Quantum correction to a new wilson-line based action for gluodynamics

Based on H. Kakkad, P. Kotko, A. Stasto, arXiv:2208.11000, arXiv:2102.11371 and arXiv:2006.16188.

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NCN GRANT 2021/41/N/ST2/02956

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"Standard Model and Beyond"
5th Symposium of the Division for Physics of Fundamental Interactions of the Polish Physical Society

October 21-23, 2022

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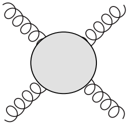
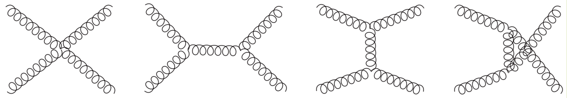
Scattering Amplitudes : Feynman Diagram Technique

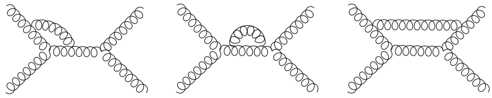
Example : Four point amplitude.



Tree Amplitude



Loop Amplitude



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Problem with Feynman diagram technique.

The number of Feynman diagrams contributing to the amplitude of a gluon tree level $(g + g \longrightarrow ng)$ grows factorily.

n	2	3	4	5	6	7	8
# of diagrams	4	25	220	2485	34300	559405	10525900

MHV Amplitudes

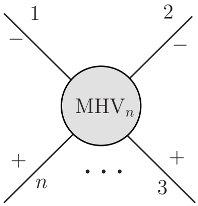
Maximally Helicity Violating

$$A_n^{tree}(\ldots, j^-, \ldots, l^-, \ldots) = \frac{\langle jl \rangle^4}{\langle 12 \rangle \cdots \langle n1 \rangle}.$$

$$\langle ij \rangle \equiv \epsilon^{\alpha\beta}(\lambda_i)_\alpha(\lambda_j)_\beta\,,$$

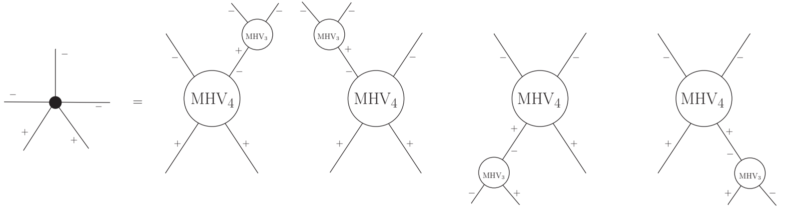
$$[ij] \equiv \epsilon^{\dot\alpha\dot\beta}(\tilde\lambda_i)_{\dot\alpha}(\tilde\lambda_j)_{\dot\beta}$$

Building blocks



5 point $(- - - + +)$ in CSW method

[F. Cachazo, P. Svrcek, E. Witten, 2004]



Lagrangian origin of MHV rules.

[P. Mansfield, 2006]

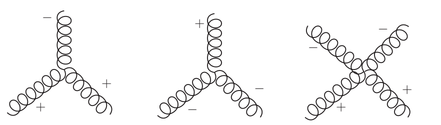
Basic Idea

$$S^{(\mathrm{LC})}_{\mathrm{Y-M}}\left[A^{+},A^{-}\right]=\left(\mathcal{L}^{(\mathrm{LC})}_{+-}+\mathcal{L}^{(\mathrm{LC})}_{++-}+\mathcal{L}^{(\mathrm{LC})}_{+--}+\mathcal{L}^{(\mathrm{LC})}_{++--}\right).$$

- Only plus-helicity and minus-helicity gluon fields.

$$\left\{A^{+},A^{-}\right\}\rightarrow\left\{B^{+},B^{-}\right\}$$

Interaction vertices



Transformation

$$\mathcal{L}^{(\mathrm{LC})}_{+-}+\mathcal{L}^{(\mathrm{LC})}_{++-}\longrightarrow\mathcal{L}^{(\mathrm{LC})}_{+-}$$

MHV action : Action with MHV vertices

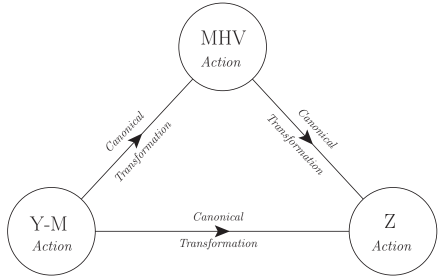
$$S^{(\mathrm{LC})}_{\mathrm{Y-M}}\left[B^{+},B^{-}\right]=\left(\mathcal{L}^{(\mathrm{LC})}_{+-}+\mathcal{L}^{(\mathrm{LC})}_{--+}+\cdots+\mathcal{L}^{(\mathrm{LC})}_{--+\cdots+}+\dots\right)$$

New action : Z-field action

$$\mathcal{L}_{+-}^{(\mathrm{LC})} + \mathcal{L}_{--+}^{(\mathrm{LC})} \longrightarrow \mathcal{L}_{+-}^{(\mathrm{LC})}$$

[H. Kakkad, P. Kotko, A. Stasto, 2021]- arXiv :2102.11371

Z Action



Solution of field transformation

At the poster !

Z Action

Structure of the new action

$$S^{(\mathrm{LC})}_{\mathrm{Y-M}}\left[Z^{+},Z^{-}\right]=\Big\{\mathcal{L}^{(\mathrm{LC})}_{-+}+\mathcal{L}^{(\mathrm{LC})}_{--++}+\mathcal{L}^{(\mathrm{LC})}_{--+++}+\mathcal{L}^{(\mathrm{LC})}_{--++++}+\dots$$
$$+\mathcal{L}^{(\mathrm{LC})}_{---++}+\mathcal{L}^{(\mathrm{LC})}_{---+++}+\mathcal{L}^{(\mathrm{LC})}_{---++++}+\dots$$
$$\vdots$$
$$+\mathcal{L}^{(\mathrm{LC})}_{---\dots-++}+\mathcal{L}^{(\mathrm{LC})}_{---\dots-+++}+\mathcal{L}^{(\mathrm{LC})}_{---\dots-++++}+\dots\Big\}$$

Important features

- There are no three point interaction vertices.
- At the classical level there are no all-plus, all-minus, as well as $(-+\cdots+)$, $(-\cdots-+)$ vertices.
- There are MHV vertices, $(--+\cdots+)$, corresponding to MHV amplitudes in the on-shell limit.
- There are $\overline{\mathrm{MHV}}$ vertices, $(-\cdots-++)$, corresponding to $\overline{\mathrm{MHV}}$ amplitudes in the on-shell limit.
- All vertices have the form which can be easily calculated.

Amplitudes Overview

# legs	helicity	# diagrams
4 point	MHV	1
	$\overline{\text{MHV}}$	1
5 point	MHV	1
	$\overline{\text{MHV}}$	1
6 point	MHV	1
	NMHV	3
	$\overline{\text{MHV}}$	1
7 point	MHV	1
	NMHV	5
	NNMHV	5
	$\overline{\text{MHV}}$	1
8 point	MHV	1
	NMHV	7
	NNMHV	13
	NNNMHV	7
	$\overline{\text{MHV}}$	1

Delannoy Numbers

The Delannoy numbers D(a,b) are the number of lattice paths from (0,0) to (b,a) in which only east (1, 0), north (0, 1), and northeast (1, 1) steps are allowed.

n \ m	0	1	2	3	4	5	6	7	8
0	1	1	1	1	1	1	1	1	1
1	1	3	5	7	9	11	13	15	17
2	1	5	13	25	41	61	85	113	145
3	1	7	25	63	129	231	377	575	833
4	1	9	41	129	321	681	1289	2241	3649
5	1	11	61	231	681	1683	3653	7183	13073
6	1	13	85	377	1289	3653	8989	19825	40081
7	1	15	113	575	2241	7183	19825	48639	108545
8	1	17	145	833	3649	13073	40081	108545	265729
9	1	19	181	1159	5641	22363	75517	224143	598417

Figure – The Delannoy numbers. $m+2$ and $n+2$ represents the total number of positive and negative helicities in a $m+n+4$ point split helicity tree amplitude.

Immediate issues

- Missing terms in the action.
- One-loop $(++\cdots+)$ amplitudes cannot be calculated within the Z-field action, since every vertex has at least two $+$ and two $-$ helicity fields.

The technique of *Effective Action* to systematically develop loop amplitudes.

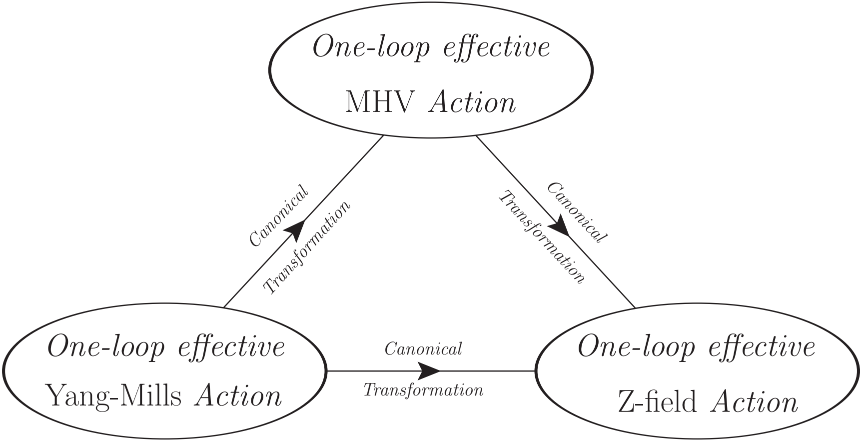
One-Loop Effective action

$$Z_{\mathrm{YM}}[J] = \int [dA]\, e^{i\left(S_{\mathrm{YM}}[A]+\int d^4x\, \mathrm{Tr}\hat{J}_i(x)\hat{A}^i(x)\right)}\,,$$

- Expand the action, up to second order in fields, around the classical solution.
- The higher order terms are necessary for corrections beyond one loop.
- The linear term vanishes due to the classical equations of motion, whereas the integration over the quadratic term gives

$$Z_{\mathrm{YM}}[J] \approx \exp\left\{ iS_{\mathrm{YM}}[A_c] + i\int d^4x\, \mathrm{Tr}\hat{J}_i(x)\hat{A}^i_c(x) - \frac{1}{2}\mathrm{Tr}\ln\left(\frac{\delta^2 S_{\mathrm{YM}}[A_c]}{\delta\hat{A}^i(x)\delta\hat{A}^j(y)}\right)\right\}$$

[H. Kakkad, P. Kotko, A. Stasto]- arXiv :2208.11000

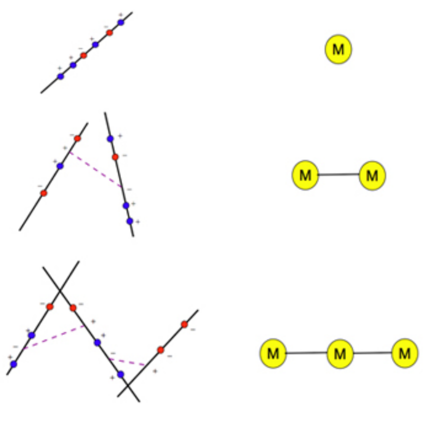


Thank You for your Time !

I invite you to my poster.

Back-up

Geometrical Origin



Correspondence

Lines in Twistor space $\Longleftrightarrow$ Points in Minkowski Space

Yang-Mills action on the light cone

Yang-Mills action

$$S_{\mathrm{Y-M}} = -\frac{1}{4}\int d^4x\, \mathrm{Tr} F^{\mu\nu} F_{\mu\nu}$$

- Two light like four-vectors : $\eta = \frac{1}{\sqrt{2}}(1,0,0,-1) \quad \tilde{\eta} = \frac{1}{\sqrt{2}}(1,0,0,1)$
- Two complex transverse four-vectors : $\varepsilon_{\perp}^{\pm} = \frac{1}{\sqrt{2}}(0,1,\pm i,0)$
- The components of a four-vector v

$$v^{+} = v\cdot\eta \quad v^{-} = v\cdot\tilde{\eta} \quad v^{\bullet} = v\cdot\varepsilon_{\perp}^{+} \quad v^{\star} = v\cdot\varepsilon_{\perp}^{-}$$

- Light-cone gauge : $A\cdot\eta = A^{+} = 0$.
- Action becomes quadratic in A^{-}, can be integrated out.

Only gluon scattering amplitudes.

[J. Scherk and J.H. Schwarz, 1975]

Yang-Mills action

$$S^{(\mathrm{LC})}_{\mathrm{Y-M}}\left[A^{\bullet}, A^{\star}\right] = \int dx^{+} \left(\mathcal{L}^{(\mathrm{LC})}_{+-} + \mathcal{L}^{(\mathrm{LC})}_{++-} + \mathcal{L}^{(\mathrm{LC})}_{+--} + \mathcal{L}^{(\mathrm{LC})}_{++--}\right)$$

$$\mathcal{L}^{(\mathrm{LC})}_{+-}\left[A^{\bullet}, A^{\star}\right] = -\int d^3\mathbf{x}\, \mathrm{Tr}\, \hat{A}^{\bullet}\square\hat{A}^{\star}$$

$$\mathcal{L}^{(\mathrm{LC})}_{++-}\left[A^{\bullet}, A^{\star}\right] = -2ig' \int d^3\mathbf{x}\, \mathrm{Tr}\, \gamma_{\mathbf{x}}\hat{A}^{\bullet}\left[\partial_{-}\hat{A}^{\star}, \hat{A}^{\bullet}\right]$$

$$\mathcal{L}^{(\mathrm{LC})}_{--+}\left[A^{\bullet}, A^{\star}\right] = -2ig' \int d^3\mathbf{x}\, \mathrm{Tr}\, \overline{\gamma}_{\mathbf{x}}\hat{A}^{\star}\left[\partial_{-}\hat{A}^{\bullet}, \hat{A}^{\star}\right]$$

$$\mathcal{L}^{(\mathrm{LC})}_{++--}\left[A^{\bullet}, A^{\star}\right] = -g^2 \int d^3\mathbf{x}\, \mathrm{Tr}\, \left[\partial_{-}\hat{A}^{\bullet}, \hat{A}^{\star}\right]\partial_{-}^{-2}\left[\partial_{-}\hat{A}^{\star}, \hat{A}^{\bullet}\right]$$

$$\gamma_{\mathbf{x}} = \partial_{-}^{-1}\partial_{\bullet}, \quad \overline{\gamma}_{\mathbf{x}} = \partial_{-}^{-1}\partial_{\star}, \qquad g' = \frac{g}{\sqrt{2}}$$

This form accommodates the so-called Coulomb instantaneous interactions.

Gluonic scattering amplitudes

- Develop the Feynman rules from the action.
- Sum of all contributing diagrams built from the Feynman rules.

Color Decomposition

[F.A. Berends and W.T. Giele, 1987] ; [M. Mangano, S. Parke and Z. Xu, 1988] ; [M. Mangano, 1988] ; [Z. Bern and D.A. Kosower, 1991]

Color Decomposition

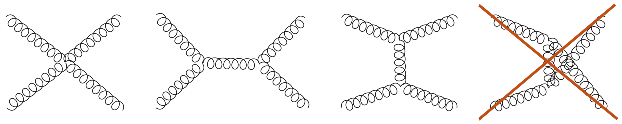
- Technique to disentangle the color and kinematical degrees of freedom in a gauge theory scattering amplitude.
- Lie Algebra structure constants in terms of generators T^a.

$$\tilde{f}^{abc} \equiv i\sqrt{2}f^{abc} = \mathrm{Tr}\left(T^a T^b T^c\right) - \mathrm{Tr}\left(T^a T^c T^b\right), \mathrm{Tr}(T^a T^b) = \delta^{ab}$$

- Fierz Identity systematically combines them into a single trace.
- n-gluon tree amplitudes :

$$\mathcal{A}_n^{tree}(\{k_i, h_i, a_i\}) = \sum_{\sigma \in S_n/Z_n} \mathrm{Tr}(T^{a_{\sigma(1)}} \cdots T^{a_{\sigma(n)}}) A_n^{tree}(\sigma(1^{h_1}), \ldots, \sigma(n^{h_n}))$$

Color ordered : Planar graphs with no leg-crossings allowed



Spinor Helicity Formalism

[P. De Causmaecker et.al. 82] ; [F. A. Berends et. al. 82] ; [R. Kleiss et. al. 85] ; [Z. Xu et. al. 87] ; [R. Gastmans et. al. 90]

Helicity Spinors

- Uniform description of the on-shell degrees of freedom (momentum and polarization).
- Spinors from massless Dirac equation.
- Kinematical DOF in terms of Spinors :
  - 4-Momentum in terms of Spinors.

$$k_i^{\mu}(\sigma_{\mu})_{\alpha\dot{\alpha}} \; = \; (\not{k}_i)_{\alpha\dot{\alpha}} \; = \; \begin{pmatrix} k_i^t + k_i^z & k_i^x - ik_i^y \\ k_i^x + ik_i^y & k_i^t - k_i^z \end{pmatrix} \; = \; (\lambda_i)_{\alpha}(\tilde{\lambda}_i)_{\dot{\alpha}} \, .$$

  - Polarization vectors also in terms of Spinors

- $$\langle ij\rangle \; \equiv \; \epsilon^{\alpha\beta}(\lambda_i)_{\alpha}(\lambda_j)_{\beta} \, , [ij] \; \equiv \; \epsilon^{\dot{\alpha}\dot{\beta}}(\tilde{\lambda}_i)_{\dot{\alpha}}(\tilde{\lambda}_j)_{\dot{\beta}} \, .$$
- Renders the analytic expressions of scattering amplitudes in an often much more compact form compared to the standard four-vector notation.
- In order to uniformize the description we shall take all particles as outgoing.

MHV Action

MHV Vertices (color stripped)

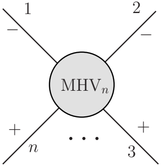
$$\mathcal{V}\left(1^-, 2^-, 3^+, \ldots, n^+\right) \equiv \left(\frac{p_1^+}{p_2^+}\right)^2 \frac{\widetilde{v}_{21}^{*4}}{\widetilde{v}_{1n}^{*}\widetilde{v}_{n(n-1)}^{*}\widetilde{v}_{(n-1)(n-2)}^{*}\ldots\widetilde{v}_{21}^{*}}$$

The $\widetilde{v}_{ij}$, $\widetilde{v}_{ij}^{\star}$ are directly related to spinor products $\langle ij\rangle$, $[ij]$.

$$\widetilde{v}_{ij}^{*} = p_i^+ \left(\frac{p_j^\bullet}{p_j^+} - \frac{p_i^\bullet}{p_i^+}\right) .$$

Important points

- The transformation results in only MHV vertices.
- The presence of one triple gluon vertex $(- - +)$.
- Interpretation of B-Fields ?



Wilson Line

$$\mathcal{W}[A](x,y) = \mathbb{P}\exp\left[ig\int_C dz_\mu\,\hat{A}^\mu(z)\right]$$

B - Fields as Wilson lines

[P. Kotko, 2014], [P. Kotko, A. Stasto, 2017]

$B^\bullet[A^\bullet]$

$$B_a^\bullet[A](x) = \int_{-\infty}^{\infty} d\alpha\,\mathrm{Tr}\left\{\frac{1}{2\pi g}t^a\partial_-\,\mathbb{P}\exp\left[ig\int_{-\infty}^{\infty} ds\,\varepsilon_\alpha^+\cdot\hat{A}\left(x+s\varepsilon_\alpha^+\right)\right]\right\}$$

$$\varepsilon_\alpha^+ = \varepsilon_\perp^+ - \alpha\eta, \quad \hat{A} = A_a t^a$$

[H. Kakkad, P. Kotko, A. Stasto, 2020]

$B^\star[A^\bullet, A^\star]$

$$B_a^\star(x) = \int d^3\mathbf{y}\left[\frac{\partial_-^2(y)}{\partial_-^2(x)}\frac{\delta B_a^\bullet(x^+;\mathbf{x})}{\delta A_c^\bullet(x^+;\mathbf{y})}\right]A_c^\star(x^+;\mathbf{y})$$

$$\widetilde{B}^{\bullet}_{a}(x^+;\mathbf{P}) = \sum_{n=1}^{\infty}\int d^3\mathbf{p}_1\ldots d^3\mathbf{p}_n\,\widetilde{\Gamma}_n^{a\{b_1\ldots b_n\}}(\mathbf{P};\{\mathbf{p}_1,\ldots,\mathbf{p}_n\})\prod_{i=1}^{n}\widetilde{A}^{\bullet}_{b_i}(x^+;\mathbf{p}_i)$$

$$\widetilde{B}^{\star}_{a}(x^+;\mathbf{P}) = \sum_{n=1}^{\infty}\int d^3\mathbf{p}_1\ldots d^3\mathbf{p}_n\,\widetilde{\Upsilon}_n^{ab_1\{b_2\ldots b_n\}}(\mathbf{P};\mathbf{p}_1,\{\mathbf{p}_2,\ldots,\mathbf{p}_n\})\widetilde{A}^{\star}_{b_1}(x^+;\mathbf{p}_1)\prod_{i=2}^{n}\widetilde{A}^{\bullet}_{b_i}(x^+;\mathbf{p}_i)$$

where

$$\widetilde{\Gamma}_n^{a\{b_1\ldots b_n\}}(\mathbf{P};\{\mathbf{p}_1,\ldots,\mathbf{p}_n\}) = (-g)^{n-1}\frac{\delta^3\left(\mathbf{p}_1+\cdots+\mathbf{p}_n-\mathbf{P}\right)\,\mathrm{Tr}\left(t^a t^{b_1}\ldots t^{b_n}\right)}{\widetilde{v}^{*}_{1(1\cdots n)}\widetilde{v}^{*}_{(12)(1\cdots n)}\cdots\widetilde{v}^{*}_{(1\cdots n-1)(1\cdots n)}}$$

$$\widetilde{\Upsilon}_n^{ab_1\{b_2\cdots b_n\}}(\mathbf{P};\mathbf{p}_1,\{\mathbf{p}_2,\ldots,\mathbf{p}_n\}) = n\left(\frac{p_1^+}{p_{1\cdots n}^+}\right)^2\widetilde{\Gamma}_n^{ab_1\ldots b_n}(\mathbf{P};\mathbf{p}_1,\ldots,\mathbf{p}_n)$$

$$\widetilde{A}^{\bullet}_{a}(x^+;\mathbf{P}) = \sum_{n=1}^{\infty} \int d^3\mathbf{p}_1 \dots d^3\mathbf{p}_n \, \widetilde{\Psi}_n^{a\{b_1\dots b_n\}}(\mathbf{P};\{\mathbf{p}_1,\dots,\mathbf{p}_n\}) \prod_{i=1}^{n} \widetilde{B}^{\bullet}_{b_i}(x^+;\mathbf{p}_i)$$

$$\widetilde{A}^{\star}_{a}(x^+;\mathbf{P}) = \sum_{n=1}^{\infty} \int d^3\mathbf{p}_1 \dots d^3\mathbf{p}_n \, \widetilde{\Omega}_n^{ab_1\{b_2\cdots b_n\}}(\mathbf{P};\mathbf{p}_1,\{\mathbf{p}_2,\dots,\mathbf{p}_n\}) \widetilde{B}^{\star}_{b_1}(x^+;\mathbf{p}_1) \prod_{i=2}^{n} \widetilde{B}^{\bullet}_{b_i}(x^+;\mathbf{p}_i)$$

where the kernels are

$$\widetilde{\Psi}_n^{a\{b_1\cdots b_n\}}(\mathbf{P};\{\mathbf{p}_1,\dots,\mathbf{p}_n\}) = -(-g)^{n-1} \, \frac{\widetilde{v}^{\star}_{(1\cdots n)1}}{\widetilde{v}^{\star}_{1(1\cdots n)}} \, \frac{\delta^3(\mathbf{p}_1+\cdots+\mathbf{p}_n-\mathbf{P}) \, \mathrm{Tr}(t^a t^{b_1}\cdots t^{b_n})}{\widetilde{v}^{\star}_{21}\widetilde{v}^{\star}_{32}\cdots\widetilde{v}^{\star}_{n(n-1)}}$$

$$\widetilde{\Omega}_n^{ab_1\{b_2\cdots b_n\}}(\mathbf{P};\mathbf{p}_1,\{\mathbf{p}_2,\dots,\mathbf{p}_n\}) = n\left(\frac{p_1^+}{p_{1\cdots n}^+}\right)^2 \widetilde{\Psi}_n^{ab_1\cdots b_n}(\mathbf{P};\mathbf{p}_1,\dots,\mathbf{p}_n)$$

Z-field Action

Structure of the new action

$$S^{(\mathrm{LC})}_{\mathrm{Y-M}}[Z^{\bullet},Z^{\star}] = \Big\{ \mathcal{L}^{(\mathrm{LC})}_{-+} + \mathcal{L}^{(\mathrm{LC})}_{--++} + \mathcal{L}^{(\mathrm{LC})}_{--+++} + \mathcal{L}^{(\mathrm{LC})}_{--++++} + \dots$$
$$+ \mathcal{L}^{(\mathrm{LC})}_{---++} + \mathcal{L}^{(\mathrm{LC})}_{---+++} + \mathcal{L}^{(\mathrm{LC})}_{---++++} + \dots$$
$$\vdots$$
$$+ \mathcal{L}^{(\mathrm{LC})}_{---\cdots-++} + \mathcal{L}^{(\mathrm{LC})}_{---\cdots-+++} + \mathcal{L}^{(\mathrm{LC})}_{---\cdots-++++} + \dots \Big\}$$

Example : $\mathcal{L}^{(\mathrm{LC})}_{----++}$

$$= \left(\frac{p_1^+}{p_2^+}\right)^2 \frac{\widetilde{v}^{*4}_{21}}{\widetilde{v}^{*}_{16}\,\widetilde{v}^{*}_{6(345)}\,\widetilde{v}^{*}_{(345)2}\,\widetilde{v}^{*}_{21}} \times \left(\frac{p_5^+}{p_{345}^+}\right)^2 \frac{\widetilde{v}_{(345)3}}{\widetilde{v}_{54}\,\widetilde{v}_{43}\,\widetilde{v}_{3(345)}}$$
$$+ \left(\frac{p_3^+}{p_4^+}\right)^2 \frac{\widetilde{v}^{*4}_{43}}{\widetilde{v}^{*}_{3(612)}\,\widetilde{v}^{*}_{(612)5}\,\widetilde{v}^{*}_{54}\,\widetilde{v}^{*}_{43}} \times \left(\frac{p_6^+}{p_{612}^+}\right)^2 \frac{\widetilde{v}_{(612)6}}{\widetilde{v}_{21}\,\widetilde{v}_{16}\,\widetilde{v}_{6(612)}} + \dots$$

Z-field Action

Important features

- There are no three point interaction vertices.
- At the classical level there are no all-plus, all-minus, as well as $(- + \cdots +)$, $(- \cdots - +)$ vertices.
- There are MHV vertices, $(- - + \cdots +)$, corresponding to MHV amplitudes in the on-shell limit.

$$\mathcal{A}\left(1^-, 2^-, 3^+, \dots, n^+\right) \equiv \left(\frac{p_1^+}{p_2^+}\right)^2 \frac{\widetilde{v}_{21}^{*4}}{\widetilde{v}_{1n}^{*}\widetilde{v}_{n(n-1)}^{*}\widetilde{v}_{(n-1)(n-2)}^{*}\dots\widetilde{v}_{21}^{*}}$$

- There are $\overline{\mathrm{MHV}}$ vertices, $(- \cdots - ++)$, corresponding to $\overline{\mathrm{MHV}}$ amplitudes in the on-shell limit.

$$\mathcal{A}\left(1^-, \dots, n-2^-, n-1^+, n^+\right) \equiv \left(\frac{p_{n-1}^+}{p_n^+}\right)^2 \frac{\widetilde{v}_{n(n-1)}^{4}}{\widetilde{v}_{1n}\widetilde{v}_{n(n-1)}\widetilde{v}_{(n-1)(n-2)}\dots\widetilde{v}_{21}}$$

- All vertices have the form which can be easily calculated.

Z field Wilson Line

Z - Fields as Wilson Line functionals

$Z^{\star}[B^{\star}]$

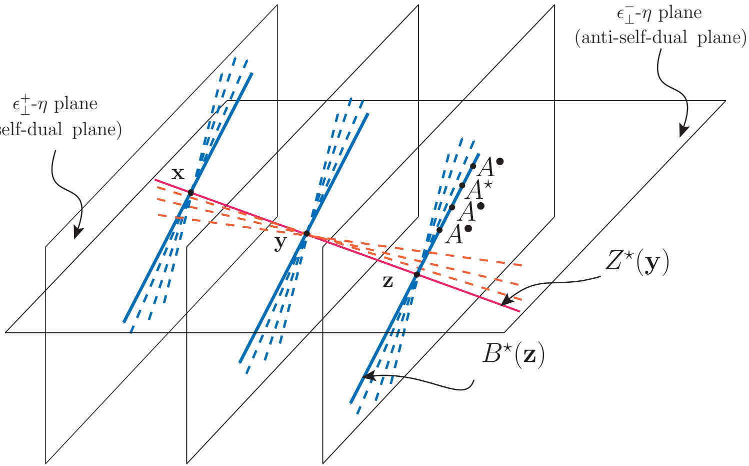
$$Z_a^{\star}[B^{\star}](x) = \int_{-\infty}^{\infty} d\alpha \, \mathrm{Tr} \left\{ \frac{1}{2\pi g} t^a \partial_- \, \mathbb{P} \exp \left[ig \int_{-\infty}^{\infty} ds \, \varepsilon_{\alpha}^{-} \cdot \hat{B} \left(x + s\varepsilon_{\alpha}^{-} \right) \right] \right\}$$

$$\varepsilon_{\alpha}^{-} = \varepsilon_{\perp}^{-} - \alpha\eta, \quad \hat{B} = B_a t^a$$

$Z^{\bullet}[B^{\bullet}, B^{\star}]$

$$Z_a^{\bullet}(x) = \int d^3\mathbf{y} \left[\frac{\partial_-^2(y)}{\partial_-^2(x)} \frac{\delta Z_a^{\star}(x^+; \mathbf{x})}{\delta B_c^{\star}(x^+; \mathbf{y})} \right] B_c^{\bullet}(x^+; \mathbf{y})$$

Geometrical Representation $Z^{\star}[B^{\star}]$



BCFW

BCFW Technique

- Complex shift the momentum of two consecutive legs such that the total momentum remains conserved.

$$\tilde{\lambda}_n \rightarrow \hat{\tilde{\lambda}}_n = \tilde{\lambda}_n - z\tilde{\lambda}_1\,, \qquad \lambda_n \rightarrow \lambda_n\,,$$
$$\lambda_1 \rightarrow \hat{\lambda}_1 = \lambda_1 + z\lambda_n\,, \qquad \tilde{\lambda}_1 \rightarrow \tilde{\lambda}_1$$

- Identify the location of complex poles.
- At poles the amplitude factorizes and the off-shell particle goes on-shell.
- Express amplitude in terms of residues.
- Apply Cauchy's residue theorem under the limit that the residue for large 'z' vanishes'.

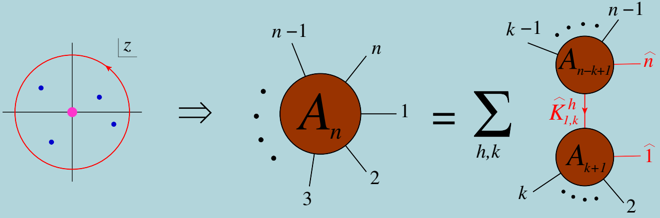


Figure – Illustration of how Cauchy's theorem leads to the BCFW recursion relation

THE END !