3HDM lepton flavor symmetry of multi-dimensional mass matrices

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Based on:

Flavor symmetries in the leptonic Yukawa sector of the 3HDM

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3HDM or Three-Higgs-Doublet model

Almost minimal extension of SM

This 3HDM:
SM + 2 HD + 3 RH neutrinos

- SM: EWSB $\rightarrow (SU(3)_F)^5$ gets broken
  Remnant flavor symmetries G may survive
- Is G viable?
- G-symmetry of 3HDM $\rightarrow$ constraints on mass matrices
- G viable $\rightarrow$ might help solve the flavor puzzle.

Scope:

- Lepton sector
- Yukawa Lagrangian – (kinetic and Higgs scalar sectors not included)

Yukawa term (charged leptons) of the Lagrangian

$$\mathcal{L}^l = \quad -(h_i^l)_{\alpha\beta}\overline{L}_{\alpha L}\tilde{\Phi}_i l_{\beta R} + \text{H.c.},$$

$i = 1..3$
27 terms + H.c.

$$
\begin{aligned}
\mathcal{L}^l = & \quad -(h_i^l)_{\alpha\beta}\overline{L}_{\alpha L}\tilde{\Phi}_i l_{\beta R} + \text{H.c.}, \\
\mathcal{L}^\nu = & \quad -(h_i^\nu)_{\alpha\beta}\overline{L}_{\alpha L}\Phi_i \nu_{\beta R} + \text{H.c.}, \\
\mathcal{L}^M = & \quad -\frac{g}{M}(h_{ij}^M)_{\alpha\beta}(\overline{L}_{\alpha L}\Phi_i)(\Phi_j^T L_{\beta R}^c) + \text{H.c.}
\end{aligned}
$$

$$
\mathcal{L}^l + \mathcal{L}^\nu
$$

$$
\mathcal{L}^l + \mathcal{L}^M
$$

for neutrinos Dirac or Majorana particle resp.

$$\begin{aligned}\mathcal{L}^l \sim \quad & \overline{L}_L \left(\tilde{\Phi}_i h_i^l\right) \, l_R \\ \stackrel{?}{=} \quad & \overline{L}_L A_L^\dagger \left(A_\Phi^* \tilde{\Phi}\right)_i h_i^l \, A_{lR} \, l_R\end{aligned}$$

Three (different) group representations A_i.
$A_i(g)$ is a 3×3 unitary matrix for each $g \in G$.

$$\begin{aligned}G\text{-symmetry} \iff & \ \mathcal{L}^l + \mathcal{L}^\nu \text{ invariant for all } g \\ \iff & \ \text{appropriate } h_i^l \text{ and } h_i^\nu \text{ exist.}\end{aligned}$$

Invariance equation:

$$((A_\Phi(g))^\dagger \otimes (A_L(g))^\dagger \otimes (A_{lR}(g))^T)\, h^l = h^l, \forall g \in G$$

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Solutions h^l, h^ν define the mass matrices

$$\begin{aligned} M^l &= -\frac{1}{\sqrt{2}} v_i^* h_i^l \\ M^\nu &= \frac{1}{\sqrt{2}} v_i h_i^\nu \end{aligned}$$

If $\mathcal{L}^l + \mathcal{L}^\nu$ is G-invariant

$\rightarrow$ M^l and M^ν as functions of the v_i

$\rightarrow$ constraints on the lepton masses and (Dirac) neutrino mixing angles

$\rightarrow$ G viable or not

Earlier results of 2HDM:

- No viable G isomorphic to a subgroup of $U(3)$, for $|G| \leq 1025$ (Chabor *et al.* 2018)

3HDM:

- Charged lepton mass ratios can be accommodated (as could be expected with 3 Higgs doublets)
- Neutrino mass spectrum can be reproduced for some groups
- Charged lepton and neutrino spectrum cannot be obtained so far

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3HDM:

- Charged lepton mass ratios can be accommodated (as could be expected with 3 Higgs doublets)
- Neutrino mass spectrum can be reproduced for some groups
- Charged lepton and neutrino spectrum cannot be obtained so far
  for 1-dimensional solutions h^l and h^ν

Why is it hard to obtain both mass spectra from a flavor symmetry?

- $\mathcal{L}^l + \mathcal{L}^\nu =$
  $\overline{L}_L A_L^\dagger \, (A_\Phi^* \tilde{\Phi})_i h_i^l \, A_{lR} \, l_R \;\; + \overline{L}_L A_L^\dagger \, (A_\Phi \Phi)_i h_i^\nu \, A_{\nu R} \, \nu_R$

- EWSB $\rightarrow$ the same 3 VEVs for M^l and M^ν

Back to the invariance equation:

$$((A_\Phi(g))^\dagger \otimes (A_L(g))^\dagger \otimes (A_{lR}(g))^T)\, h^l = h^l, \forall g \in G$$

- Up to 3 inequivalent solutions (h^l_a, h^l_b and h^l_c) depending on G and choice of A
- $\rightarrow M^l = \lambda_a M^l_a + \lambda_b M^l_b + \lambda_c M^l_c$
- $\rightarrow M^\nu = \mu_a M^\nu_a + \mu_b M^\nu_b + \mu_c M^\nu_c$

Example group $\Sigma(81)$ or $(\mathbb{Z}_3 \times \mathbb{Z}_3 \times \mathbb{Z}_3) \rtimes \mathbb{Z}_3$

$$M^l = -\frac{c_l}{\sqrt{2}} \begin{pmatrix} \lambda_c v_1^\star & \lambda_a v_3^\star & \lambda_b v_2^\star \\ \lambda_b v_3^\star & \lambda_c v_2^\star & \lambda_a v_1^\star \\ \lambda_a v_2^\star & \lambda_b v_1^\star & \lambda_c v_3^\star \end{pmatrix}$$

3D irrep $A_i = \mathbf{3}_1$ gives rise to $\mathbf{3}_1 \times \mathbf{3}_1 = \mathbf{3}_1 + \mathbf{3}_1 + \mathbf{3}_1$

v_i, λ_i, μ_i: 9 complex parameters.

386 groups $|G| \leq 600$ generate 3-dim mass matrices.
Viable G "easily" attainable?

Chuliá *et al.* (2022) identifies $\Sigma(81)$ as viable in 3HDM, however with different particle content.

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In our 3HDM 3-dim mass matrices do not occur in <u>joint</u> solutions of $\Sigma(81)$

They do not occur at all! (for $|G| \leq 600$)

3HDM analysis of groups with $|G| \leq 600$

36 groups provide 27,283 distinct (M^l, M^ν) pairs.

		dim M^ν		
		1	2	3
dim M^l	1	18922	3816	0
	2	3816	729	0
	3	0	0	0

(1,1) $\rightarrow$ correct $\frac{m_\mu}{m_e}$, $\frac{m_\tau}{m_e}$ only

For dimensionality (1,2), (2,1) and (2,2):

$$M^l = \lambda_a M^l_a + \lambda_b M^l_b, \qquad M^\nu = \mu_a M^\nu_a + \mu_b M^\nu_b$$

Fitting strategies depend on pattern types of M^l_a, M^l_b, M^ν_a and M^ν_b.

Six patterns of mass matrices M_a^l and M_a^ν

$$1: \begin{pmatrix} . & . & x \\ y & . & . \\ . & z & . \end{pmatrix} \quad 2: \begin{pmatrix} . & z & y \\ z & . & x \\ y & x & . \end{pmatrix} \quad 3: \begin{pmatrix} y & y & y \\ z & z & z \\ x & x & x \end{pmatrix}$$

$$4: \begin{pmatrix} y & z & x \\ y & z & x \\ y & z & x \end{pmatrix} \quad 5: \begin{pmatrix} x & y & z \\ z & x & y \\ y & z & x \end{pmatrix}$$

$$6: \begin{pmatrix} . & x+y+z & . \\ . & . & x+y+z \\ x+y+z & . & . \end{pmatrix}$$

Patterns involved in (1,1) solutions:

		M^ν					
		1	2	3	4	5	6
	1	9559	0	238	0	0	0
	2	0	4	0	0	0	0
M^l	3	237	0	108	0	0	0
	4	0	0	0	494	0	0
	5	0	0	0	0	7784	0
	6	0	0	0	0	0	498

Multi-dim $\rightarrow \Sigma$ of patterns.

Dim(1,2) solutions of (M^l, M^ν) = (1, (1,1)):

$$M^l \propto \begin{pmatrix} 0 & \lambda_a v_3^\star & 0 \\ 0 & 0 & \lambda_a v_1^\star \\ \lambda_a v_2^\star & 0 & 0 \end{pmatrix},$$

$$M^\nu \propto \begin{pmatrix} \mu_a v_3 & 0 & \mu_b v_1 \\ \mu_b v_2 & \mu_a v_1 & 0 \\ 0 & \mu_b v_3 & \mu_a v_2 \end{pmatrix}$$

$M^l \to |v_i|$ proportional to CL masses

$M^l \to |v_i|$ known (proportional to CL masses)

$M^\nu \to \frac{m_2}{m_1}$ and $\frac{m_3}{m_1}$, (denote by a and b)

$a, b \to R := \frac{(a^2-1)}{(b^2-a^2}$

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$M^\nu \to \frac{m_2}{m_1}$ and $\frac{m_3}{m_1}$, (denote by a and b)

$a, b \to R := \frac{(a^2-1)}{(b^2-a^2}$

$R_{exp} = \frac{\Delta m_{21}^2}{\Delta m_{32}^2} = 0.0307 \pm 0.0009$

$R \stackrel{?}{=} R_{exp}$

One free parameter only: $R = R(|\mu_b|)$

Group $\mathbb{Z}_7 \rtimes \mathbb{Z}_3 \rightarrow$ good fit (Normal Ordering)

$\rightarrow (m_1, m_2, m_3) =$

$= (0.05 \times 10^{-4}, 0.86 \times 10^{-2}, 5.03 \times 10^{-2})$ eV.

First successful fit of minimal 3HDM to all lepton masses.

Dim(1,2) solutions of $(M^l, M^\nu) = (3, (1,1))$:

pattern 3 $\rightarrow$ wrong CL masses.

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Further occurring cases:

Dim(2,1) solutions of $(M^l, M^\nu) = ((1,1),1)$:

Dim(2,1) solutions of $(M^l, M^\nu) = ((1,1),3)$:

Dim(2,2) solutions of $(M^l, M^\nu) = ((1,1),(1,1))$:

G viable?
Indications only, so far.

Summary and preliminary conclusions:

	Viability of G-symmetry		
	charged lepton masses	Dirac neutrino masses	flavor mixing
2HDM (1,1)	✗	✗	✗
3HDM (1,1)	✓	✗	✗
3HDM (1,(1,1))	✓	✓	?
3HDM (3,(1,1))	✗	✗	✗
3HDM ((1,1),1)	✓	✗	✗
3HDM ((1,1),3)	✓	✗	✗
3HDM ((1,1),(1,1))	?	?	?

Following steps:

Complete 3HDM analysis, analytically, numerically

Scan groups up to $|G|= 1032$

Assume neutrinos are Majorana particles

Verify completeness of "effective" group (sub-, factor-, automorphisme-group, $G \not\leq U(3)$)

.....

Evaluate against:

Symmetries known in Higgs sector

FCNC

....

Thank you

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