Maximally entangled proton and entropy in high energy collisions

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HORIZON 2020

Motivation

bounds and properties of EE may provide some new insight on behavior of pdfs

links to other areas (thermodynamics, gravity, quantum information, conformal field theory)

Interesting in context of parton saturation and thermalization problem of Quark Gluon Plasma

Various approaches to entropy in the low x limit: entropy of gluon density, thermodynamic entropy momentum space entanglement, coordinate space entanglement, Wehrl entropy,...

K. Kutak '11, R. Peschanski '13,

A. Kovner, M. Lublinsky '15, A. Kovner, M. Lublinsky, M. Serino '18,

Hagiwara, Y, Hatta, B. Xiao,Yuan'18, N. Armesto, F. Dominguez, A. Kovner, M. Lublinsky, V. Skokov'19

Z. Tu, D. Kharzeev, T. Ulrich '20, C. Akkaya, H. Duan , A. Kovner, V. Skokov '20

K. Zhang, K. Hao, D. Kharzeev, V. Korepin' 21, E. Levin, D. Kharzeev '21,

D. Kharzeev '21, M. Hentschinski, K. Kutak '21,

Dvali, Venugopalan' 21 Liu, Nowak, Zahed, 21;

A. Dumitriu,Kolbusz '22, H. Duan , A. Kovner, V. Skokov '22

Boltzman and von Neuman entropy formulas – reminder

The entropy S of macrostate is given by the log of number W of distinct microstates that compose it

$$S = -\sum_{i=1}^{W} p(i) \ln p(i)$$

Gibbs entropy

For uniform distribution $p(i) = \frac{1}{W}$ the entropy is maximal

Boltzmann entropy

$$S = \ln W$$

Since partons are introduced as the microscopic constituents that compose the macroscopic state of the proton, it is natural to evaluate the corresponding entropy or entropy corresponding to parton density.

K. Kutak '11, Peschanski'12
A. Kovner, M. Lublinsky '15
D. Kharzeev, E. Levin '17,...

But proton as a whole is a pure state and the von Neuman entropy is 0. Can one get any nontrivial result?

For pure state (one state) density matrix is:

$$\rho = |\psi\rangle\langle\psi|$$

$$S_{VN} = -Tr[\rho \ln \rho] = -1 \ln 1 = 0$$

For mixed state i.e. classical statistical mixture

$$\rho = \sum p(i) |\psi_i\rangle\langle\psi_i|$$

$$S_{VN} \neq 0$$

Kharzeev, Levin '17

Entanglement entropy in DIS

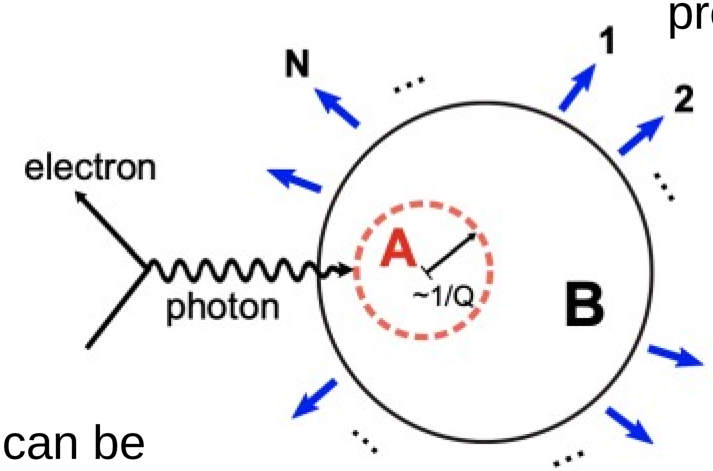
The composite system is described by $\mathcal{H}_A \otimes \mathcal{H}_B$

$|\Psi_{AB}\rangle$ in $A \cap B$

physical state in A
physical state in B

$$|\Psi_{AB}\rangle = \sum_{i,j} c_{ij} |\varphi_i^A\rangle \otimes |\varphi_j^B\rangle$$

proton's rest frame



$\mathcal{H}_B$ of dimension n_B.

$\mathcal{H}_A$ of dimension n_A

entangled
if the product can not be expressed as separable product state

$$|\Psi_{AB}\rangle = \sum_{i,j} c_{ij} |\varphi_i^A\rangle \otimes |\varphi_j^B\rangle$$

separable
if the product can be expressed as separable product state

$$|\Psi_{AB}\rangle = |\varphi^A\rangle \otimes |\varphi^B\rangle$$

Kharzeev, Levin '17

We perform Schmidt decomposition

$$|\Psi_{AB}\rangle = \sum \alpha_n |\Psi_n^A\rangle |\Psi_n^B\rangle$$

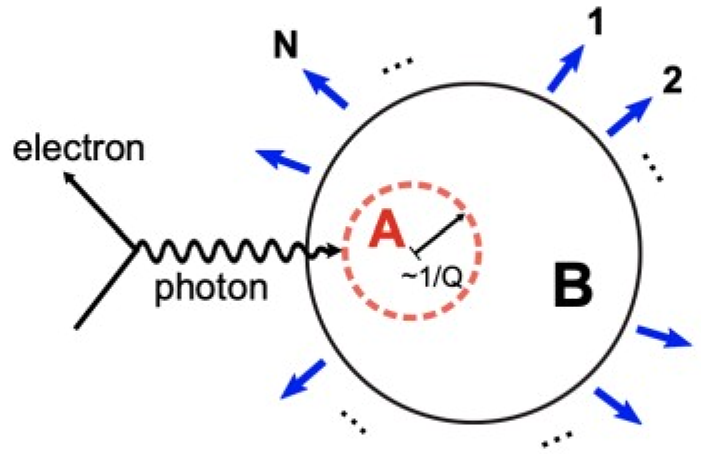
orthonormal states belonging to A

orthonormal states bolonging to B

related to matrix C

Entanglement entropy in DIS

$$|\Psi_{AB}\rangle = \sum_n \alpha_n |\Psi_n^A\rangle |\Psi_n^B\rangle$$



proton's rest frame

$$\rho_{AB} = |\Psi_{AB}\rangle\langle\Psi_{AB}|$$

$$\rho_A = \mathrm{tr}_B \rho_{AB} = \sum_n \alpha_n^2 |\Psi_n^A\rangle\langle\Psi_n^A|$$

The density matrix of the mixed state probed in region A

$\alpha_n^2 \equiv p_n$ probability of state with n partons

Kharzeev, Levin '17

$$S = -\sum_n p_n \ln p_n$$

entropy results from the entanglement between the regions A and B, and can thus be interpreted as the entanglement entropy. Entropy of region A is the same as entropy in region B.

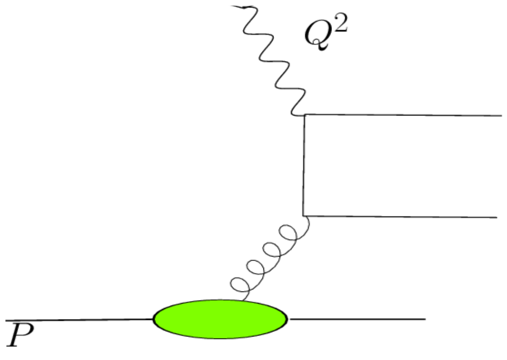
Proton structure function and dipole cross section

$$F_2(x,Q^2) = \frac{Q^2}{4\pi^2}\alpha_s \sum_q e_q^2 \int d^2k\, \mathcal{F}(x,k^2)\left(S_L(k^2,Q^2,m_q^2) + S_T(k^2,Q^2,m_q^2)\right)$$

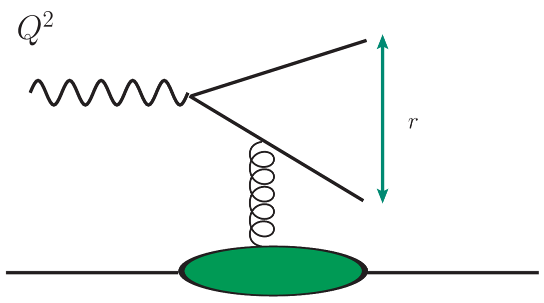
Enters also into inclusive gluon production in adjoint representation recently called dipole gluon density

Impact factors from Feynman diagrams in momentum space

In the kt factorization



In the dipole formalism



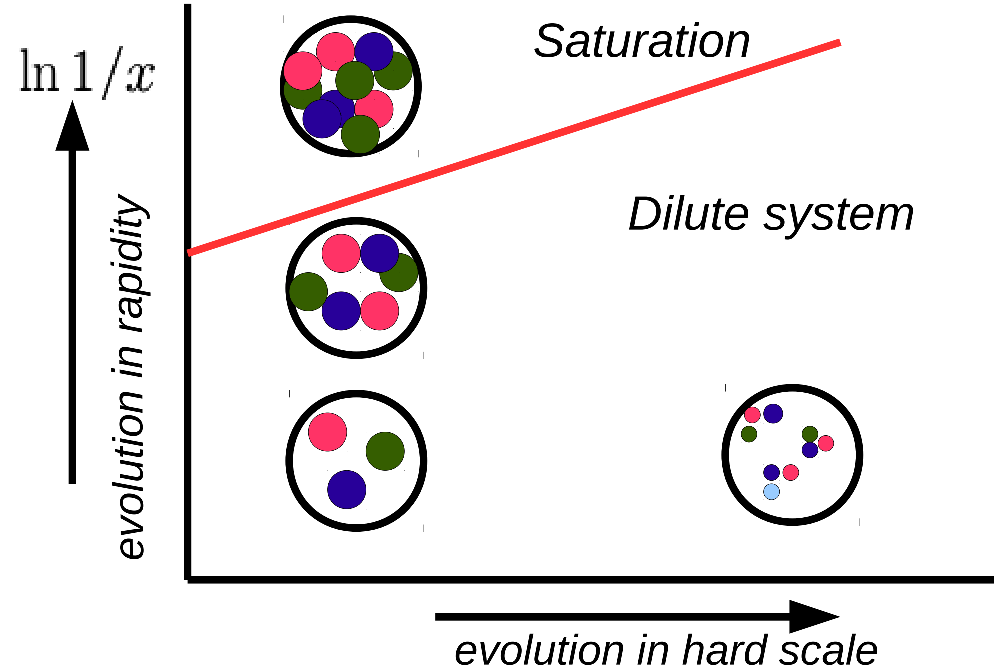
$$F_2(x,Q^2) = \frac{Q^2}{4\pi^2\alpha_{em}}\int d^2b \int_0^1 dz \int d^2r \left(|\psi_L(z,r)|^2 + |\psi_T(z,r)|^2\right) N(x,r,b)$$

wave function

Dipole cross section

Gluons at high energies

Saturation – state where number of gluons stops growing due to high occupation number. Way to fulfill unitarity requirements in high energy limit of QCD.

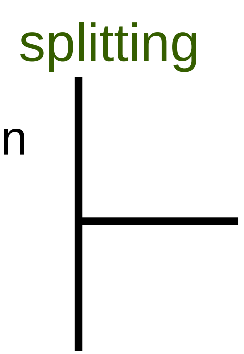


L.V. Gribov, E.M. Levin, M.G. Ryskin
Phys.Rept. 100 (1983) 1-150

Larry D. McLerran, Raju Venugopalan
Phys.Rev. D49 (1994) 3352-3355

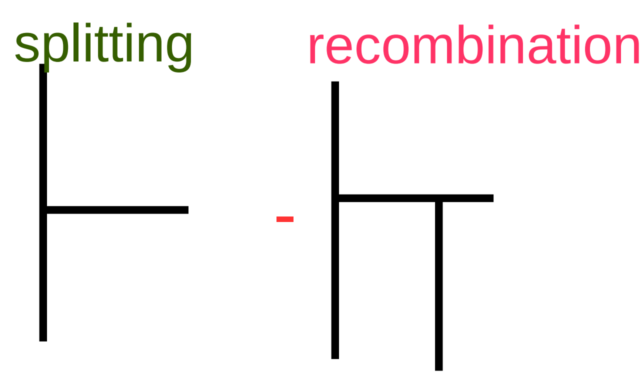
On microscopic level it means that
gluon apart splitting recombine

Linear evolution
Equation
BFKL



Nonlinear evolution equations
BK, JIMWLK
Balitcky-Kovchegov,

Jailian-Marian,Iancu
McLerran,Weigert,Leonidov,Kovner



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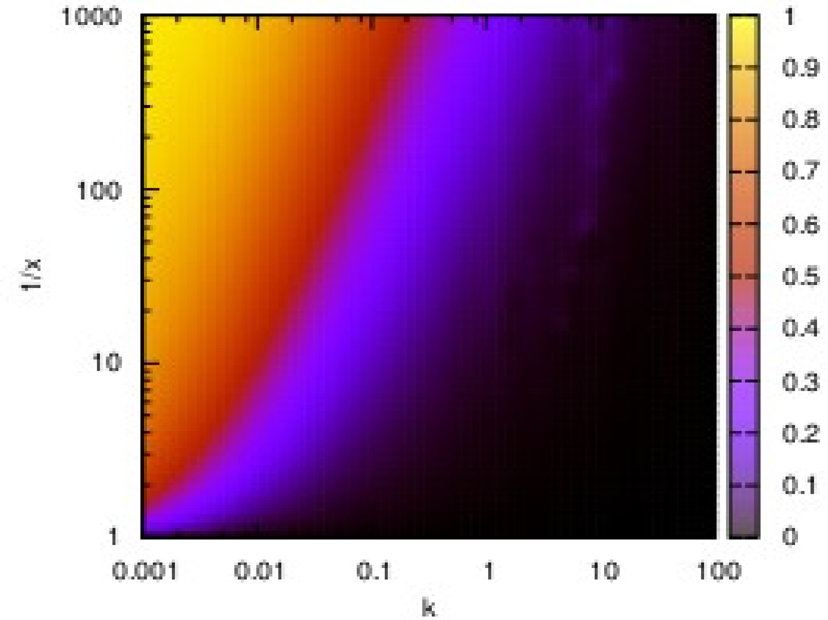
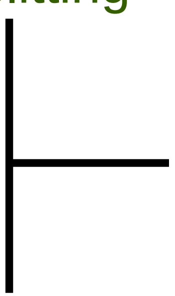


On microscopic level it means that
gluon apart splitting recombine

splitting

Linear evolution
Equation
BFKL



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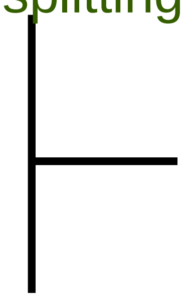

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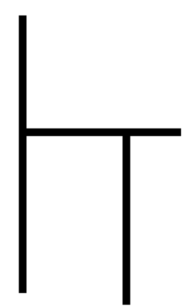
recombination

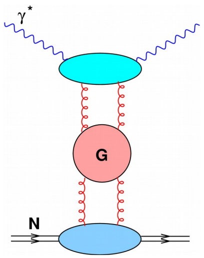


Momentum space vs coordinate space

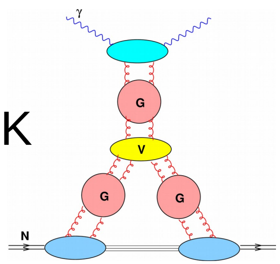
momentum space - Bjorken frame

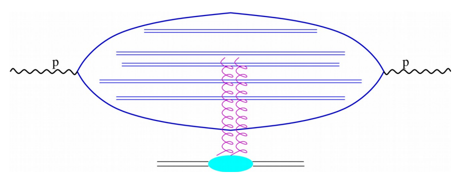
position space - Mueller frame

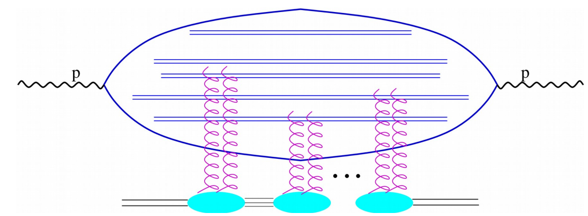
BFKL

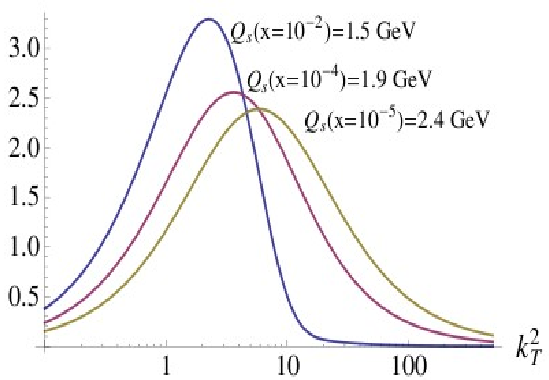


BK

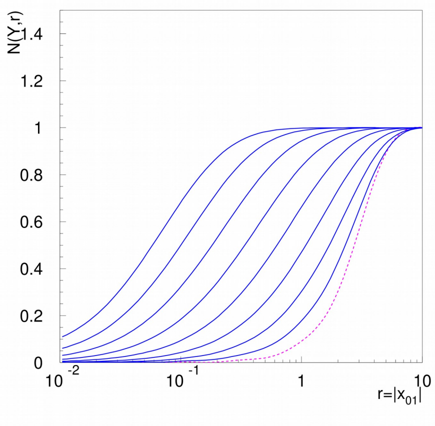








gluon ~ color dipole



from *A. Stasto Acta Phys.Polon. B35 (2004) 3069-3102*

$$\mathcal{F}(x,k) = \mathcal{F} + K_{m\,s} \otimes \mathcal{F}(x,k) - \frac{1}{R^2} TPV \otimes \mathcal{F}(x,k)^2 \qquad N(x,r,b) = N_0 + K_{p\,s} \otimes (N(x,r,b) - N(x,r,b)^2)$$

dipole unintegrated gluon density

Evolved with BK dipole amplitude – expectation value of product of Wilson lines in fundamental representation

related by Fourier transform

The dipole cross section and integrated gluon

$$\sigma(x,r) = \frac{4\pi^2}{N_c}\int \frac{dk^2}{k^2}(1 - J_0(kr))\mathcal{F}(x,k^2)$$

$$\sigma(x,r) \approx \frac{4\pi^2}{N_c}\int \frac{dk^2}{k^2}\left(1 - \left(1 - \frac{k^2r^2}{4}\right)\right)\mathcal{F}(x,k^2)$$

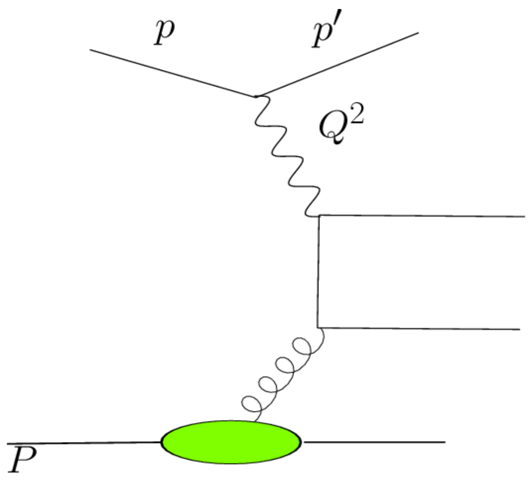
$$\sigma(x,r) \approx \frac{\pi^2}{N_c} r^2 xg(x, 1/r^2)$$

$$\sigma(x,r) = \sigma_0 N(x,r)$$

$$N(x,r) \approx xg(x, 1/r^2)$$

$$xg(x,Q^2) \equiv \int^{Q^2} dk^2 \mathcal{F}(x,k^2)$$

integrated gluon density



For fixed dipole size one has.

$$N(x) = 1 - Z(x)$$

$$Z(x) \propto \sum_n P_n$$

generating function for dipoles

In the context of the scale dependent GBW model this approximation is viewed as linear approximation

Partonic, dipole cascade

$$p_n = P_n \qquad Z(x) \propto \sum_n P_n$$

$$\frac{dP_n(Y)}{dY} = -\lambda n P_n(Y) + (n-1)\lambda P_{n-1}(Y)$$

set of partons is described by set of dipoles with fixed sizes ,Y is rapidity and is related to energy

Lublinsky, Levin ‘03

$$P_n(Y) = e^{-\lambda Y}\left(1 - e^{-\lambda Y}\right)^{n-1}$$

depletion of the probability to find n dipoles due to the splitting into (n + 1) dipoles.

$$S = -\sum_n p_n \ln p_n$$

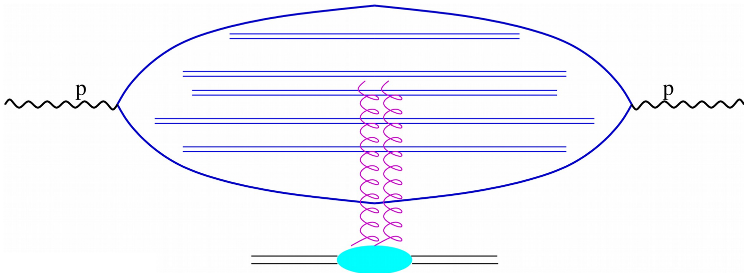
the growth due to the splitting of (n − 1) dipoles into n dipoles.

$$S(Y) = \ln\left(e^{\lambda Y} - 1\right) + e^{\lambda Y}\ln\left(\frac{1}{1 - e^{-\lambda Y}}\right)$$

$S(Y) \approx \lambda Y$ where $Y = \ln 1/x$

BFKL intercept = $4\ \ln 2\ \bar{\alpha}_S$

$$\langle n \rangle = \sum_n n P_n(Y) = \left(\frac{1}{x}\right)^{\lambda}$$



Kharzeev, Levin ‘17

Assumption $xg(x) = \langle n \rangle$

$$S(x) = \ln(xg(x))$$

The model can be generalized within 3+1 BK and one can argue how to account for hard scale dependence dependence.

$$S(x,Q) = \ln(xg(x,Q))$$

KL entropy formula - interpretation

At low x partonic microstates have equal probabilities $P_n(Y) = e^{-\lambda Y}\cancel{\left(1 - e^{-\lambda Y}\right)^{n-1}}$

In this equipartitioned state the entropy is maximal – the partonic state at small x is maximally entangled.

In terms of information theory as Shanon entropy:
- equipartitioning in the maximally entangled state means that all “signals” with different number of partons are equally likely
- it is impossible to predict how many partons will be detected in a give event.
- structure function at small x should become universal for all hadrons.

From strict bounds on entanglement entropy (from conformal field theory) one can obtain that at low x (in conformal regime) one has

$$xg(x) \leq \text{const}\, x^{-1/3}$$

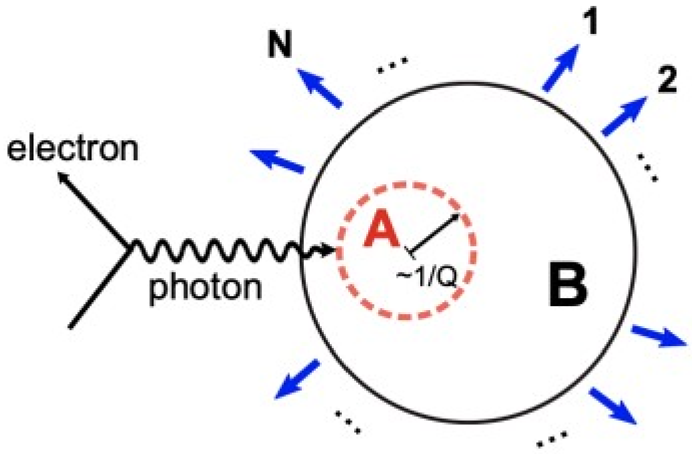
Kharzeev, Levin ‘17

Furthermore entropy of the final state hadrons can not be smaller than entropy of partons.

Entanglement entropy – calculation and measurement

For DIS at high energies, this entanglement entropy can be calculated using

$$S(x, Q^2) = \ln \left\langle n \left(\ln \frac{1}{x}, Q \right) \right\rangle$$



$$S_{hadron} = \sum P(N) \ln P(N)$$

The charged particle multiplicity distribution measured in either the current fragmentation region or the target fragmentation region.

Fraction of events with charged hadron

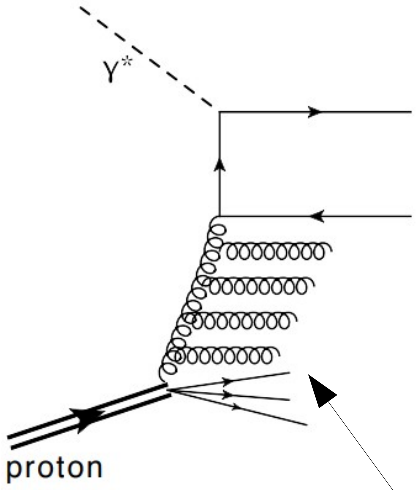
Extension of KL entropy formula

Hentschinski, Kutak ‘21

$$\left\langle n\left(\ln\frac{1}{x},Q\right)\right\rangle = xg(x,Q) + x\Sigma(x,Q)$$

To get the entropy of system of partons one needs to account for both quarks and gluons. One can view this as a higher order correction to KL formula. Furthermore it is impossible to isolate quarks from gluons therefore the compete entropy formula should receive contributions from quarks and gluons

Gluon and quark distribution



In the linear regime obeys BFKL equation. In our calculations we use NLO BFKL with kinematical improvements and running coupling. The gluon density has been fitted to F_2 data (exact kinematics was used)

Hentschinski, Sabio-Vera, Salas.
Phys.Rev.D 87 (2013) 7, 076005
Phys.Rev.Lett. 110 (2013) 4, 041601

We calculate the sea quarks distribution using

$$x\Sigma(x,Q) = P_{qg}(Q,\mathbf{k}) \otimes \mathcal{F}(x,\mathbf{k}^2)$$

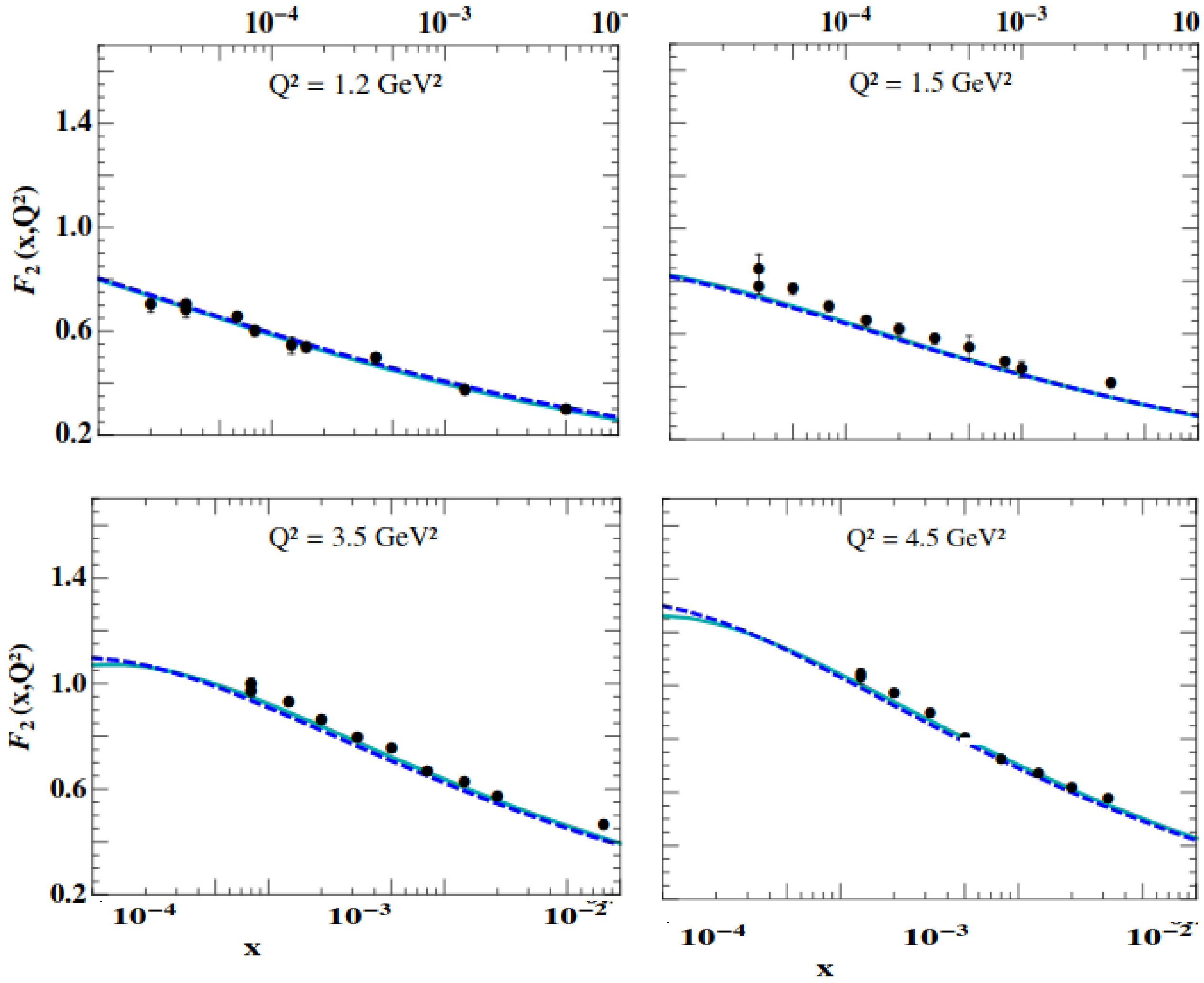
$$xg(x,Q) = \int_0^{Q^2} d\mathbf{k}^2 \mathcal{F}(x,\mathbf{k}^2)$$

Transverse momentum dependent splitting function
Catani, Hautmann
Nucl.Phys. B427 (1994) 475-524

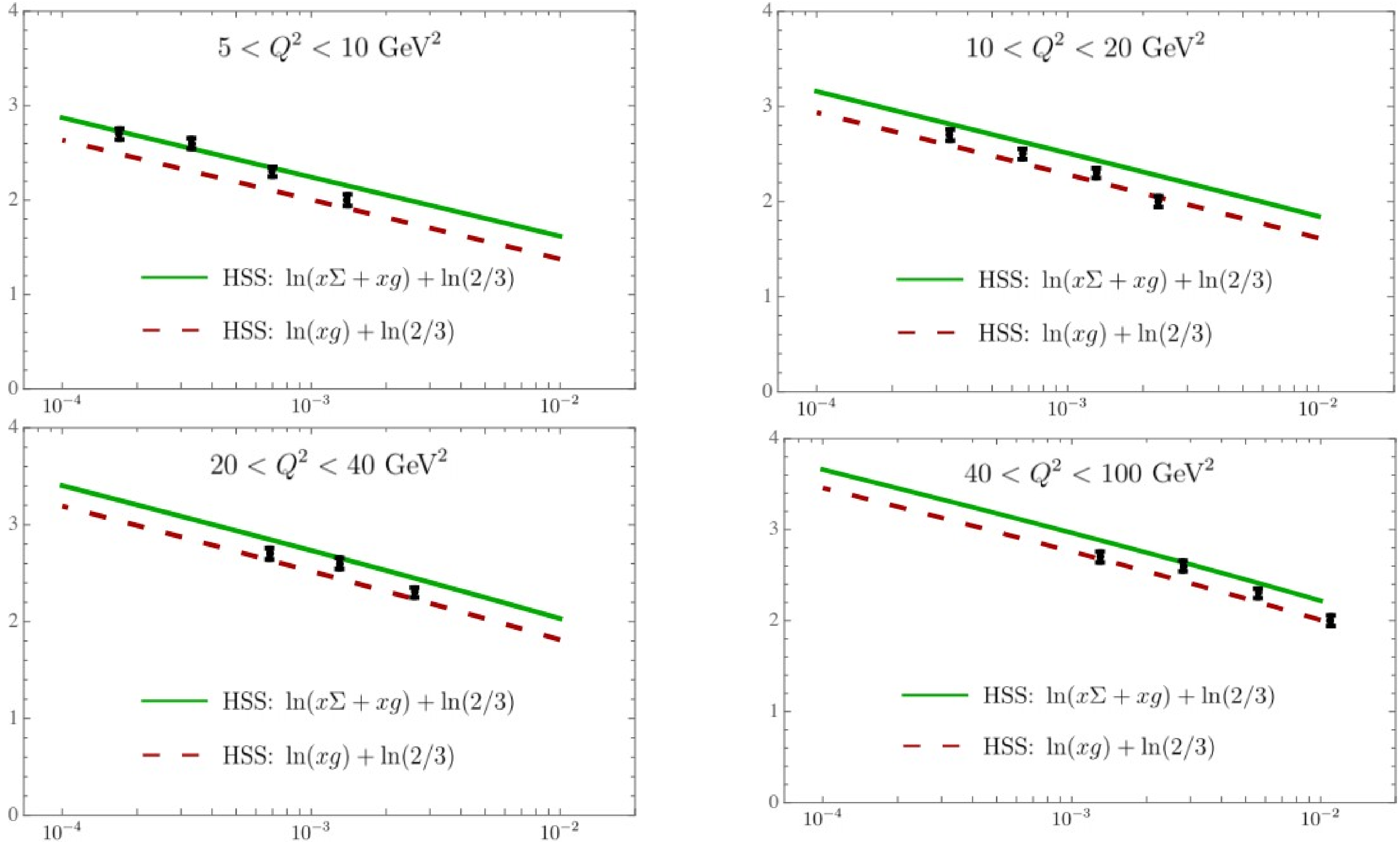
Other methods for resummation:
KMS (Kwiecinski, Martin, Stasto);
CCSS (Colferai, Ciafaloni, Staśto, Salam)

Proton structure function from HSS fit

F_2 data description

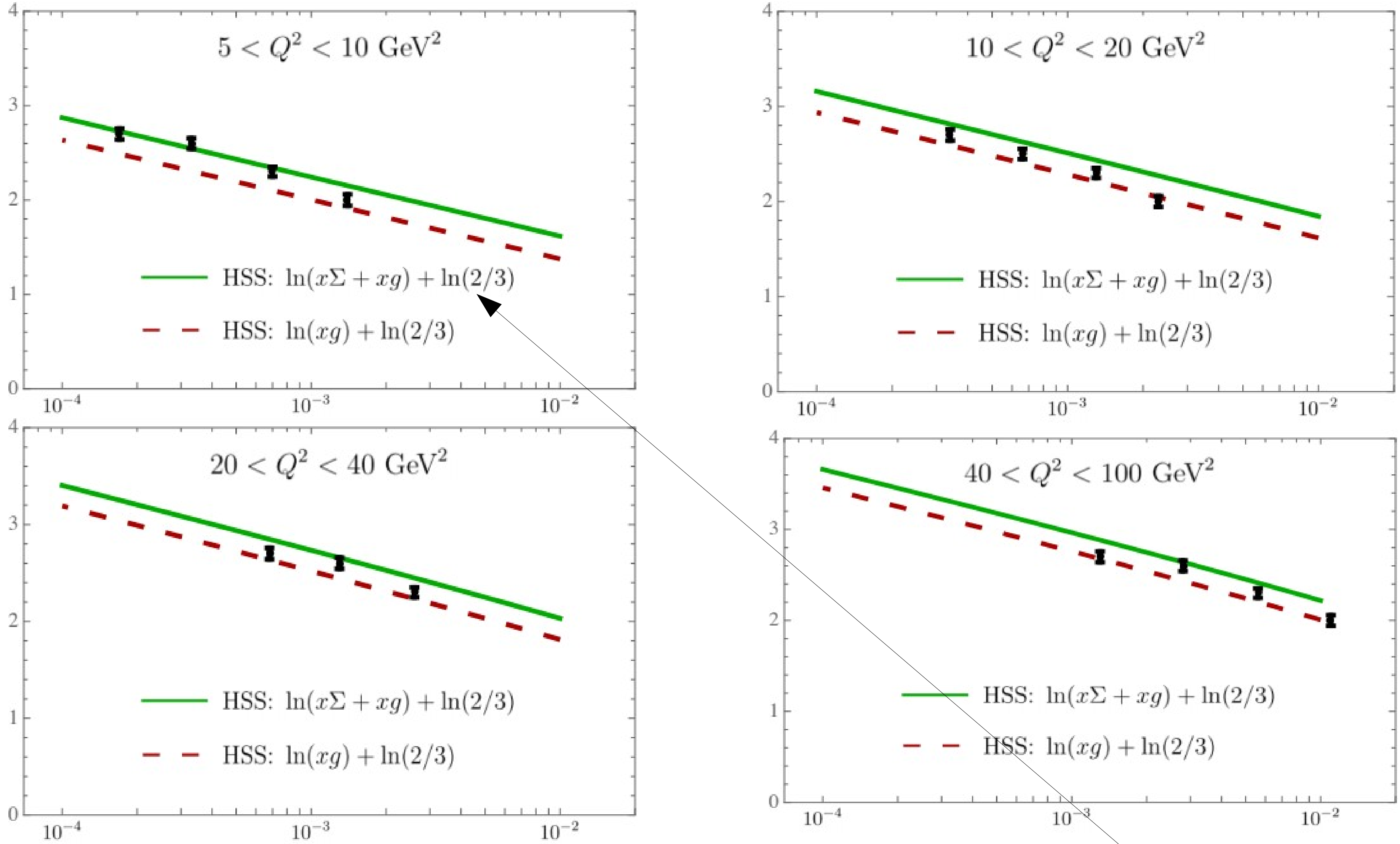


Results



Hint that the general idea works. Gluon dominates over quarks.
One has to also take into account that only charged hadrons were measured.

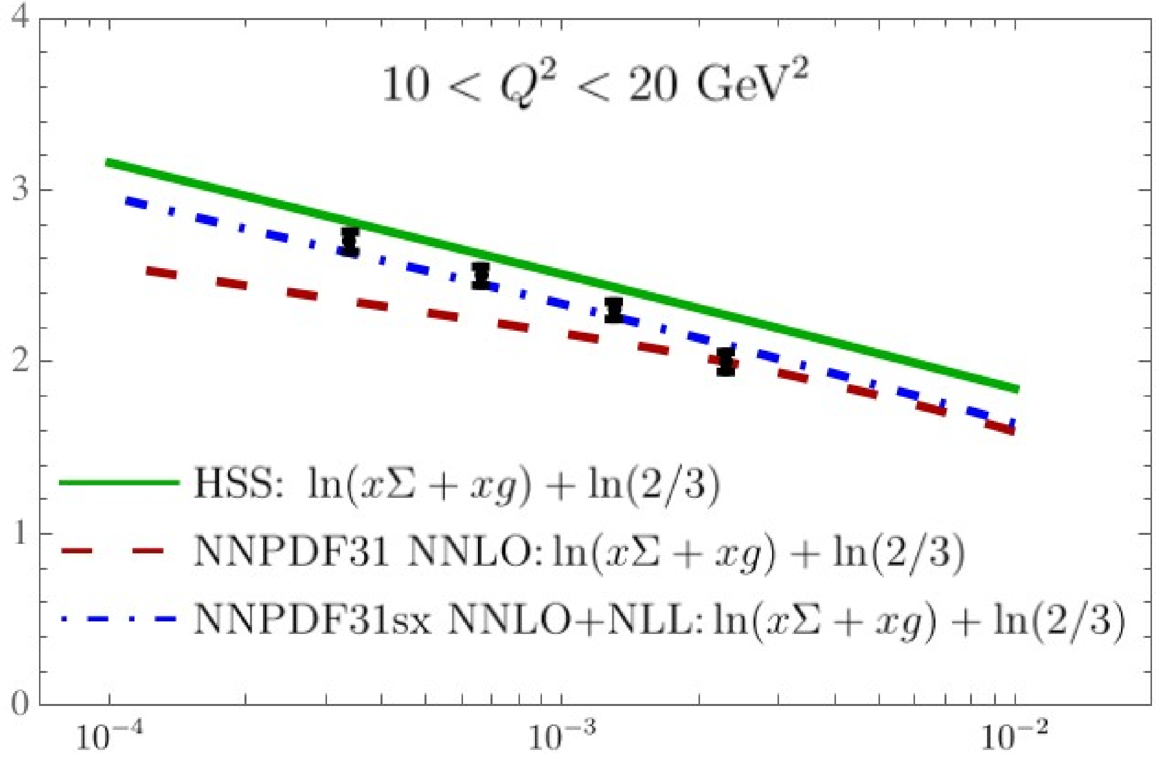
Results



Hint that the general idea works. Gluon dominates over quarks.
One has to also take into account that only charged hadrons were measured i.e 2/3 of partons contribute

Results

Eur.Phys.J.C 82 (2022) 2, 111 Hentschinski, Kutak



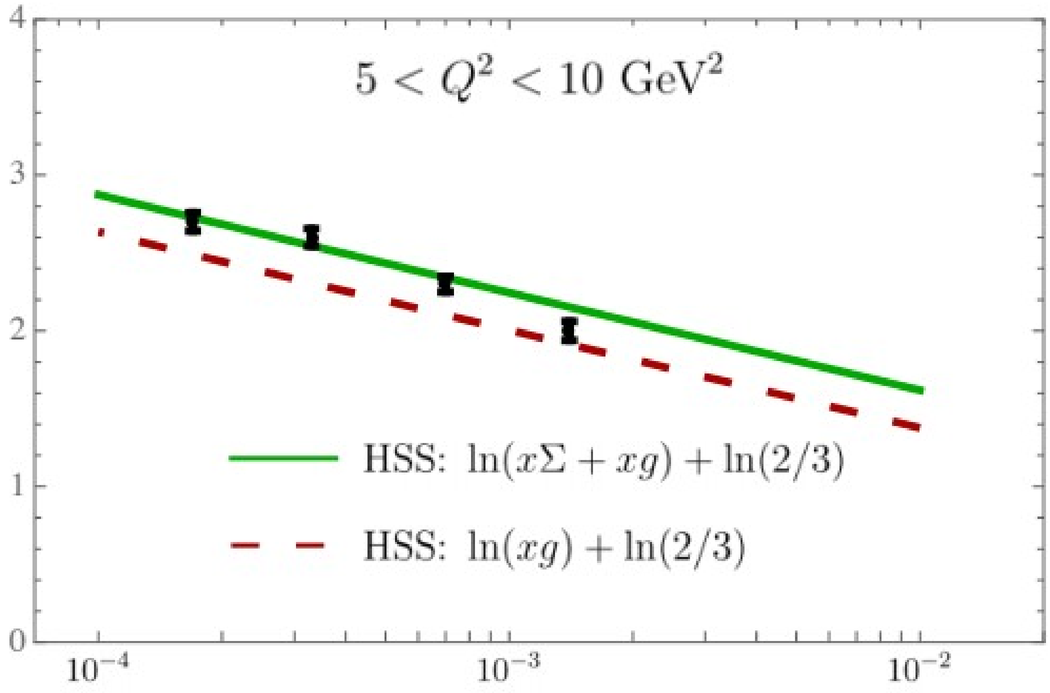
Low x resummation is essential

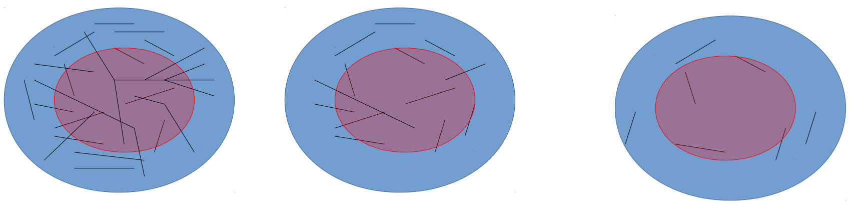
HSS gluon density used
i.e. NLO BFKL + kinematical Improvements
Hentschinski, Sabio-Vera, Salas.
Phys.Rev.D 87 (2013) 7, 076005
Phys.Rev.Lett. 110 (2013) 4, 041601

Large uncertainities of pdfs. In this study we did not take them into account.

NNPDF 31 → DGLAP
NNPDF 31sx → DGLAP + low x resummation

Dipoles and mechanism of entanglement





segments – dipoles, color singlets
maximally entangled states

red circle – resolved area defined by photon

entanglement arises because of dipoles that are partially in the red circle and partially in blue.

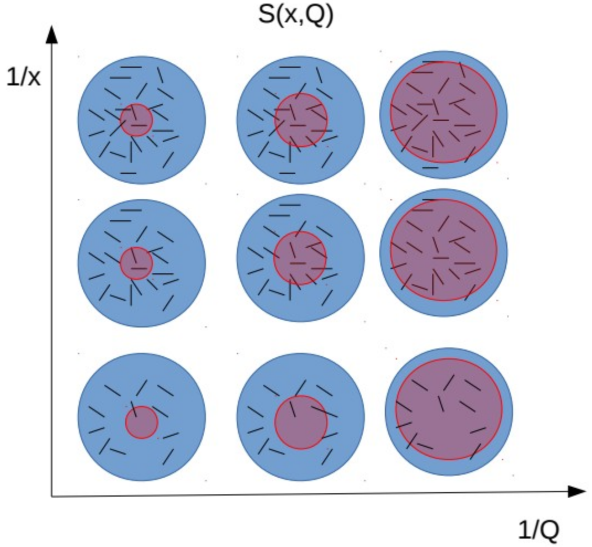
The broken dipoles contribute to final state hadron multiplicity and entropy of proton

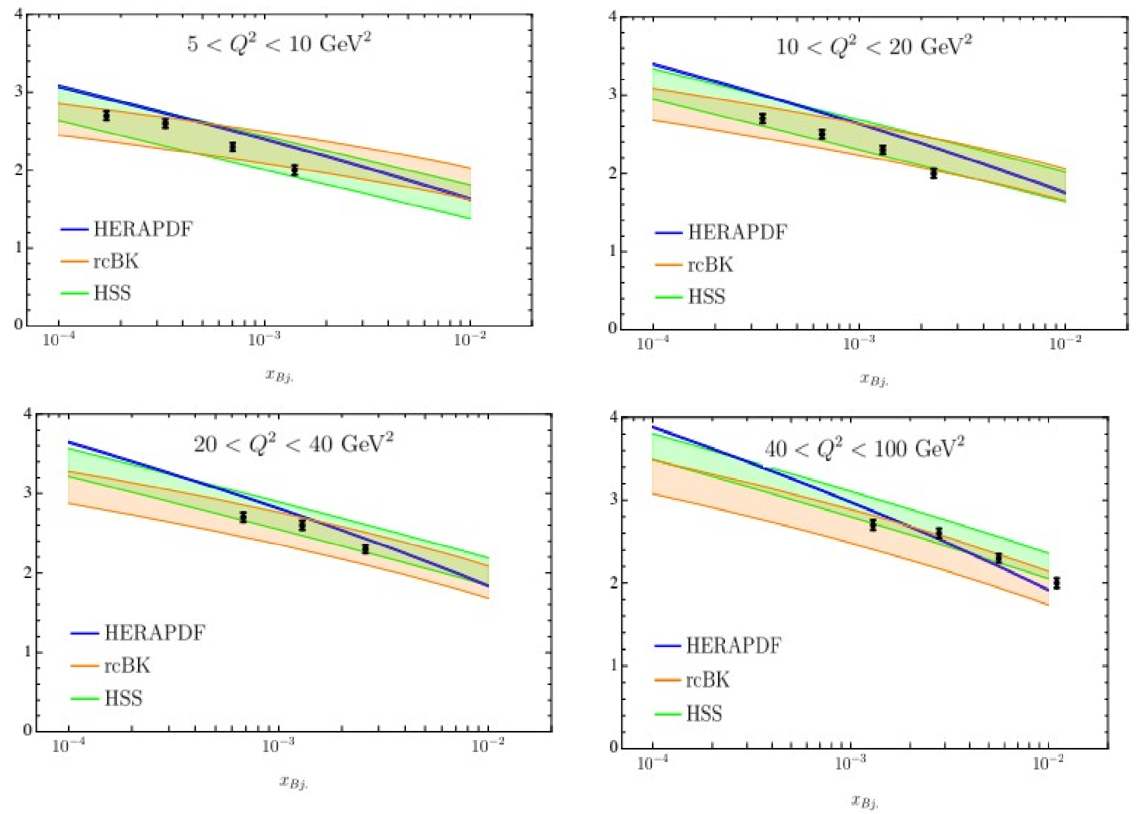
If we go to lower x we have more and more dipoles that cross the red line and entanglement grows

"Entanglement of predictions arises from the fact that the two bodies at some earlier time from in the true sense one system that is were interacting and have left behind choices on each other."

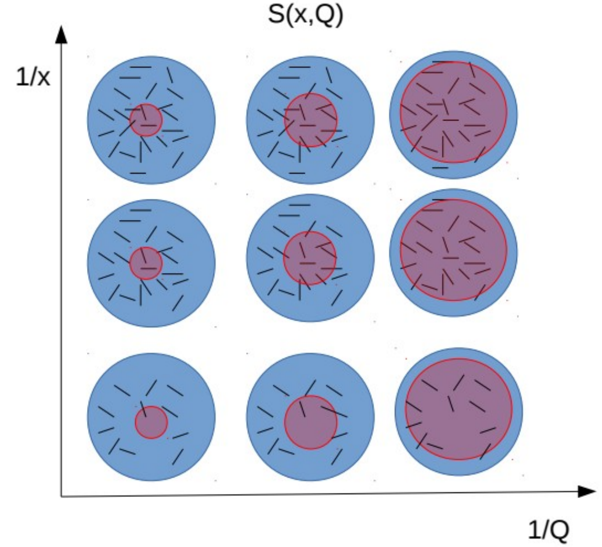
E. Schrodinger

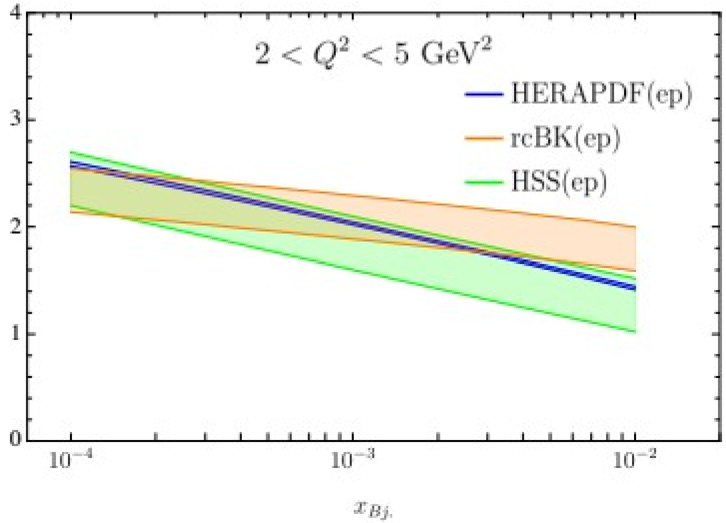
Large scales - description

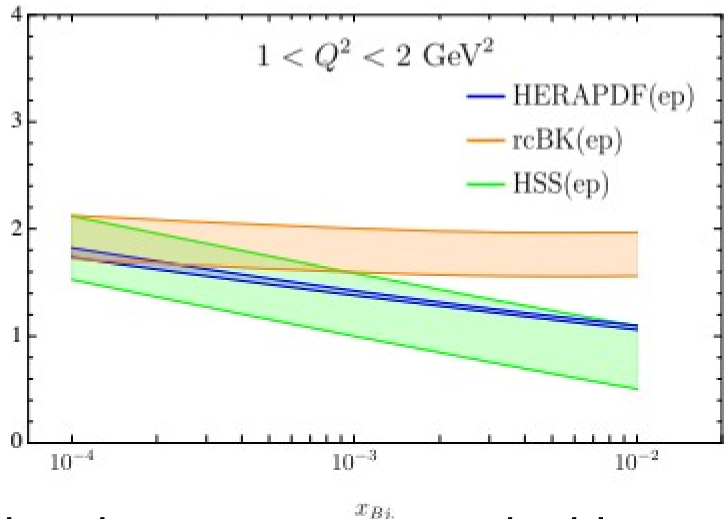




Small scales - prediction

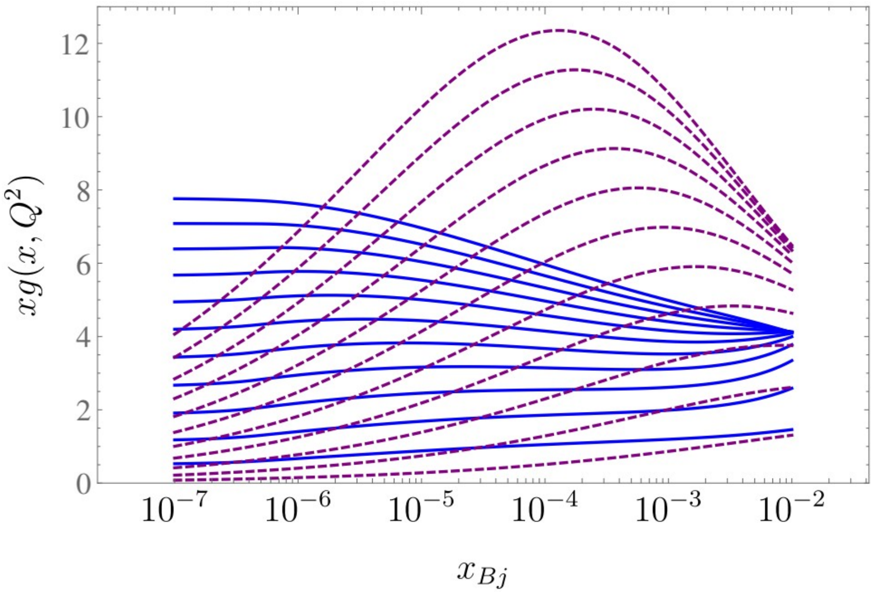


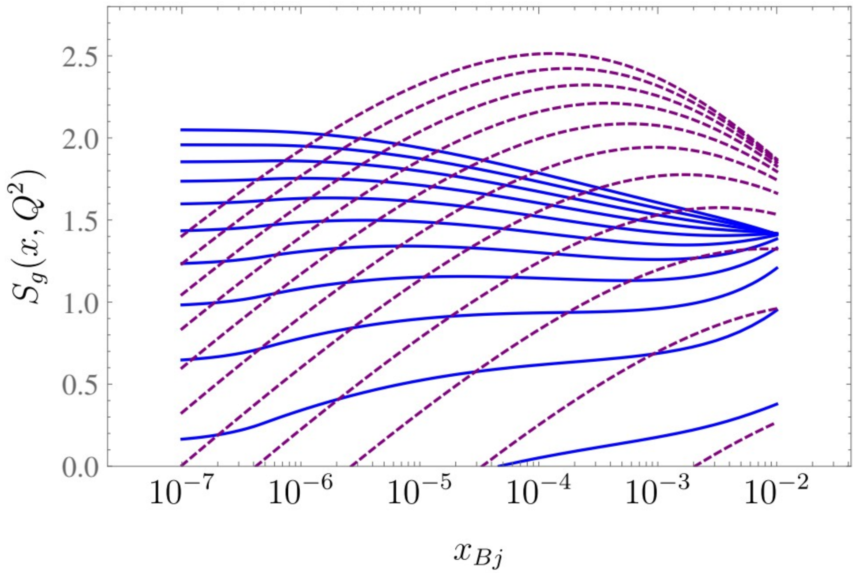




See also Hagivara, Hatta, Xiao ' 18
The genaralized KL model is used and entropy saturates in this approach
and Nowak, Liu, Zahed '22

Integrated gluon and entropy





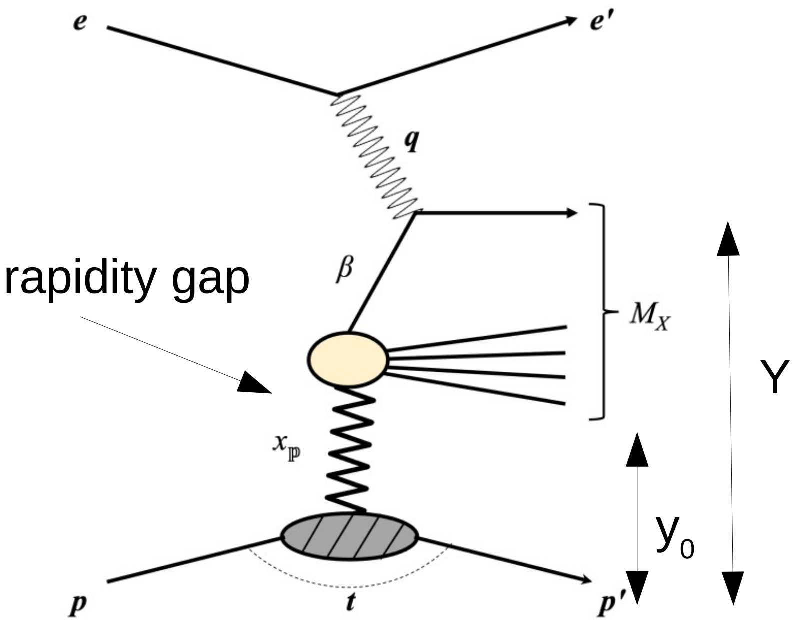
$$\lim_{Q^2 \gg Q_s^2} S(x,Q^2) = \ln\left(S_\perp Q_s^2(x)\right) + \ln\frac{N_c}{8\alpha_s\pi^2} = \lambda \ln\frac{1}{x} + \text{const}$$

$$\lim_{Q^2 \ll Q_s^2} S(x,Q^2) = \ln\left(\frac{S_\perp Q^4}{Q_s^2(x)}\right) + \ln\frac{N_c}{16\alpha_s\pi^2}$$

Photon can not resolve proton anymore therefore the EE vanishes.
But it might be that the formalism breaks down for low scales.
There might be another source of entropy that keep the total entropy not vanishing → generalized second law Bekenstein

H. Hentschinski, D. Kharzeev. K. Kutak, Z. Tu ‘23

EE in Diffractive Deep Inelastic Scattering



$x_{\mathbb{P}}$ proton’s momentum fraction carried by the Pomeron

β denotes the Pomeron’s momentum fraction carried by the quark interacting with the virtual photon

$x = \beta \cdot x_{\mathbb{P}}$ Bjorken x

$y_0 \simeq \ln 1/x_{\mathbb{P}}$ size of rpidity gap

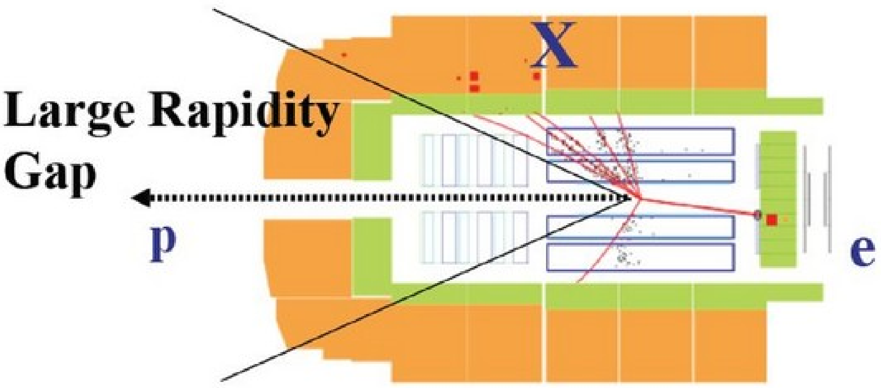
$Y = \ln 1/x$

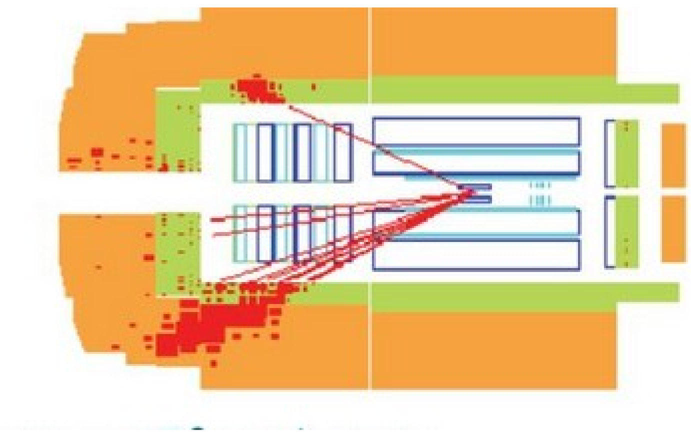
$y_X = Y - y_0 \simeq \ln 1/\beta$

Analogous evolution equation as for non-diffractive case but Initial conditions are different and there is delay because of rapidity gap.
Munier, Mueller Phys. Rev. D 98, 034021 (2018)

See also Peschanski, Seki’19 for entanglement in diffraction in p-p

Diffraction vs. nondiffraction

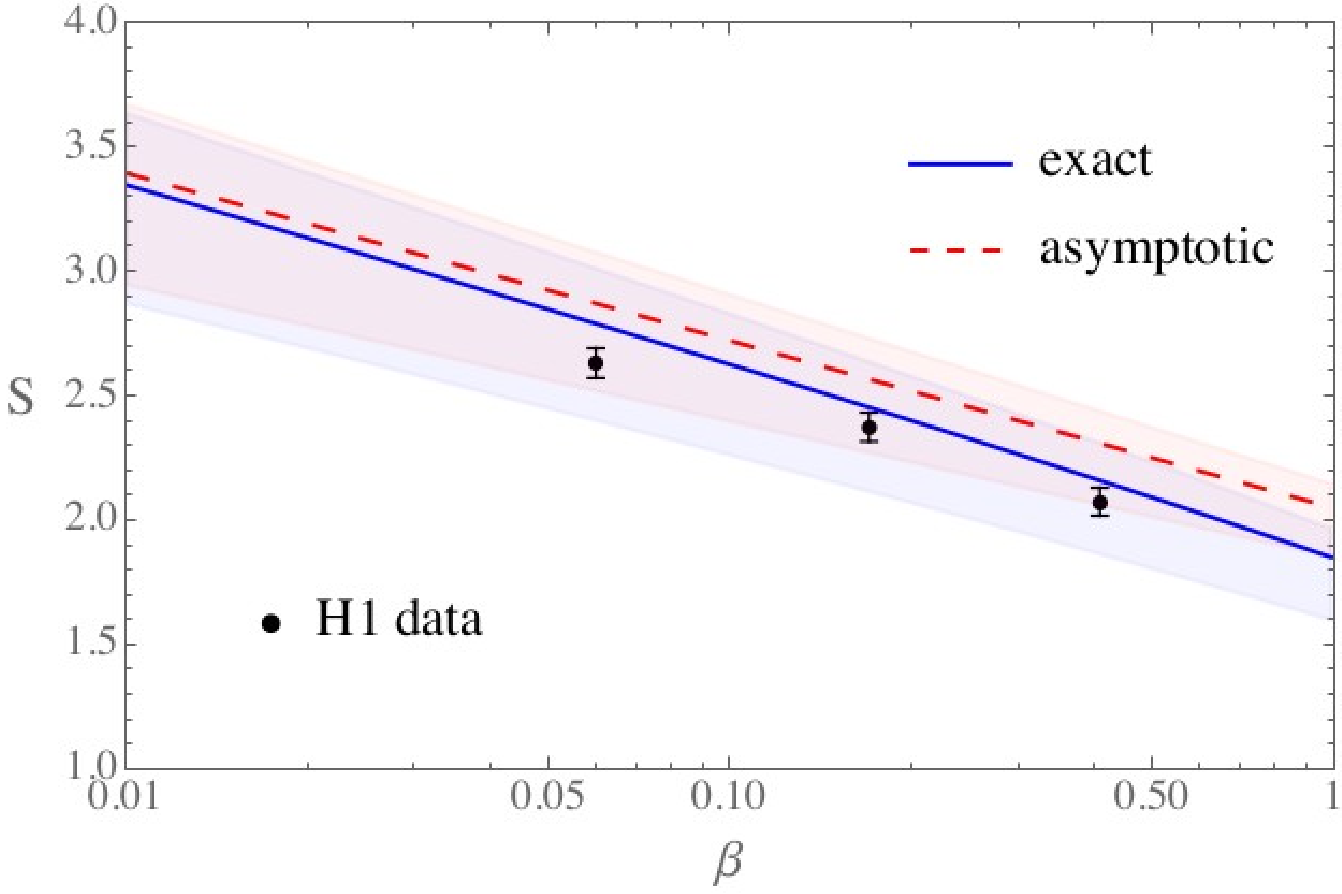




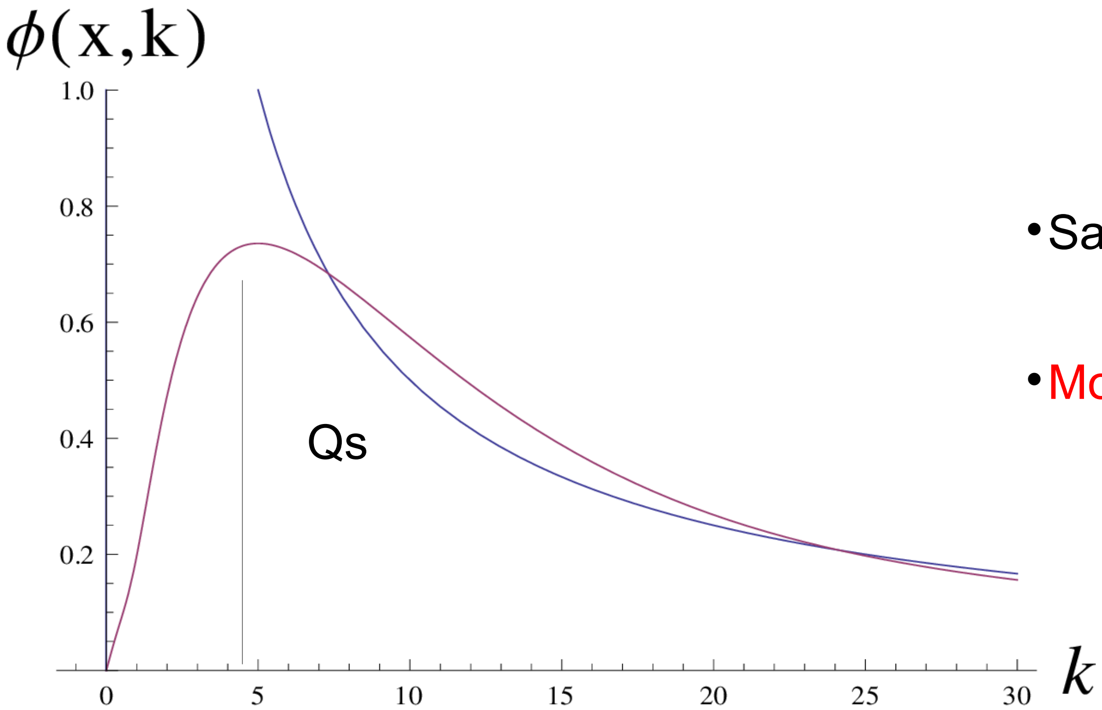
H1 detector

H. Hentschinski, D. Kharzeev. K. Kutak, Z. Tu ‘23

EE in DDIS

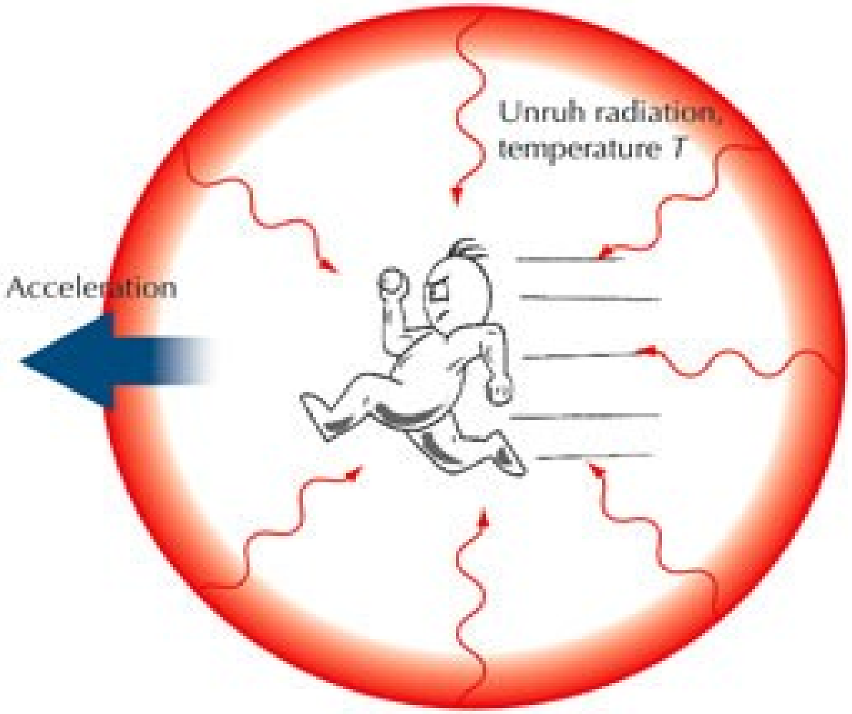


Saturation and gluon density



- Saturation scale regulates the divergence
- Most of gluons have momentum of the order of Qs

Unruh effect



Accelerated observer in its rest frame feels thermal radiation or Bose-Einstein distribution with temperature

$$T = \frac{|a|}{2\pi}$$

Entropy

The relation $T = \frac{Q_s(x)}{2\pi}$ Can be understood in a generalized sense i.e. that saturation scale defines some temperature.

Equilibrium thermodynamics relations $\longrightarrow$ Lower bound on produced entropy

It can be shown that the saturation line has an interpretation of a characteristics i.e. line along which the gluon density has a constant value.

Kutak 2011,
arxiv v1 and v2

$$dE = TdS$$

$$dE = dM$$

$$dM = TdS$$

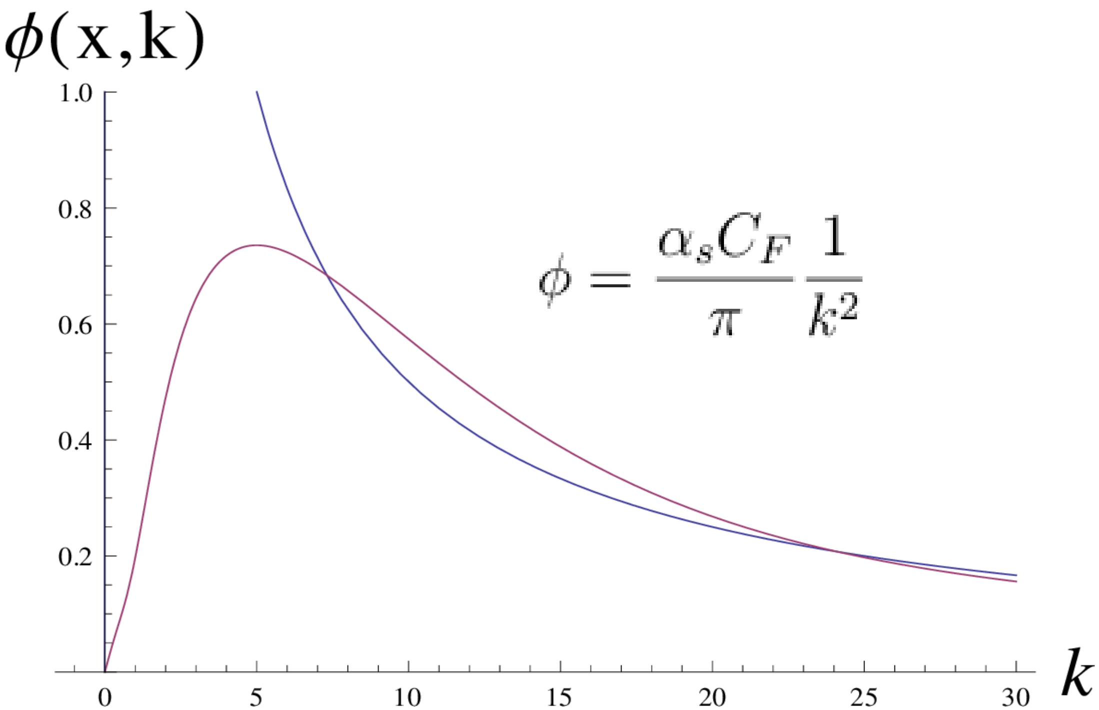
$$dM = dQ_s(x)$$

mass of system of gluons

$$\frac{dQ_s(x)}{Q_s(x)} = \frac{dS}{2\pi}$$

Gluon density and entropy

Kutak '11



$$\Delta S = \pi\lambda\Delta y$$

$$\Delta y = \ln(x_0/x)$$

$$Q_s^2 = Q_0^2(x_0/x)^\lambda$$

Many-body interactions

Medium generated mass of gluon.
Framework of Hard Thermal Loops.

Similarly in QED. Cut on photon`s kt
Is equivalent to introducing mass.

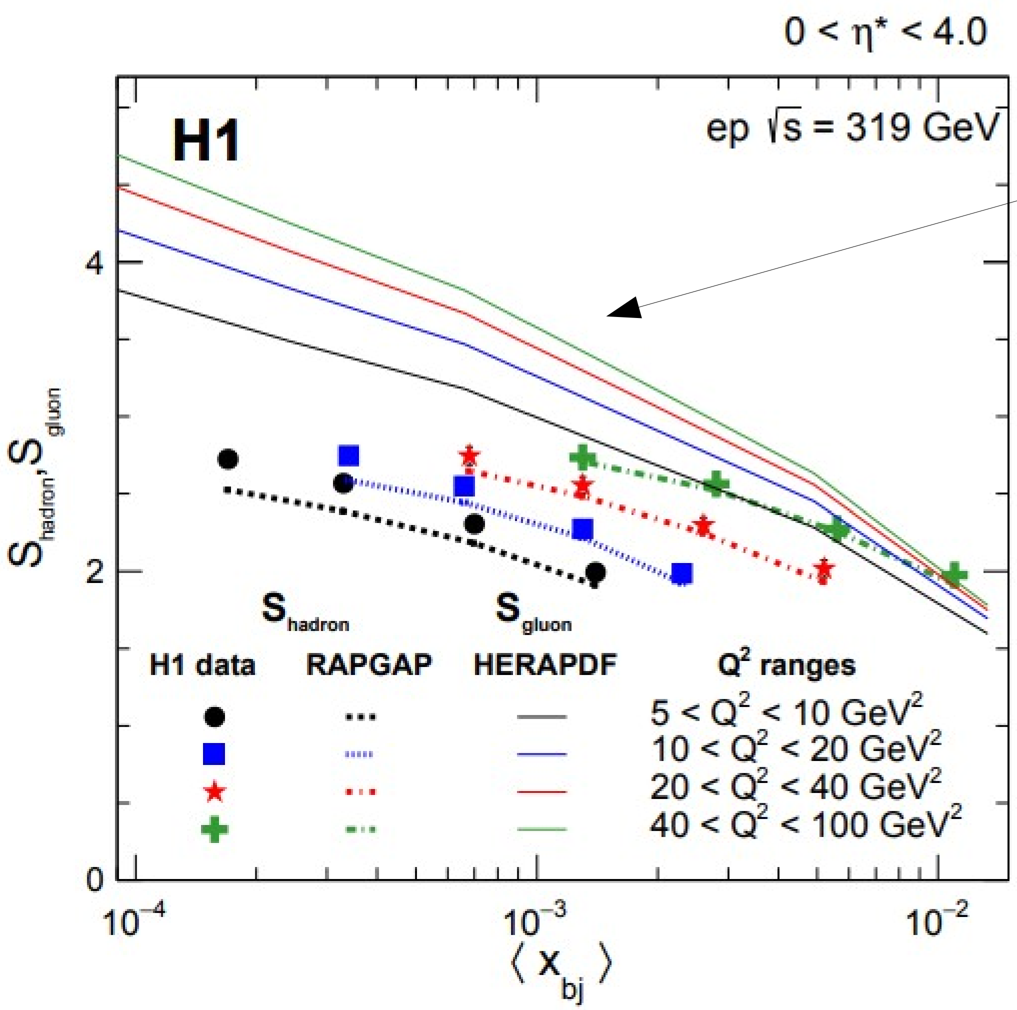
In presented approach mass is not fixed it is x dependent

Conclusions and outlook

- We show evidences for the proposal for low x maximal entanglement entropy of proton constituents .
- It can be systematically improved (quark contributions, NLO BFKL, rc BK) and can describe successfully H1 data.
- We obtain saturation of entropy at small resolution scales.
- We demonstrate that the proposal works for DDIS and that it can be used to study onset of maximal entanglement
- The thermodynamic based approach agrees with KL approach

Backup

Monte Carlo, KL formula, and data



HERA pdf used

$$S(x,Q) = \ln(xg(x,Q))$$

Also attempt by Kharzeev and Levin to use quarks instead of gluons Phys. Rev. D 104, 031503 (2021)

$$S(x,Q) = \ln(x\Sigma(x,Q))$$

This argument is however based on incorrect formula...but it is a illuminating mistake

H1

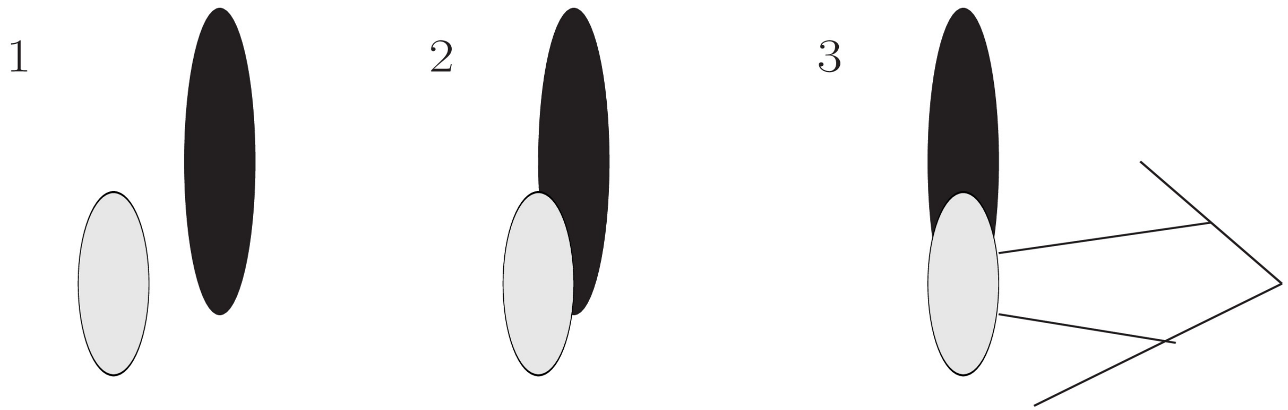
Eur.Phys.J.C 81 (2021) 3, 212

See also Z. Tu, D. Kharzeev, T. Ulrich '20 for calculations of EE in p-p.

Colliding hadrons and Unruh effect

Stages of collision

Kharzeev, Tuchin '05



$$P(M \leftarrow m) = 2\pi |\mathcal{T}(M \leftarrow m)|^2 \, \rho(M),$$

$$\int dM \, P(M \leftarrow m)$$

Probability for transition to final state

should be finite

density of states determined by typical momentum. Qs emerges

$$|\mathcal{T}(M \leftarrow m)|^2 \sim \exp(-2\pi M/a)$$

$$\frac{a}{2\pi} \equiv T \leq \frac{\sqrt{6}}{4\pi} \frac{1}{\sqrt{b}} \equiv T_{Hag}$$

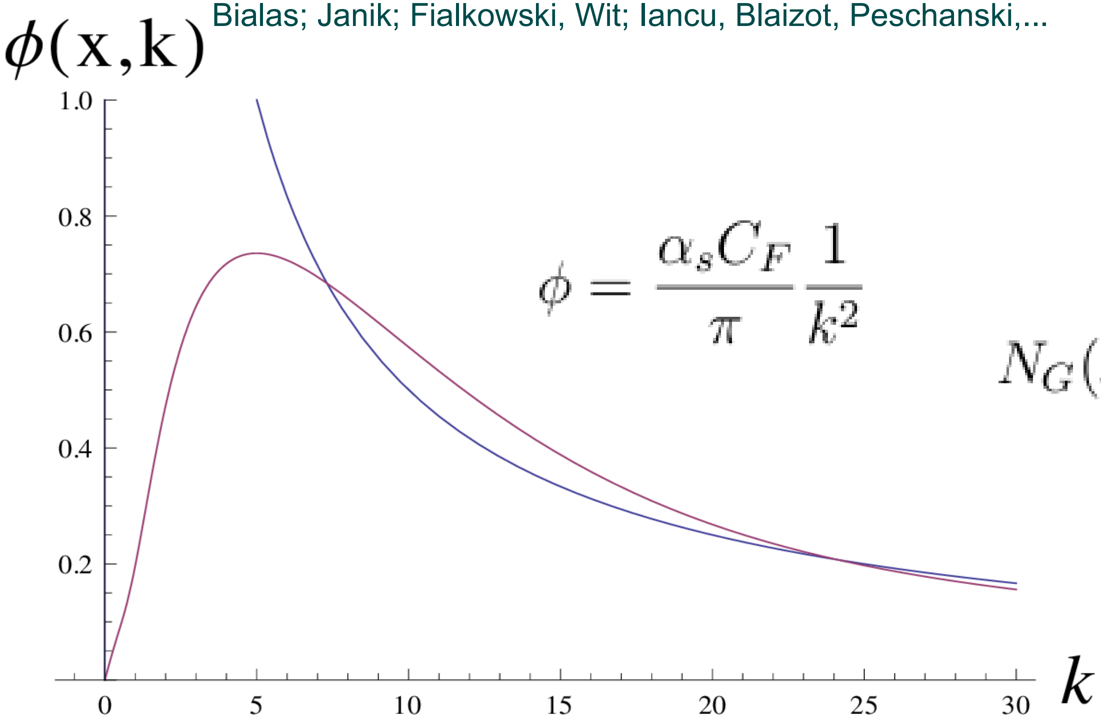
$$T = \frac{Q_s(x)}{2\pi}$$

transition amplitude

Gluon production and entropy – another assumptions

Kutak '11

Bialas; Janik; Fialkowski, Wit; Iancu, Blaizot, Peschanski,...



$$\phi = \frac{\alpha_s C_F}{\pi}\frac{1}{k^2}$$

$$M_G(x) = Q_s(x)$$ energy dependent gluon's mass

$$M(x) = N_G(x) M_G(x)$$ mass of system of gluons

$$N_G(x) \equiv \frac{dN}{dy} = \frac{1}{S_\perp}\frac{d\sigma}{dy}$$ number of gluons

$$dE = TdS$$

$$dM = TdS$$

$$d\left[N_G(x)\, M_G(x)\right] = \frac{Q_s(x)}{2\pi} dS$$

Many-body interactions

Medium generated mass of gluon.
Framework of Hard Thermal Loops.

Entropy due to less dense hadron

$$S = \frac{6 C_F A_\perp}{\pi \alpha_s} Q_s^2(x) + S_0$$

$$S = 3\pi\left[N_G(x) + N_{G0}\right]$$

Similarly in QED. Cut on photon`s kt
Is equivalent to introducing mass.

In presented approach mass is not fixed it is x dependent