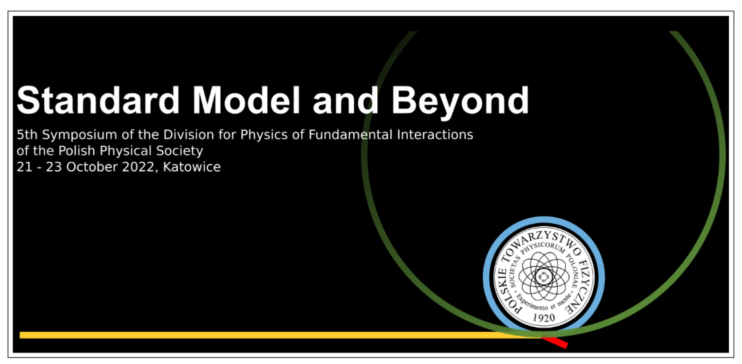


Phenomenological Aspects of Discrete Flavor Symmetries

Biswajit Karmakar

Institute of Physics, University Silesia in Katowice, Poland

Abstract

We discuss neutrino mass and mixing models based on discrete flavor symmetries. These models can include a variety of new interactions and non-standard particles such as sterile neutrinos, scalar Higgs singlets and multiplets. We point at connections of the models with leptogenesis and dark matter and the ways to detect the corresponding non-standard particles at intensity and energy frontier experiments.

Introduction

- Almost twenty-five years ago SNO and Super-Kamiokande experiments proved that neutrinos have mass and mixing.
- The theory behind the observed neutrino masses and mixing still remains a puzzle to us.
- Parameters involved in three flavor neutrino oscillation are: atmospheric mixing angle θ_{23}, solar mixing angle θ_{12}, reactor mixing angle θ_{13}, solar mass-squared difference $\Delta m^2_{\rm sol}$, atmospheric mass-squared difference $\Delta m^2_{\rm atm}$ and Dirac CP phase $\delta_{\rm CP}$.

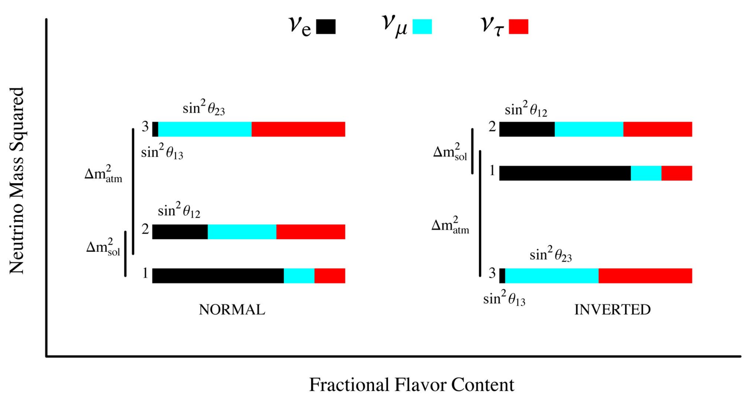
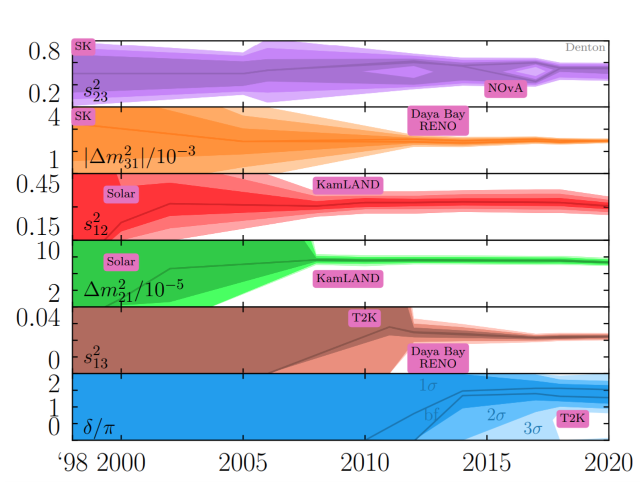


Figure 1: Possible ways the neutrino flavors ν_e, ν_μ, ν_τ mix to form three massive neutrinos. Left : Normal Hierarchy, Right : Inverted Hierarchy (Image: cerncourier.com).

Evolution and preset status of oscillation parameters



From talk by Peter B. Denton at Neutrino 2022

Parameters	Best-fit	3σ range
$\theta_{12}/^\circ$	33.45	31.27 - 35.87
$\theta_{13}/^\circ$	8.62	8.25 - 9.98
	8.61	8.24 - 9.02
$\theta_{23}/^\circ$	42.1	39.7 - 50.9
	49.0	39.8 - 51.6
$\frac{\Delta m^2_{21}}{10^{-5}{\rm eV}^2}$	7.72	6.82 - 8.04
$\frac{\lvert\Delta m^2_{31}\rvert}{10^{-3}{\rm eV}^2}$	2.51	2.43 - 2.60
	2.49	2.41 - 2.57

Data by nu-fit.org

- Standard Model is unable to dictate the flavor structure of the Yukawa couplings.
- The mixing in the lepton sector exhibits a completely different pattern compared to the quark sector.
- Can discrete symmetry play any role here?

Neutrino mixing schemes prior 2010

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix}$$

$$\Downarrow$$

$$\theta_{23} = 45^\circ,\ \theta_{13} = 0$$

$$\Downarrow$$

$$U_0 = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}.$$

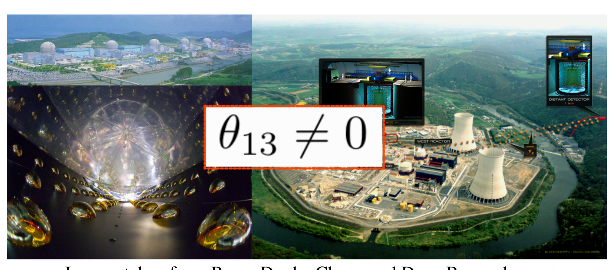
$\theta_{12} = 45^\circ$	$\theta_{12} = 35.26^\circ$	$\theta_{12} = 31.7^\circ$	$\theta_{12} = 30^\circ$
Bimaximal	Tribimaximal	Golden Ratio	Hexagonal

$$U_0 :: \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \frac{\varphi}{\sqrt{2+\varphi}} & \frac{1}{\sqrt{2+\varphi}} & 0 \\ \frac{-1}{\sqrt{4+2\varphi}} & \frac{\varphi}{\sqrt{4+2\varphi}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{4+2\varphi}} & \frac{-\varphi}{\sqrt{4+2\varphi}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \sqrt{\frac{3}{4}} & \frac{1}{2} & 0 \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

- **Example:** A $\mu-\tau$ symmetric neutrino mass matrix can be diagonalized using $m_\nu = U_0^\star {\rm diag}\,(m_1, m_2, m_3) U_0^\dagger$ of the form

$$m_\nu = \begin{pmatrix} A & B & B \\ B & C & D \\ B & D & C \end{pmatrix},$$

where the elements A, B, C and D in Eq. (1) are in general complex. With $A+B = C+D$ this matrix yields Tribimaximal mixing pattern where $\theta_{23} = 45^\circ$, $\theta_{13} = 0$.



Images taken from Reno, Doube Chooz and Daya Bay webpages

- Finite non-Abelian discrete groups such as S_3, A_4, S_4, A_5, T', $\Delta(27)$, D_n, T_7, $\Delta(6n^2)$ have been extensively used to explain various fixed mixing schemes.
- How to go beyond fixed mixing schemes? What implication do they have in cosmology, collider physics, etc?

General Framework

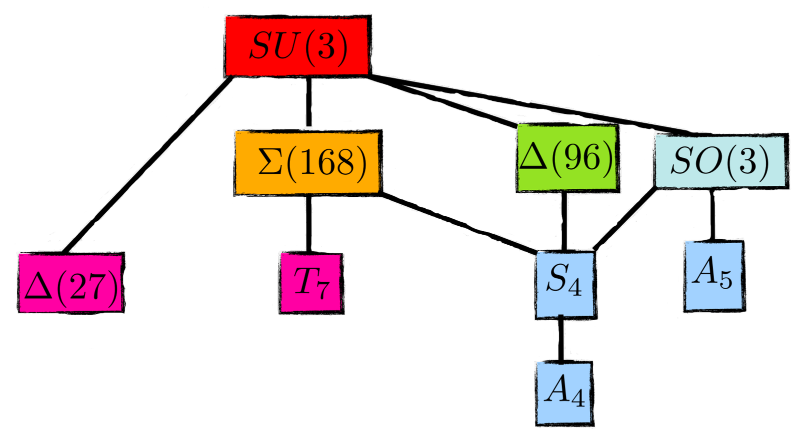


Figure 2: Examples of subgroups of SU (3) with triplet representations [3]

- Let G be the underlying flavor symmetry where the group elements are generated by the generators (say) S, T, and U.

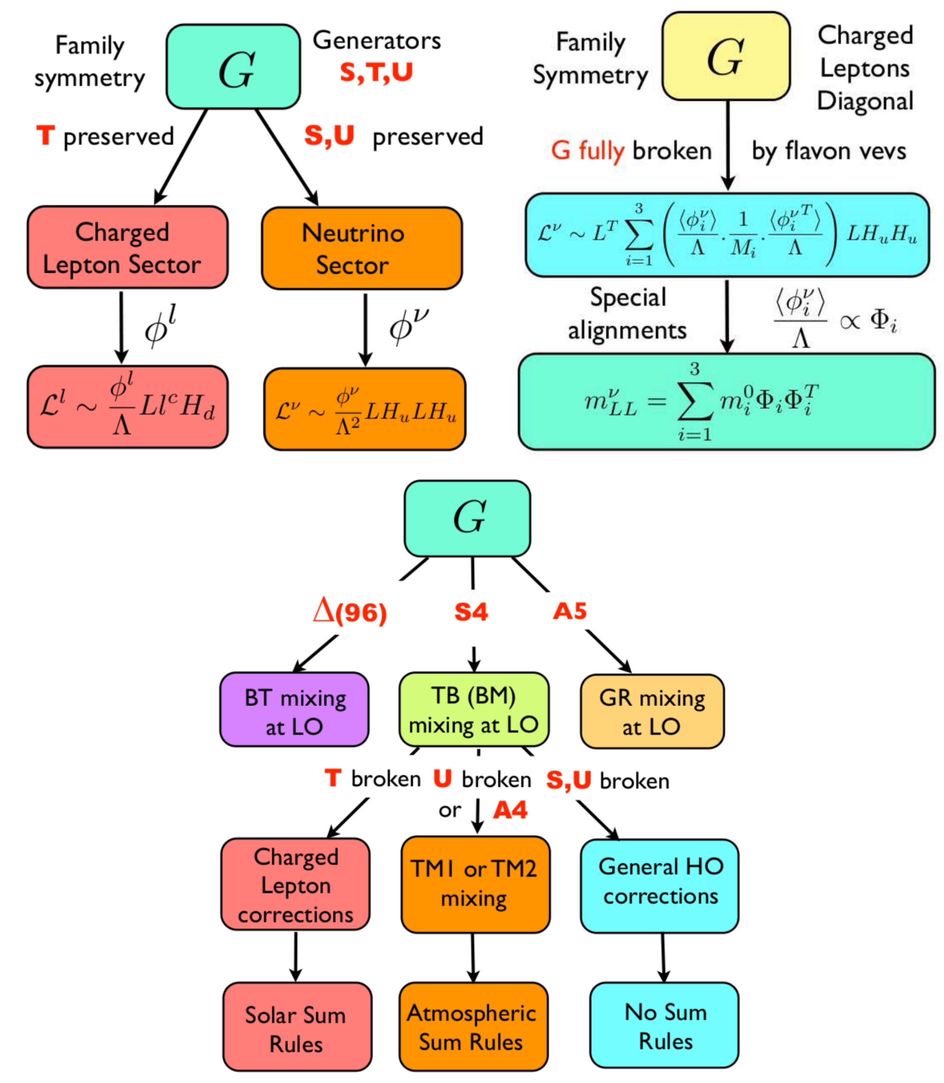


Figure 3: Direct vs indirect vs semi-direct model building [3]

- Surviving schemes with $\theta_{13} \neq 0$: Trimaximal, Cobimaxial mixings

$$|U_{\rm TM_1}| = \begin{pmatrix} \frac{2}{\sqrt{6}} & * & * \\ \frac{1}{\sqrt{6}} & * & * \\ \frac{1}{\sqrt{6}} & * & * \end{pmatrix} \quad |U_{\rm TM_2}| = \begin{pmatrix} * & \frac{1}{\sqrt{3}} & * \\ * & \frac{1}{\sqrt{3}} & * \\ * & \frac{1}{\sqrt{3}} & * \end{pmatrix}, U_{\rm CMB} = \begin{pmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ v_1^* & v_2^* & v_3^* \end{pmatrix}$$

,

- Example : Let us consider $G_f = S_4$. The generators S, T and U satisfies the relation

$$S^2 = T^3 = U^2 = 1 \ \text{ and } \ ST^3 = (SU)^2 = (TU)^2 = 1.$$

In their irreducible triplet representations, these three generators can be written as

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}; T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix} \ \text{ and } \ U = \mp \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}.$$

If S_4 is considered to be broken spontaneously into $Z_3 = \{1, T, T^2\}$ (for the charged lepton sector) $Z_2 = \{1, SU\}$ (for the neutrino sector) such that it satisfies

$$[T, M_\ell^\dagger M_\ell] = [SU, M_\nu] = 0,$$

where M_ℓ and M_ν are charged lepton and neutrino mass matrix, and the effective mixing matrix can be written as

$$U_{\rm TM_1} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{c_\theta}{\sqrt{3}} & \frac{s_\theta}{\sqrt{3}}e^{-i\gamma} \\ -\frac{1}{\sqrt{6}} & \frac{c_\theta}{\sqrt{3}} - \frac{s_\theta}{\sqrt{2}}e^{i\gamma} & -\frac{s_\theta}{\sqrt{3}}e^{-i\gamma} - \frac{c_\theta}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{c_\theta}{\sqrt{3}} - \frac{s}{\sqrt{2}}e^{i\gamma} & -\frac{s_\theta}{\sqrt{3}}e^{-i\gamma} + \frac{c_\theta}{\sqrt{2}} \end{pmatrix},$$

where $c_\theta = \cos\theta$ and $s_\theta = \sin\theta$.

A_4 flavor Symmetric Scoto-Seesaw:

- Type-I Seesaw Contribution:

$$\mathcal{L}_N = \frac{y_{N_1}}{\Lambda}(\bar{L}\phi_s)\tilde{H}N_{R_1} + \frac{y_{N_2}}{\Lambda}(\bar{L}\phi_a)\tilde{H}N_{R_2} + \frac{1}{2}M_{N_1}\bar{N}^c_{R_1}N_{R_1} + \frac{1}{2}M_{N_2}\bar{N}^c_{R_2}N_{R_2}$$

- The scotogenic contribution with the fermion f and scalar field η :

$$\mathcal{L}_S = \frac{y_s}{\Lambda^2}(\bar{L}\phi_s)\xi i\sigma_2\eta^* f + \frac{1}{2}M_f\bar{f}^c f + h.c.,$$

- Effective mass matrix reads:

$$M_\nu = (M_\nu)_{\rm TREE} + (M_\nu)_{\rm LOOP} = \begin{pmatrix} -B+C & -B & -B-C \\ -B & -(A+B) & A-B \\ -B-C & A-B & -A-B+C \end{pmatrix}.$$

- After rotation by tribimaximal mixing matrix

$$M'_\nu = U^T_{TB}M_\nu U_{TB} = \frac{1}{2}\begin{pmatrix} 3C & 0 & -\sqrt{3}C \\ 0 & -6B & 0 \\ -\sqrt{3}C & 0 & -4A+C \end{pmatrix}.$$

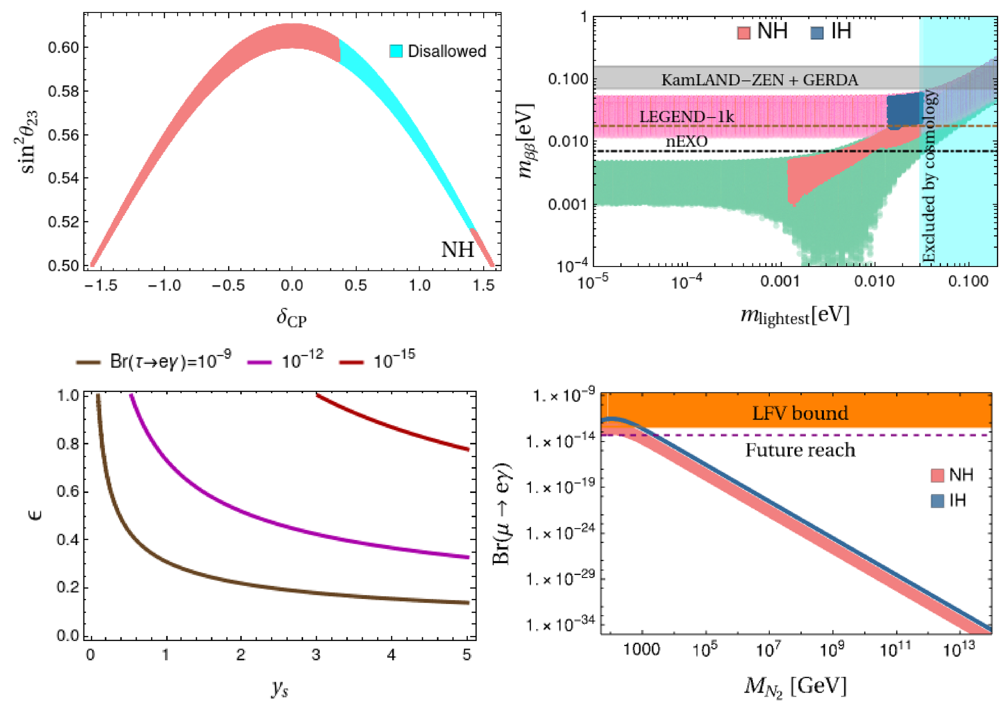
- The full diagonalization relation of the mass matrix M_ν can be written as

$$(U_{TB}U_{13})^T M_\nu U_{13}U_{TB} = {\rm diag}(m_1e^{i\gamma_1}, m_2e^{i\gamma_2}, m_3e^{i\gamma_3}),$$

- Effective mixing matrix takes ${\rm TM}_2$ form

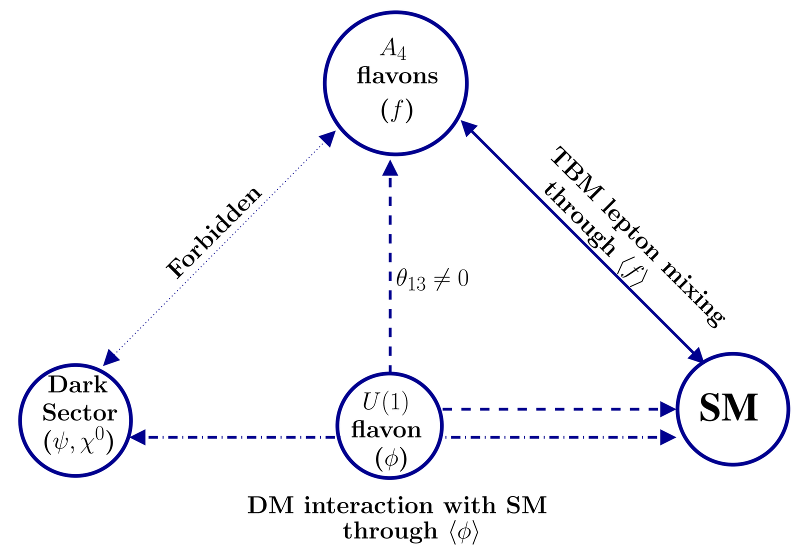
$$U_{\rm TM_2} \sim \begin{pmatrix} \sqrt{\frac{2}{3}}\cos\theta & \frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}}e^{i\phi}\sin\theta \\ -\frac{\cos\theta}{\sqrt{6}} + \frac{e^{i\phi}\sin\theta}{\sqrt{2}} & \frac{1}{\sqrt{3}} & -\frac{\cos\theta}{\sqrt{2}} - \frac{e^{i\phi}\sin\theta}{\sqrt{6}} \\ -\frac{\cos\theta}{\sqrt{6}} - \frac{e^{i\phi}\sin\theta}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{\cos\theta}{\sqrt{2}} - \frac{e^{i\phi}\sin\theta}{\sqrt{6}} \end{pmatrix}$$

- Predictions:



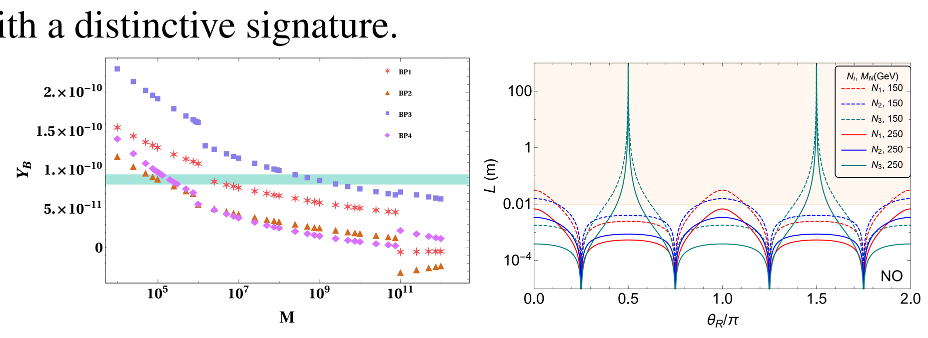
Flavor symmetry and dark matter

- Discrete flavor symmetric constructions can explain neutrino masses and mixing as well as can ensure the stability of dark matter.



Leptogenesis and collider physics

- Discrete flavor symmetry dictates the structures of fermion mass matrices hence leaves an imprint on leptogenesis.
- Flavor symmetries can be probed in collider experiments with a distinctive signature.



Conclusions and Future Directions

- Discrete flavor symmetries may be crucial in understanding the puzzle associated with Standard Model flavor.
- A key challenge lies in distinguishing the wide variety of flavor symmetric models.
- Discrete flavor symmetries may also have consequential implications in intensity, collider, and cosmic frontiers.
- Non-Abelian discrete symmetries can emerge from extra dimensions, as a subgroup of the symmetry of the extra dimensional lattice vectors, commonly referred to as modular symmetry. Neutrino masses might be of modular forms, with constraints on the Yukawa couplings, which lead to various phenomenological possibilities.

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