Damping of neutrino oscillations and decoherence in reactor and radioactive source experiments

Evgeny Akhmedov

Max-Planck Institute für Kernphysik, Heidelberg

In collaboration with Alexei Smirnov

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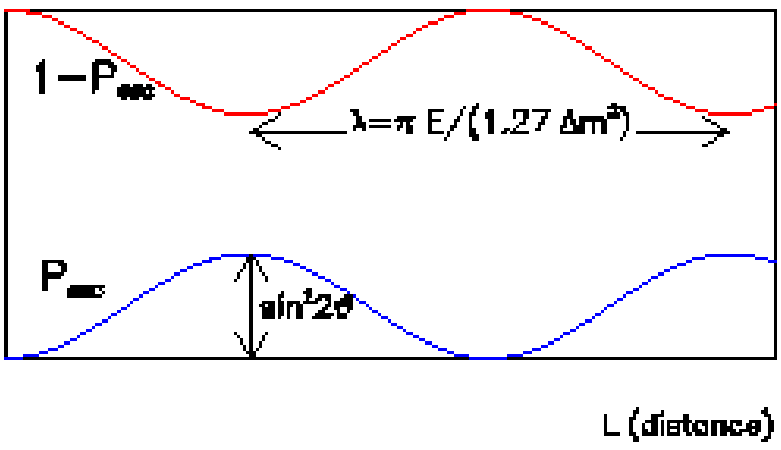
MAX-PLANCK-INSTITUT FÜR KERNPHYSIK

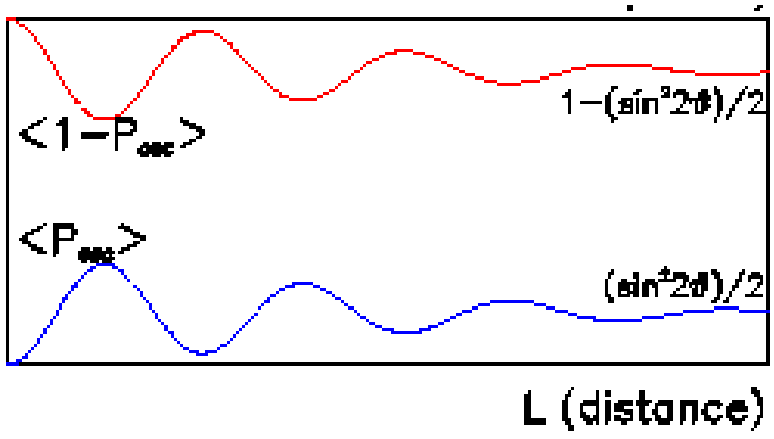
Damping of neutrino oscillations

Can occur due to averaging effects (e.g. due to finite size of ν source/detector or finite E-resolution of detectors). For 2f oscillations:

$$P_{\rm tr} = \sin^2 2\theta \sin^2\left(\frac{\Delta m^2}{4E}L\right) \quad \to \quad \frac{1}{2}\sin^2 2\theta$$

$$P_{\rm surv} = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2}{4E}L\right) \quad \to \quad 1 - \frac{1}{2}\sin^2 2\theta$$

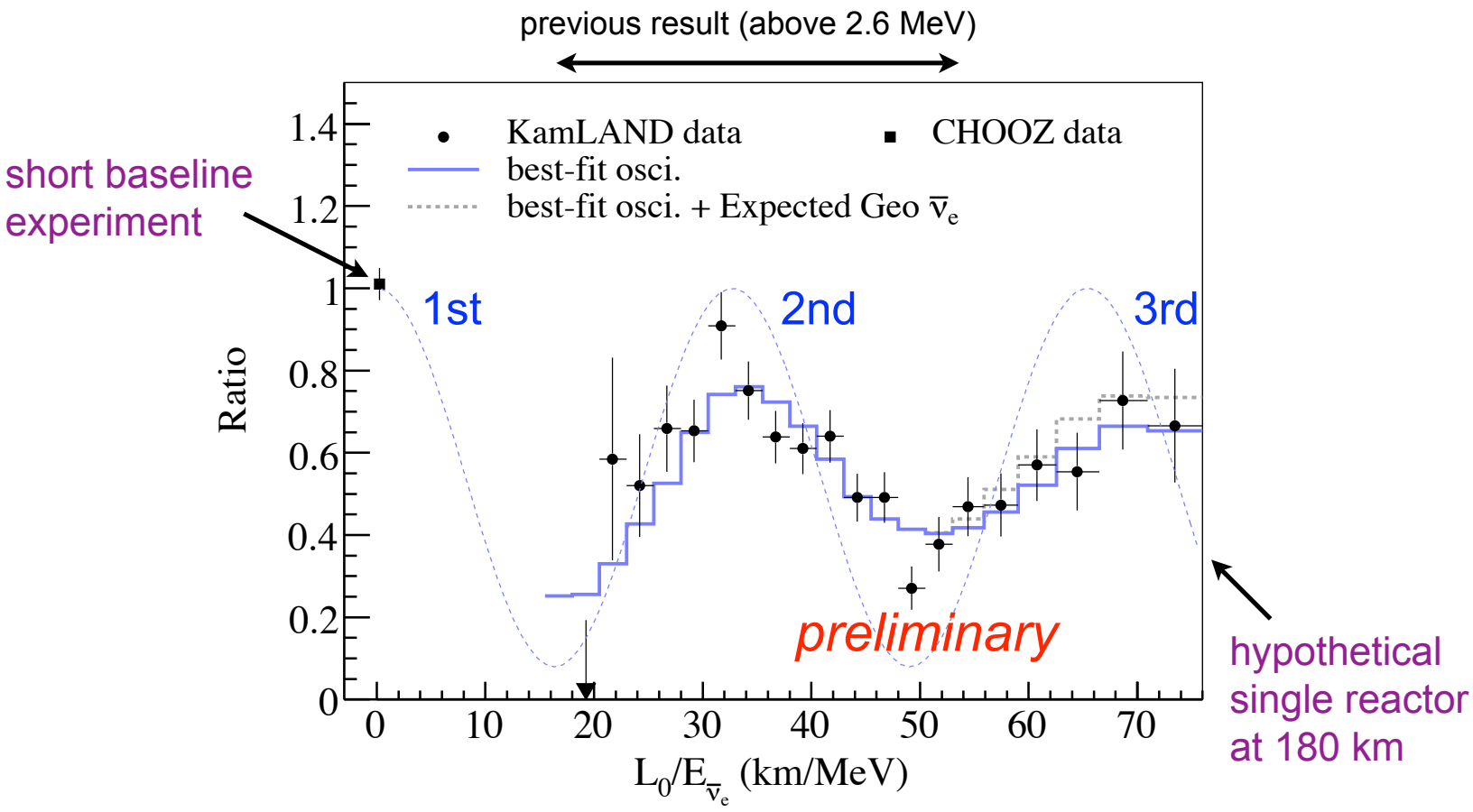




Can also occur due to new physics, e.g. non-standard QM or Q. gravity – not discussed in this talk.

Effect of spread of baselines: KamLAND

Neutrino Oscillation



KamLAND covers the 2nd and 3rd maximum

→ **characteristic of neutrino oscillation**

Oscillation phase:

$$\phi(E) = \frac{\Delta m^2}{4E} L = \pi \frac{L}{l_{\rm osc}}\,, \qquad\qquad l_{\rm osc} \equiv \frac{4\pi E}{\Delta m^2}\,.$$

For most terrestrial (reactor, accelerator and ν source) expts: $\delta L \ll L_{\rm osc} \quad \Rightarrow$ averaging due to the finite sizes of source and detector is negligible.

Finite E-resolution of detector (or finite linewidth of neutrino line in source expts with discrete neutrino spectrum): neutrino energy uncertainty δ_E.

Requiring that variations of ϕ be small (absence of averaging effects): $|\phi(E) - \phi(E + \delta_E)| < 1 \Rightarrow \quad \delta_E$ must satisfy

$$\frac{\delta_E}{E} < \frac{1}{2\pi} \frac{l_{\rm osc}}{L}\,.$$

The longer the baseline, the more stringent the constraint on the energy resolution of the detector.

Damping due to QM decoherence

Neutrino oscillations – a QM interference effect. Flavour states (ν_e, ν_μ, ...) – coherent superpositions of mass eigenstates (ν_1, ν_2, ...):

$$\diamondsuit \quad |\nu_{\alpha L}\rangle = \sum_i U^*_{\alpha i} |\nu_{iL}\rangle \qquad (\alpha = e, \mu, \tau, \quad i = 1, 2, 3)$$

$$\Downarrow$$

$$\diamondsuit \quad \boxed{P^0_{\alpha\beta}(E,L) \equiv \sum_{i,k} U^*_{\alpha i} U_{\beta i} U_{\alpha k} U^*_{\beta k}\, e^{-i\frac{\Delta m^2_{ik}}{2E}L}}$$

If coherence of the contributions different neutrino mass eigenstates to the transition amplitude is destroyed, terms with $i \neq k$ are suppressed $\Rightarrow$

$$P^0_{\alpha\beta}(E,L) \to \sum_i |U_{\alpha i}|^2 |U_{\beta i}|^2$$

The same result as due to averaging out the oscillation terms in $P^0_{\alpha\beta}(E,L)$. Independent of E and L!

When is coherence destroyed?

Different neutrino mass eigenstates propagate with slightly different group velocities:

$$\frac{\Delta v_g}{v_g} \simeq \frac{\Delta m^2}{2E^2} \qquad \Rightarrow$$

The overlap of their wave packets decreases with time, suppressing their coherence. After the separation exceeds the length σ_x of their WPs, coherence is lost. Oscillations can only be observed when

$$L \; < \; L_{\rm coh} \equiv \frac{v_g}{\Delta v_g}\sigma_x$$

The WP length σ_x is related to the intrinsic QM uncertainty of neutrino energy σ_E by $\sigma_x \simeq v_g/\sigma_E$; condition $L < L_{\rm coh}$ yields

$$\frac{\sigma_E}{E} < \frac{1}{2\pi}\frac{l_{\rm osc}}{L}$$

Cf. condition of no averaging due to finite detector resolution: $\frac{\delta_E}{E} < \frac{1}{2\pi}\frac{l_{osc}}{L}$.

Detector E resolution vs. ν WP separation

Oscillation probability with possible decoherence effects taken into account:

$$P_{\alpha\beta}(\bar{E}, L) = \sum_{i,k} U^*_{\alpha i} U_{\beta i} U_{\alpha k} U^*_{\beta k} \, \exp\Big(- i \frac{\Delta m^2_{ik}}{2\bar{E}} L \Big) \, D_{ik}(\bar{E}, L)$$

($\bar{E}$ – mean energy of neutrino WP). $D_{ik}(\bar{E}, L)$ is the damping factor, depends on the properties of neutrino WPs.

$$D_{ik}(\bar{E}, L) \simeq \int dE |f(E, \bar{E})|^2 e^{i \frac{\Delta m^2_{ik}}{2\bar{E}^2}(E - \bar{E})L} \, .$$

Here: $f(E, \bar{E})$ is neutrino WP in energy representation.

For Gaussian WPs:

$$D_{ik}(\bar{E}, L) = e^{-\frac{1}{2}\left(\frac{L}{L_{\mathrm{coh},ik}}\right)^2}$$

Observational equivalence

Effects of QM decoherence by WP separation can be incorporated into modification of experimental energy resolution.

$$\diamondsuit \quad N(E_r) = \mathcal{N} \int d\bar{E} \phi_\alpha(\bar{E}) P_{\alpha\beta}(\bar{E}, L) \sigma_\beta(\bar{E}) R(E_r, \bar{E})$$

$\bar{E}$ – mean energy of the neutrino WP, E_r – reconstructed neutrino energy.

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Damping of neutrino oscillations

Effective energy resolution function:

$$\tilde{R}(E_r, E) = \int d\bar{E}\, R(E_r, \bar{E}) |f(E, \bar{E})|^2 \,.$$

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For Gaussian neutrino WP $f(E, \bar{E})$ and experim. energy resolution $R(E_r, \bar{E})$:

$$f(E, \bar{E}) = \frac{1}{(2\pi\sigma_E^2)^{1/4}} e^{-\frac{(\bar{E}-E)^2}{4\sigma_E^2}} \,, \qquad R(E_r, \bar{E}) = \frac{1}{\sqrt{2\pi}\delta_E} e^{-\frac{(E_r-\bar{E})^2}{2\delta_E^2}}$$

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For $\delta_E \gg \sigma_E$: $\tilde{R}(E_r, E)$ essentially coincides with the true resolution $\Rightarrow$ quantum decoherence by WP separation can be completely neglected. Whether or not the oscillations are damped will then depend on whether or not condition $\frac{\delta_E}{E} < \frac{1}{2\pi}\frac{l_{osc}}{L}$ is satisfied.

E resolution vs. ν WP separation – contd.

$\Rightarrow$ Effects of QM damping by WP separation can only be probed by the experiment if $\sigma_E \gtrsim \delta_E$, i.e. if the neutrino WPs are short enough: $\sigma_x \sim \frac{1}{\sigma_E} \lesssim \delta_E^{-1}$.

An example: for $\delta_E \sim 100\,\mathrm{keV}$ (JUNO), decoherence by WP separation can be probed if $\sigma_x \lesssim 2 \times 10^{-10}\,\mathrm{cm}$.

Similar considerations apply to expts. with artificial neutrino sources like ^{51}Cr (GALLEX, SAGE, BEST). Neutrino production by atomic electron capture $\Rightarrow$ quasi-discrete neutrino spectrum: neutrino lines of small but finite width.

Substitute $\phi_e(E) \to S_e(E)$, where $S_e(E)$ is the line shape function. Careful analysis of various line broadening effects necessary.

Probing WP separation experimentally

Recently: an increased interest to the possibility of probing quantum decoherence by WP separation in reactor and source expts.

Daya Bay (2016): analyzed their data treating σ_p/p, along with $\sin^2 2\theta_{13}$ and Δm^2_{32}, as a free parameter. Result:

$$\sigma_p/p < 0.23 \;\; \text{at } 95\% \text{ C.L.} \quad (\Rightarrow \text{for p} \simeq 3 \text{ MeV}: \; \sigma_x \gtrsim 2.8 \times 10^{-11} \text{ cm}).$$

de Gouvêa et al. (2020, 2021): analyzed Daya Bay, RENO and KamLAND data using σ_x rather than σ_p/p as a fit parameter. From the combined fit:

$$\sigma_x > 2.1 \times 10^{-11} \text{ cm} \;\; (90\% \text{ C.L.}).$$

Also found that JUNO would be able to improve this bound by an order of magnitude.

JUNO (2021): expected sensitivity to WP separation $\Rightarrow$ constraints

$$\sigma_p/p < 1.04 \times 10^{-2}\,, \qquad \sigma_x > 2.3 \times 10^{-10}\ \mathrm{cm} \quad (95\%\mathrm{C.L.})$$

Argüelles et al. 2022:

QM damping effects due to WP separation in oscillations of ν_e and $\bar{\nu}_e$ to sterile neutrinos ν_s can reconcile negative results from reactor experiments with the positive signal claimed in the BEST radioactive source experiment.
Assumption: the actual value of σ_x coincides with the lower bound 2.1×10^{-11} cm found by de Gouvêa et al.

Hardin et al. 2022: Similar analysis but with global fit of SBL data. Tensions can be significantly relaxed for $\sigma_x \sim (0.7 - 1) \times 10^{-11}$ cm.

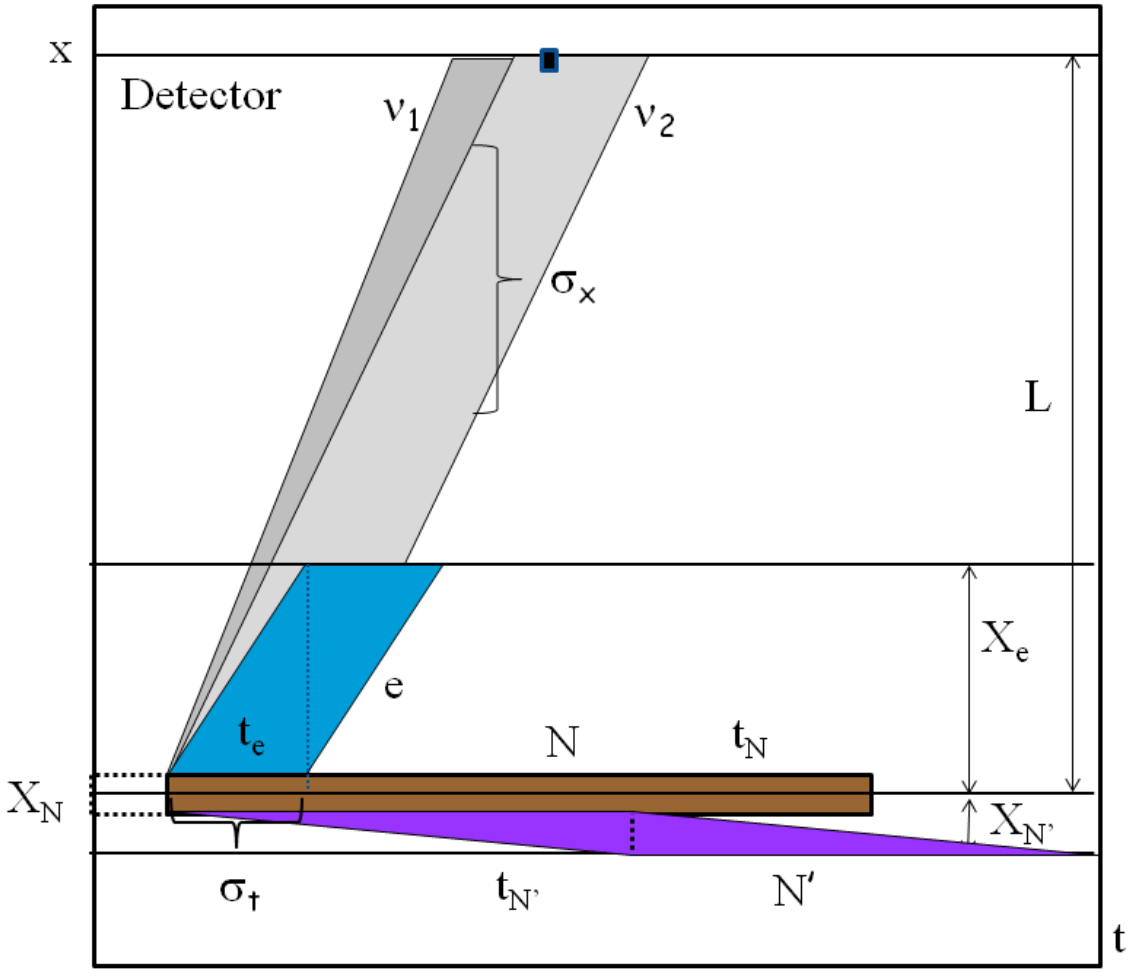
Our results: Such values of σ_x are actually unrealistic.

WP lengths estimates

- The lengths of neutrino WPs are determined by the space-time localization of their production and detection processes. In turn, they depend on the lifetimes of the (unstable) parent particles and the velocities and WP lengths of the participating particles
- The space-time localization of the production and detection processes are essentially given by the overlap of the WPs of paricles taking part in neutrino production and detection
- In the cases we consider the the properties of the neutrino WPs are dominated by the production processes
- Our consideration of the localization of the particles partcipation in neutrino production is based on the collisional broadening effects (analogous to those in atomic physics) and essentially means that we take their WP lengths to be given by their mean free paths.

WP picture

Neutrino production in $N \to N' + e + \bar{\nu}_e$ process, propagation and detection



QM decoherence in terrestrial experiments?

Finding σ_x (or $\sigma_E \simeq 1/\sigma_x$) – difficult task! No first principle calculations.

Our estimates: based on consideration collisional broadening effects for particles taking part in neutrino production. The lengths of their WPs are taken to be given by their mean free paths. Results:

$$\sigma_E \simeq 1\,\mathrm{eV}\,, \qquad \sigma_x \simeq 2\times 10^{-5}\,\mathrm{cm} \qquad (\mathrm{reactor})\,,$$

$$\sigma_E \simeq 0.14\ \mathrm{eV}\,, \qquad \sigma_x \simeq 1.4\times 10^{-4}\,\mathrm{cm} \qquad (\mathrm{source})\,.$$

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In strong disagreement w/ results of Jones et al. (2022) who assumed production localization on an inter-nucleon scale.

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It is also necessary that the baseline be sufficiently large:

$$L \gtrsim L_{\mathrm{coh}} = (2E^2/\Delta m^2)\sigma_x$$

Coherence lengths $L_{\mathrm{coh},ik}$ for reactor neutrino expts.

(for $\Delta m^2_{21} \simeq 7.5 \times 10^{-5}\,\mathrm{eV}^2,\ \Delta m^2_{31} \simeq 2.5 \times 10^{-3}\,\mathrm{eV}^2,\ \Delta m^2_{41} \simeq 1\,\mathrm{eV}^2$)

- ♢ $L_{\mathrm{coh},21} \simeq 4.8 \times 10^7\,\mathrm{km}\,, \quad L_{\mathrm{coh},31} \simeq 1.4 \times 10^6\,\mathrm{km}\,, \quad L_{\mathrm{coh},41} \simeq 3600\,\mathrm{km}\,.$

For chromium source experiments ($E = 0.75$ MeV):

- ♢ $L_{\mathrm{coh},21} \simeq 2.1 \times 10^7\,\mathrm{km}\,, \quad L_{\mathrm{coh},31} \simeq 6.3 \times 10^5\,\mathrm{km}\,, \quad L_{\mathrm{coh},41} \simeq 1600\,\mathrm{km}\,.$

No reactor or neutrino source experiments with such baselines are possible.

$L_{\mathrm{coh}} \propto 1/\Delta m^2$; is it easier to probe WP separation effects in experiments sensitive to larger Δm^2 (like active-sterile neutrino osc. expts.)?

Not really! Experiments are usually devised such that L is of the order of the expected l_{osc}. But l_{osc} is also $\propto 1/\Delta m^2$!

The ratio

$$\frac{L_{\rm coh}}{l_{\rm osc}} = \frac{\sigma_x E}{2\pi}$$

is independent of Δm^2.

For reactor experiments $L_{\rm coh}/l_{\rm osc} \sim 5 \times 10^5 \quad \Rightarrow \quad$ decoherence by WP separation would start to be seen only after neutrinos have propagated half a million oscillation lengths (similarly for neutrino source expts.).

Even if experiments with such huge L were possible, effects of averaging caused by finite detector energy resolution would reveal themselves much before.

WP separation effects should become more pronounced with decreasing E; but it is not easy to detect neutrinos with energies much below $\sim$ MeV range.

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Effects of decoherence by WP separation cannot be observed in reactor and neutrino source experiments.

If osc. damping exceeding what can be expected from (accurately known) finite energy resolution is still observed, this would be a sign of new physics.

Backup slides

QM decoherence in coord. vs. energy space

Coordinate space: spatial separation of ν_i WPs due to their finite lengths and different group velocities. Oscillations observability condition:

$$L < L_{\rm coh} \equiv \frac{v_g}{\Delta v_g}\sigma_x = \frac{2E^2}{\Delta m^2}\sigma_x .$$

$\sigma_x \simeq v_g/\sigma_E \quad \Rightarrow \quad$ condition $L < L_{\rm coh}$ can be written as

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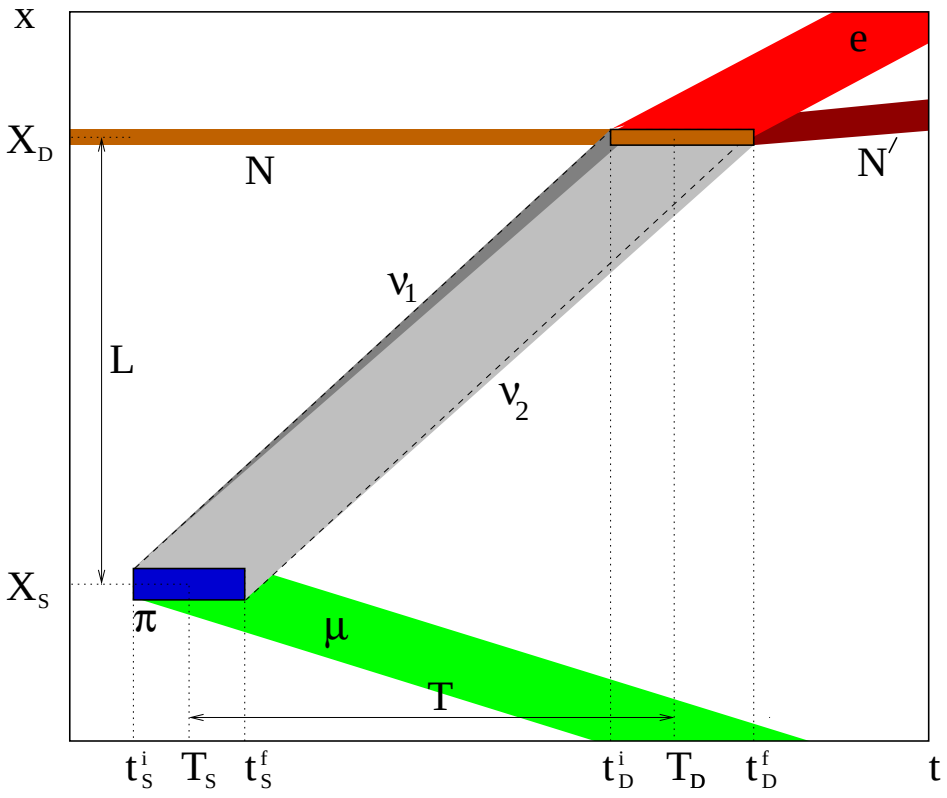
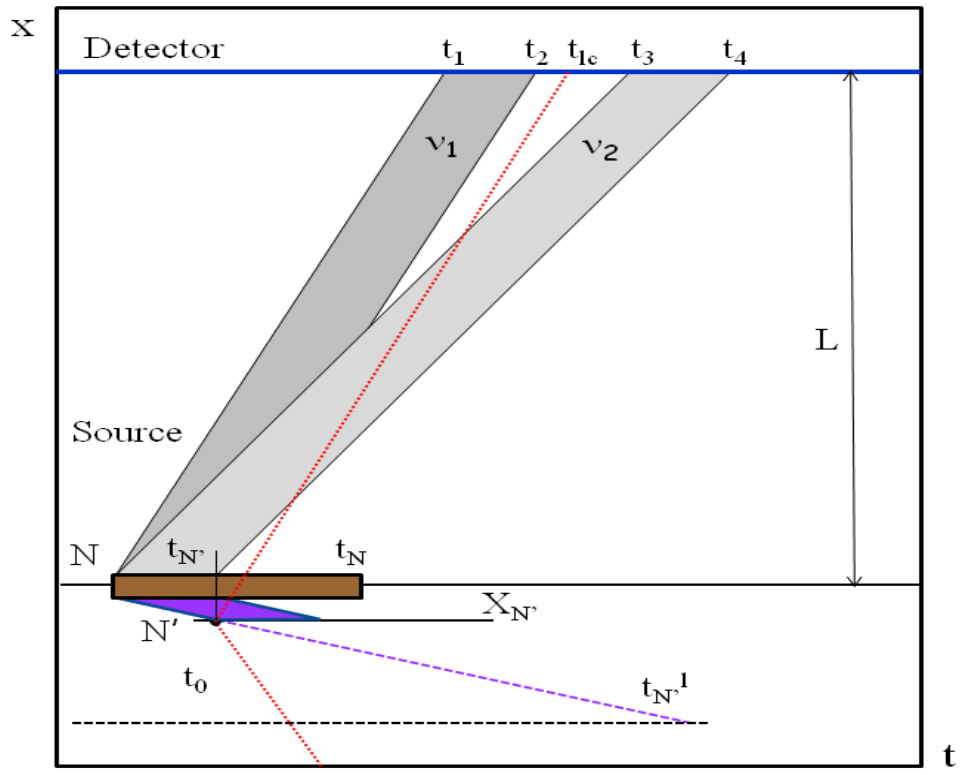
WP picture

Neutrino production in $\pi \to \mu + \nu_\mu$ decay, propagation and detection of oscillated ν_e through IBD



Causality?

Neutrino production in $N \to N' + e + \bar{\nu}_e$ process, propagation and detection



For $L > L_{\rm coh}$ the slower neutrino ν_2 , arrives at detector inside future light cone (shown by red dotted line). Violet dashed: uncompressed gas in source.

Collisional broadening

Analogy with atomic physics. Spectrum of photons emitted by excited atoms in the absence of broadening effects and neglecting natural linewidth:

$$I(\omega) = \lim_{T\to\infty} \frac{1}{2\pi T} \left| \int_{-T/2}^{T/2} e^{i(\omega-\omega_0)t} dt \right|^2 = \delta(\omega - \omega_0) \qquad (\omega_0 = E_i - E_f)$$

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Let at times t_i, $t_i + \tau_i$, ... interactions with surrounding medium occur, introducing random uncontrollable phases in the amplitude $\Rightarrow$ contributions of different intervals $(t_i, t_i + \tau_i)$ are incoherent:

$$I(\omega) = \lim_{T\to\infty} \frac{1}{2\pi T} \sum_i \left| \int_{t_i}^{t_i+\tau_i} e^{i(\omega-\omega_0)t} dt \right|^2 = \lim_{T\to\infty} \frac{1}{\pi T} \sum_i \frac{1 - \cos(\omega - \omega_0)\tau_i}{(\omega - \omega_0)^2}$$

Replace $\sum_i$ by $(T/\tau_0) \int d\tau W(\tau)$ with $W(\tau)$ the normalized distrib. function

$\diamondsuit$ $\quad W(\tau) = (1/\tau_0) e^{-\frac{\tau}{\tau_0}} \qquad (\tau_0 - \text{mean time between collisions})$

$$I(\omega) = \frac{1}{\pi\tau_0}\int_0^\infty \frac{1-\cos(\omega-\omega_0)\tau}{(\omega-\omega_0)^2}\frac{1}{\tau_0}e^{-\frac{\tau}{\tau_0}}d\tau$$

$$= \frac{1}{\pi\tau_0}\frac{1}{(\omega-\omega_0)^2+\frac{1}{\tau_0^2}}$$

The line has the Lorentzian shape with the linewidth (energy uncertainty) given by the inverse mean time between the collisions $\Rightarrow$ WP length given by the mean free path of the particle.

N.B.: The (negative) exponential distribution is the probability distribution of the time between events in a Poisson point process, i.e., a process in which events occur continuously and independently at a constant average rate.