



THE HENRYK NIEWODNICZAŃSKI
INSTITUTE OF NUCLEAR PHYSICS
POLISH ACADEMY OF SCIENCES

NLO matching with KrkNLO

theory and progress

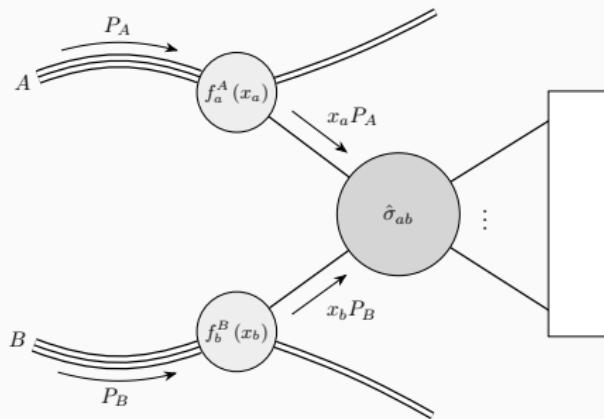
James Whitehead (IFJ PAN, Kraków)

with Wiesław Płaczek, Pratixan Sarmah, Andrzej Sióderek (UJ, Kraków)

MTTD 2023

Matching at NLO

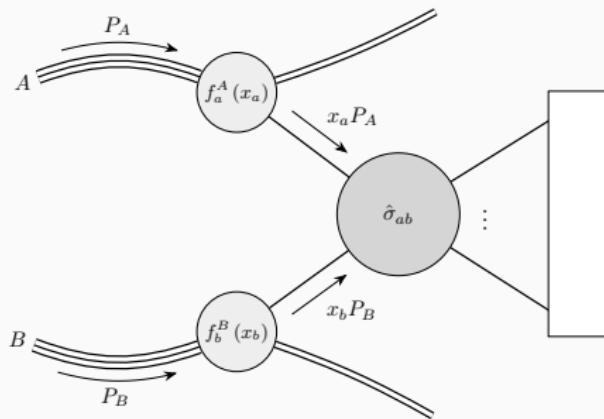
NLO matching



Interested in some¹ function \mathcal{O} of phase-space:

$$d\sigma_{AB}[\mathcal{O}](\mu_F, \mu_R) = \sum_{a,b} f_a^A(\xi_1; \mu_F) \otimes_{\xi_1} d\hat{\sigma}_{ab}[\mathcal{O}](\xi_1, \xi_2; \mu_F, \mu_R) \otimes_{\xi_2} f_b^B(\xi_2; \mu_F).$$

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¹infrared and collinear safe

Building a cross-section

For fixed-order calculations: expand perturbatively (and subtract)

$$\begin{aligned} d\sigma_{ab}^{\text{NLO}} [\mathcal{O}] (\xi_1, \xi_2) = & \left(\frac{\alpha_s}{2\pi} \right)^k \left\{ d\Phi_m(\xi_1 P_1, \xi_2 P_2) \left[B(\Phi_m) \right] \mathcal{O}(\Phi_m) \right\} \\ & + \left(\frac{\alpha_s}{2\pi} \right)^{k+1} \left\{ d\Phi_m(\xi_1 P_1, \xi_2 P_2) \left[V(\Phi_m) \right] \mathcal{O}(\Phi_m) \right. \\ & \quad \left. + d\Phi_{m+1}(\xi_1 P_1, \xi_2 P_2) \left[R(\Phi_{m+1}) \right] \mathcal{O}(\Phi_{m+1}) \right\} \end{aligned}$$

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...look familiar?

New legs from old²

What is a parton shower?

At its heart:

$$\text{PS} [\mathcal{O}(\Phi_m)] = \Delta_{\mu_s}^{Q(\Phi_m)} \mathcal{O}(\Phi_m)$$

²Based on ongoing work with Andrzej Siódtek and Simon Plätzer.

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where the Sudakov form factor is

$$\Delta_{\mu_s}^{Q(\Phi_m)} = \exp \left[- \sum_{\alpha} \int d\Phi_{+1} \Theta[\mu_s < \mu(\Phi_{+1}) < Q(\Phi_m)] P_m^{(\alpha)}(\Phi_{+1}) \Theta_{\text{PS}}^{(\alpha)} \right]$$

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NLO matching criterion

The matched NLO cross-section shouldn't spoil the fixed-order result:

$$\hat{\sigma}^{\text{NLO+PS}}[\mathcal{O}] = \hat{\sigma}^{\text{NLO}}[\mathcal{O}]$$

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NLO (α_s^{k+1})	$V(\Phi_m)$	$R(\Phi_{m+1})$
shower (α_s^{k+1})	$-B \cdot \int d\Phi_{+1} P_m^{(\alpha)}(\Phi_{+1})$	$+B \cdot d\Phi_{+1} P_m^{(\alpha)}(\Phi_{+1})$
factorisation scheme (α_s^{k+1})	$\Delta f_a \otimes_{\xi_1} B + B \otimes_{\xi_2} \Delta f_b$	

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- but weight is always positive.
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- need full phase-space coverage from shower!
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$\mathcal{O}(\Phi_m)$: restore the cancellation required by the matching condition by modifying the PDF factorisation scheme

- collinear convolution terms can only go into the PDF
- where to put end-point contributions $\propto \delta(1-x)$?

Familiar ingredients

What is $-B \cdot \int d\Phi_{+1} P_m^{(\alpha)}(\Phi_{+1})$?

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For dipoles, we already know the answer from dipole subtraction:³

$$-\sum_{\alpha} \int d\Phi_{+1} \Theta[\mu_s < \mu(\Phi_{+1}) < Q(\Phi_m)] P_m^{(\alpha)}(\Phi_{+1}) \Theta_{PS}^{(\alpha)} = \sum_{(\alpha)} I^{(\alpha)} + dx(P^{(\alpha)} + K^{(\alpha)})$$

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This provides the recipe for the PDF transformation (more later).

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$$\begin{aligned} d\Phi_m \Theta_{\text{cut}}[\Phi_m] & \left[\left\{ B(\Phi_m) + V(\Phi_m) + I(\Phi_m) + \Delta_0^{\text{FS}} \right\} \Delta_{\mu_s}^{Q(\Phi_m)} \mathcal{O}(\Phi_m) \right. \\ & \left. + \sum_{\alpha} d\Phi_{+1}^{(\alpha)} \left\{ \frac{R^{(\alpha)}(\Phi_{m+1}^{(\alpha)})}{PS^{(\alpha)}(\Phi_{m+1}^{(\alpha)})} \Theta_{PS}^{(\alpha)}[\Phi_{m+1}^{(\alpha)}] PS^{(\alpha)}[\Phi_{m+1}^{(\alpha)}] \Theta_{\mu_s}^{(\alpha)} \mathcal{O}(\Phi_{m+1}^{(\alpha)}) \right\} \right] \end{aligned}$$

1. generate a Born phase-space point, ME and shower:
 - if an emission is generated, reweight to R
 - if not, reweight to B + V
2. matching complete; allow the shower to proceed!

⁴ S. Jadach et al. "Matching NLO QCD with parton shower in Monte Carlo scheme — the KrkNLO method". arXiv: 1503.06849 [hep-ph], Stanislaw Jadach et al. "New simpler methods of matching NLO corrections with parton shower Monte Carlo". arXiv: 1607.00919 [hep-ph].

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This is NLO accurate, but differs from other methods at higher orders.

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Krk PDF scheme

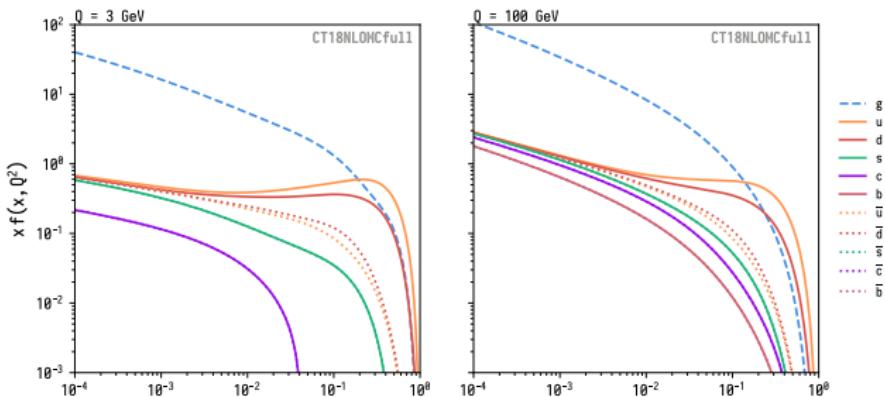
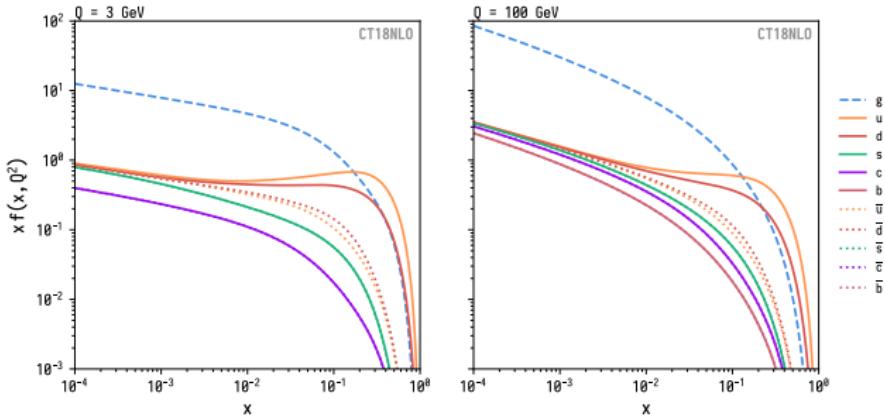
Krk (/MC/CS) factorisation scheme⁵

From the dipole operators, we can write down the convolution terms:

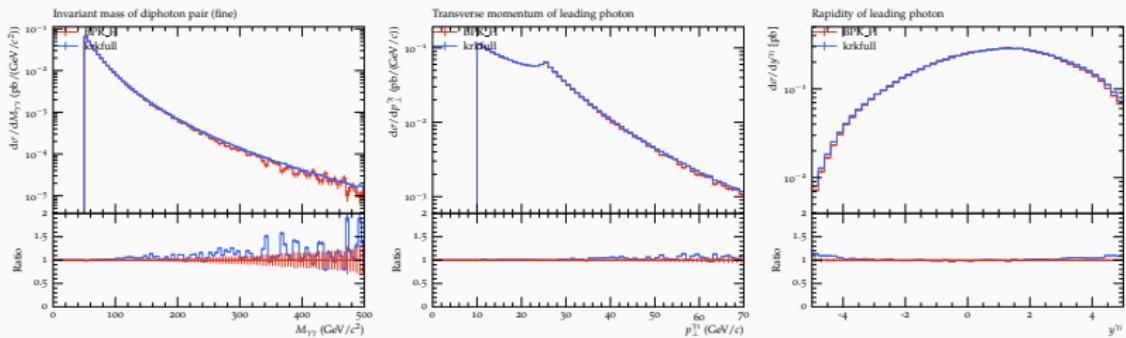
$$\begin{aligned}
 f_q^{\text{Krk}}(x, \mu_F) &= f_q^{\overline{\text{MS}}} (x, \mu_F) \\
 &\quad - \frac{\alpha_s(\mu_F)}{2\pi} \frac{3}{2} C_F f_q^{\overline{\text{MS}}} (x, \mu_F) \\
 &\quad + \frac{\alpha_s(\mu_F)}{2\pi} C_F \left[\int_x^1 \frac{dz}{z} f_q^{\overline{\text{MS}}} \left(\frac{x}{z}, \mu_F \right) \left[\frac{1+z^2}{1-z} \log \frac{(1-z)^2}{z} + 1-z \right]_+ \right] \\
 &\quad + \frac{\alpha_s(\mu_F)}{2\pi} C_A \left[\int_x^1 \frac{dz}{z} f_g^{\overline{\text{MS}}} \left(\frac{x}{z}, \mu_F \right) \left[z^2 + (1-z)^2 \right] \log \frac{(1-z)^2}{z} + 2z(1-z) \right] \\
 f_g^{\text{Krk}}(x, \mu_F) &= f_g^{\overline{\text{MS}}} (x, \mu_F) \\
 &\quad - \frac{\alpha_s(\mu_F)}{2\pi} C_A \left[\frac{\pi^2}{3} + \frac{341}{72} - \frac{59}{36} \frac{N_f T_R}{C_A} \right] f_g^{\overline{\text{MS}}} (x, \mu_F) \\
 &\quad + \frac{\alpha_s(\mu_F)}{2\pi} C_A \left[\int_x^1 \frac{dz}{z} f_g^{\overline{\text{MS}}} \left(\frac{x}{z}, \mu_F \right) \left[4 \left[\frac{\log(1-z)}{1-z} \right]_+ - 2 \frac{\log z}{1-z} \right. \right. \\
 &\quad \quad \quad \left. \left. + 2 \left(\frac{1}{z} - 2 + z(1-z) \right) \ln \frac{(1-z)^2}{z} \right] \right] \\
 &\quad + \frac{\alpha_s(\mu_F)}{2\pi} C_F \sum_{q_f, \bar{q}_f} \left[\int_x^1 \frac{dz}{z} f_q^{\overline{\text{MS}}} \left(\frac{x}{z}, \mu_F \right) \left[\frac{1+(1-z)^2}{z} \log \frac{(1-z)^2}{z} + z \right] \right]
 \end{aligned}$$

PDFs in MC scheme

Applied to LHAPDF6 grids:

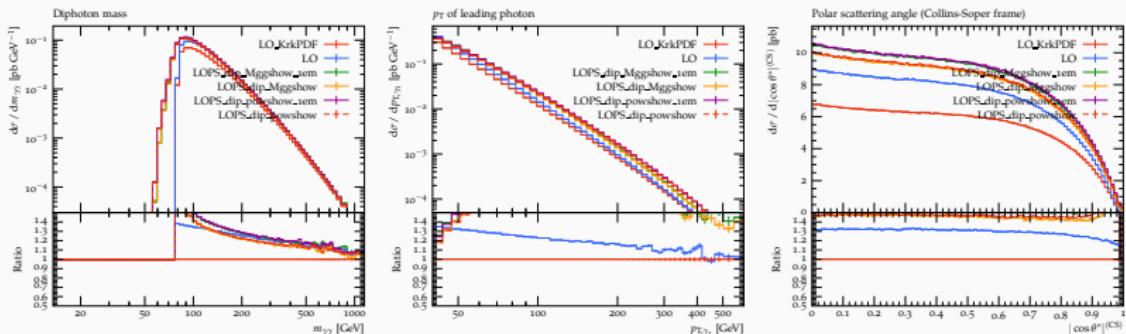


Do we reproduce the Herwig (Matchbox) automated P and K operators?



Normalisation

What is the numerical impact of the Krk scheme?



Validation

To verify the real weight, we must *unweight* the Sudakov:

- numerical integration of dipole kernels considered in shower algorithm;
- over the same splitting phase-space/kinematic region used in the shower algorithm;
- with the same scales, PDF arguments, α_s etc

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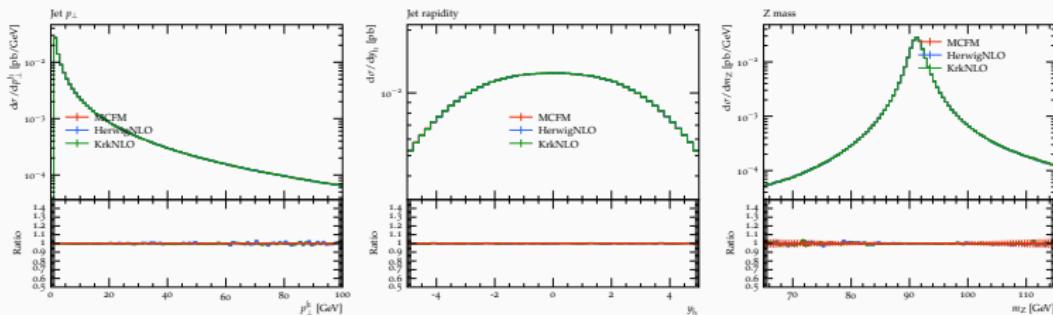
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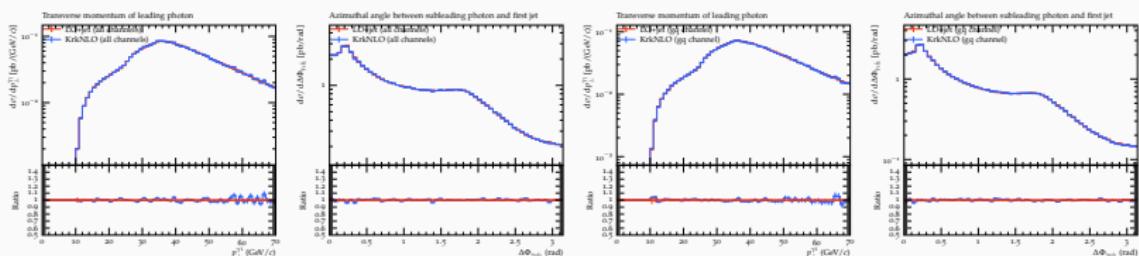
This is non-trivial!

Does it work?

Drell-Yan:

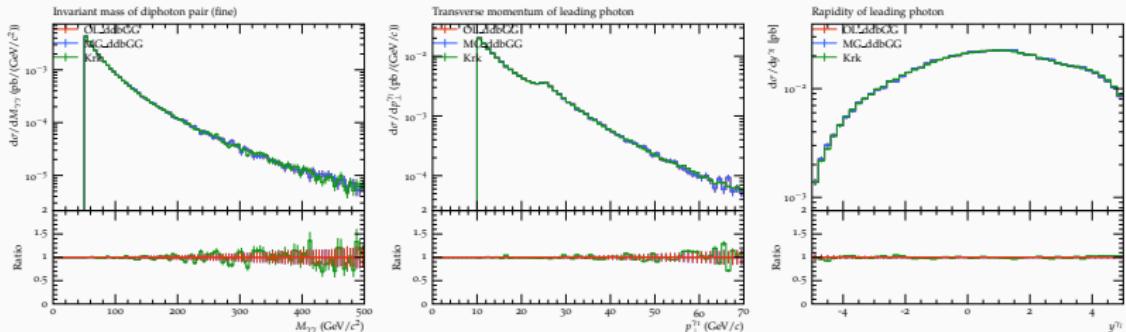


Diphoton:



What about the virtuals?

Diphoton:



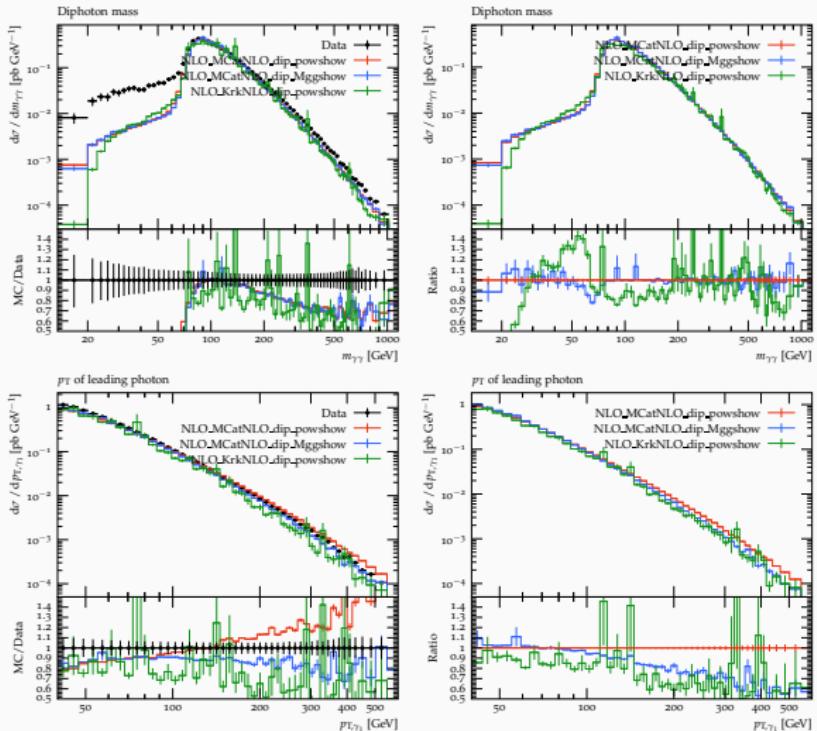
- new processes
- PDF factorisation scheme⁶
- automation!

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...and physics results (very soon)

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First results:⁷⁷ very preliminary! statistics improving as we speak

Thank you!