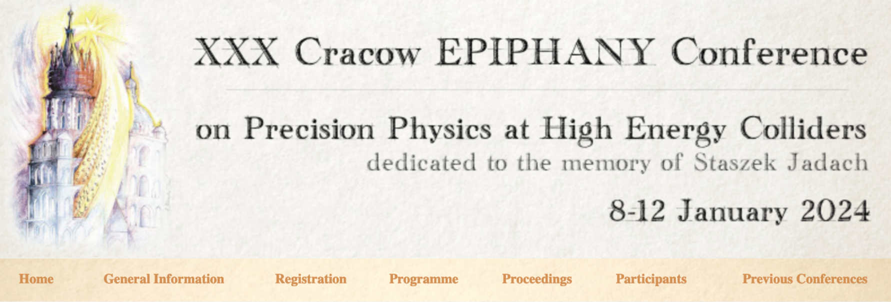
Partly supported by the CERN FCC Design Study Programme.

MTTD Ustron, Poland, 17-22 Sept. 2023

Dedicated to the memory of Staszek Jadach




XXX Cracow EPIPHANY Conference
on Precision Physics at High Energy Colliders
dedicated to the memory of Staszek Jadach
8-12 January 2024
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Intro – lumi basics

Bhabha cross sect. depends on detector acceptance angles

$$\sigma_{Bh} \simeq 4\pi\alpha^2 \left(\frac{1}{t_{\min}} - \frac{1}{t_{\max}} \right) = 4\pi\alpha^2 \left(\frac{t_{\max} - t_{\min}}{\bar{t}^2} \right), \quad \bar{t} = \sqrt{t_{\min} t_{\max}}$$

$\bar{t}$ is the characteristic scale of the process

$\bar{t}/s$ is the suppression factor between s- and t-channel contributions

Machine	$\theta_{\min}\div\theta_{\max}$ [mrad]	$\sqrt{s}$ [GeV]	$\bar{t}/s \simeq \bar{\theta}^2/4$	$\sqrt{\bar{t}}$ [GeV]
LEP	28÷50	M_Z	3.5×10^{-4}	1.70
FCCee	64÷86	M_Z	13.7×10^{-4}	3.37
FCCee	64÷86	240	13.7×10^{-4}	8.9
FCCee	64÷86	350	13.7×10^{-4}	13.0
ILC	31÷77	500	6.0×10^{-4}	12.2
ILC	31÷77	1000	6.0×10^{-4}	24.4
CLIC	39÷134	3000	13.0×10^{-4}	108
CEPC	26÷105	M_Z	6.8×10^{-4}	2.38
CEPC	26÷105	240	6.8×10^{-4}	6.27

Luminosity today – BHLUMI status

The 2019 update comes from P. Janot & S. Jadach Phys.Lett.B 803 (2020) 135319

Type of correction / Error	1999	Update 2019
(a) Photonic $\mathcal{O}(L_e\alpha^2)$	0.027%	0.027%
(b) Photonic $\mathcal{O}(L_e^3\alpha^3)$	0.015%	0.015%
(c) Vacuum polariz.	0.040%	0.009%
(d) Light pairs	0.030%	0.010%
(e) Z and s-channel γ exchange	0.015%	0.015%
(f) Up-down interference	0.0014%	0.0014%
(f) Technical Precision	–	(0.027)%
Total	6.1×10^{-4}	3.7×10^{-4}

Table: Summary of the total (physical+technical) theoretical uncertainty for a typical calorimetric LEP luminosity detector within the generic angular range of 18–52 mrad. Total error is summed in quadrature.

- Hadronic vacuum polarisation from F. Jegerlehner (fortran code `hadr5x.f`) 2019
- Light pairs: real – `FERMISV` MC by J. Hilgart et.al. 1993 and `KoralW` by S. Jadach et.al.; virtual – S. Actis et.al. 2008

Current BHLUMI precision forecast for FCCee

Current BHLUMI precision forecast for FCCee			
Type of correction / Error	M_Z (2019) [1]	240 GeV	350 GeV [2]
(a) Photonic $\mathcal{O}(L_e\alpha^2)$	0.027%	0.032%	0.033%
(b) Photonic $\mathcal{O}(L_e^3\alpha^3)$	0.015%	0.026%	0.028%
(c) Vacuum polariz.	0.009%	0.020%	0.022%
(d) Light pairs	0.010%	0.015%	0.015%
(e) Z and s-channel γ exchange	0.09%	0.25% (0.034%)	0.5% (0.07%)
(f) Up-down interference	0.009%	0.010%	0.010%
(g) Technical Precision	[0.027%]		
Total	10×10^{-4}	25×10^{-4} (6×10^{-4})	50×10^{-4} (8.7×10^{-4})

Table: Entries in curly brackets represent hypothetic situation with all Born-level interferences included in BHLUMI

Entry (c) for M_Z optimistic (SiCal), 0.015% more realistic – [3] LCal setup

Few times worse than at LEP !!

[1] S. Jadach *et.al.* Phys. Lett B790 (2019) 314
[2] S. Jadach *et.al.* Eur. Phys. J. C (2021) 81:1047

[3] P. Janot, S. Jadach Phys. Lett B803 (2020) 135319

Photonic corrections

- Included in BHLUMI: $\mathcal{O}(\alpha+\alpha^2 L^2)$-YFS exponentiated
- To be added: to BHLUMI $\mathcal{O}(\alpha^3 L^3)$ and $\mathcal{O}(\alpha^2 L^1)$ – known
- Errors: $\mathcal{O}(\alpha^4 L^4)$ and $\mathcal{O}(\alpha^3 L^2)$
  - reference points – LEP:
    $\mathcal{O}(\alpha^3 L^3) \simeq 1.5 \times 10^{-4}$ and $\mathcal{O}(\alpha^2 L^1) \simeq 2.7 \times 10^{-4}$
  - estimated based on LEP analysis and scale $(\alpha/\pi)^n L^m$
  - scale with energy/angles as $\ln^m(\bar{t}_{xx}/m_e^2)$
- Likely not needed: $\mathcal{O}(\alpha^2 L^0)$ – known
  $\sim \mathcal{O}(\alpha^2 L^1)/L \simeq 2.7 \times 10^{-4}/16.3 \simeq 0.17 \times 10^{-4}$

$(\gamma_s + Z_s + \gamma_t + Z_t)^{\otimes 2}$ EW interferences

- Included in BHLUMI: $(\gamma_s + Z_s) \otimes \gamma_t$
- To be added:
  - complete Born – trivial
  - complete $\mathcal{O}(\alpha_{EW})$ – known, e.g. BHWIDE
- Error: $\mathcal{O}(\alpha_{EW}^2)$
  - estimated at FCCee(M_Z) based on analysis of S. Jadach *et.al.* Phys. Lett B790 (2019) 314 – from BHWIDE
  - estimated at other energies/angles based on analysis done with $\mathcal{O}(\alpha_{EW})$ DIZET/ZFITTER (by changing switch NPAR(2) from 2 to 3) M. Battaglia, S. Jadach, D. Bardin, *eConf* **C010630** (2001) E3015, http://www.slac.stanford.edu/econf/C010630/papers/E3015.PDF for the energies of 800 GeV and 3 TeV.
    Extrapolation from 800 to 350/240 GeV not done $\Rightarrow$ error likely overestimated (factor of 2-3 ???)
  - Error at higher $\bar{t}/M_Z^2$ almost entirelly from $\gamma_t \otimes Z_t$ interference
- Amplitude-level exponentiation (KKMC-style) needed to account for leading $\mathcal{O}(\alpha_{EW}^2)$ corrs.

QED photonic up-down interference

- Missing in BHLUMI
  size at $\mathcal{O}(\alpha)$: $0.07 \times \bar{t}_{xx}/s$ – easy to include,
  $\bar{t}_{xx}/s$ depends only on angles
  LEP $\rightarrow$ FCCee: t/s grows 4 times (LEP $\rightarrow$ ILC: 2 times)
- Error: h.o.t. – suppressed by $(\alpha/\pi)\ln(\bar{t}_{xx}/m_e^2)$ times safety factor of 2 ($\mathcal{O}(\alpha^2_{QED})$ calculations exist) – almost negligible

Vacuum polarisation

- Uncertainty due to vacuum polarisation:
  $\delta_{VP}\sigma/\sigma = 2\delta\alpha_{eff}(\bar{t})/\alpha_{eff}(\bar{t})$
  - $\delta\alpha_{eff}(\bar{t})$ from (based on R-ratio measured at low energies)
    F. Jegerlehner, *CERN Yellow Reports: Monographs* **3** (2020) 9–37

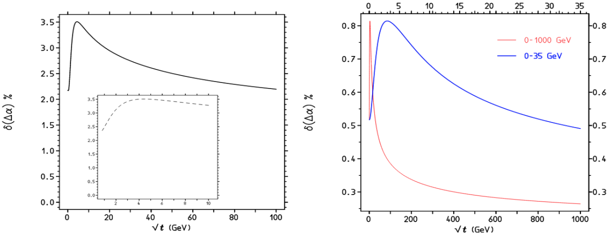

Fig. B.1.15: Hadronic uncertainty $\delta\Delta\alpha_{\rm had}(\sqrt{t})$. The progress since LEP times, from 1996 (left) to now (right) is remarkable. A great deal of much more precise low-energy data, $\pi\pi$, etc., are now available.

  - $\alpha_{eff}(\bar{t})$ from
    F. Jegerlehner, *Nucl. Phys. Proc. Suppl.* **162** (2006) 22–32
- By FCCee operation time factor of 2 improvement expected (F. Jegerlehner)

Light pairs

- Current state of the art: BHLUMI + external four-fermion code + virtual semianalytical corrections
  – P. Janot and S. Jadach, *Phys. Lett. B* **803** (2020) 135319
- included components:
  - *ee*-pair, $\mu\mu$-pair, $\tau\tau$-pair, *qq*-pair with *s*-channel photonic emissions (FERMISV, KORALW)
  - result for LEP: $4 \times 10^{-4} \pm 1 \times 10^{-4}$
- future prospects for external 4*fermion* code scenario
  – error components:
  - $4f + \gamma$ (25% of $4f$) – *s* vs. *t* mismatch $\sim$ 30%
    $\mathcal{O}(\alpha)$ 4*fermion* calculations exist for selected final states
  - $4f + 2\gamma$, $6f$
- future prospects for BHLUMI upgrade scenario
  – error components:
  - $4f + \gamma$ – absent – correct *t*-channel behavior (LL+soft),
    $\mathcal{O}(\alpha)$ 4*fermion* likely not needed
  - $4f + 2\gamma$ – included via exponentiation + LL,
  - $6f$

Light pairs

Extrapolation to other energies/angles

- use LEP result for *ff*: $4 \times 10^{-4} \pm 1 \times 10^{-4}$ and scale with $\ln^2(\bar{t}_{xx}/m_{yy}^2)/\ln^2(\bar{t}_{LEP}/m_{yy}^2)$ (pairs)
- use LEP result for $ff\gamma$ terms: $20\% \times 4 \times 10^{-4}$ (G. Montagna, M. Moretti, O. Nicrosini, A. Pallavicini, and F. Piccinini, *Nucl. Phys.* **B547** (1999) 39–59), and scale with $\ln(\bar{t}_{xx}/m_e^2)/\ln(\bar{t}_{LEP}/m_e^2)$ (photons)
- τ-pair (negligible at LEP) estimated relative to muon-pair as $\ln^2(\bar{t}_{xx}/m_\tau^2)/\ln^2(\bar{t}_{xx}/m_\mu^2)$
- *hadron*-pair estimated relative to muon-pair as $R_{had} \times \ln^2(\bar{t}_{xx}/(0.5GeV)^2)/\ln^2(\bar{t}_{xx}/m_\mu^2)$

Lumi at FCCee: Technical precision

- At LEP BHLUMI technical prec. was tested in two ways:
  - Comparison with semian. integration of $\mathcal{O}(\alpha^2)_{exp}$ matrix el. of BHLUMI: agreement 2.7×10^{-4}
  - Comparison with LUMLOG+OLDBIS hybrid MC and with SABSPV MC. All of these MCs have incomplete soft resummation: agreement 2.7×10^{-4} (for sharp photon energy cut-offs 1.7×10^{-3})
- Now another MC code BabaYaga [Balossini et.al.] with complete soft-photon resummation is available. After upgrade to NNLO in hard process it could be ideal for technical comparison with BHLUMI

Lumi at FCCee – Forecast

Forecast			
Type of correction / Error	FCCee$_{M_Z}$[1]	FCCee$_{240}$[2]	FCCee$_{350}$[2]
(a) Photonic $\mathcal{O}(L_e^2\alpha^3)$	0.10×10^{-4}	0.10×10^{-4}	0.13×10^{-4}
(b) Photonic $\mathcal{O}(L_e^4\alpha^4)$	0.06×10^{-4}	$0.26 \times 10^{-4(a)}$	$0.27 \times 10^{-4(a)}$
(c) Vacuum polariz.	0.6×10^{-4}	1.0×10^{-4}	1.1×10^{-4}
(d) Light pairs	0.5×10^{-4}	0.4×10^{-4}	0.4×10^{-4}
(e) Z and s-channel γ exch.	$0.1 \times 10^{-4(\diamond)}$	$1.0 \times 10^{-4(*)}$	$1.0 \times 10^{-4(*)}$
(f) Up-down interference	0.1×10^{-4}	0.09×10^{-4}	0.1×10^{-4}
Total	1.0×10^{-4}	1.5×10^{-4}	1.6×10^{-4}

Numbers: ($\star$) likely overestimated, (a) include safety factor 2. Technical error is not included

Precision dominated by:

- Vacuum polarisation (c) – seems irreducible.
- The EW $\mathcal{O}(\alpha^2)$ uncertainty (e): Numbers ($\star$) are likely overestimated (taken from 800 GeV estimate) – factor 2 too big ? Number ($\diamond$) possibly underestimated (0.3×10^{-4} ?)

Precision loss at higher energies reasonable

factor of 2 loss w.r.t. M_Z

[1] S. Jadach, W. Płaczek, M. Skrzypek, B. F. L. Ward, S. A. Yost, *Phys. Lett. B* **790** (2019) 314

[2] S. Jadach, W. Płaczek, M. Skrzypek, B. F. L. Ward, Eur. Phys. J. C (2021) 81:1047

Forecast study for FCCee$_{M_Z}$		
Type of correction / Error	Published [1]	Redone
(a) Photonic $\mathcal{O}(L_e^2\alpha^3)$	0.10×10^{-4}	0.10×10^{-4}
(b) Photonic $\mathcal{O}(L_e^4\alpha^4)$	0.06×10^{-4}	0.06×10^{-4}
(b') Photonic $\mathcal{O}(\alpha^2 L_e^0)$		0.17×10^{-4}
(c) Vacuum polariz.	0.6×10^{-4}	0.6×10^{-4}
(d) Light pairs	0.5×10^{-4}	0.27×10^{-4}
(e) Z and s-channel γ exch.	0.1×10^{-4}	0.1×10^{-4}
(f) Up-down interference	0.1×10^{-4}	0.08×10^{-4}
Total	1.0×10^{-4}	0.70×10^{-4}

Lumi at FCCee$_{M_Z}$ – Forecast study

- (d) light pairs are re-analysed w.r.t. [1] (safety factor 1.25 is removed; $ff\gamma$ non-leading contrib. less conservative: $z_{cut} \leq .5$ can help; *hadr*-pair uncertainty is set to few % as in [2])
- (f) value not rounded up is used as compared to Ref. [1]
- "Total" value not rounded up is used as compared to Ref. [1] (the above three entries corrected at 240 and 350 GeV as well)
- (b') missing non-logarithmic $\mathcal{O}(\alpha^2 L_e^0)$ correction added for completeness
- (e): size of $\mathcal{O}(\alpha^2)_{EW}$ corrs. to be revisited – available BHWIDE (conservative scaling 0.3×10^{-4}) & DIZET (switches, at higher energy)

  CEEX amplitude level exponentiation instrumental (KKMC style) ?

[1] S. Jadach, W. Płaczek, M. Skrzypek, B. F. L. Ward, S. A. Yost, *Phys. Lett. B* **790** (2019) 314
[2] ALEPH Collaboration, D. Buskulic *et al.*, *Z. Phys. C* **66** (1995) 3–18

Possible precision $\sim 0.7 \times 10^{-4}$ within the reach ??

Lumi precision at 0.1×10^{-4} level?

at the Z-peak

Lumi via gamma-gamma

Another option for Lumi measurement: $e^+e^- \to \gamma\gamma$ [1],[2]
(KLOE, CLEO, BESIII)

- statistical error $\sim 2 \times 10^{-5}$
- not sensitive to hadronic corrections (vac. pol. contributes at NNLO) $\leq 10^{-5}$
- not sensitive to detector geometry (measurement at wide angles), low angle Bhabha needs μ-meter alignment
- huge background due to large angle Bhabhas ($\sim$ 100 times bigger than signal)
- channel sensitive to new physics, must be well controled
- precision below 10^{-4} requires full $\mathcal{O}(\alpha^2)$ QED and EW corrections

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[1] Electroweak corrections to $e^+e^- \to \gamma\gamma$ as a luminosity process at FCC-ee Carlo M. Carloni Calame, Mauro Chiesa, Guido Montagna Oreste Nicrosinia, Fulvio Piccinini, arXiv:1906.08056

[2] Precision studies of quantum electrodynamics at future e^+e^- colliders

J. Alcaraz Maestre, arXiv:2206.07564

Bhabha lumi: 0.1×10^{-4} at Z-peak?

Vacuum polarisation

From BFL Ward's presentation at RADCOR 2023:

Note: Lattice methods with jegerlehner's results allow, in principle, (c) -> (c)/6

$$\Delta\alpha_{had}(t) = \Delta\alpha_{had}(-Q_0^2)|_{lat} + [\Delta\alpha_{had}(t) - \Delta\alpha_{had}(-Q_0^2)]|_{pQCDAdler}$$

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Lattice results are mainly limited now by statistics (?), so if enoug computing resources are available, the 0.1×10^{-4} precision at –few GeV^2 may be feasible.

The above is more optimistic than the 3.5σ tension with estimates based on exp. data of R-ratio reported in arXiv: 2203.08676, 2211.11401 [hep-lat] for $\Delta\alpha^{(5)}_{had}(-Q^2)$, $Q^2 = 3 \div 7$ GeV^2.

The precision of lattice results given in the above papers is $\Delta\alpha_{had}(-5GeV^2) = 0.00716 \pm 0.9 \times 10^{-4}$ – on par with R-ratio method.

Bhabha lumi: 0.1×10^{-4} at Z-peak?

EW corrections

In *S. Jadach, W. Płaczek, M. Skrzypek, B. F. L. Ward, and S. A. Yost, Phys. Lett. B 790 (2019) 314–321* we estimated

- $\mathcal{O}(\alpha_{EW}^2)$ uncertainties in BHWIDE at Z-peak:
  Conservatively estimated as $\frac{\alpha}{\pi} \ln \frac{\bar{t}}{m_e^2} \times \mathcal{O}(\alpha^2_{from\ exponentiation})$ times safety factor of 2.
  This gives 0.7×10^{-4} for QED part and 0.3×10^{-4} for EW part.
  Added linearly one obtains 1×10^{-4}.
  But we are interested only in EW part !
- More aggresive estimate (no safety factor, added in quadratures) would give 0.4×10^{-4} for total and 0.15×10^{-4} for EW part.

Bhabha lumi: 0.1×10^{-4} at Z-peak?

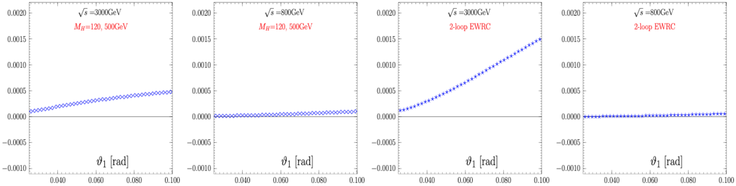
DIZET analysis of EW corrs. done above Z-peak. At the peak different graphs contribute ($\gamma_t \otimes Z_s$ vs $\gamma_t \otimes Z_t$), but rough idea could be valid?

M. Battaglia, S. Jadach, and D. Bardin, eConf C010630 (2001) E3015

S. Jadach, "MC tools for extracting luminosity spectra. What do we need?".

https://jadach.web.cern.ch/jadach/public/LumLCslac.pdf, 2002

How big is, therefore, uncertainty of due to EW corrections?



DIZET: Varied M_H =120→500GeV, M_t =165→185GeV and NPAR(2)=3→NPAR(2)=4 which manipulates non-leading 2-loop EW corrections $\mathcal{O}(G_F^2 M_t^2 M_Z^2)$, Degrassi et.al., keeping 2-loop EW corrections $\mathcal{O}(G_F^2 M_t^4)$.

At 800 GeV $\mathcal{O}(\alpha_{EW}^2)$ contributes below 0.4×10^{-4} and decreases with energy decrease ($15. \times 10^{-4}$ at 3 TeV).

Bottom line: leading α_{EW}^2 contribs may be needed

Bhabha lumi: 0.1×10^{-4} at Z-peak?

Fermion pairs

One will probably need $\mathcal{O}(\alpha)$ corrections to four fermion final state.

- Calculations of Denner et.al. *(PLB 612(2005) 223)* exist for charged current final states. Claimed physical precision (due to higher orders) at WW threshold is few$\times$0.1% of the 4f Born.
- The whole pair contribution to Bhabha is $\sim 4 \times 10^{-4}$. Assuming precision of 1% for NC final states we are well below 0.1×10^{-4} target, provided *t*-channel multiphotons are properly resummed.

Note, that above $\sim$ 500 GeV Sudakov logs must be resummed.

Bottom line

0.1×10^{-4} precision *a priori* not excluded

Lumi forecast at ILC and CLIC

Forecast			
Type of correction / Error	ILC_{500}	ILC_{1000}	CLIC_{3000}
(a) Photonic $\mathcal{O}(L_e^2\alpha^3)$	0.13×10^{-4}	0.15×10^{-4}	0.20×10^{-4}
(b) Photonic $\mathcal{O}(L_e^4\alpha^4)$	0.27×10^{-4}	0.37×10^{-4}	0.63×10^{-4}
(c) Vacuum polariz.	1.1×10^{-4}	1.1×10^{-4}	1.2×10^{-4}
(d) Light pairs	0.4×10^{-4}	0.5×10^{-4}	0.7×10^{-4}
(e) Z and s-channel γ exch.	$1.0 \times 10^{-4(*)}$	2.4×10^{-4}	16×10^{-4}
(f) Up-down interference	$< 0.1 \times 10^{-4}$	$< 0.1 \times 10^{-4}$	0.1×10^{-4}
Total	1.6×10^{-4}	2.7×10^{-4}	16×10^{-4}

Number (*) is somewhat overestimated (taken from 800 GeV estimate)

- Precision at high energies totally due to the EW $\mathcal{O}(\alpha^2)$ hard process uncertainty (e).
- EW interferences are dominated by $\gamma_t \otimes Z_t$ (15% of $\gamma_t \otimes \gamma_t$ at CLIC) and $Z_t \otimes Z_t$ (2% of $\gamma_t \otimes \gamma_t$ at CLIC)
  – usefull for $\mathcal{O}(\alpha_{EW}^2)$ calculation ?
  CEEX amplitude level exponentiation mandatory ?

At 3 TeV loss of precision is dramatic, dominant $\mathcal{O}(\alpha_{EW}^2)$ and CEEX are a must!

Lumi forecast at CEPC

Forecast		
Type of correction / Error	CEPC_{M_Z}	CEPC_{240}
(a) Photonic $\mathcal{O}(L_e^2\alpha^3)$	0.08×10^{-4}	0.10×10^{-4}
(b) Photonic $\mathcal{O}(L_e^4\alpha^4)$	0.14×10^{-4}	0.21×10^{-4}
(c) Vacuum polariz.	0.6×10^{-4}	1.2×10^{-4}
(d) Light pairs	0.24×10^{-4}	0.34×10^{-4}
(e) Z and s-channel γ exch.	0.5×10^{-4}	$1.0 \times 10^{-4(*)}$
(f) Up-down interference	0.03×10^{-4}	0.04×10^{-4}
Total	0.83×10^{-4}	1.62×10^{-4}

Number $(^*)$ is likely overestimated.

- The total error is summed in quadrature. A technical error is not included.
- In the lines (d), (f) and "Total" of the column "CEPC_{M_Z}" safety factors were removed as compared to [1].
- In line (e) estimate based on BHWIDE in [1] is used with a 1/2 factor applied due to reduced transfer. That number differs from the 0.1×10^{-4} used for $FCCee_{M_Z}$.

[1] S. Jadach, W. Płaczek, M. Skrzypek, B. F. L. Ward, S. A. Yost, *Phys. Lett. B* **790** (2019) 314

Summary

- Our starting point is BHLUMI 4.04 with the inherited from LEP precision of 0.06%
- 2019 development of Janot&Jadach reduced this error to 0.037%
- The precision of BHLUMI for FCCee_{240} as of now is 25×10^{-4} and forecasted one is 1.5×10^{-4}, factor of 2 worse than at FCCee_{M_Z}
- At high energies forecasted precision deteriorates drastically, up to 16×10^{-4} for CLIC at 3 TeV, due to missing $\mathcal{O}(\alpha^2)_{EW}$ corrections
- Forecasted in Jadach et.al. (2019) precision 1×10^{-4} at FCCee_{M_Z} seems to be reducible to 0.7×10^{-4} by reducing error on pair emission and loosening conservative approach to safety factors; $\mathcal{O}(\alpha^2_{EW})$ corrs must be revisited. Further precision improvement seems to be blocked by the error on vacuum polarisation contrib.
- Precision 0.1×10^{-4} at Z-peak could be discussed provided lattice QCD delivers vacuum polarisation with precision 0.1×10^{-4} (matter of CPU?), dominant $\mathcal{O}(\alpha^2_{EW})$ corrs to Bhabha and $\mathcal{O}(\alpha_{EW})$ corrs to 4-fermions are available.
- Technical precision requires second MC code, e.g. BABAYAGA