

# Optimal probe of New Physics at future $e^+e^-$ Colliders

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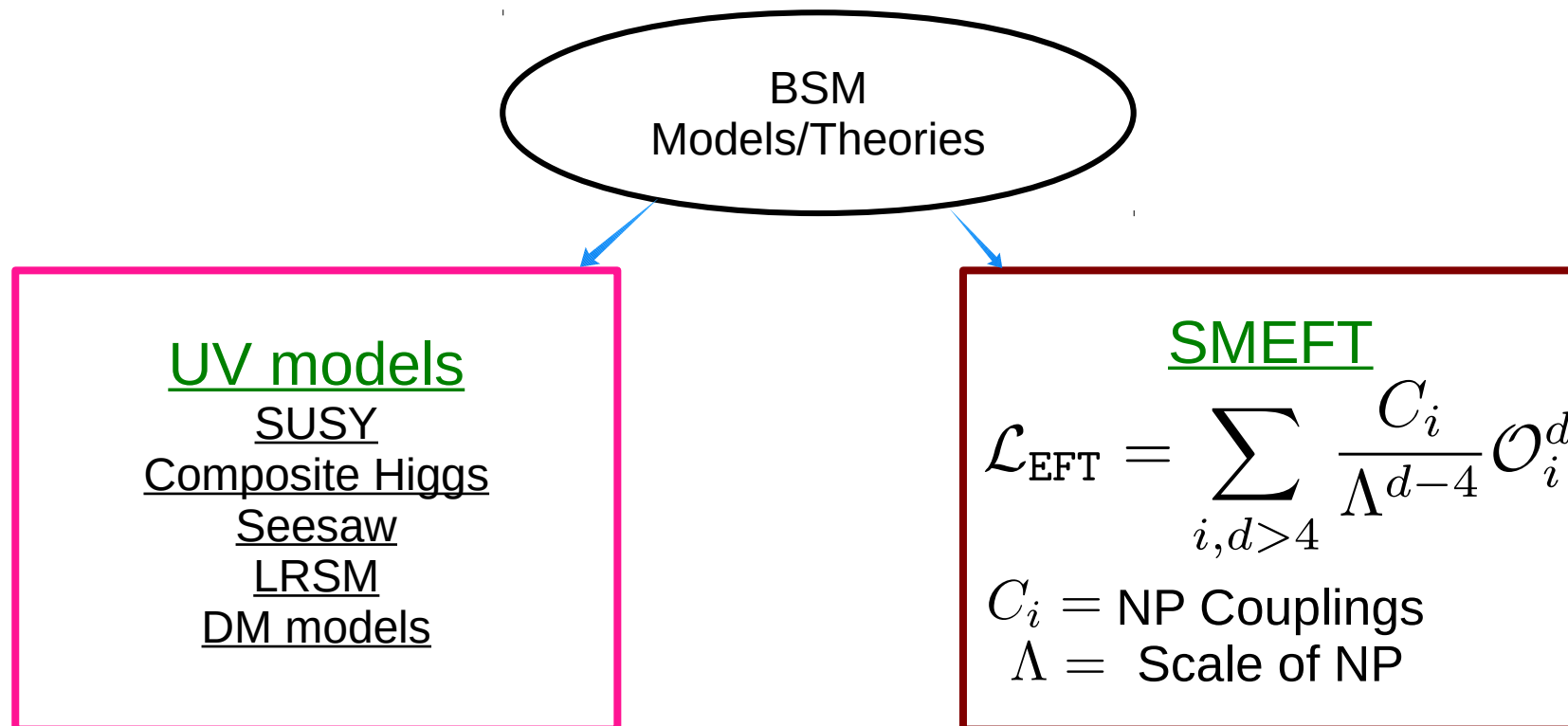
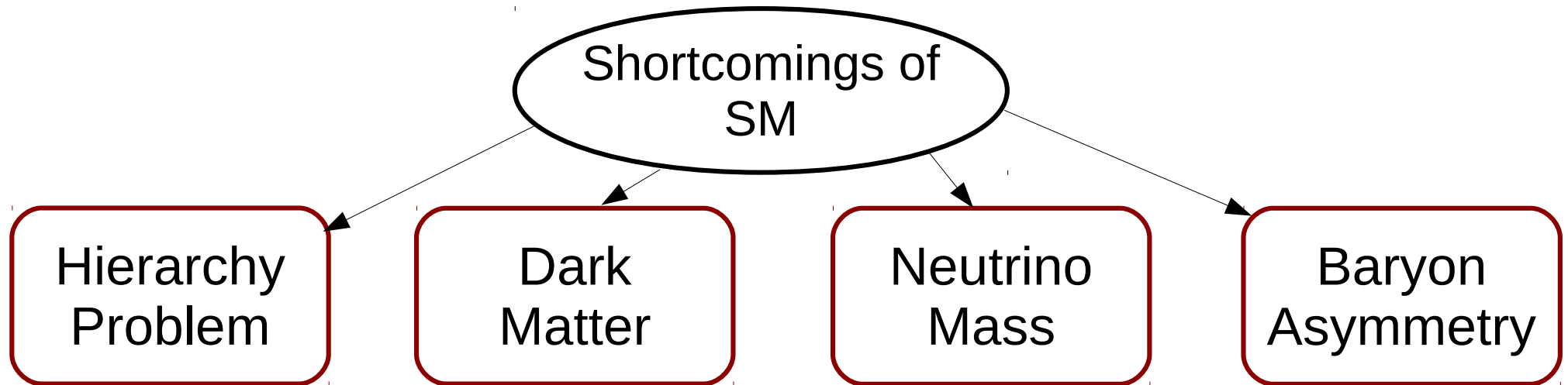
Date : 18<sup>th</sup> September, 2023, Ustroń, Poland

# Outline:

- \* Chapter-I: Optimal Observable Technique (OOT)
- \* Chapter-II: Example of OOT in NP dominance
- \* Chapter-III: Example of OOT in SM dominance
- \* Chapter-IV: OOT in presence of SM background
- \* Summary

# Chapter-I (Optimal Observable Technique)

# Beyond Standard Model:

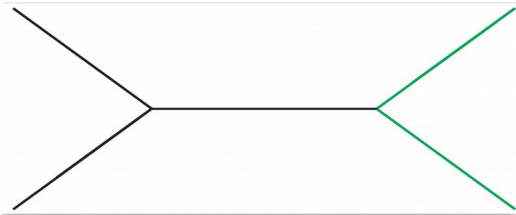


# BSM search at colliders:

Build a Large Collider

Increase CM energy

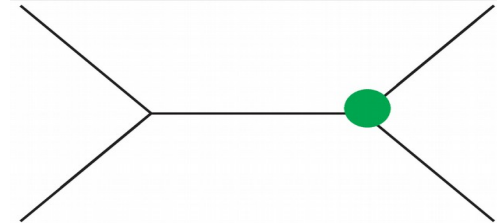
Discover new particles!



(BSM dominated scenario)

Measure the deviation  
via SM processes

Discover new physics  
indirectly!



(SM dominated scenario)

**Statistical analysis** plays a crucial role to elucidate both scenarios

## $\chi^2$ analysis:

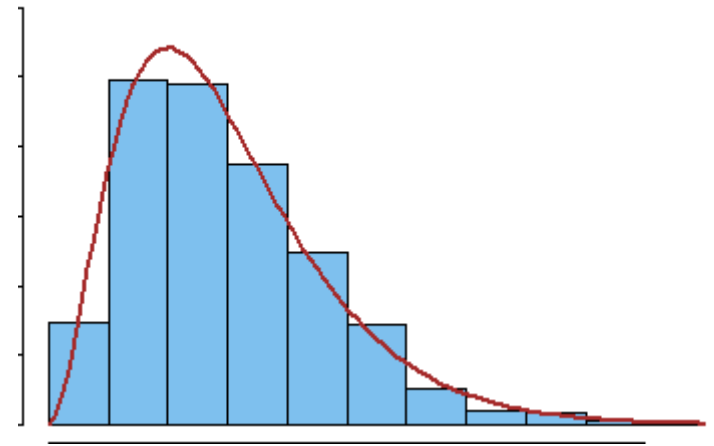
Definition:  $\chi^2 = \frac{(\mathcal{O}^{\text{data}} - \mathcal{O}(a, b)^{\text{theory}})^2}{\sigma^2}$

$\Delta\chi^2 = \chi^2 - \chi_{\min}^2 = n$ , provides  $\sqrt{n}\sigma$  deviation.

## Binned analysis:

$$\chi^2 = \sum_i \frac{\left(N_i^{\text{data}} - N_i^{\text{theory}}\right)^2}{N_i^{\text{data}}}$$

$$\sigma = \sqrt{N_i^{\text{data}}}$$



# Optimal Observable Technique (OOT):

Case-I : BSM dominates over SM

(Phys. Rev. Lett. 77 (1996) 5172)

Observable:  $\mathcal{O}_{BSM} = \frac{d\sigma}{d\Omega} = \sum_i g_i f_i(\phi)$

Contains the  
information of **NP**  
parameters

**Linearly  
independent  
functions**

Covariance Matrix :

$$V_{ij} = \frac{M_{ij}^{-1}}{\mathcal{L}_{int}};$$

Integrated  
Luminosity of  
the Collider

Optimal in a  
sense that  
 $\frac{\partial V_{ij}}{\partial g_i} = 0$

$$M_{ij} = \int \frac{f_i(\phi) f_j(\phi)}{\mathcal{O}_{BSM}} d\phi$$

# Optimal Observable Technique (OOT):

Case-II : SM dominates over BSM

(Z. Phys. C 62 (1994))

Observable :  $\mathcal{O} = \mathcal{O}_{SM} + \sum_i g_i f_i$

$$M_{ij} = \int \frac{f_i(\phi) f_j(\phi)}{\mathcal{O}_{SM}} d\phi$$

Case-III : Non interfering SM background

Observable :  $\mathcal{O}_{tot} = \mathcal{O}_{sig} + \mathcal{O}_{bkg} = \sum_i g_i f_i$

$$M_{ij} = \int \frac{f_i(\phi) f_j(\phi)}{\mathcal{O}_{tot}} d\phi$$

$$\chi^2 = \sum_i (g_i - g_i^0)(g_j - g_j^0) V_{ij}^{-1}; \quad g_i^0 = \text{'seed values'}$$

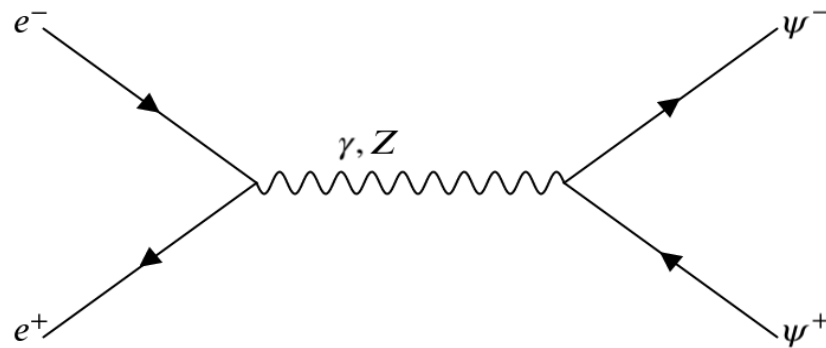
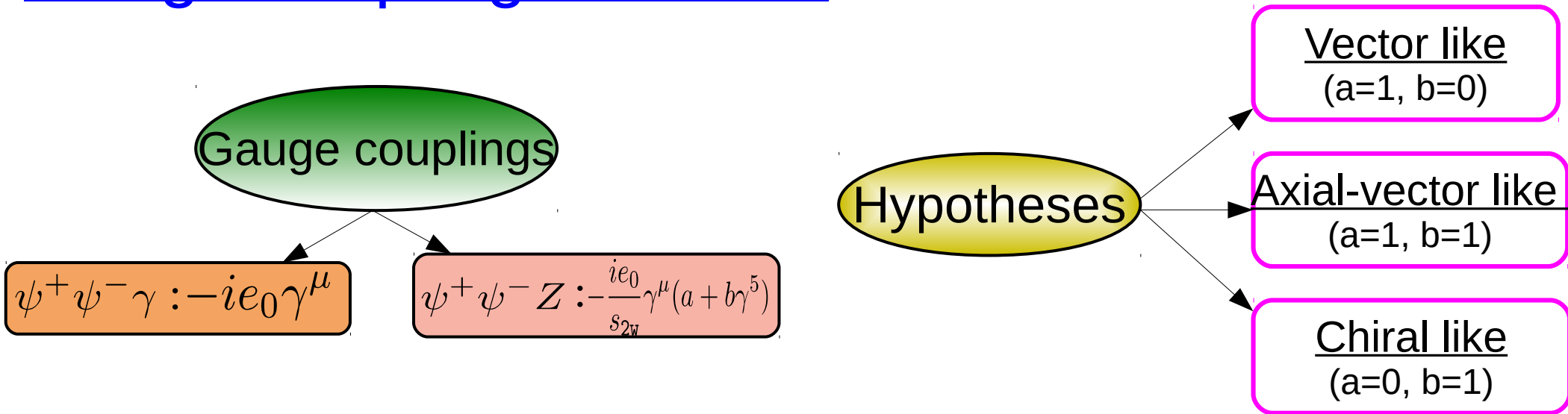


## Chapter-II (Example of NP dominance)

based on

Probing heavy charged fermions at  $e^+e^-$  collider using Optimal  
Observable Technique, S. Bhattacharya, **S. Jahedi**, J. Wudka  
(JHEP 05 (2022) 009)

# Gauge couplings with $\psi^\pm$ :



## Helicity Amplitudes:

$$\mathcal{M}(\lambda_{e^-}, -\lambda_{e^-}, \lambda_\psi, -\lambda_\psi) = -e_0^2 (\lambda_{e^-} \lambda_\psi + \cos \theta) [1 + \xi (a + b\lambda_\psi \beta_\psi)] ; \quad \xi = \xi_1 + \lambda_{e^-} \xi_2$$

$$\mathcal{M}(\lambda_{e^-}, -\lambda_{e^-}, \lambda_\psi, \lambda_\psi) = -e_0^2 \left( \frac{2m_{\psi^\pm} \lambda_\psi \sin \theta}{\sqrt{s}} \right) (1 + \xi a)$$

# Cross-section:

$$\frac{d\sigma(P_{e+}, P_{e-})}{d\Omega} = \sum_{\lambda_e^+ = \pm 1} \sum_{\lambda_e^- = \pm 1} \frac{(1 + \lambda_{e-} P_{e-})(1 + \lambda_{e+} P_{e+})}{4} \left( \frac{d\sigma}{d\Omega} \right)_{\lambda_{e-}, \lambda_{e+}} = \sum_i g_i f_i$$

$$g_1 = \alpha_0^2 \frac{1 - P_{e-} P_{e+}}{2} \left[ 1 + 2\xi_1 a + (\xi_1^2 + \xi_2^2) \left( a^2 + \frac{\beta_\psi^2}{2 - \beta_\psi^2} b^2 \right) - 2P_{\text{eff}} \left\{ \xi_2 a + \xi_1 \xi_2 a^2 + \frac{\beta_\psi^2}{2 - \beta_\psi^2} \xi_1 \xi_2 b^2 \right\} \right]$$

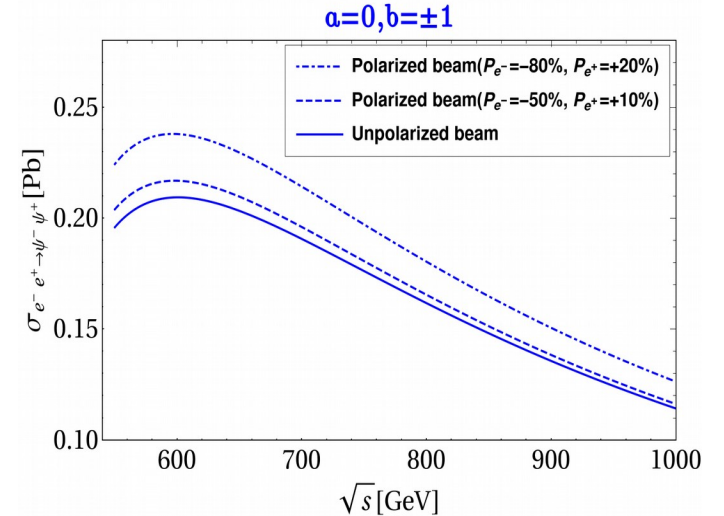
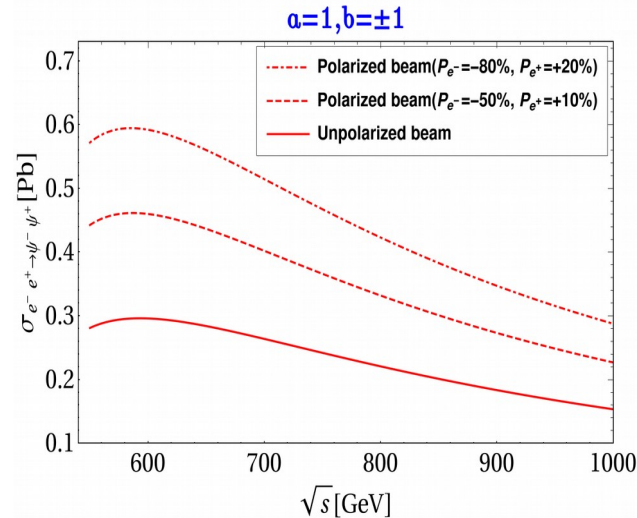
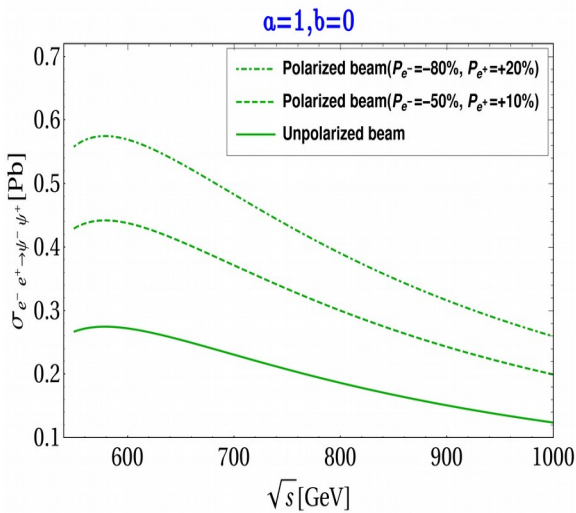
$$g_2 = \alpha_0^2 \frac{1 - P_{e-} P_{e+}}{2} \left[ 2\xi_2 b + 4\xi_1 \xi_2 ab - P_{\text{eff}} \left\{ 2\xi_1 b + (\xi_1^2 + \xi_2^2) ab \right\} \right]$$

$$g_3 = \alpha_0^2 \frac{1 - P_{e-} P_{e+}}{2} \left[ 1 + 2\xi_1 a + (\xi_1^2 + \xi_2^2)(a^2 + b^2) - 2P_{\text{eff}} \left\{ \xi_2 a + \xi_1 \xi_2 (a^2 + b^2) \right\} \right]$$

$\xi_{1,2}$  = SM couplings

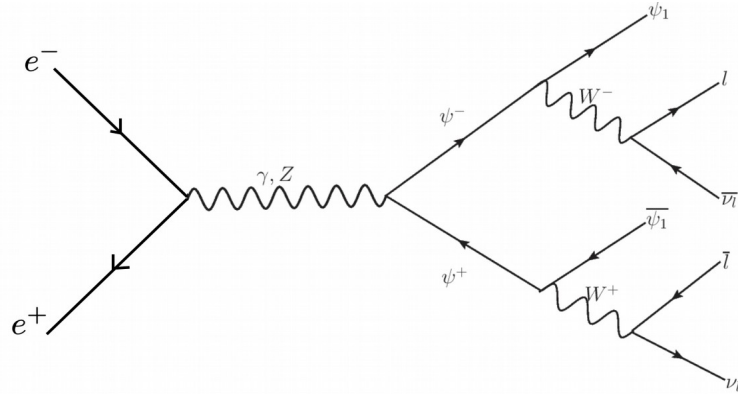
$$\{f_1, f_2, f_3\} = \frac{\beta_\psi}{2s} \{ (2 - \beta_\psi^2), \beta_\psi \cos \theta, \beta_\psi^2 \cos^2 \theta \} \quad \beta_\psi = \sqrt{1 - \frac{4m_\psi^2}{s}}$$

$$\begin{aligned} P_{e-} &= -80\% \\ P_{e+} &= +20\% \end{aligned}$$

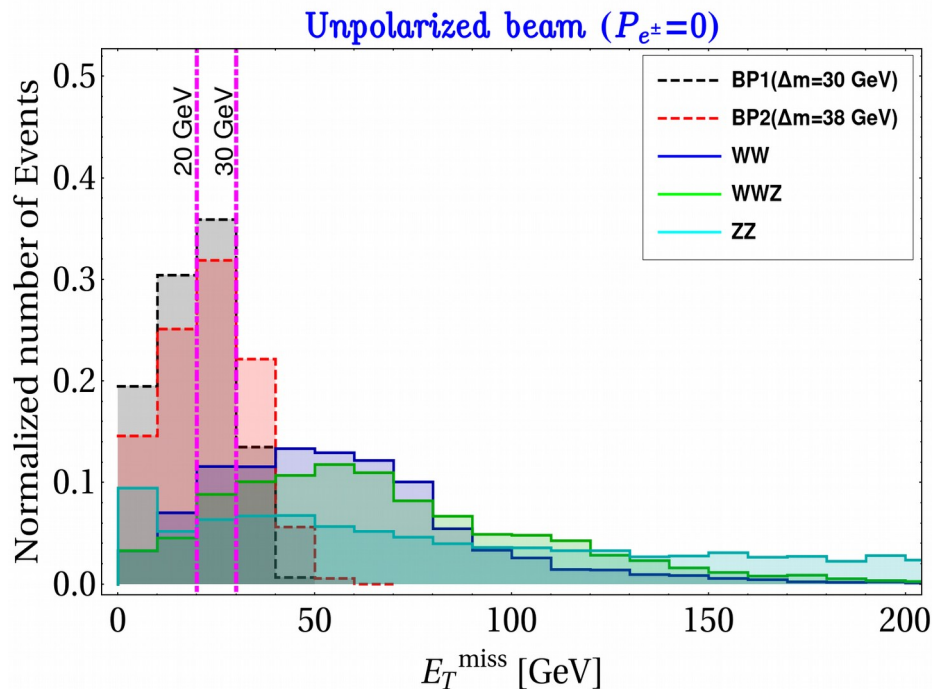


# Event analysis:

Singlet-doublet model:  $\psi^\pm$ ,  $\psi_1$  and  $\psi_2$



Missing transverse energy distribution



Final state signal:

$l^+ l^- + \text{missing energy}$

Cut flows:

$\mathcal{C}_1$ : Events with 2 leptons

$\mathcal{C}_2$ :  $m_{\ell\ell} < |m_Z \pm 15| \text{ GeV}$

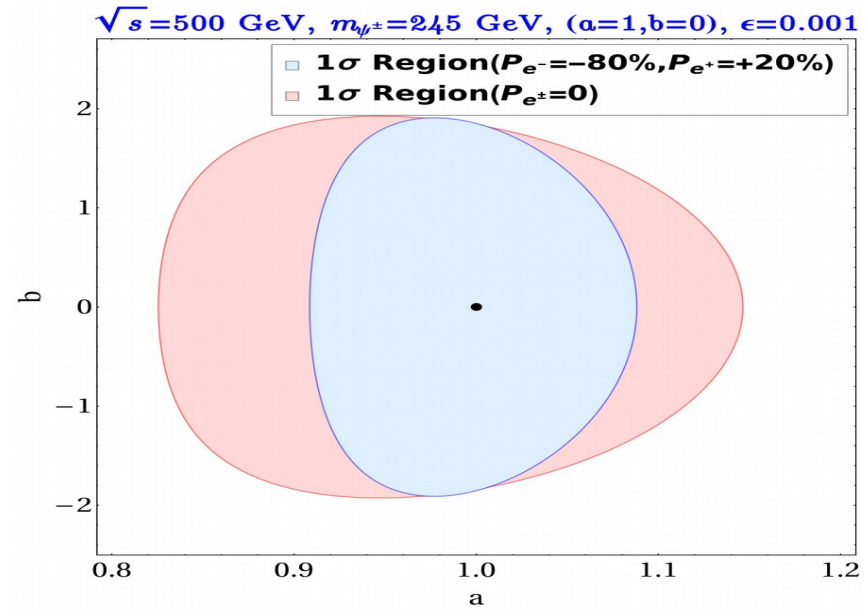
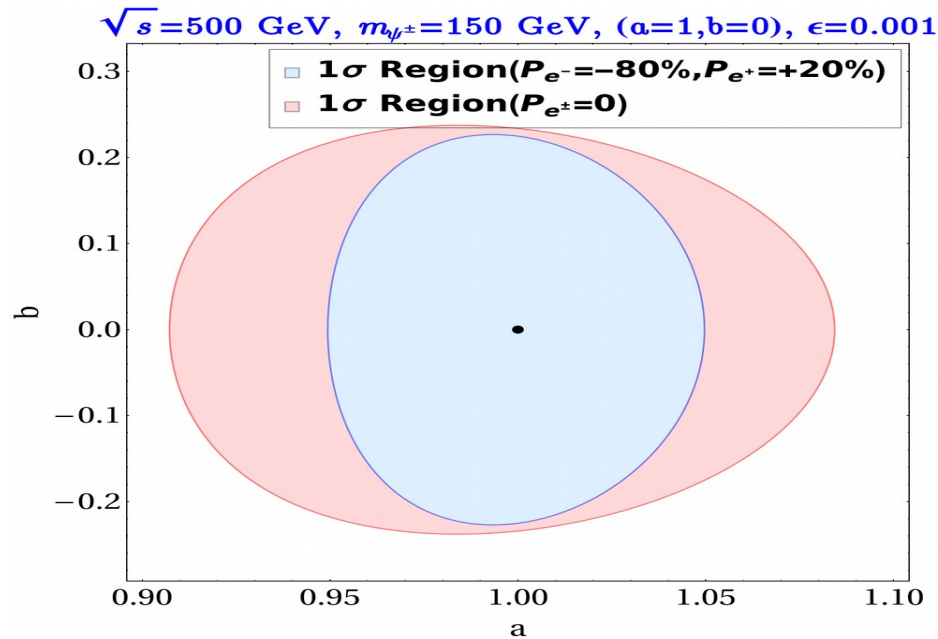
$\mathcal{C}_3$ :  $E_T^{\text{miss}} < 30 \text{ GeV}$

$$\epsilon = \frac{\sigma^{\text{sig}}}{\sigma^{\text{prod}}} = 0.006$$

Our choice

$$\epsilon = 0.001$$

# 1 $\sigma$ sigma regions:



- ◆ For lower charged fermion mass, the NP couplings are more precise.
- ◆ Judicious choice of beam polarization provides more stringent parameter space.

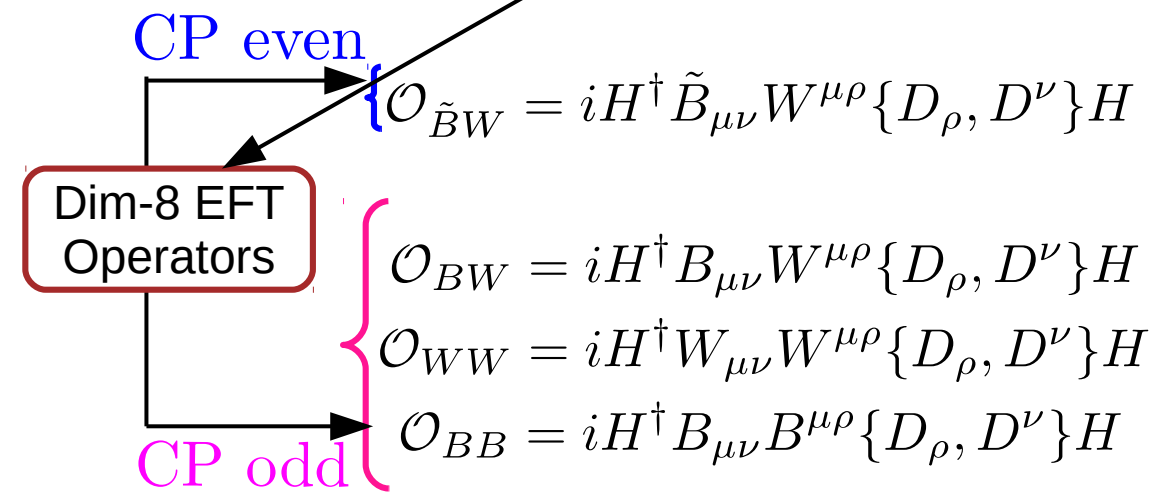
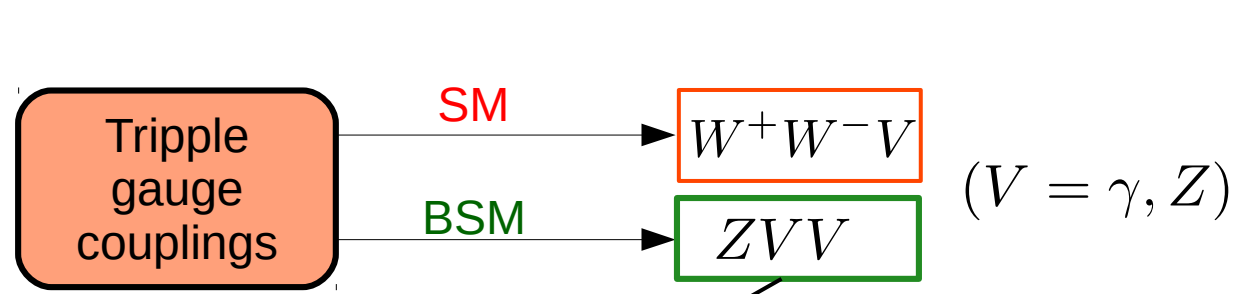
## Chapter-III (Example of SM dominance)

based on

Probing anomalous  $ZZ\gamma$  and  $Z\gamma\gamma$  couplings at the  $e^+e^-$  collider using  
Optimal Observable Technique, **S. Jahedi**, J. Lahiri  
(JHEP 04 (2023) 085)

Optimal determination of New Physics couplings: A comparative study,  
S. Bhattacharya, **S. Jahedi**, J. Wudka  
(arXiv:2301.07721, In communication with JHEP)

# Neutral triple gauge couplings:

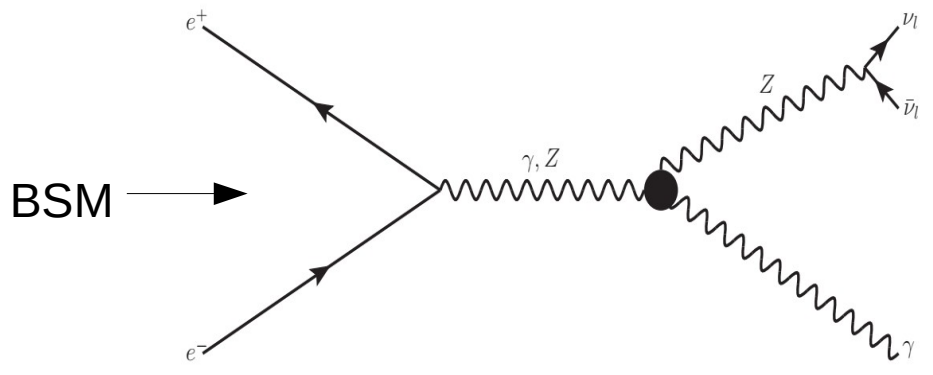
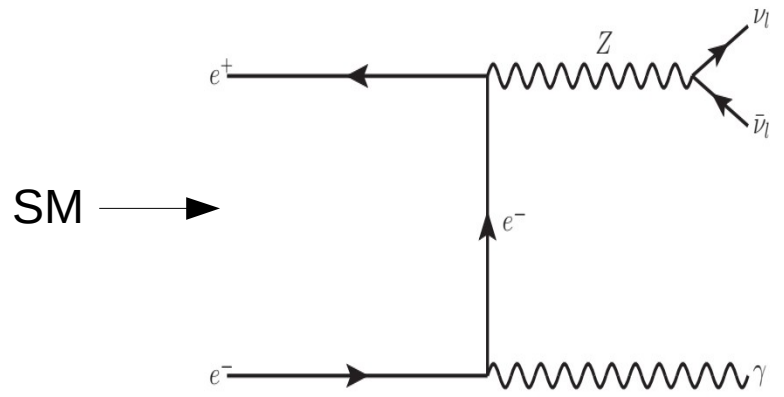


ATLAS constrains

$$pp \rightarrow Z(\nu\bar{\nu})\gamma$$

95% C.L.

$$\begin{aligned} -1.1 \text{ TeV}^{-4} &< \frac{C_{\tilde{B}W}}{\Lambda^4} < 1.1 \text{ TeV}^{-4} \\ -0.65 \text{ TeV}^{-4} &< \frac{C_{BW}}{\Lambda^4} < 0.64 \text{ TeV}^{-4} \\ -2.3 \text{ TeV}^{-4} &< \frac{C_{WW}}{\Lambda^4} < 2.3 \text{ TeV}^{-4} \\ -0.24 \text{ TeV}^{-4} &< \frac{C_{BB}}{\Lambda^4} < 0.24 \text{ TeV}^{-4} \end{aligned}$$



# Event analysis:

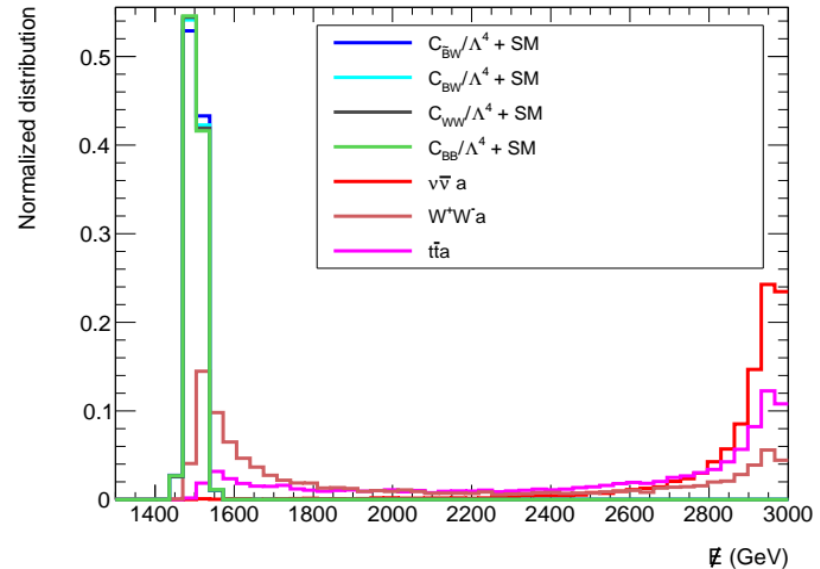
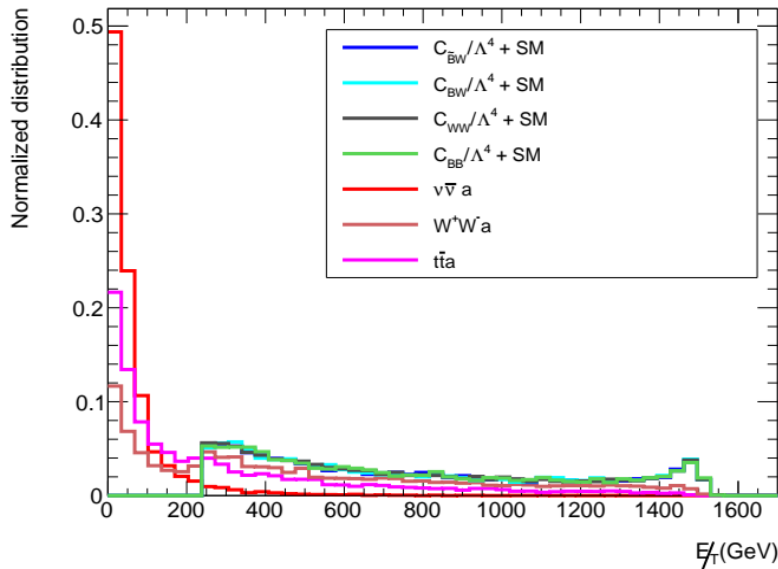
Final state signal:

Mono-photon+  
missing energy

Collider  
Variables

Missing energy  $= \cancel{E} = \sqrt{s} - \sum_{\ell, j, \gamma} E$

Missing transverse energy  $= \cancel{E}_T = -\sqrt{(\sum_{\ell, j} p_x)^2 + (\sum_{\ell, j} p_y)^2}$



Cut-employed:  $\rightarrow$

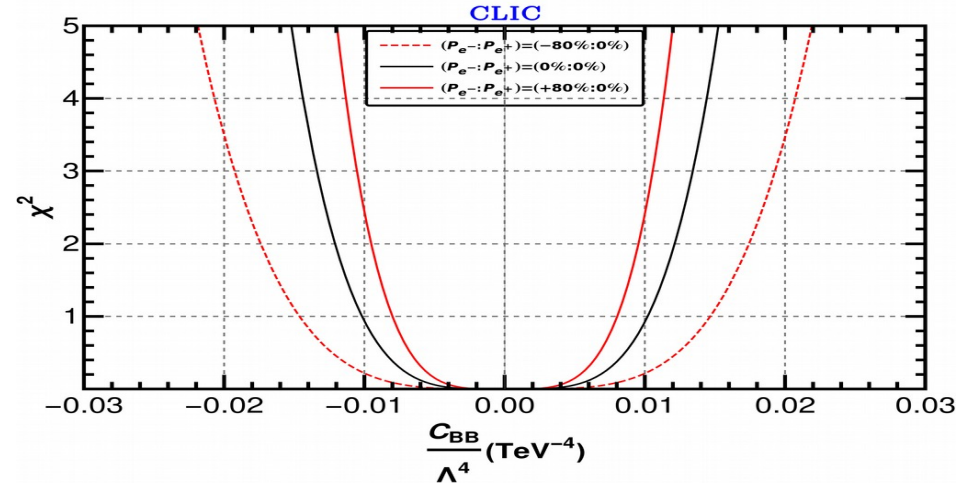
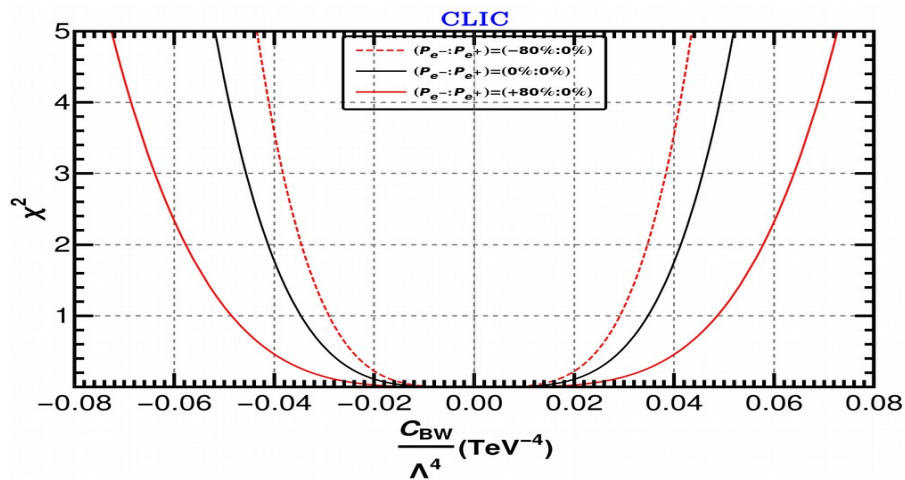
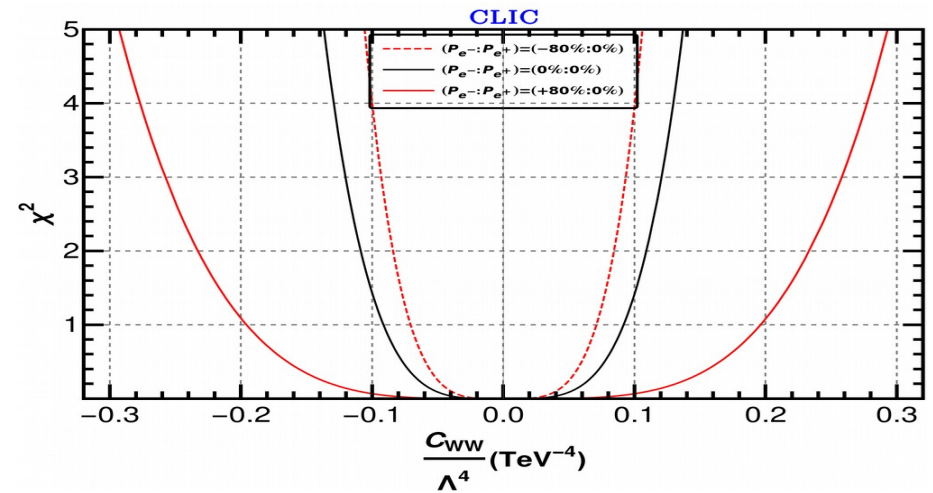
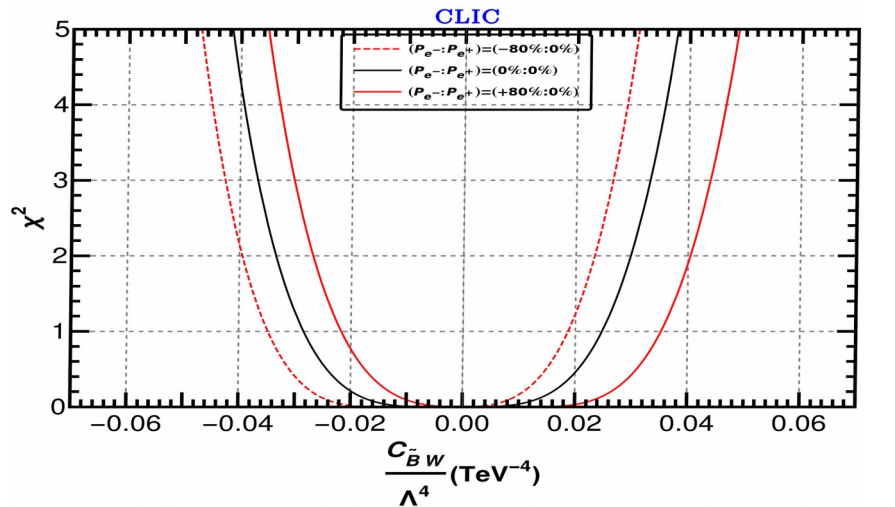
- $C_1$ : Events with one photon
- $C_2$ :  $\cancel{E}_T > 500$  GeV
- $C_3$ :  $1440 \text{ GeV} < \cancel{E} < 1560 \text{ GeV}$

$$\epsilon = \frac{\sigma^{sig}}{\sigma^{prod}} \sim 0.1$$



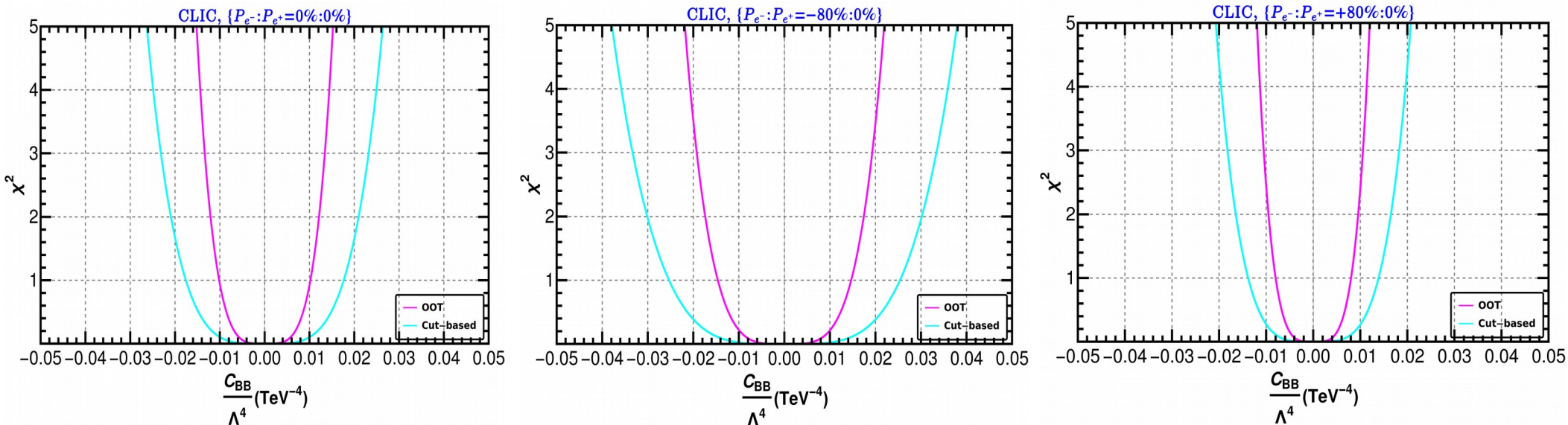
# $\chi^2$ variation:

$$\sqrt{S} = 3 \text{ TeV}, \mathcal{L}_{\text{int}} = 1000 \text{ fb}^{-1}$$



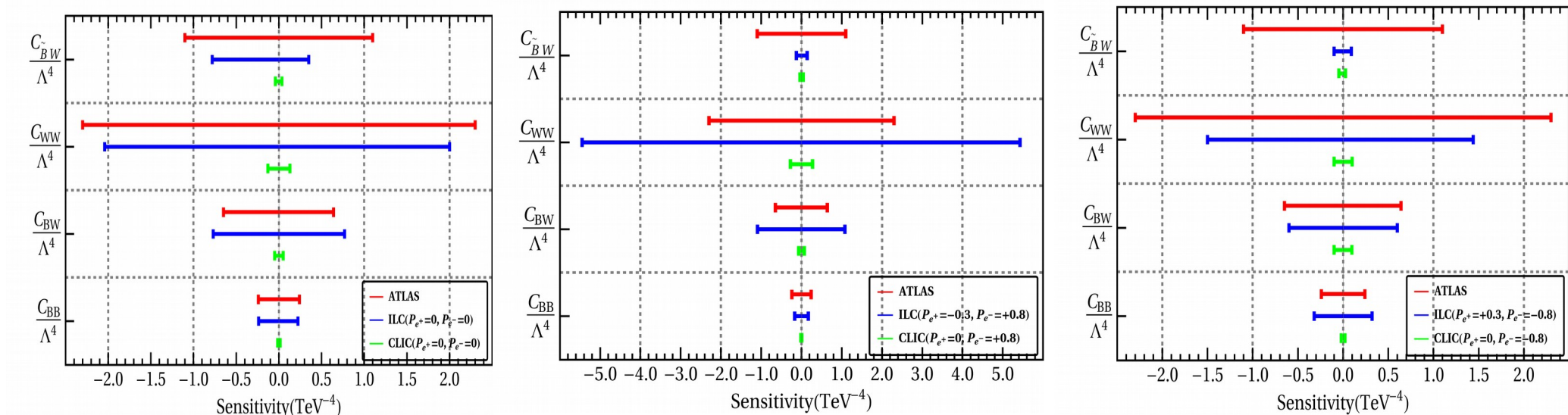
- $\left(\frac{C_{\tilde{B}W}}{\Lambda^4}\right), \left(\frac{C_{BW}}{\Lambda^4}\right), \left(\frac{C_{WW}}{\Lambda^4}\right) \longrightarrow \{P_{e^-} : P_{e^+} = -80\% : 0\% \}$
- $\left(\frac{C_{BB}}{\Lambda^4}\right) \longrightarrow \{P_{e^-} : P_{e^+} = +80\% : 0\% \}$

# OOT vs binned analysis:



- OOT limits are more stringent than binned limits by a factor of 1.7.

## ATLAS VS ILC VS CLIC:



- CLIC outperforms ILC and ATLAS by a factor of 10.

# Top quark pair production:

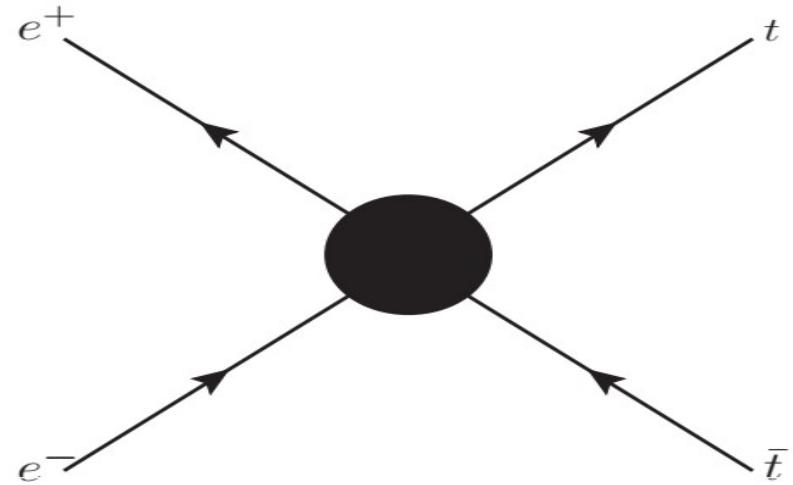
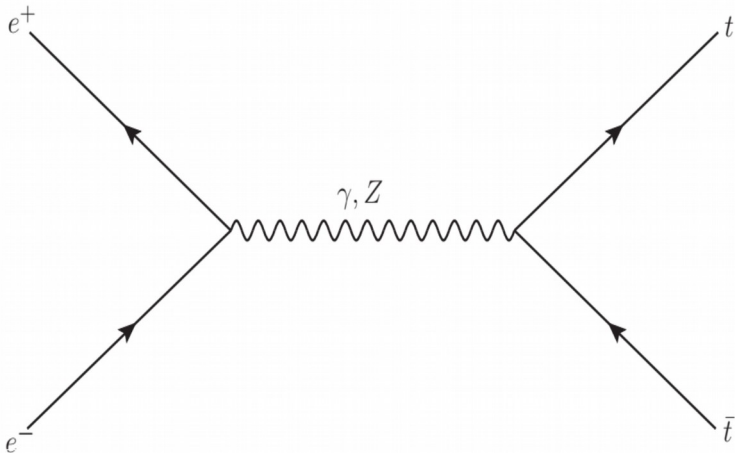
$$e^+ e^- \rightarrow t \bar{t}$$

## SM Contribution

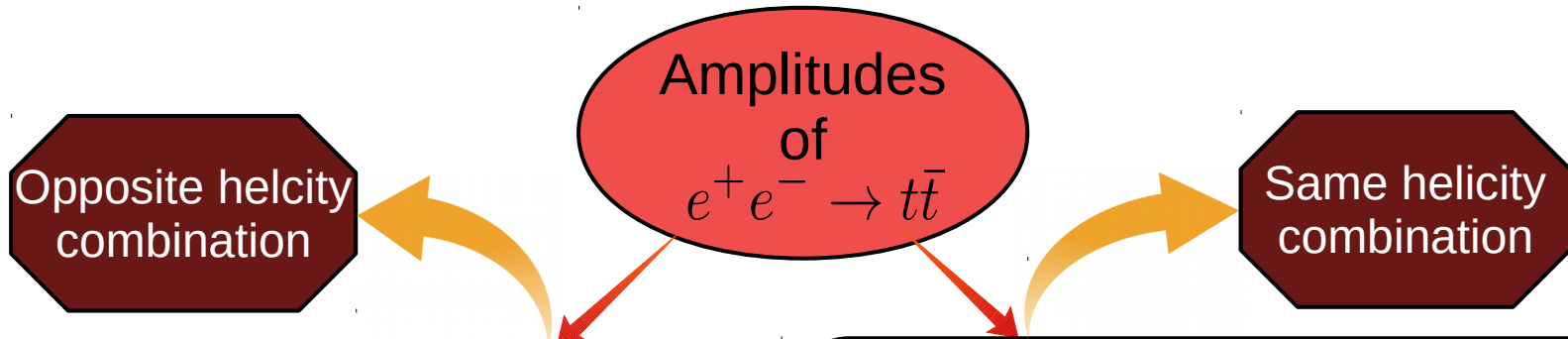
- $e\bar{e}\gamma = -ie_0\gamma^\mu$
- $t\bar{t}Z = -\frac{ig}{c_w}(c_v - c_a\gamma^5)\gamma^\mu$

## BSM contribution

- $\mathcal{O}_{lelq}^{(1)} = \frac{c_1}{\Lambda^2} (\bar{l}_p^j e_r) \epsilon_{jk} (\bar{q}_s^k u_t) + h.c.$
- $\mathcal{O}_{lelq}^{(3)} = \frac{c_2}{\Lambda^2} (\bar{l}_p^j \sigma^{\mu\nu} e_r) \epsilon_{jk} (\bar{q}_s^k \sigma_{\mu\nu} u_t) + h.c.$



# Helicity amplitudes and beam polarization:



## SM contribution

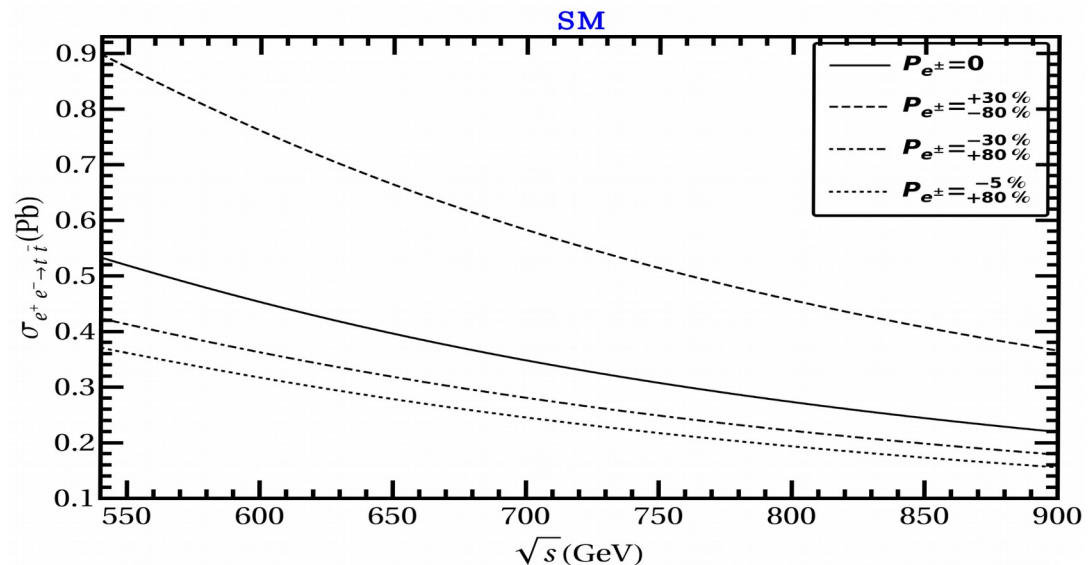
$$\begin{aligned} \blacktriangleright M(\lambda, -\lambda; \lambda', -\lambda') &= e_0^2 \left( \frac{\lambda\lambda' + \cos\theta}{\beta_z^2} \right) \left[ \frac{2}{3}\beta_z^2 - \left( \frac{4^2 - 1 + \lambda}{4^2} \right) \left( 1 - \frac{8^2}{3} - \beta_t\lambda' \right) \right] \\ \blacktriangleright M(\lambda, -\lambda; \lambda', \lambda') &= \frac{2e_0^2 m_t \lambda' \sin\theta}{\sqrt{s}} \left[ \frac{2}{3} - \left( \frac{4^2 - 1 + \lambda}{4^2 \beta_z^2} \right) \left( 1 - \frac{8^2}{3} \right) \right] \end{aligned}$$

## BSM contribution

$$\begin{aligned} \blacktriangleright M(\lambda, \lambda; \lambda', -\lambda') &= \frac{4c_2 m_t \sqrt{s}}{\Lambda^2} \lambda \sin\theta \\ \blacktriangleright M(\lambda, \lambda; \lambda', \lambda') &= \frac{s}{2\Lambda^2} (\lambda\lambda'\beta_t - 1) (c_1 + 4c_2\lambda\lambda' \cos\theta) \end{aligned}$$

Choice of  
beam polarization

$$\begin{aligned} P_{e^-} &= +80\% \\ P_{e^+} &= -5\% \end{aligned}$$



# Differential cross-section:

$$\mathcal{O} = \frac{d\sigma_{\text{tot}}}{d\Omega} = \frac{d\sigma_{\text{SM}}}{d\Omega} + \sum_i g_i f_i$$

$$P_{\text{eff}} = \frac{P_{e^-} - P_{e^+}}{1 - P_{e^-} P_{e^+}}$$

$$\frac{d\sigma_{\text{SM}}}{d\Omega}$$

$$\begin{aligned} & \frac{\alpha_0^2(1 - P_{e^-} P_{e^+})}{3s} \left\{ 1 + \mathcal{C}(\xi_1 - P_{\text{eff}} \xi_2) + 4(\xi_1^2 - 2P_{\text{eff}} \xi_1 \xi_2 + \xi_2^2) \left( \mathcal{C}^2 + \frac{\beta_t^2}{2 - \beta_t^2} \right) \right. \\ & - \left[ \xi_2(1 + \mathcal{C} \xi_1) - 4P_{\text{eff}} (4\xi_1 - (2\xi_1^2 - \xi_2^2)\mathcal{C}) \right] \beta_t \cos \theta + \\ & \left. \left[ 1 + \mathcal{C}(\xi_1 - P_{\text{eff}} \xi_2) + \frac{(2\mathcal{C} + 1)}{4}(\xi_1^2 - 2P_{\text{eff}} \xi_1 \xi_2 + \xi_2^2) + \frac{\mathcal{C}(2 - \mathcal{C})}{2} P_{\text{eff}} \xi_1 \xi_2 \right] \beta_t^2 \cos^2 \theta \right\} \end{aligned}$$

$$\xi_1 = \frac{1}{2s_{2w}^2 \beta_z^2}$$

$$\xi_2 = \frac{4s_w^2 - 1}{2s_{2w}^2 \beta^2}$$

$$g'_i s$$

$$\begin{aligned} g_1 &= (1 + P_{e^-} P_{e^+}) \left( c_1^2 + 16 \frac{1 - \beta_t^2}{1 + \beta_t^2} c_2^2 \right) \\ g_2 &= -(1 + P_{e^-} P_{e^+}) c_1 c_2 \\ g_3 &= (1 + P_{e^-} P_{e^+}) c_2^2 \end{aligned}$$

$$\mathcal{C} = 3 - 12s_w^2$$

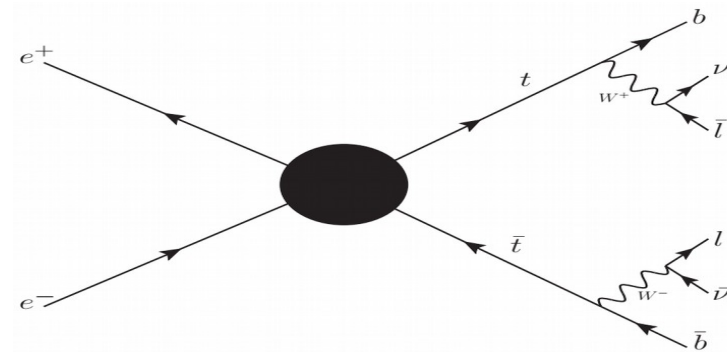
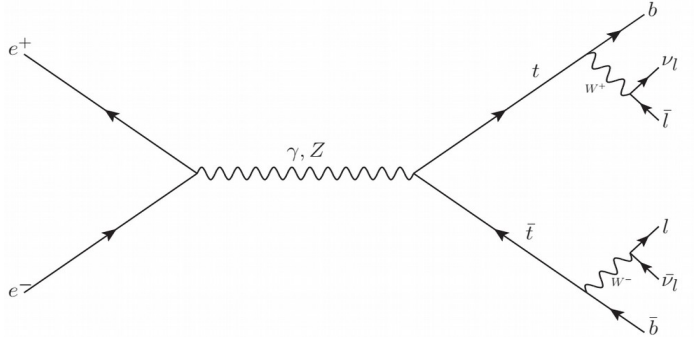
$$f'_i s$$

$$\begin{aligned} f_1 &= \frac{3\beta_t s}{256\pi^2 \Lambda^4} (1 + \beta_t^2) \\ f_2 &= \frac{3\beta_t^2 s}{16\pi^2 \Lambda^4} \cos \theta \\ f_3 &= \frac{3\beta_t^3 s}{8\pi^2 \Lambda^4} \cos^2 \theta \end{aligned}$$

Linearly Independent

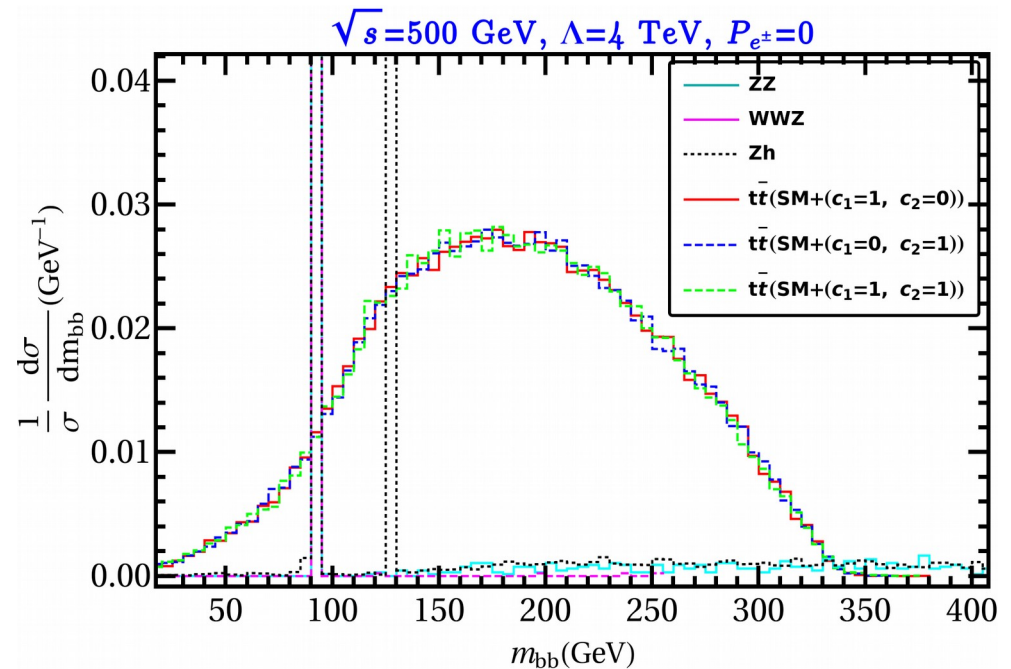
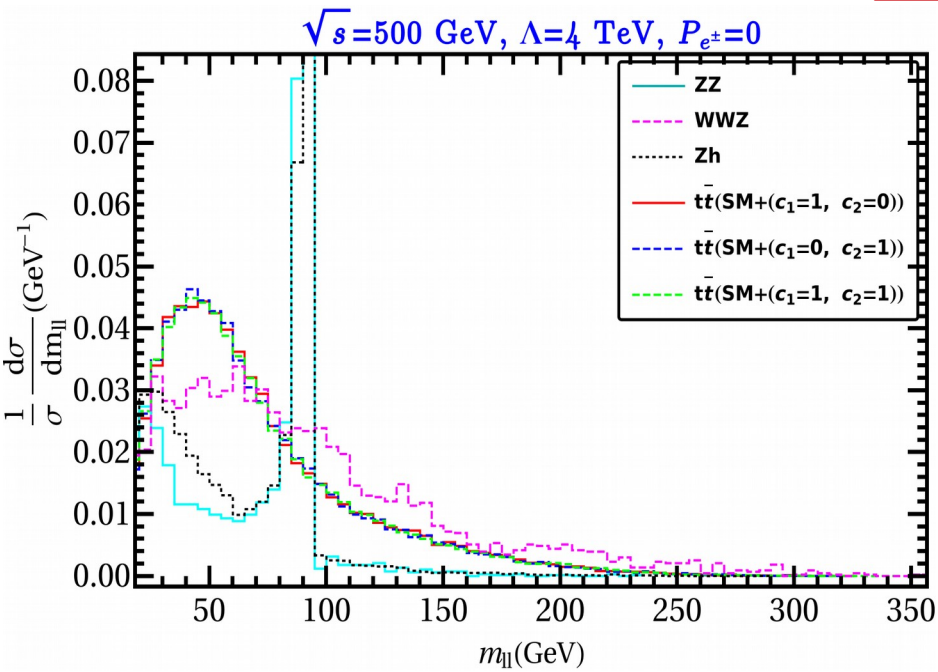


# Event analysis:



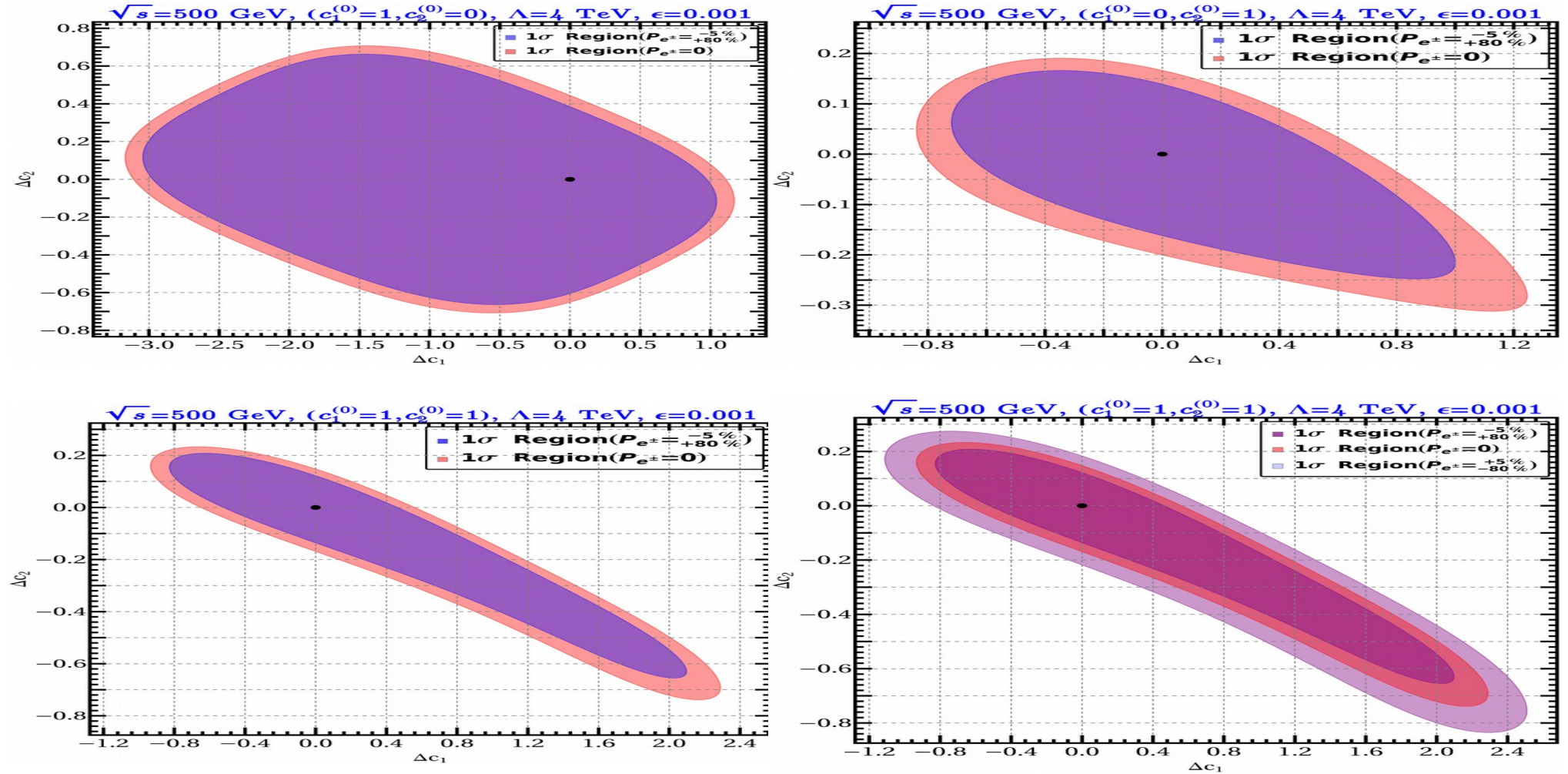
Final state signal

$2\ell 2b + \text{missing energy}$



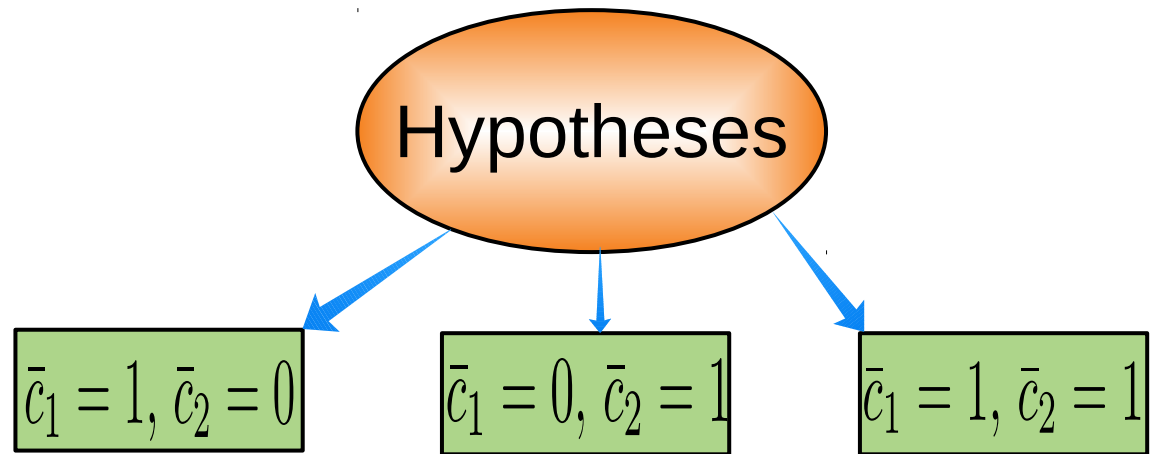
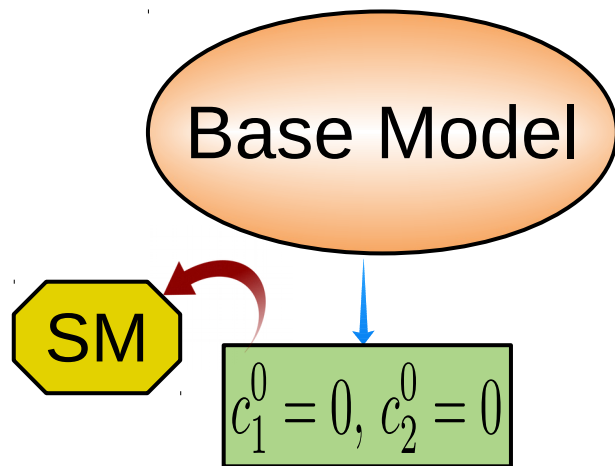
Efficiency factor ( $\epsilon$ ):  $\sim 0.001$

# 1 $\sigma$ regions:

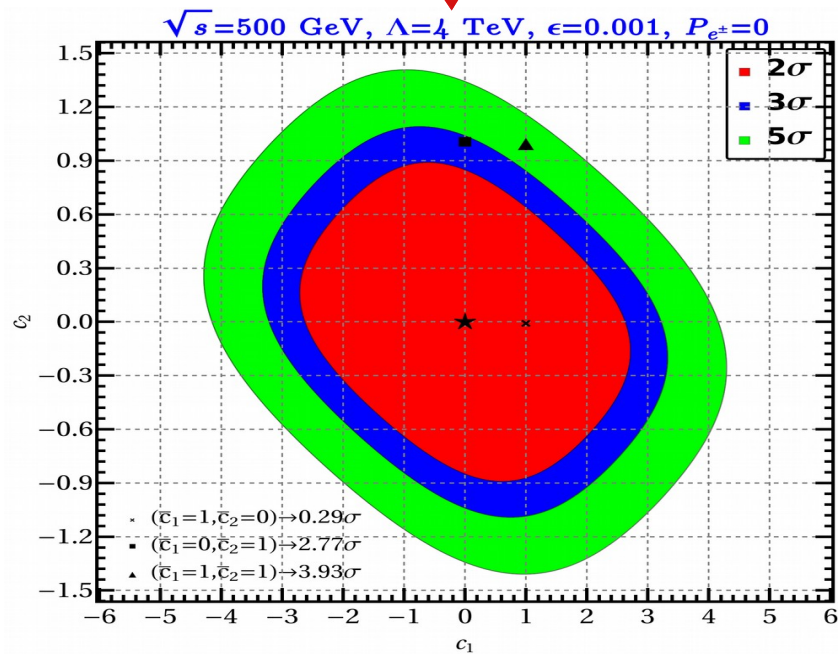


- Tensor coupling is more precise than scalar couplings.
- Beam polarizations help to estimate the coupling more precisely.

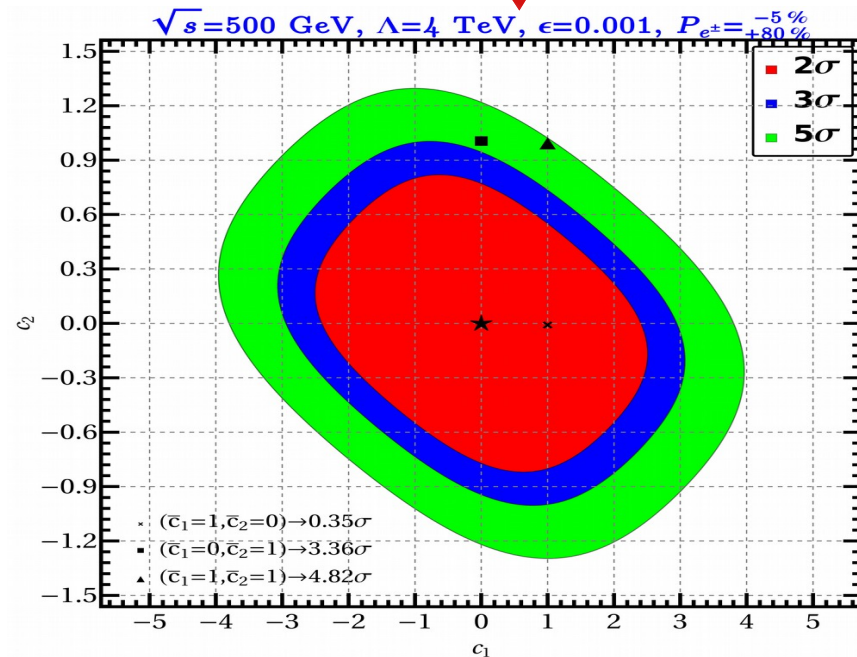
# Distinction of hypotheses:



Unpolarized beam



Polarized beam





# UV Completion of EFT operator:

- We have discussed the probe of EFT operators for  $O(1)$  couplings with NP scale  $\sim 4$  TeV with CM energy 500 GeV.
- If CM energy is **greater** than the NP scale then we can probe the NP directly.
- Scalar operator can be generated by integrating out the heavy scalars in 2 Higgs Doublet Model.
- Next, we probe charged higgs couplings through charged scalar pair production that falls into **BSM dominance scenario**.

# Gauge and Yukawa couplings with $H^\pm$ :

Gauge Couplings:  $\rightarrow \begin{cases} H^+ H^- \gamma : -ie_0(p_{H^+}^\mu - p_{H^-}^\mu) \\ H^+ H^- Z : -ia(P_{H^+}^\mu - P_{H^-}^\mu) \end{cases}$

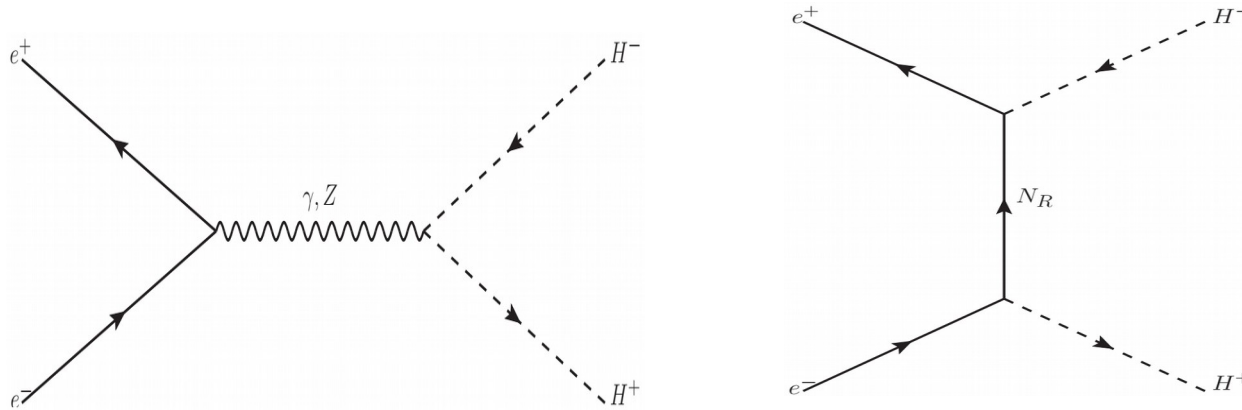
Yukawa Coupling:  $\rightarrow e^+ N_R H^- : b$

Models

Inert doublet  
(a=0.21, b=0)

Type-II seesaw  
(a=0.17, b=0)

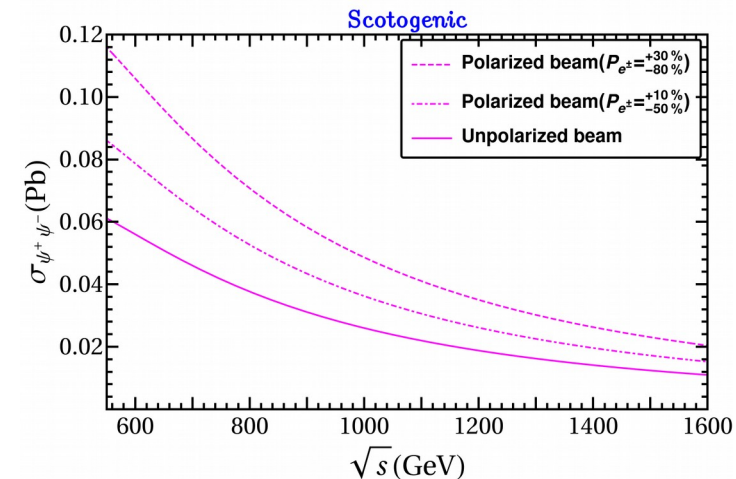
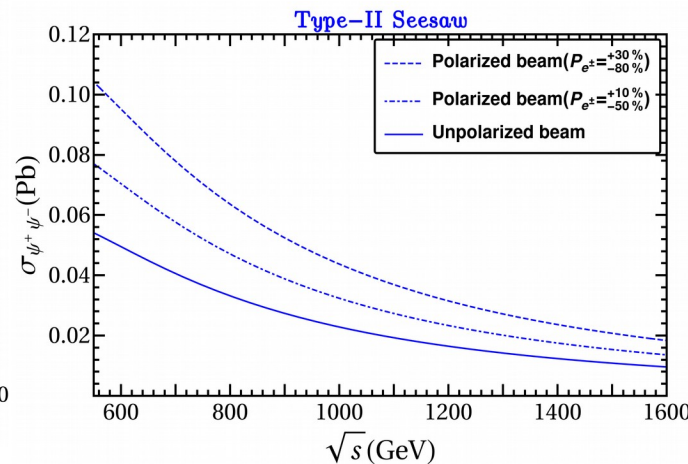
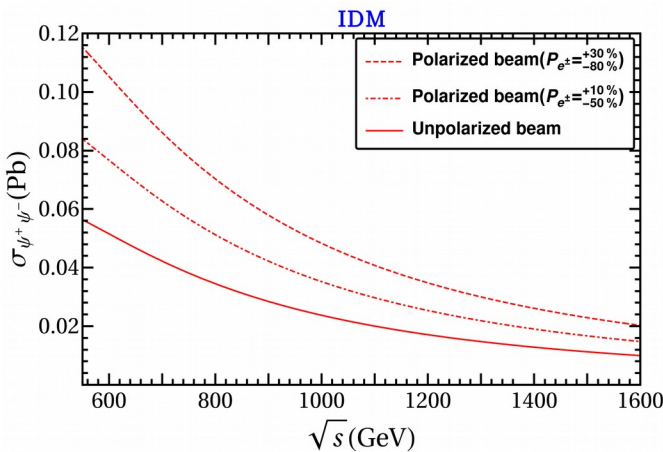
Scotogenic  
(a=0.21, b=0.1)



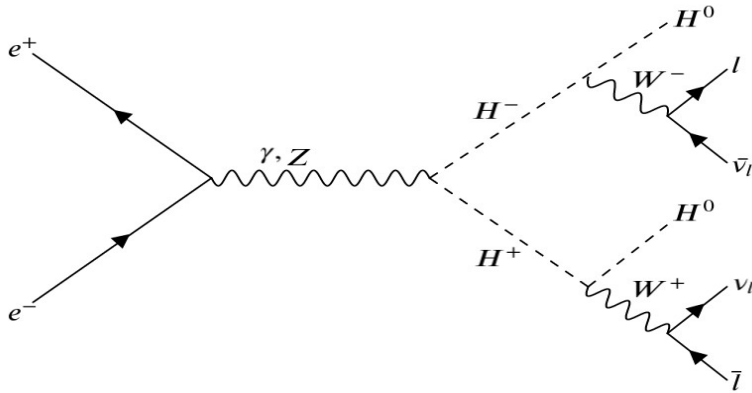
$$P_{e^-} = -80\%$$

$$P_{e^+} = +30\%$$

Helicity amplitude:  $\rightarrow M(\lambda, -\lambda) = i \left[ e_0^2 + ae_0 \left( \frac{4s_w^2 - 1}{2s_{2w}} - \lambda \frac{1}{2s_{2w}} \right) \frac{s}{s - m_Z^2} + \frac{(1 + \lambda)b^2}{4} \frac{s}{t - m_N^2} \right] \beta_{H^+} \sin \theta$

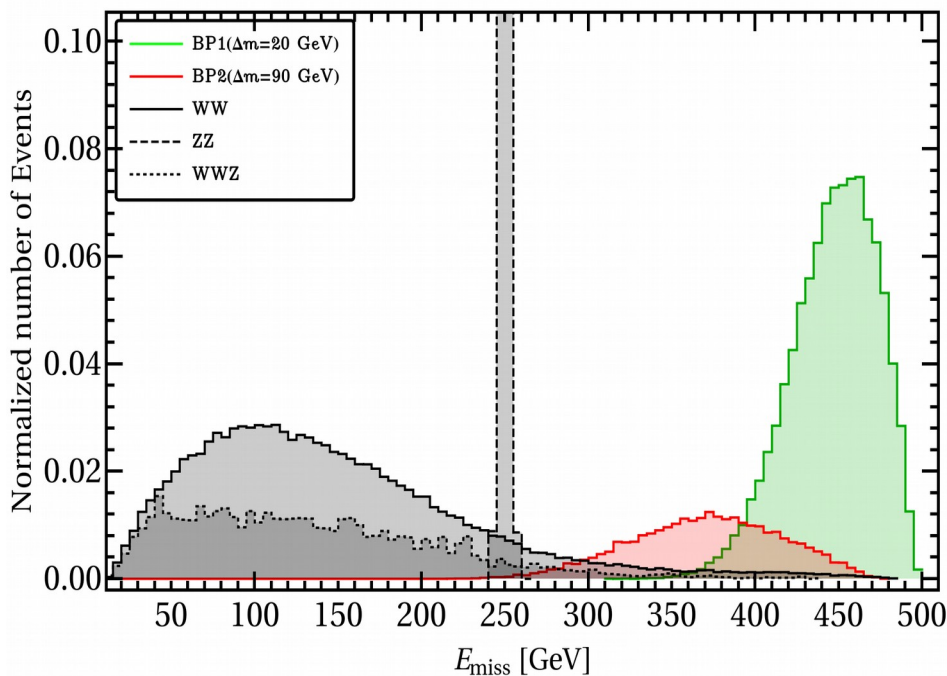


# Event analysis:



▶ Missing Energy ( $\cancel{E}$ ) =  $\sqrt{s} - \sum_i E_i^{\text{vis}}$

## Missing energy distribution



Final state: →

$\ell^+ \ell^- + \text{missing energy}$

Cut flows:

$\mathcal{C}_1$ : Events with 2 leptons



$\mathcal{C}_2$ :  $\cancel{E} \leq 370$  (300) for BP1 (BP2)

Efficiency factor ( $\epsilon$ )

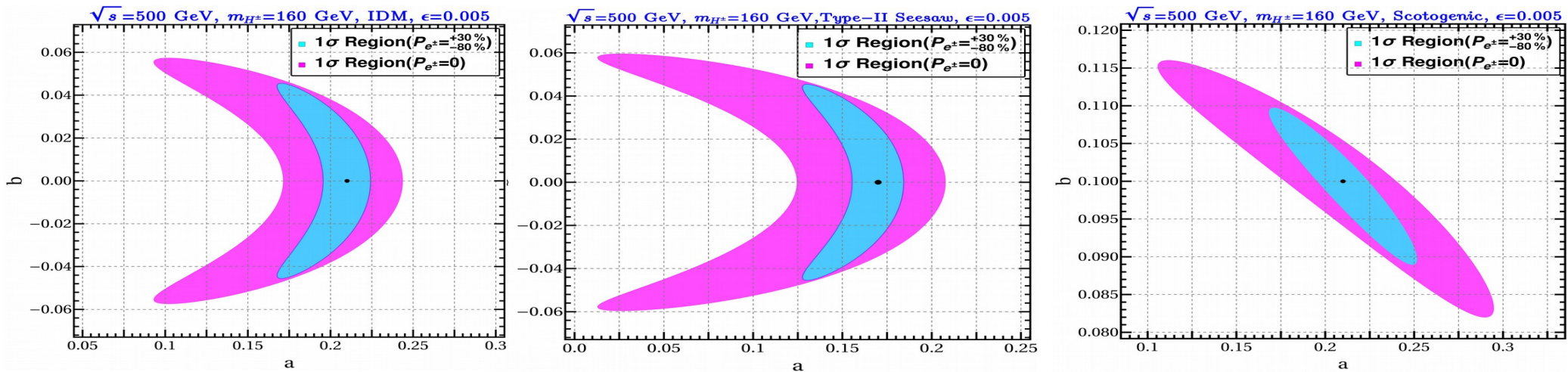


$$\epsilon = \frac{\sigma^{\text{FS}}}{\sigma^{\text{prod}}} = 0.015$$

Our  
choice

$\epsilon \sim 0.005$

# 1 $\sigma$ regions:



- Beam polarization plays crucial role to reduce the uncertainty in NP couplings.
- In case of BSM dominance, estimation of NP couplings are much better.

## Chapter-IV (OOT in presence of SM background)

based on

Optimal New Physics estimation in presence of Standard Model background,

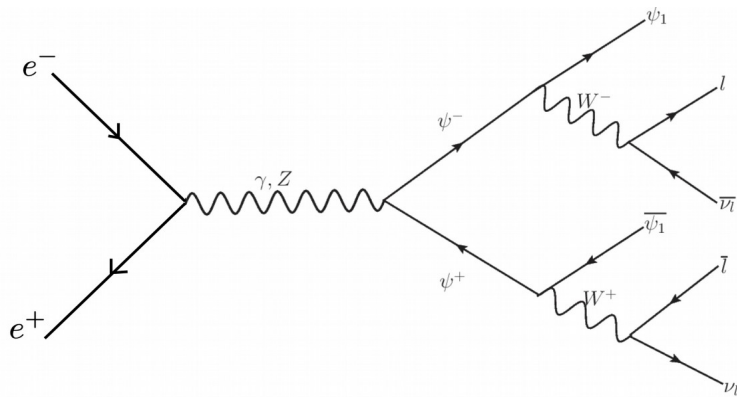
S. Bhattacharya, S. Jahedi, J. Lahiri, J. Wudka

(In progress)

# Signal and backgrounds:

- ▶ Previous works are based on signal only hypothesis.
- ▶ We propose OOT in presence of SM backgrounds.

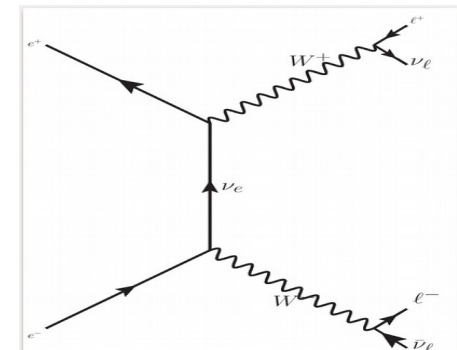
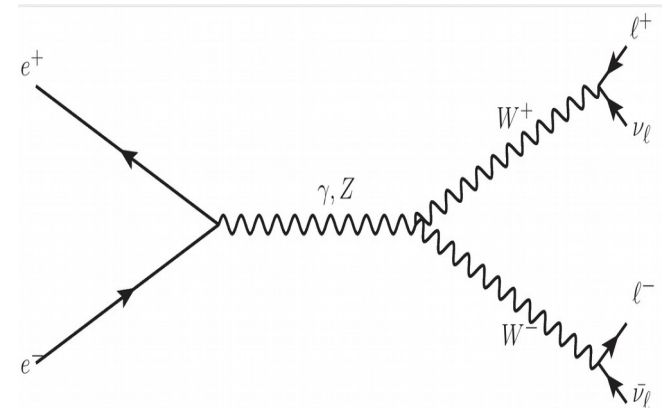
**Singlet-doublet model:**  $\psi^\pm$ ,  $\psi_1$  and  $\psi_2$



Final state signal: →

$l^+ l^- + \text{missing energy}$

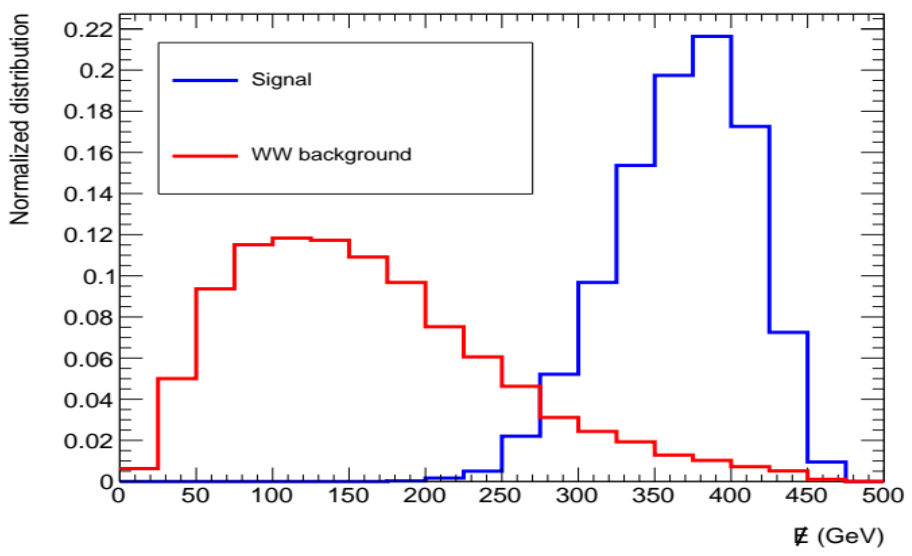
**WW background**



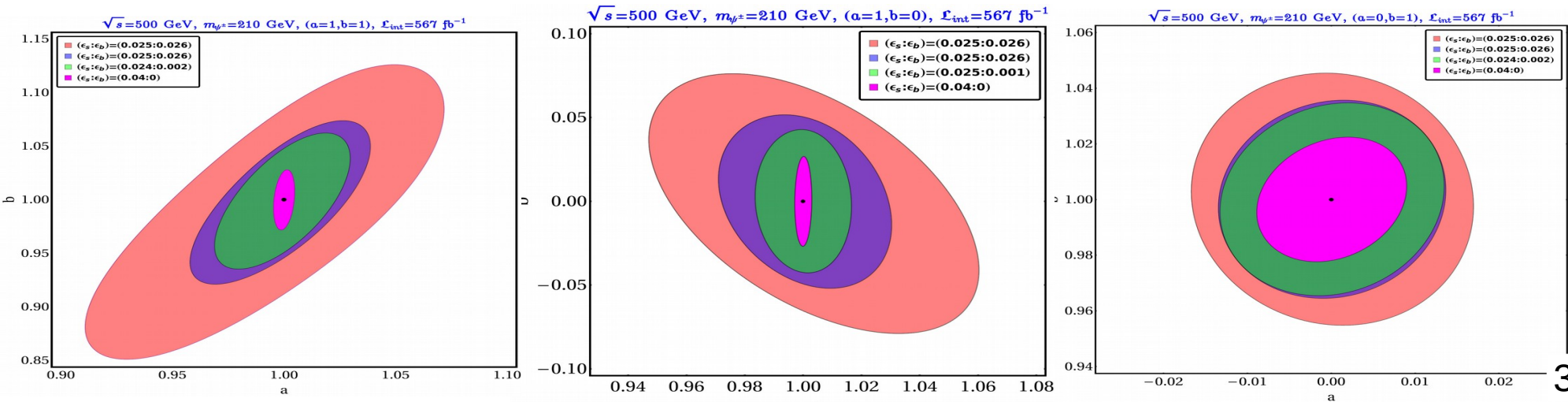
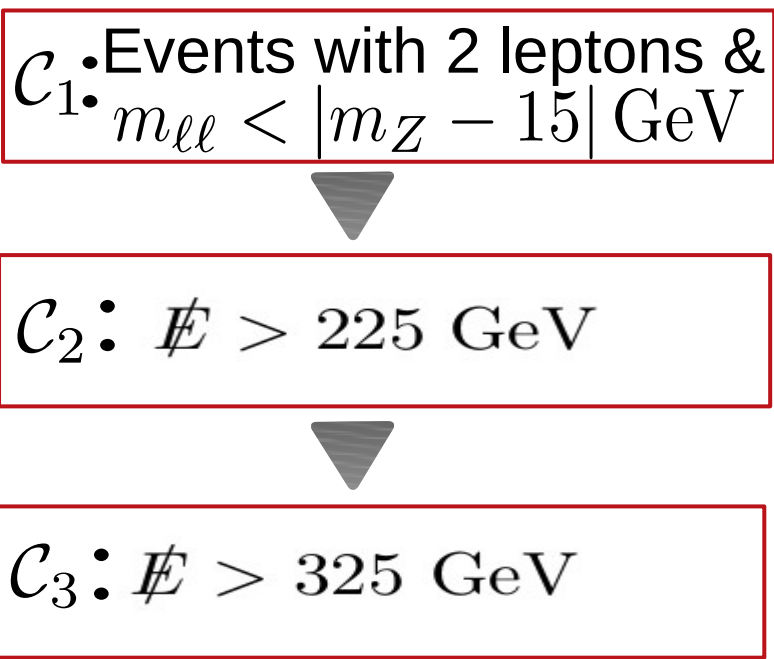


# Gradual background reduction:

## Missing energy distribution



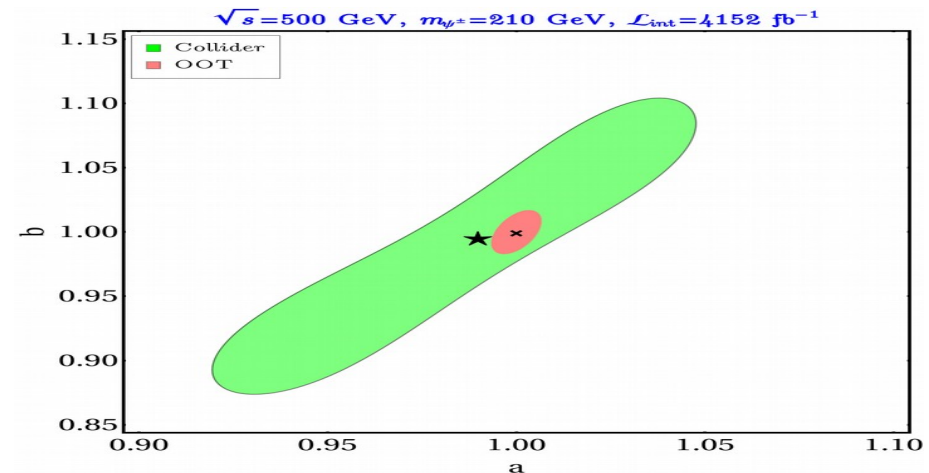
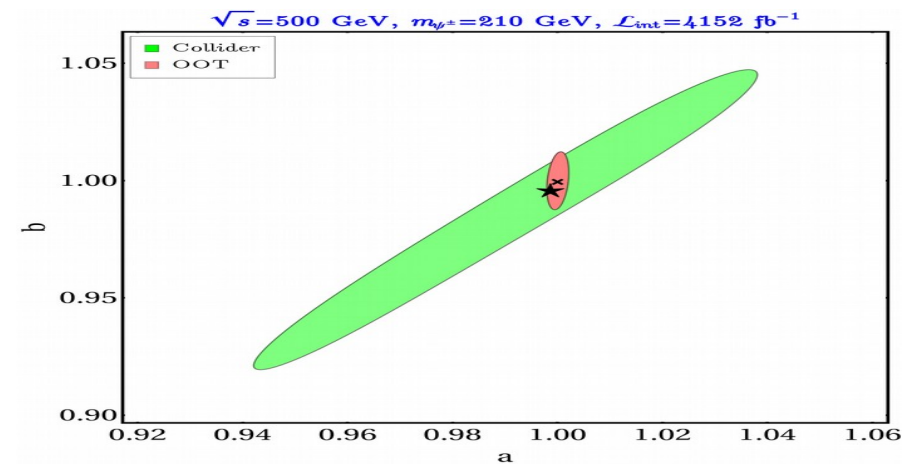
## Cut flows:



# How OOT is better than Binned analysis:

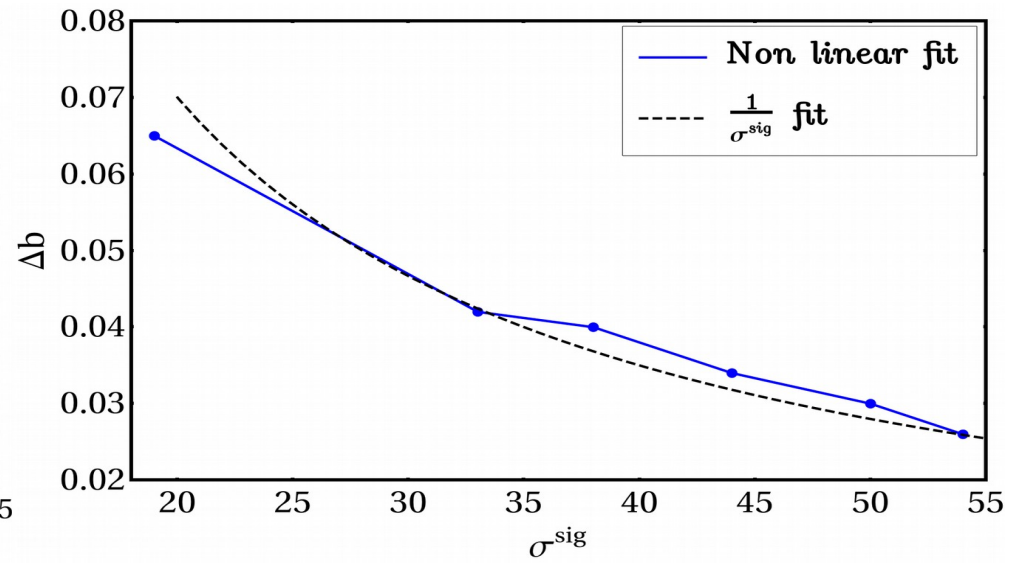
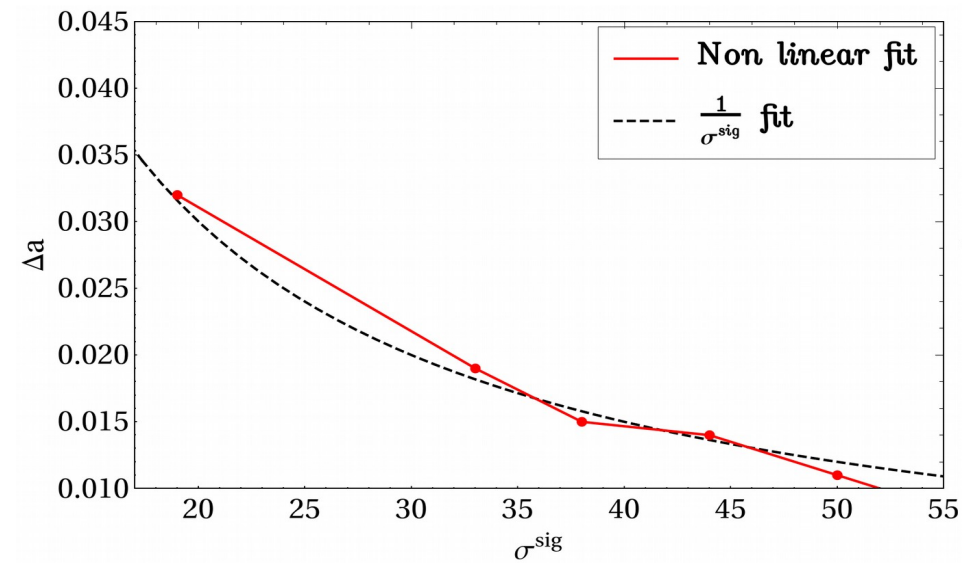
i) Only signal

ii) signal+bkg



## OOT sensitivity vs collider significance:

$$\frac{\Delta g_1}{g_1} = \frac{\sqrt{S+B}}{S} = \frac{1}{\sigma^{\text{sig}}}.$$





# Summary:

- ★ OOT guides us to extract the **optimal** uncertainties of NP parameters and helps us to distinguish one model from another model.
- ★ Beam polarization and luminosity play a crucial role to estimate NP couplings and segregate a hypothesis from a base model.
- ★ In case of BSM dominated scenario, estimation of NP couplings are better.
- ★ The more efficient the background reduction is, the more precise the uncertainties.
- ★ OOT provides a better estimation of NP physics couplings compared to binned analysis.



**THANK**

**YOU !**