NATIONAL SCIENCE CENTRE
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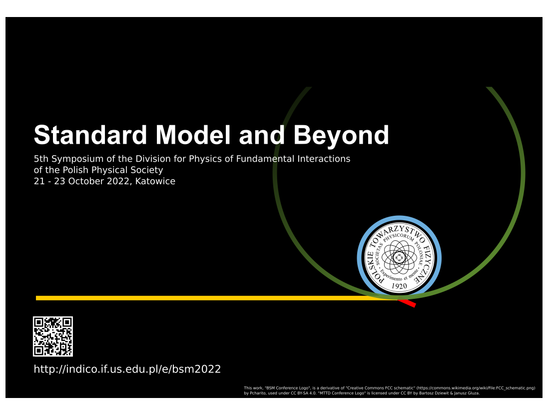
ADVANCES IN DARK MATTER PRODUCTION THEORY

Andrzej Hryczuk

NCBJ



A personal selection of recent ideas in the field

+ some results based on:

T. Binder, T. Bringmann, M. Gustafsson & A.H. 1706.07433, 2103.01944

A.H. & M. Laletin 2204.07078

A.H. & M. Laletin 2104.05684

Symposium PTF, Katowice

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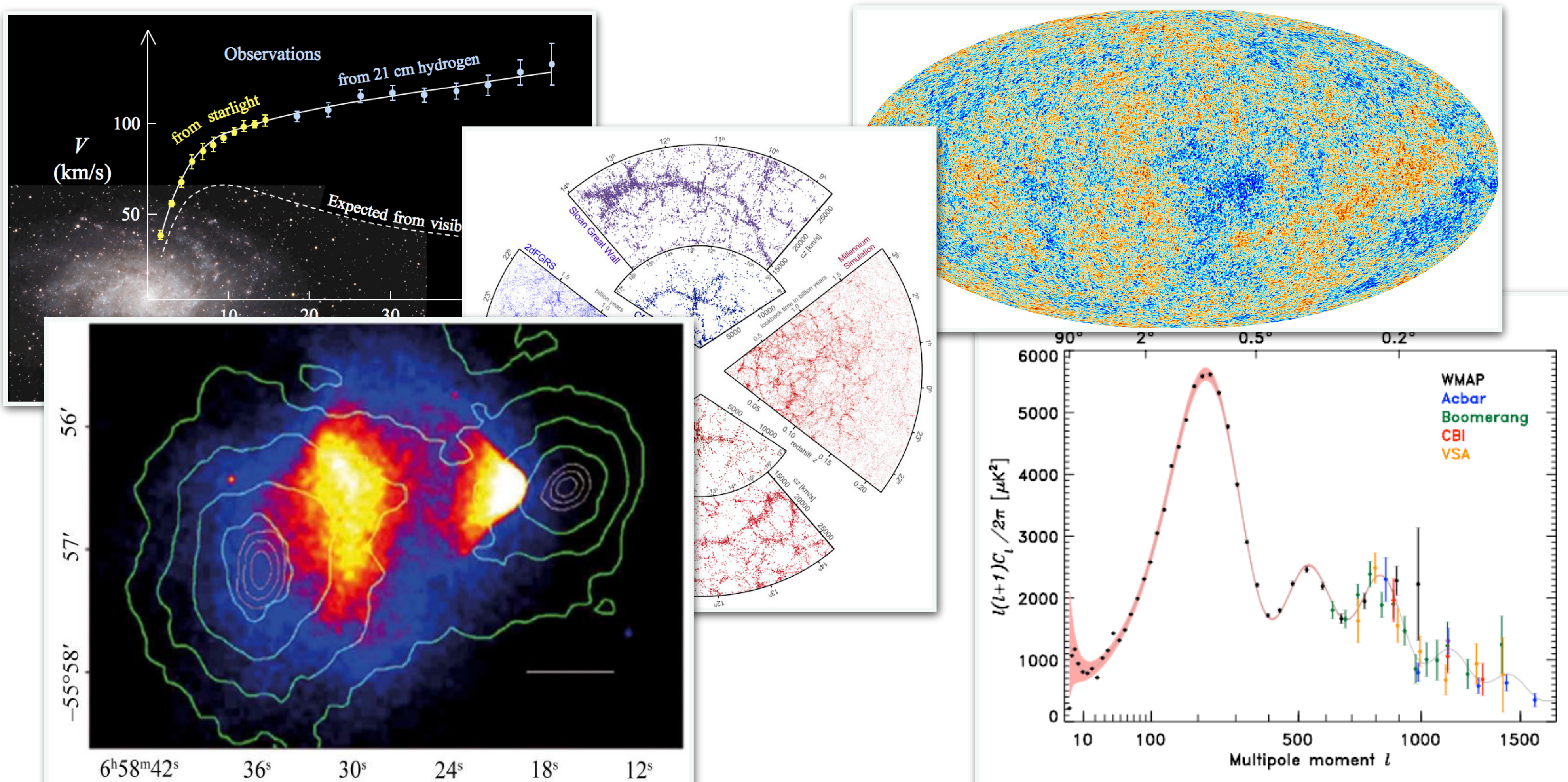
22nd October 2022

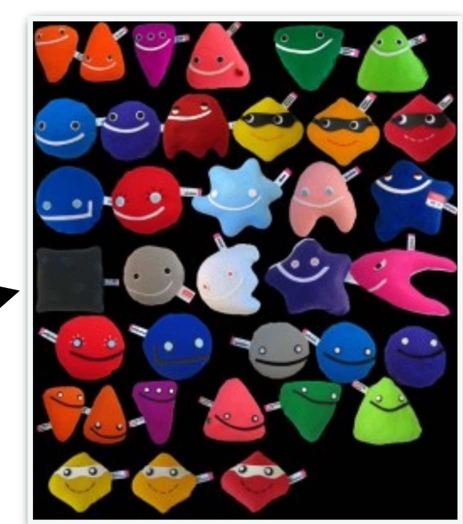
Dark Matter

I don't think there is any need for convincing you that DM exists…

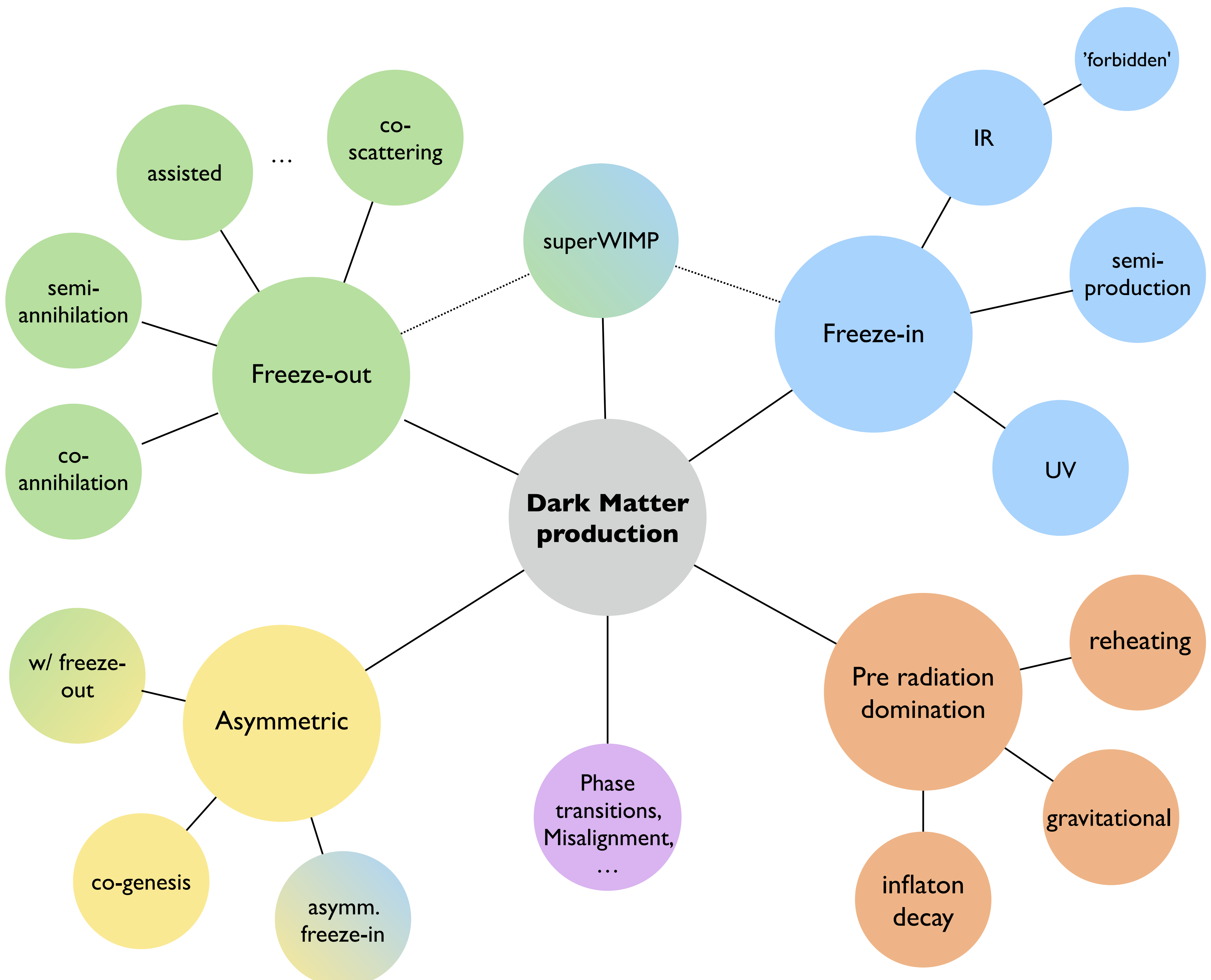


$\Longrightarrow$ Evidence on all scales!

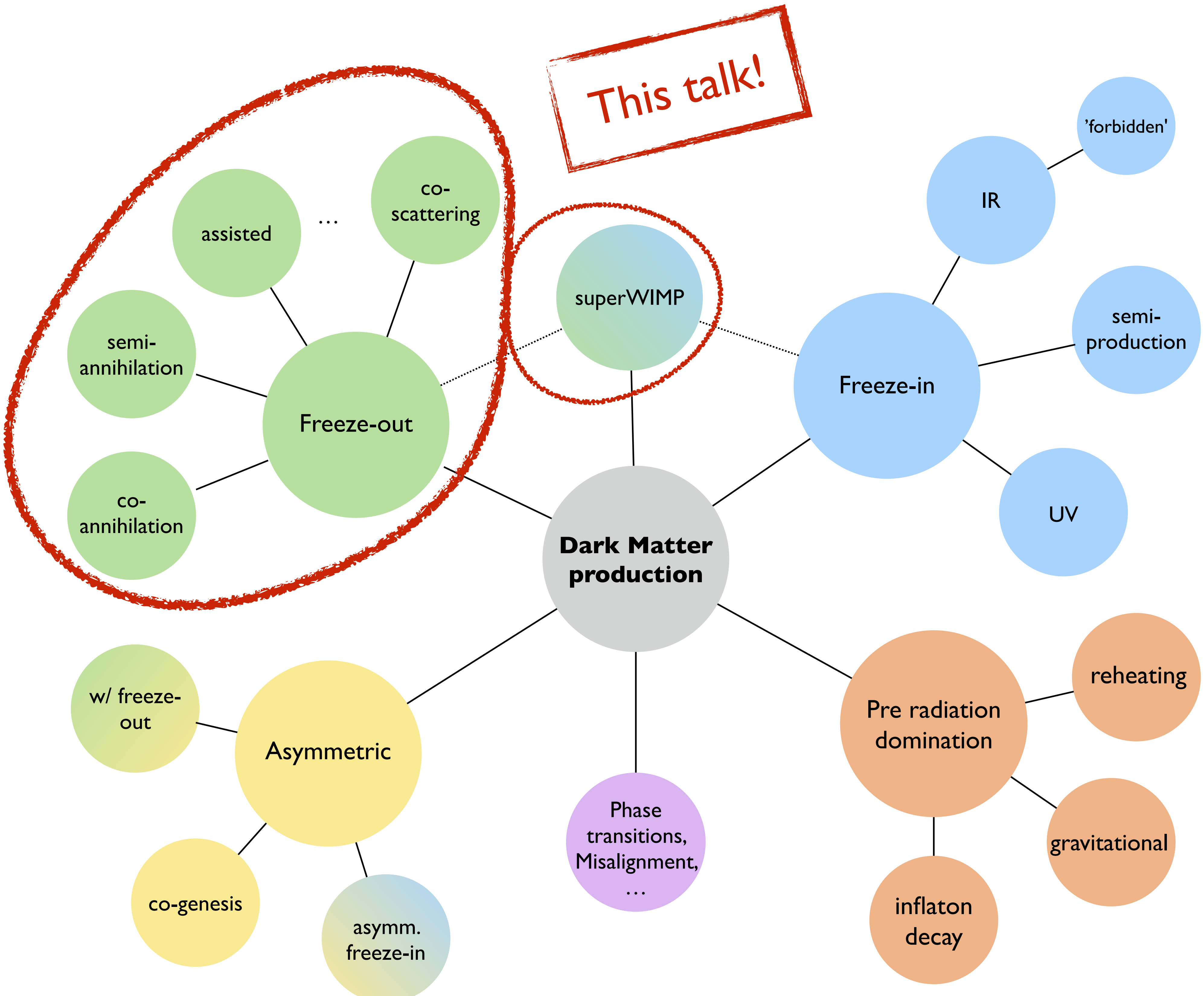
… perhaps I should argue why particle DM



Dark Matter origin



DARK MATTER ORIGIN



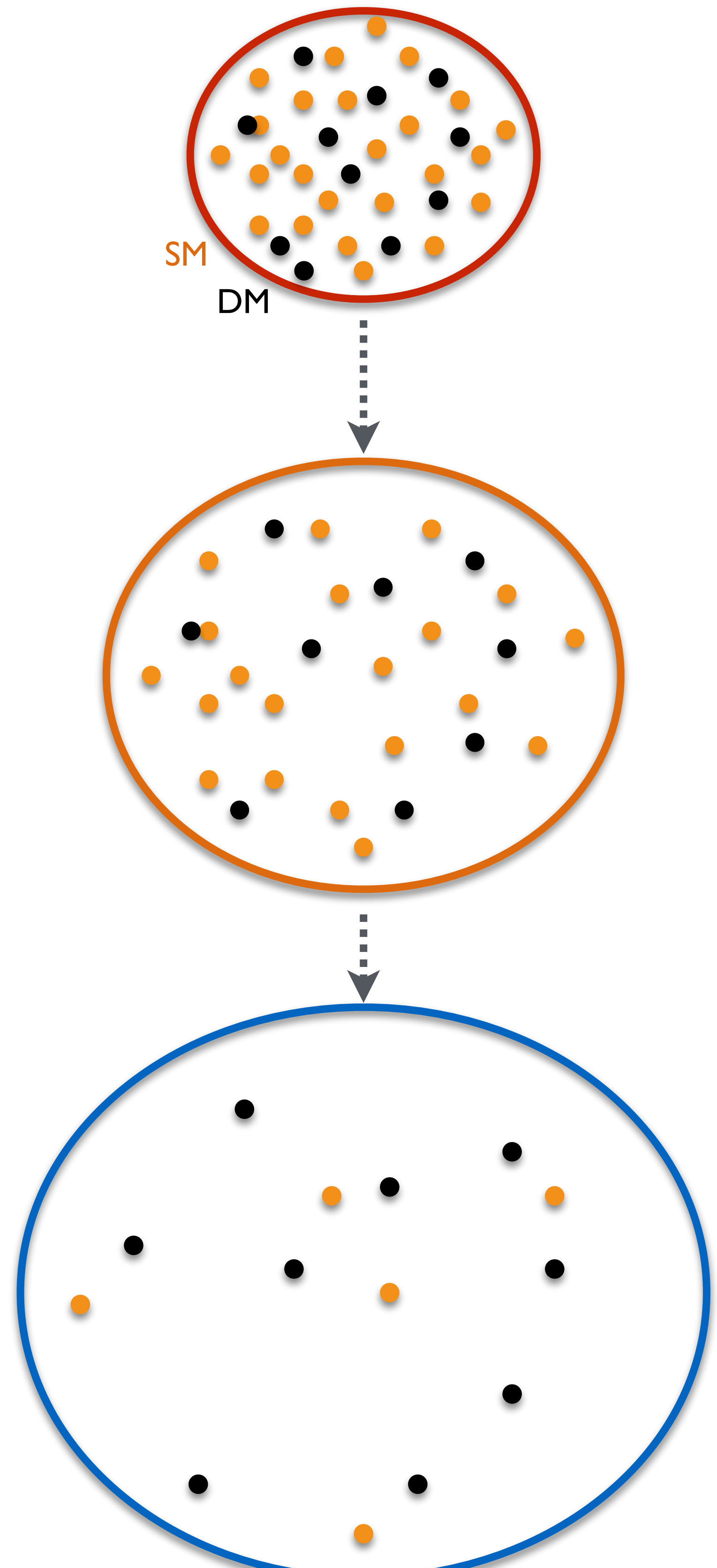
Thermal Relic Density
a.k.a. Freeze-Out

T

$\Gamma_{\text{ann}} > H$ — DM in full equilibrium

$\Gamma_{\text{ann}} \sim H$ — chemical decoupling

$\Gamma_{\text{ann}} < H$ — freeze-out



time

Thermal Relic Density
Standard Scenario

numerical codes e.g.,
DarkSUSY, micrOMEGAs, MadDM, SuperISOrelic, ...

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle \sigma_{\chi\bar{\chi}\to ij}\sigma_{\rm rel}\rangle^{\rm eq}\left(n_\chi n_{\bar{\chi}} - n_\chi^{\rm eq} n_{\bar{\chi}}^{\rm eq}\right)$$

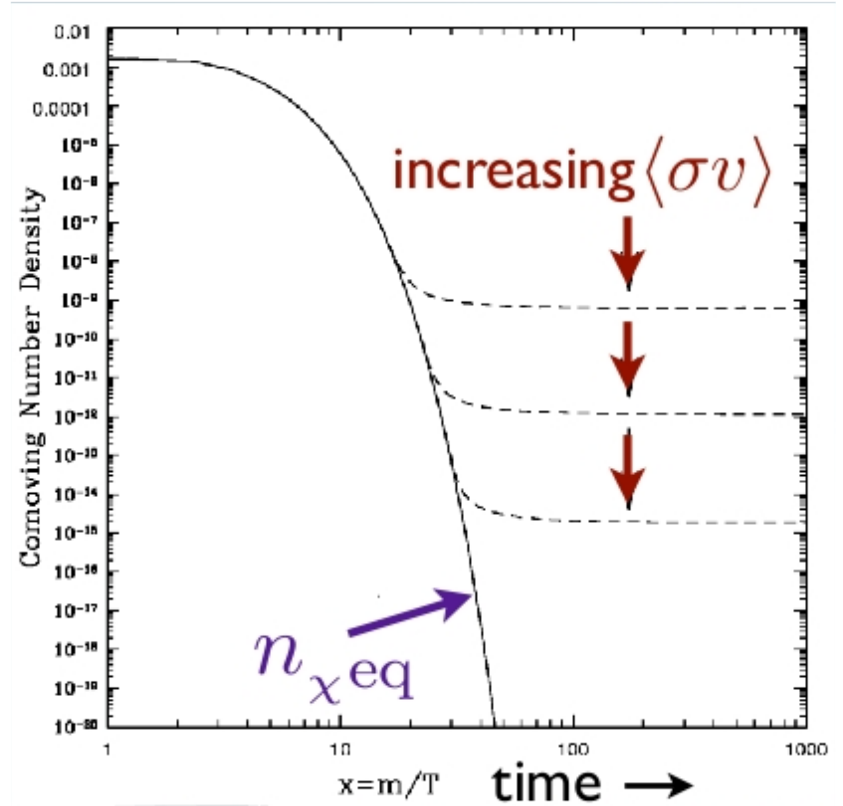


Fig.: Jungman, Kamionkowski & Griest, PR'96

where the thermally averaged cross section:

$$\langle \sigma_{\chi\bar{\chi}\to ij} v_{\rm rel}\rangle^{\rm eq} = -\frac{h_\chi^2}{n_\chi^{\rm eq} n_{\bar{\chi}}^{\rm eq}} \int \frac{d^3\vec{p}_\chi}{(2\pi)^3}\frac{d^3\vec{p}_{\bar{\chi}}}{(2\pi)^3}\, \sigma_{\chi\bar{\chi}\to ij} v_{\rm rel}\; f_\chi^{\rm eq} f_{\bar{\chi}}^{\rm eq}$$

Thermal Relic Density

STANDARD SCENARIO

modified expansion rate

e.g., relentless DM, D'Eramo et al. '17, …

numerical codes e.g.,
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modified cross section

Sommerfeld enhancement

Bound State formation

NLO

finite T effects

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Bound State formation

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breakdown of necessary assumptions leading to different form of the equation, <u>e.g. violation of kinetic equilibrium</u>

where the thermally averaged cross section:

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general multi-component dark sector

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle \sigma_{\chi\bar{\chi}\to ij}\sigma_{\rm rel}\rangle^{\rm eq}\left(n_\chi n_{\bar{\chi}} - n_\chi^{\rm eq} n_{\bar{\chi}}^{\rm eq}\right)$$

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modified cross section

- Sommerfeld enhancement
- Bound State formation
- NLO
- finite T effects

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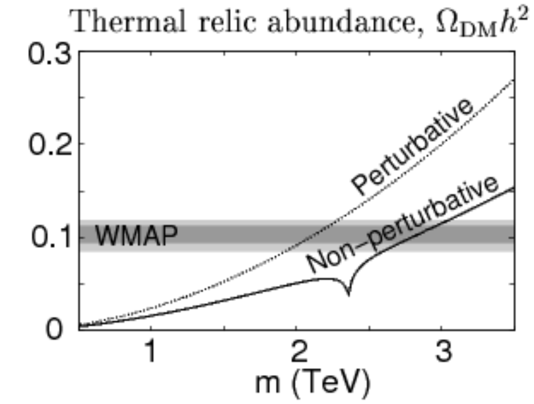
Chapter I:
Particle Physics Effects

The Sommerfeld effect
from EW interactions

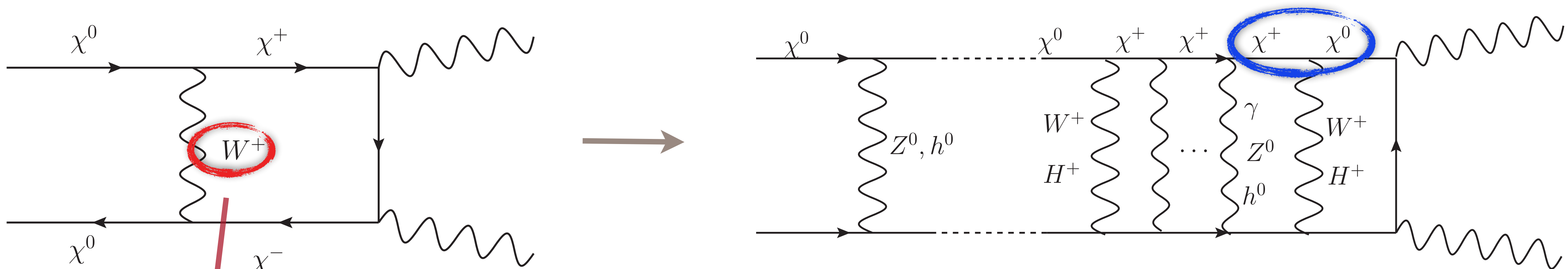


force carriers in the MSSM:

$\cancel{\gamma}$, $W^{\pm}$, Z^0, h_1^0, h_2^0, $H^{\pm}$

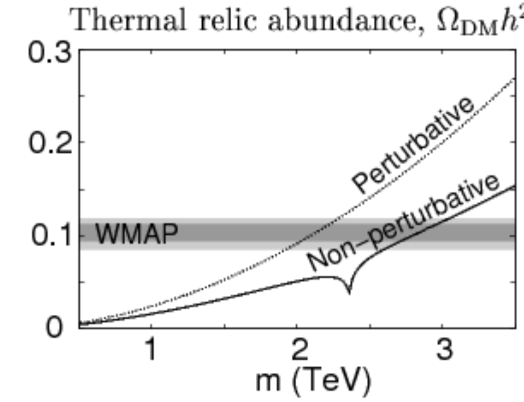
seminal papers
Hisano *et al.* '04,'06,...

$\delta m \ll m_\chi$



$m_\chi \gg m_W$

The Sommerfeld effect
from EW interactions



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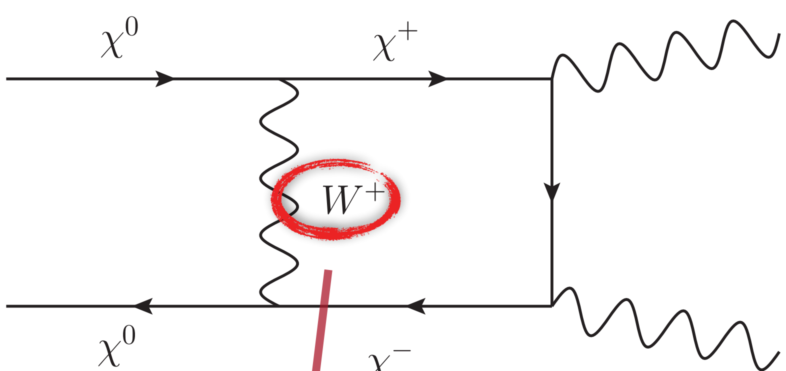

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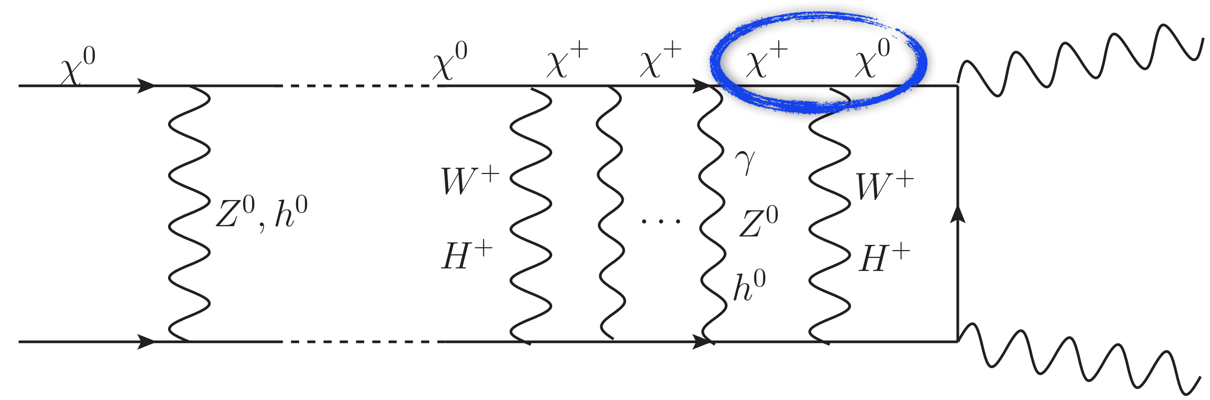


at TeV scale $\Rightarrow$ generically effect of $\mathcal{O}(1-100\%)$

on top of that resonance structure

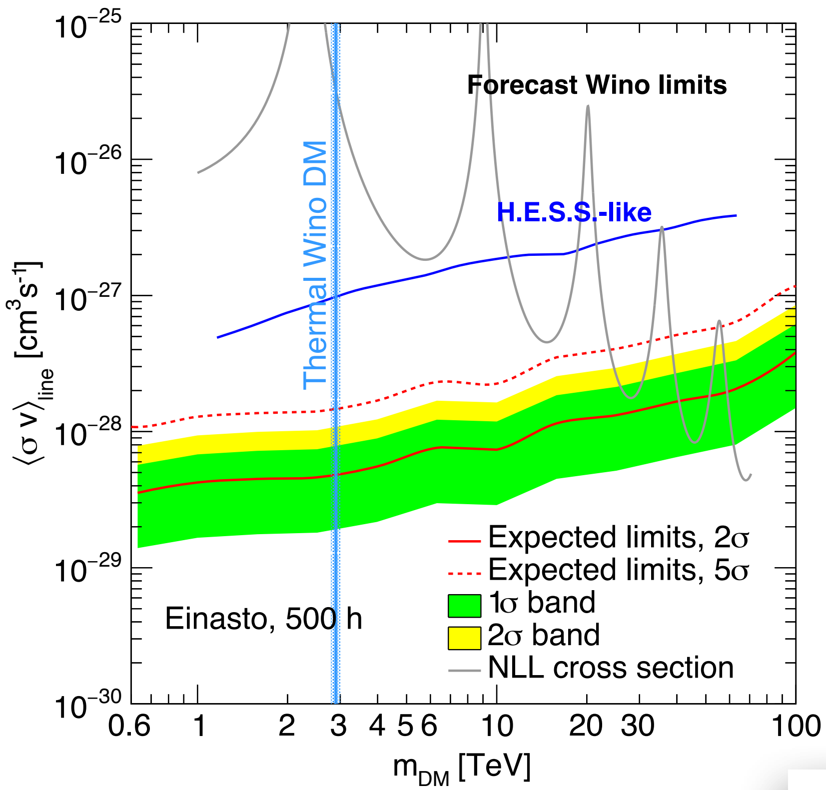
effect of $\mathcal{O}(\text{few})$
for the relic density

AH, R. Iengo, P. Ullio. '10
AH '11
AH *et al.* '17, M. Beneke *et al.*; '16

can be understood as being close to
a threshold of lowest bound state

The Sommerfeld effect

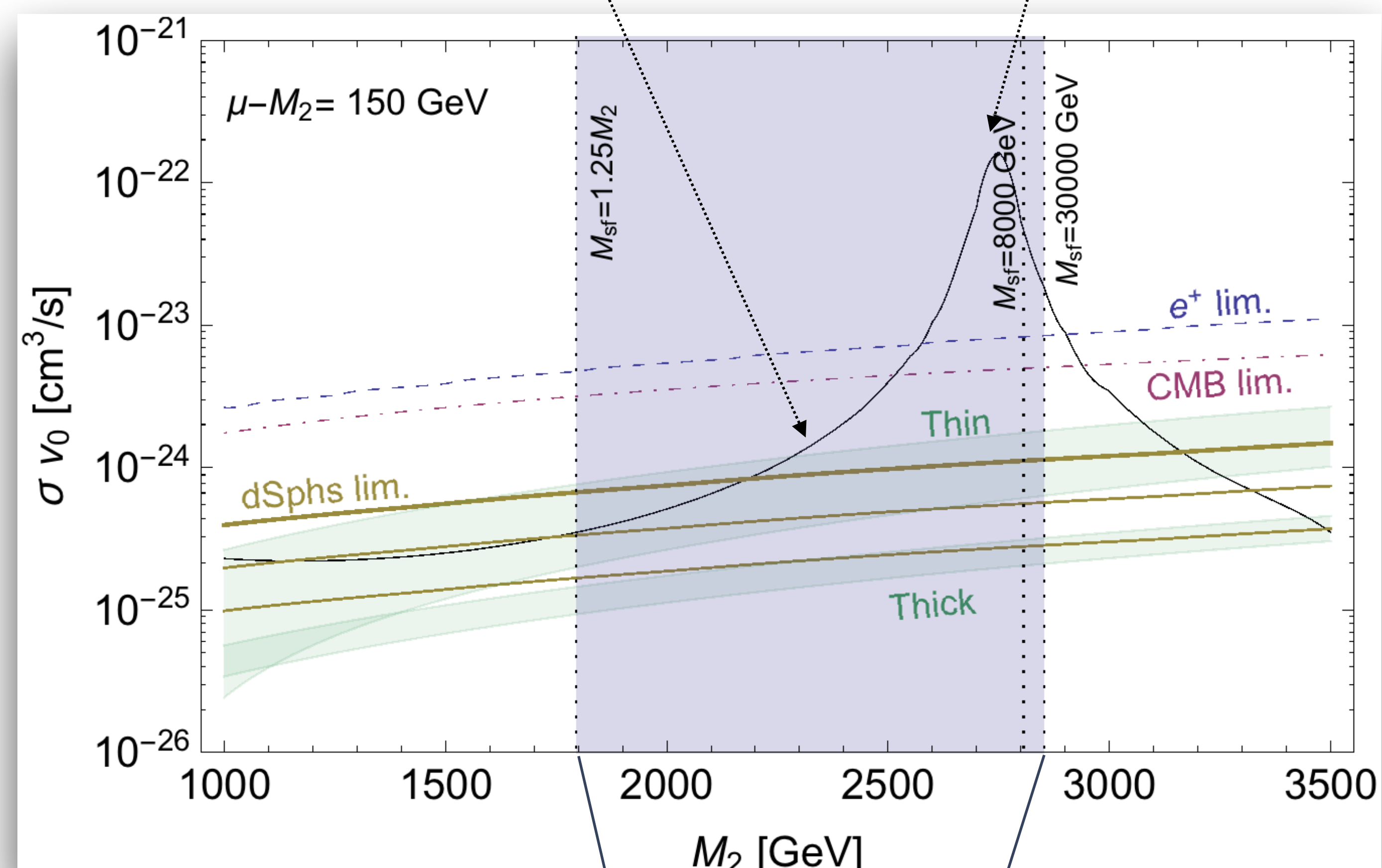
Indirect Detection



Slatyer *et al.*, '21

actual cross section

resonance moves to the right w.r.t. pure wino



Beneke, …AH, … *et al.*, '16

correct RD can be achieved: when varying sfermion masses

similar study, pure Wino case: Ibe *et al.* '15

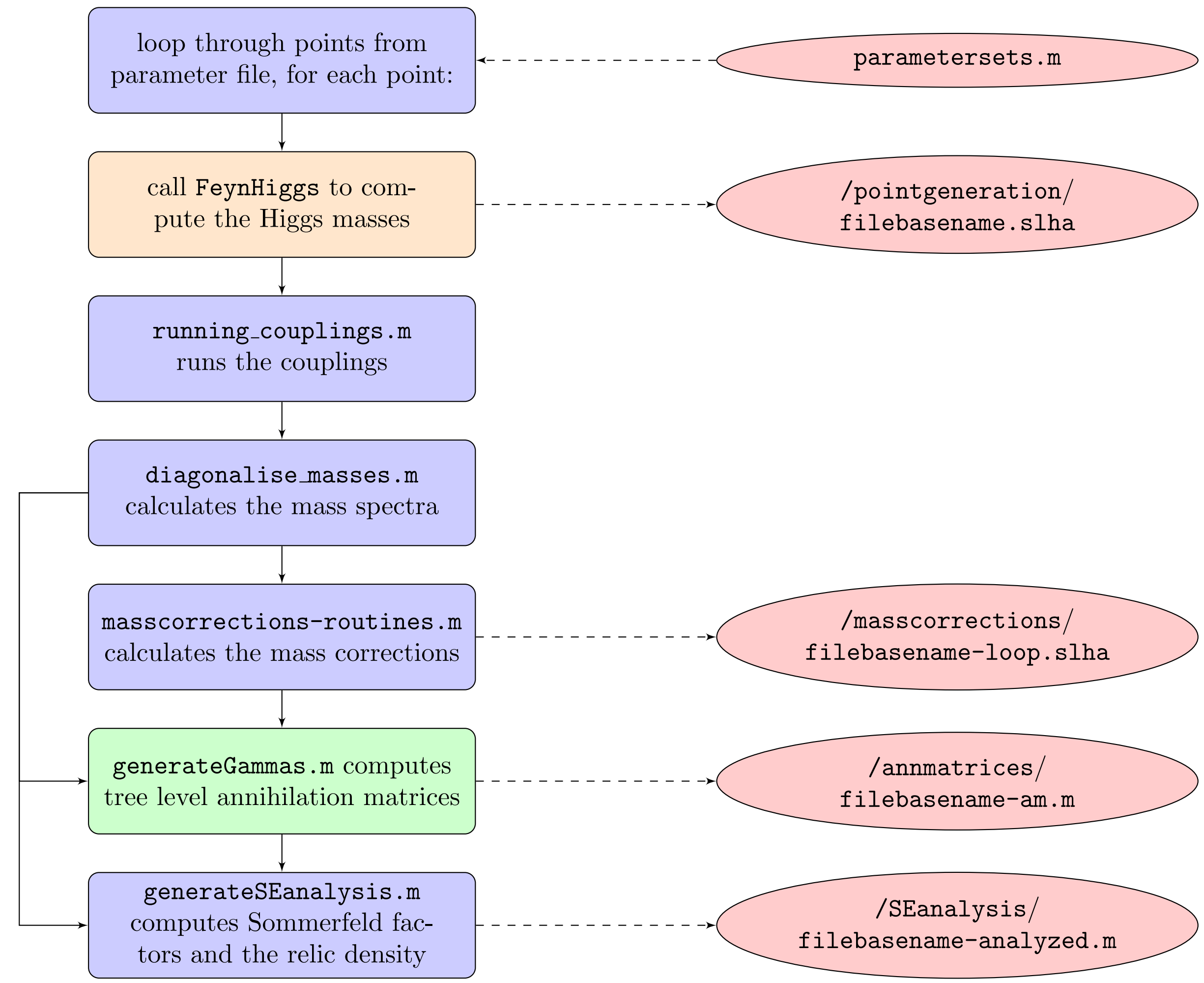
NEW NUMERICAL TOOL

based on EFT, improving accuracy in numerous ways

- suitable for (large scale) scans
- implemented full MSSM
- one-loop on-shell mass splittings and running couplings
- the Sommerfeld effect for P- and $O(v^2)$ S-wave
- off-diagonal annihilation matrices

not present in DarkSE
AH, ′11

- present day annihilation in the halo (for ID)
- possibility of including thermal corrections
- …
- accuracy at O(%), dominated by theoretical uncertainties of EFT



Status: all works as intended, making the code ready for public release

Beneke,…, AH,… *et al.* in preparation

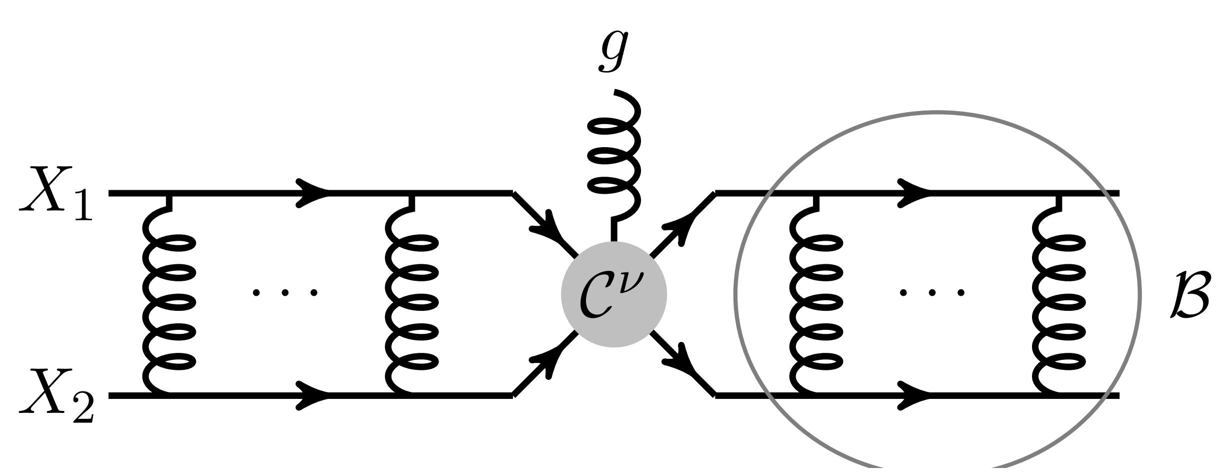
Bound State Formation

As noticed before Sommerfeld effect has resonances when Bohr radius ~ potential range, i.e. when close to a bound state threshold $\longrightarrow$ Can DM form actual bound states from such long range interactions?

$\downarrow$

Yes, it can!

Q: How to describe such bound states and their formation?



free DM states DM bound state

*the effect was first studied in simplified models with light mediators, then gradually extended to non-Abelian interactions, double emissions, co-annihilations, etc.
see papers by **K. Petraki** *et al.* '14-19

**vide also "WIMPonium"
March-Russel, West '10

DARK MATTER AT NLO

Bergstrom '89; Drees et al., 9306325;
Ullio & Bergstrom, 9707333
} helicity suppression lifting

⋮

Bergstrom et al., 0507229;
Bringmann et al., 0710.3169
} spectral features in indirect searches

⋮

Ciafaloni et al., 1009.0224
Cirelli et al., 1012.4515
Ciafaloni et al., 1202.0692
AH & Iengo, 1111.2916
} large EW corrections

⋮

Chatterjee et al., 1209.2328
Harz et al., 1212.5241
Ciafaloni et al., 1305.6391
Hermann et al., 1404.2931
Boudjema et al., 1403.7459
Bringmann et al., 1510.02473
Klasen et al., 1607.06396
} ***thermal relic density***

$$\Omega_{DM} h^2 = 0.1187 \pm 0.0017.$$

<1.5% uncertainty!

Planck+WMAP pol.+highL+BAO; 1303.5062

⋮

SloopS, DM@NLO, PPC4DMID } NLO codes

Relic Density at NLO

Recall at LO:

$$C_{\mathrm{LO}} = -h_\chi^2 \int \frac{d^3\vec{p}_\chi}{(2\pi)^3}\frac{d^3\vec{p}_{\bar{\chi}}}{(2\pi)^3}\ \sigma_{\chi\bar{\chi}\to ij} v_{\mathrm{rel}}\ \left[f_\chi f_{\bar{\chi}}(1\pm f_i)(1\pm f_j) - f_i f_j (1\pm f_\chi)(1\pm f_{\bar{\chi}})\right]$$

crucial point: $$p_\chi + p_{\bar{\chi}} = p_i + p_j \Rightarrow f_\chi^{\mathrm{eq}} f_{\bar{\chi}}^{\mathrm{eq}} \approx f_i^{\mathrm{eq}} f_j^{\mathrm{eq}}$$

in Maxwell approx.

at NLO both virtual one-loop and 3-body processes contribute:

$$C_{1-\mathrm{loop}} = -h_\chi^2 \int \frac{d^3\vec{p}_\chi}{(2\pi)^3}\frac{d^3\vec{p}_{\bar{\chi}}}{(2\pi)^3}\ \sigma^{1-\mathrm{loop}}_{\chi\bar{\chi}\to ij} v_{\mathrm{rel}}\ \left[f_\chi f_{\bar{\chi}}(1\pm f_i)(1\pm f_j) - f_i f_j (1\pm f_\chi)(1\pm f_{\bar{\chi}})\right]$$

$$C_{\mathrm{real}} = -h_\chi^2 \int \frac{d^3\vec{p}_\chi}{(2\pi)^3}\frac{d^3\vec{p}_{\bar{\chi}}}{(2\pi)^3}\ \sigma_{\chi\bar{\chi}\to ij\gamma} v_{\mathrm{rel}}\ \left[f_\chi f_{\bar{\chi}}(1\pm f_i)(1\pm f_j)(1+f_\gamma) - f_i f_j f_\gamma (1\pm f_\chi)(1\pm f_{\bar{\chi}})\right]$$

Relic Density at NLO

Recall at LO:

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$$p_\chi + p_{\bar\chi} = p_i + p_j \pm p_\gamma \Rightarrow$$

photon can be arbitrarily soft

$f_\gamma \sim \omega^{-1}$

Maxwell approx. not valid anymore...

...problem: *T*-dependent IR divergence!

Relic Density

what Really happens at NLO?

Beneke, Dighera, AH, 1409.3049

$$
\begin{aligned}
C_{\rm NLO} \sim \int d\Pi_{\chi\bar\chi ij}\;\; f_\chi f_{\bar\chi}\;\; &\Big\{ |\mathcal{M}^{\rm LO}_{\chi\bar\chi\to ij}|^2 + |\mathcal{M}^{{\rm NLO}\;T=0}_{\chi\bar\chi\to ij}|^2 + \int d\Pi_\gamma |\mathcal{M}_{\chi\bar\chi\to ij\gamma}|^2 + \\
&|\mathcal{M}^{{\rm NLO}\;T\neq 0}_{\chi\bar\chi\to ij}|^2 + \int d\Pi_\gamma \Big[f_\gamma \left(|\mathcal{M}_{\chi\bar\chi\to ij\gamma}|^2 + |\mathcal{M}_{\chi\bar\chi\gamma\to ij}|^2 \right) \\
&- f_i \left(|\mathcal{M}_{\chi\bar\chi\to ij\gamma}|^2 + |\mathcal{M}_{\chi\bar\chi i\to j\gamma}|^2 \right) - f_j \left(|\mathcal{M}_{\chi\bar\chi\to ij\gamma}|^2 + |\mathcal{M}_{\chi\bar\chi j\to i\gamma}|^2 \right) \Big] \Big\} \\
- f_i f_j\;\; &\Big\{ |\mathcal{M}^{\rm LO}_{ij\to\chi\bar\chi}|^2 + |\mathcal{M}^{{\rm NLO}\;T=0}_{ij\to\chi\bar\chi}|^2 + \int d\Pi_\gamma |\mathcal{M}_{ij\to\chi\bar\chi\gamma}|^2 + \\
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typically only this used in NLO literature

$$C_{\rm NLO} \sim \int d\Pi_{\chi\bar\chi ij}\;\; f_\chi f_{\bar\chi} \;\boxed{\Big\{ |\mathcal{M}^{\rm LO}_{\chi\bar\chi\to ij}|^2 + |\mathcal{M}^{{\rm NLO}\; T=0}_{\chi\bar\chi\to ij}|^2 + \int d\Pi_\gamma |\mathcal{M}_{\chi\bar\chi\to ij\gamma}|^2 +}$$

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$$\left. - f_i \left(|\mathcal{M}_{\chi\bar\chi\to ij\gamma}|^2 + |\mathcal{M}_{\chi\bar\chi i\to j\gamma}|^2 \right) - f_j \left(|\mathcal{M}_{\chi\bar\chi\to ij\gamma}|^2 + |\mathcal{M}_{\chi\bar\chi j\to i\gamma}|^2 \right)\right] \Big\}$$

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thermal 1-loop

photon absorption

photon emission

$$-f_i f_j \Big\{ |\mathcal{M}^{\rm LO}_{ij\to\chi\bar\chi}|^2 + |\mathcal{M}^{{\rm NLO}\ T=0}_{ij\to\chi\bar\chi}|^2 + \int d\Pi_\gamma |\mathcal{M}_{ij\to\chi\bar\chi\gamma}|^2 +$$

$$|\mathcal{M}^{{\rm NLO}\ T\neq 0}_{ij\to\chi\bar\chi}|^2 + \int d\Pi_\gamma \Big[f_\gamma \left(|\mathcal{M}_{ij\to\chi\bar\chi\gamma}|^2 + |\mathcal{M}_{ij\gamma\to\chi\bar\chi}|^2 \right)$$

$$- f_\chi \left(|\mathcal{M}_{ij\to\chi\bar\chi\gamma}|^2 + |\mathcal{M}_{ij\chi\to\chi\gamma}|^2 \right) - f_{\bar\chi} \left(|\mathcal{M}_{ij\to\chi\bar\chi\gamma}|^2 + |\mathcal{M}_{ij\bar\chi\to\bar\chi\gamma}|^2 \right) \Big] \Big\}$$

Relic Density

what Really happens at NLO?

Beneke, Dighera, AH, 1409.3049

typically only this used in NLO literature

$$C_{\rm NLO} \sim \int d\Pi_{\chi\bar\chi ij}\; f_\chi f_{\bar\chi}\; \boxed{\Big\{ |\mathcal{M}^{\rm LO}_{\chi\bar\chi\to ij}|^2 + |\mathcal{M}^{{\rm NLO}\;T=0}_{\chi\bar\chi\to ij}|^2 + \int d\Pi_\gamma |\mathcal{M}_{\chi\bar\chi\to ij\gamma}|^2 +}$$

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thermal 1-loop

photon absorption

photon emission

SM fermions emission

SM fermions absorption

RELIC DENSITY

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thermal 1-loop

photon absorption

photon emission

SM fermions emission

SM fermions absorption

SOLUTION: non-equilibrium thermal field theory

in the DM context some results available, lot more to be done…
but typically not that relevant for phenomenology

Chapter II:
Non-Equilibrium effects

Thermal Relic Density

Standard Scenario

(chemical and kinetic)

T			time
	$\Gamma_{\rm ann} > H$	DM in full equilibrium	
	$\Gamma_{\rm ann} \sim H$	chemical decoupling	
	$\Gamma_{\rm ann} < H$	freeze-out	
	$\Gamma_{\rm scatt} \sim H$	kinetic decoupling	

THERMAL RELIC DENSITY

STANDARD SCENARIO

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	$\Gamma_{\text{scatt}} \sim H$	kinetic decoupling	

non-equilibrium

time evolution of $f_\chi(p)$ in kinetic theory:

$$\boxed{E\left(\partial_t - H\vec{p}\cdot\nabla_{\vec{p}}\right)} f_\chi = \mathcal{C}[f_\chi]$$

Liouville operator in FRW background

the collision term

THERMAL RELIC DENSITY
STANDARD APPROACH

Boltzmann equation for $f_\chi(p)$:

$$E\left(\partial_t - H\vec{p}\cdot\nabla_{\vec{p}}\right) f_\chi = \mathcal{C}[f_\chi]$$

*assumptions for using Boltzmann eq: classical limit, molecular chaos,...

…for derivation from thermal QFT see e.g., 1409.3049

THERMAL RELIC DENSITY

STANDARD APPROACH

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$\Downarrow$ integrate over p (i.e. take 0th moment)

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle\sigma_{\chi\bar{\chi}\to ij}\sigma_{\rm rel}\rangle^{\rm eq}\left(n_\chi n_{\bar{\chi}} - n_\chi^{\rm eq} n_{\bar{\chi}}^{\rm eq}\right)$$

Thermal Relic Density

standard approach

Boltzmann equation for $f_\chi(p)$:

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$\Downarrow$

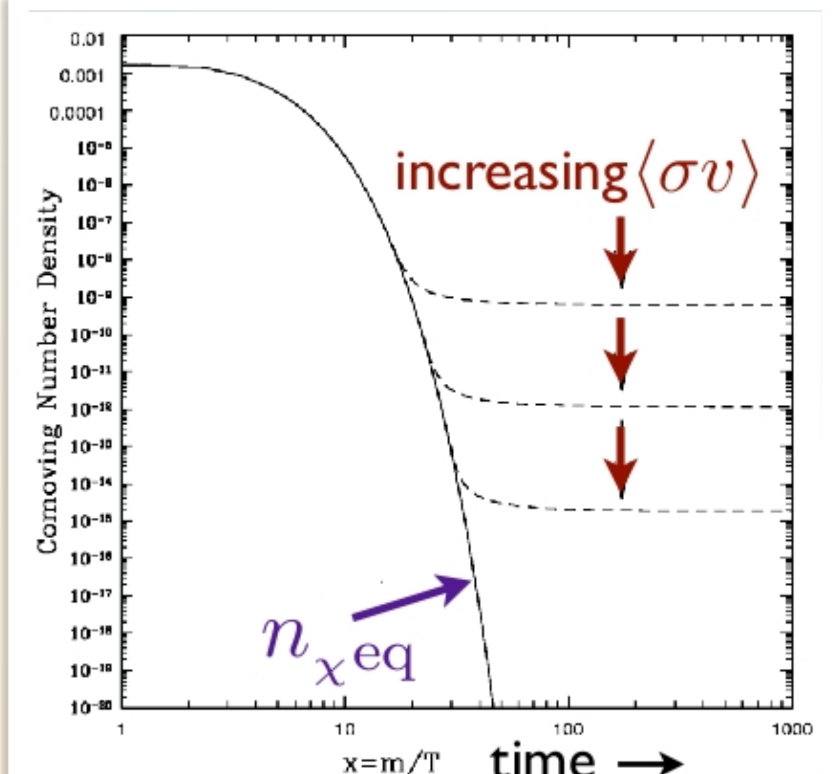

Fig.: Jungman, Kamionkowski & Griest, PR'96

THERMAL RELIC DENSITY

STANDARD APPROACH

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Critical assumption:
kinetic equilibrium at chemical decoupling

$$f_\chi \sim a(T) f_\chi^{\rm eq}$$

$\Downarrow$

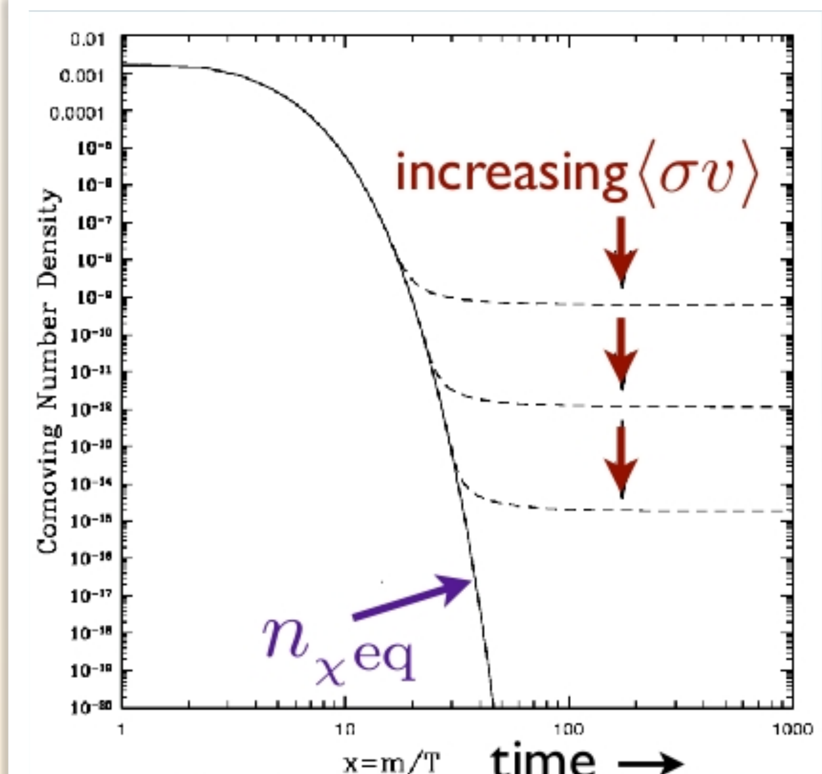


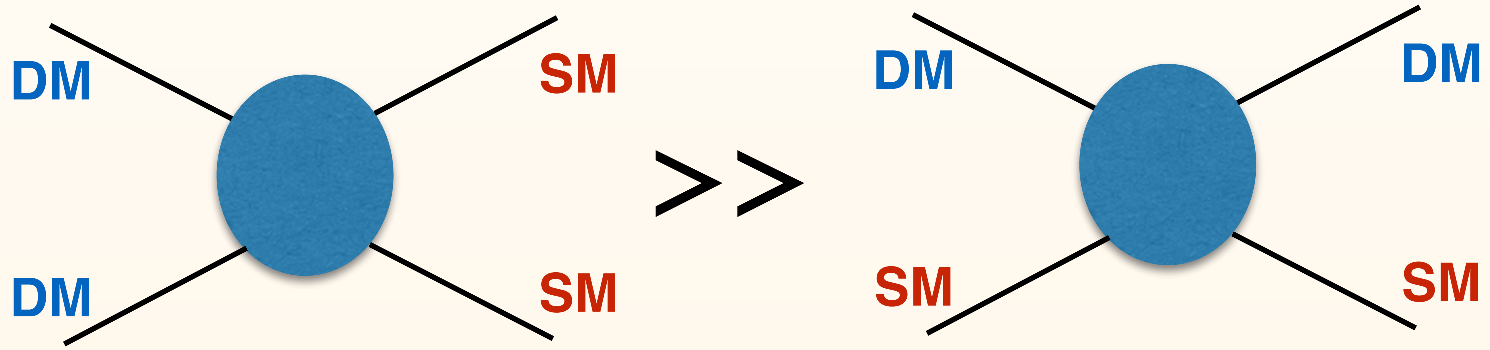
Fig.: Jungman, Kamionkowski & Griest, PR'96

EARLY KINETIC DECOUPLING?

A necessary and sufficient condition: <u>scatterings weaker than annihilation</u>

i.e. rates around freeze-out: $H \sim \Gamma_{\rm ann} \gtrsim \Gamma_{\rm el}$

Possibilities:

A) 

e.g., resonant annihilation

B) Boltzmann suppression of SM as strong as for DM

e.g., below threshold annihilation (forbidden-like DM)

C) Scatterings and annihilation have different structure

e.g., semi-annihilation, 3 to 2 models,…

D) Multi-component dark sectors

e.g., additional sources of DM from late decays, …

How to go beyond kinetic equilibrium?

All information is in the full BE:

both about chemical ("normalization") and kinetic ("shape") equilibrium/decoupling

$$E\left(\partial_t - H\vec{p}\cdot\nabla_{\vec{p}}\right) f_\chi = \mathcal{C}[f_\chi]$$

contains both scatterings and annihilations

Two possible approaches:

fBE

cBE

0-th moment: n_χ
2-nd moment: T_χ
$\dots$

solve numerically for full $f_\chi(p)$

have insight on the distribution
no constraining assumptions

numerically challenging
often an overkill

consider system of equations for moments of $f_\chi(p)$

partially analytic/much easier numerically
manifestly captures all of the relevant physics

finite range of validity
no insight on the distribution

NEW TOOL!

GOING BEYOND THE STANDARD APPROACH

- Home
- Downloads
- Contact

DRAKE

Dark matter Relic Abundance beyond Kinetic Equilibrium

Authors: Tobias Binder, Torsten Bringmann, Michael Gustafsson and Andrzej Hryczuk

DRAKE is a numerical precision tool for predicting the dark matter relic abundance also in situations where the standard assumption of kinetic equilibrium during the freeze-out process may not be satisfied. The code comes with a set of three dedicated Boltzmann equation solvers that implement, respectively, the traditionally adopted equation for the dark matter number density, fluid-like equations that couple the evolution of number density and velocity dispersion, and a full numerical evolution of the phase-space distribution. The code is written in Wolfram Language and includes a Mathematica notebook example program, a template script for terminal usage with the free Wolfram Engine, as well as several concrete example models.
DRAKE is a free software licensed under GPL3.

If you use DRAKE for your scientific publications, please cite

- **DRAKE: Dark matter Relic Abundance beyond Kinetic Equilibrium,**
  Tobias Binder, Torsten Bringmann, Michael Gustafsson and Andrzej Hryczuk, [arXiv:2103.01944]

Currently, an user guide can be found in the Appendix A of this reference.
Please cite also quoted other works applying for specific cases.

v1.0 « Click here to download DRAKE

(March 3, 2021)

https://drake.hepforge.org

Applications:

DM relic density for
any (user defined) model*

Interplay between chemical and kinetic decoupling

Prediction for the DM
phase space distribution

Late kinetic decoupling
and impact on cosmology

see e.g., 1202.5456

…

(only) prerequisite:
Wolfram Language (or *Mathematica*)

*at the moment for a single DM species and w/o co-annihlations… but stay tuned for extensions!

EXAMPLE A:
SCALAR SINGLET DM

A)

Example A
scalar singlet DM

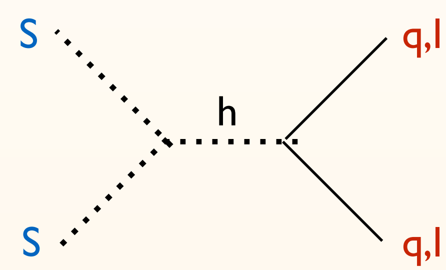
To the SM Lagrangian add one singlet scalar field *S* with interactions with the Higgs:

$$\mathcal{L}_S = \frac{1}{2}\partial_\mu S \partial^\mu S - \frac{1}{2}\mu_S^2 S^2 - \frac{1}{2}\lambda_s S^2 |H|^2$$

$$m_s = \sqrt{\mu_S^2 + \frac{1}{2}\lambda_s v_0^2}$$

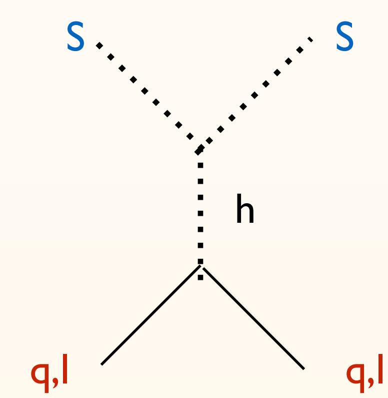
Annihilation processes:

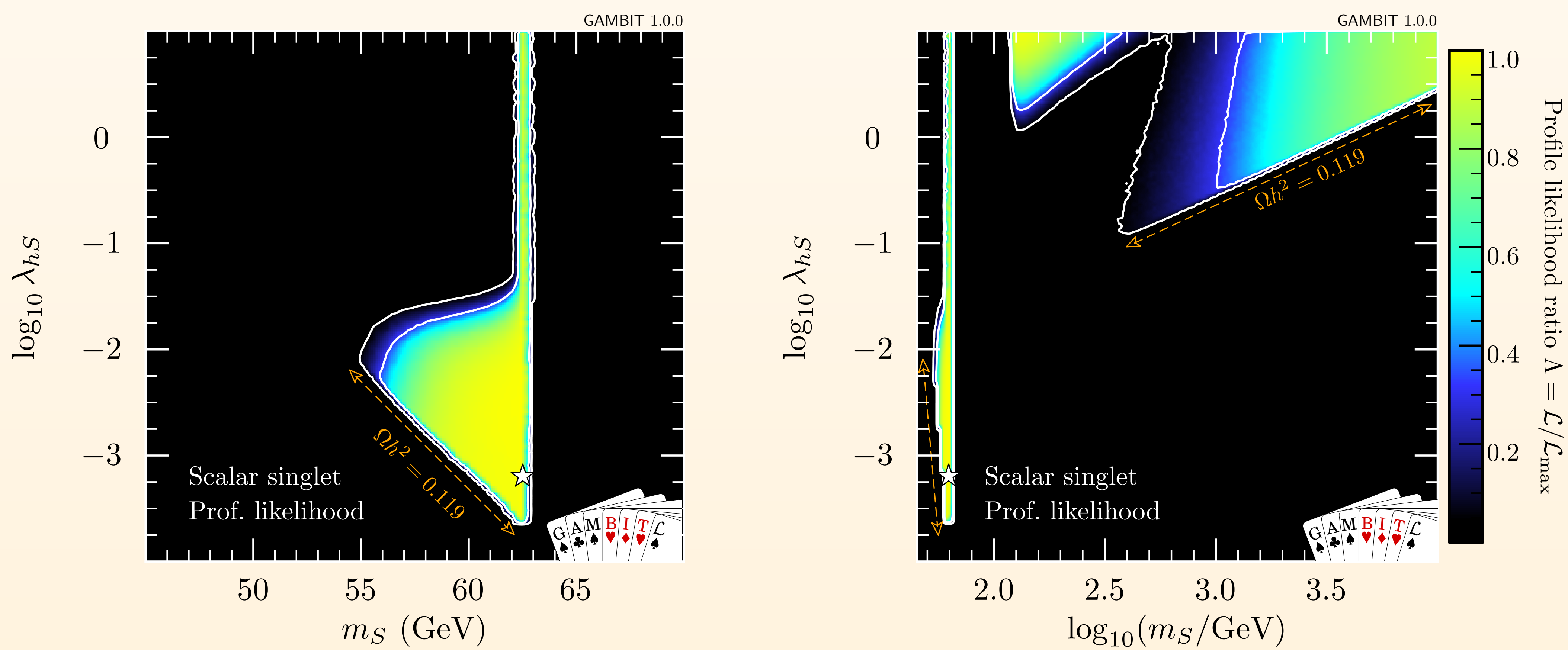
resonant



El. scattering processes:

non-resonant



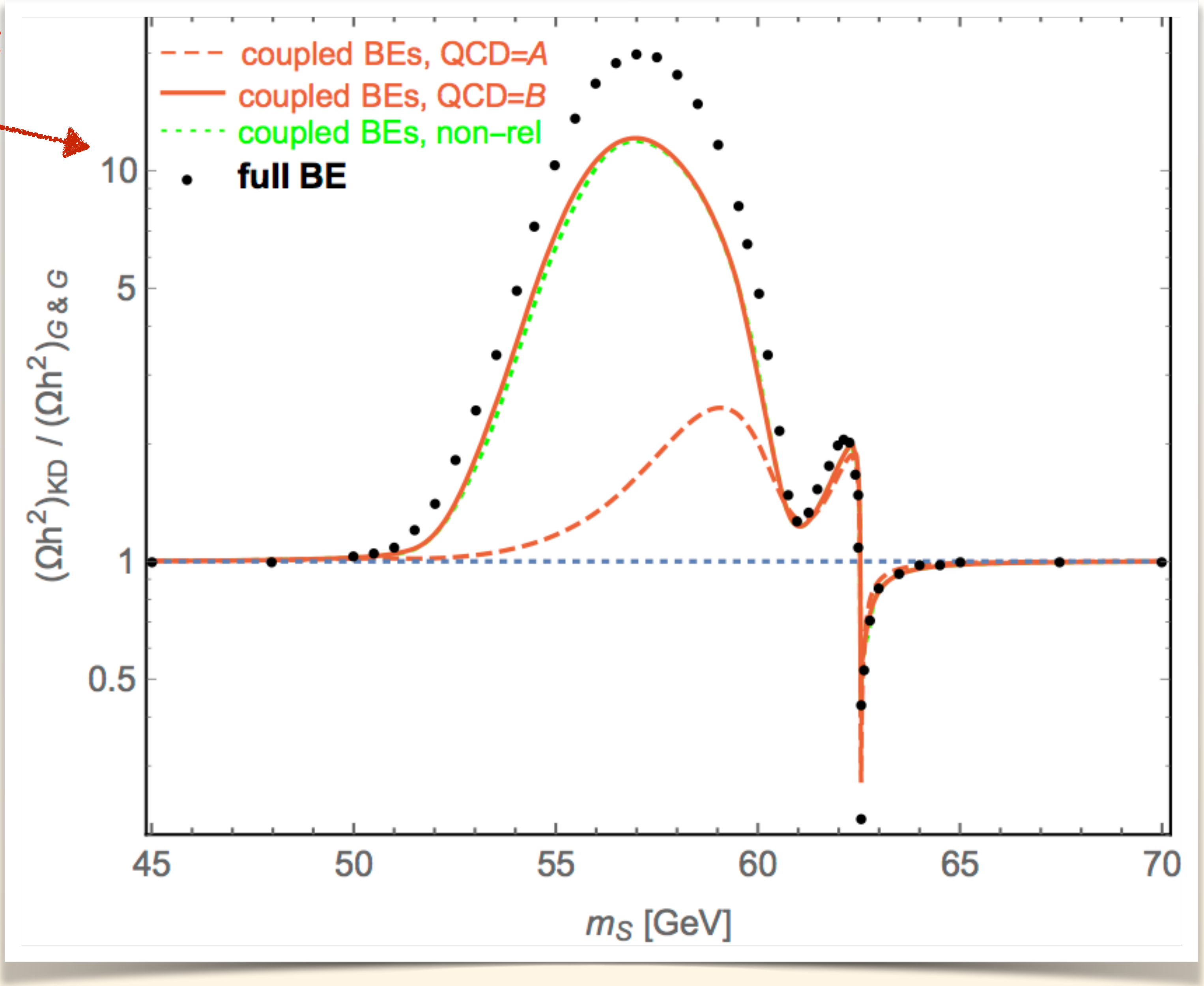


GAMBIT collaboration
1705.07931

RESULTS

EFFECT ON THE Ωh^2

effect on relic density:
up to O(~10)



[… Freeze-out at few GeV ⟶ what is the abundance of heavy quarks in QCD plasma?

two scenarios: QCD = A - all quarks are free and present in the plasma down to T_c =154 MeV
QCD = B - only light quarks contribute to scattering and only down to $4T_c$ …]

RESULTS

EFFECT ON THE Ωh^2

effect on relic density:
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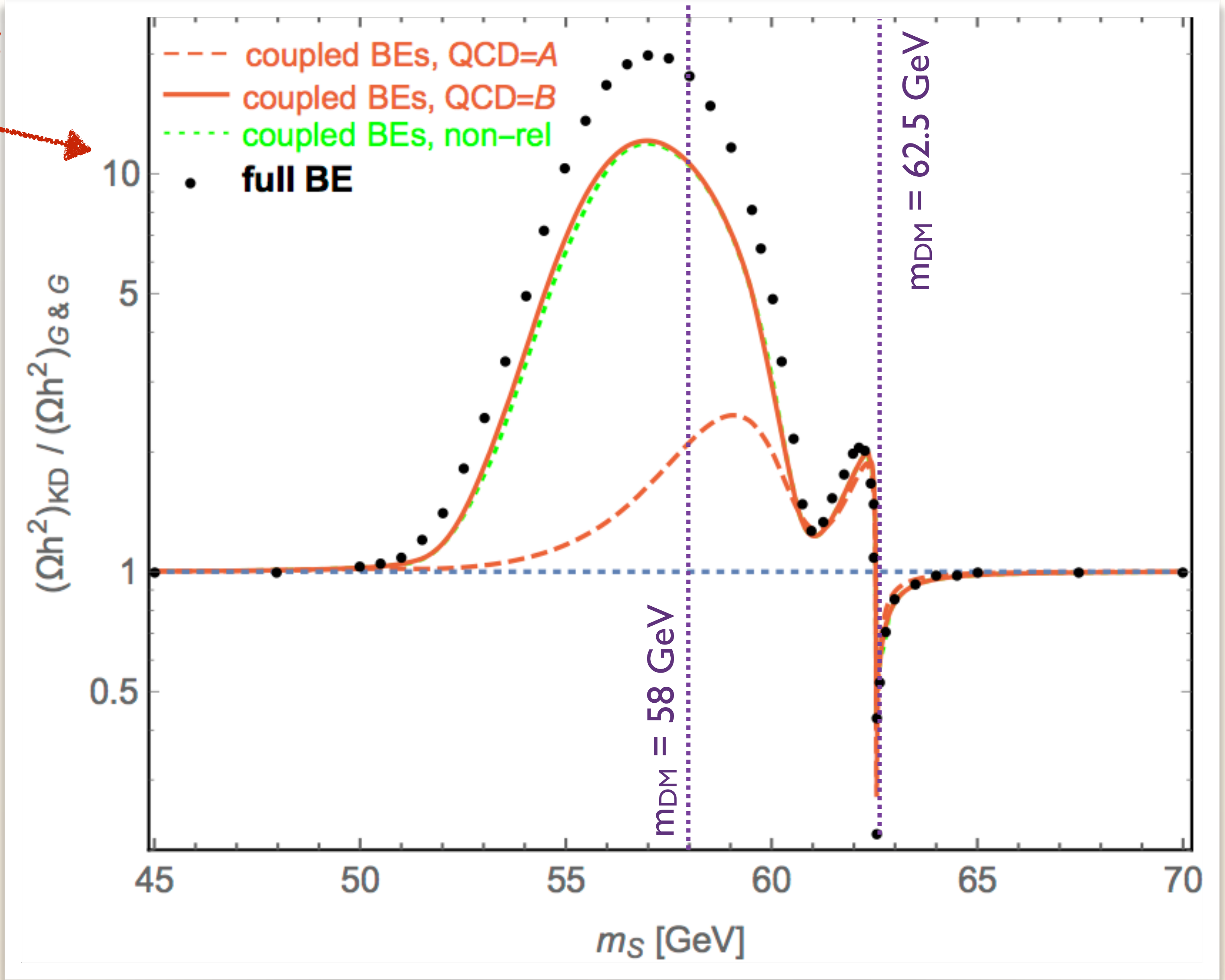

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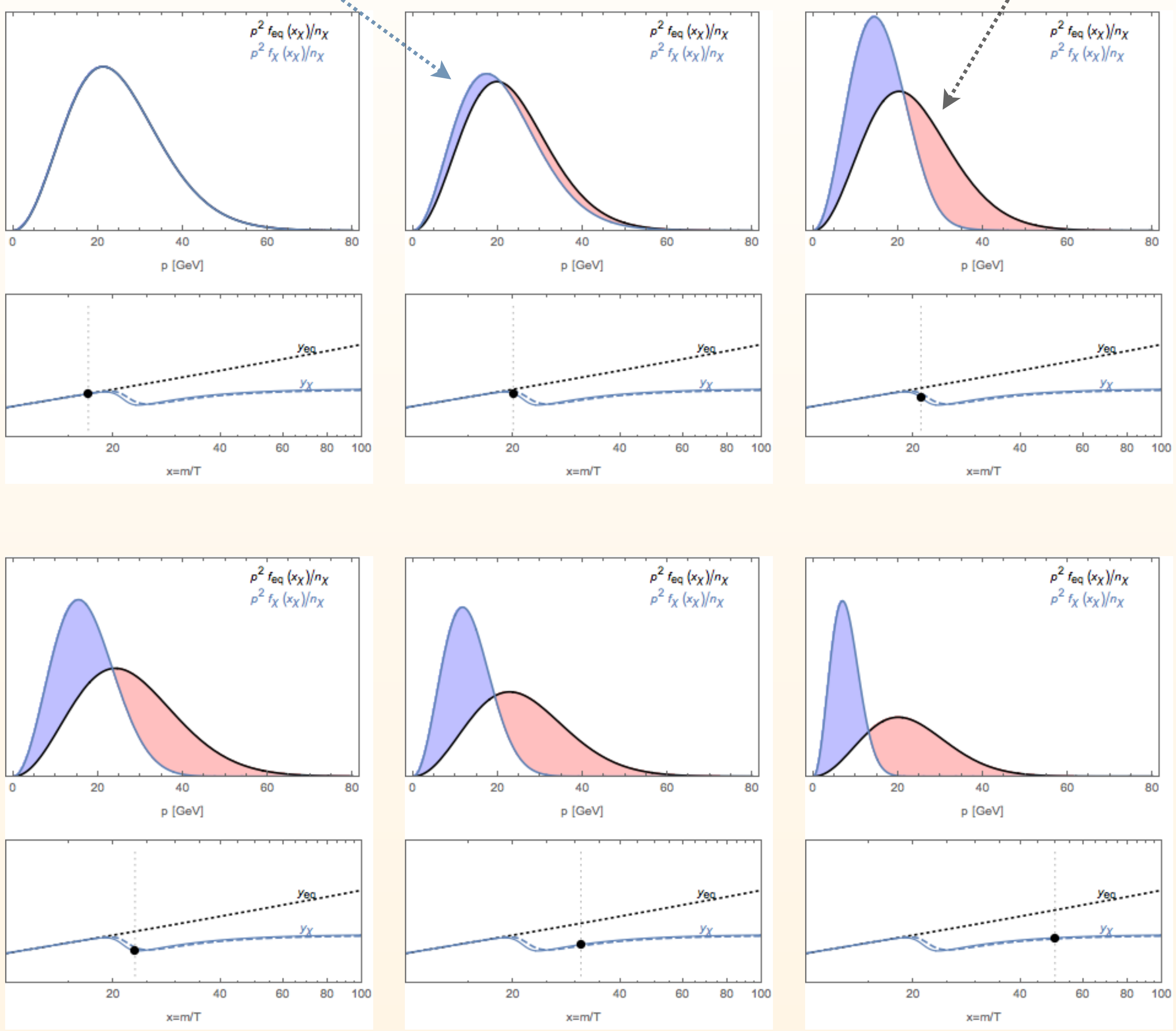
FULL PHASE-SPACE EVOLUTION

blue - full solution for f_{DM} at T_{DM}

m_{DM} = 58 GeV

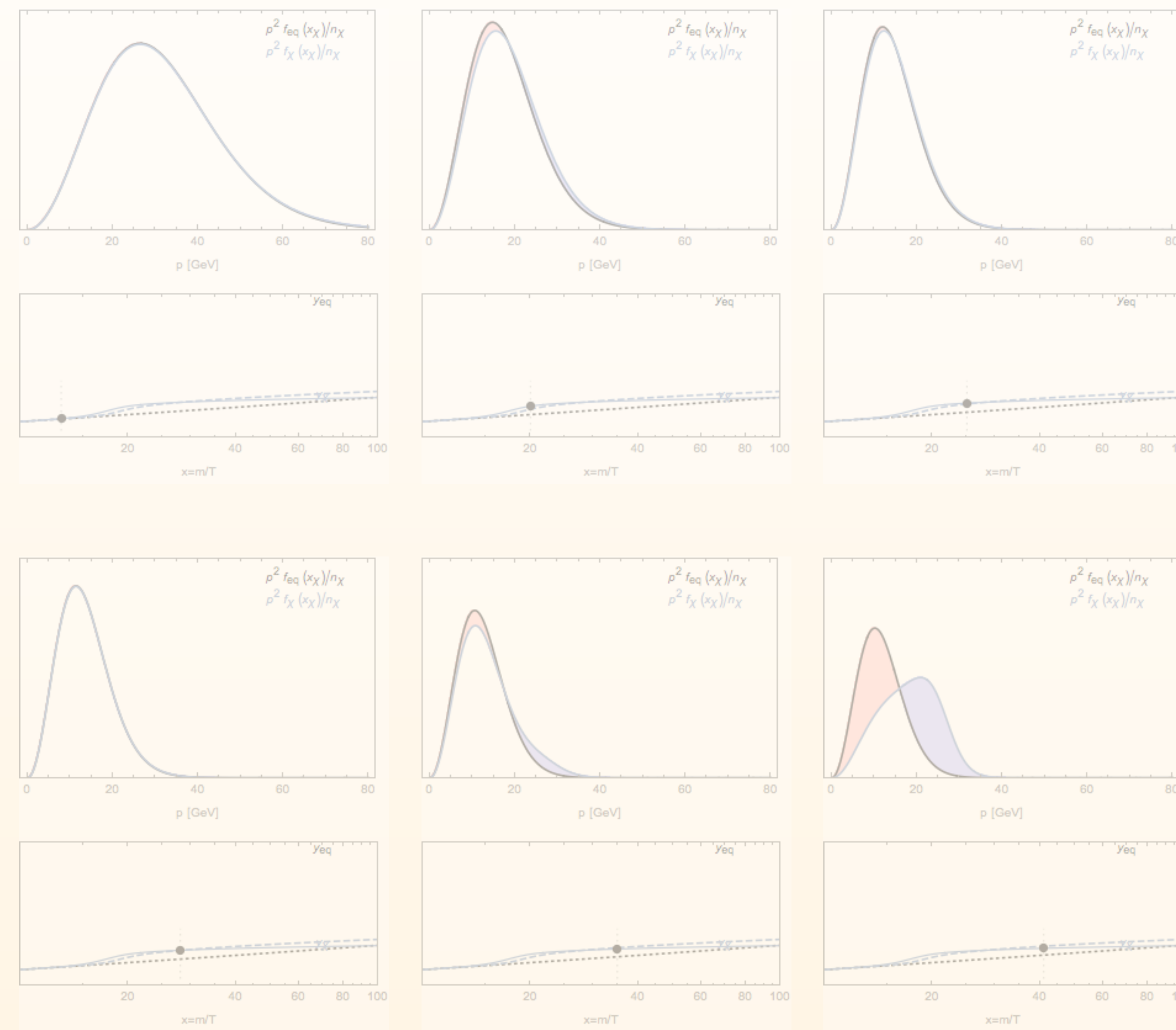
black - equilibrium at T_{DM}



significant deviation from equilibrium shape already around freeze-out

⟶ effect on relic density largest, both from different T and f_{DM}

m_{DM} = 62.5 GeV



large deviations only at later times, around freeze-out not far from eq. shape

⟶ effect on relic density ~only from different T

Full phase-space Evolution

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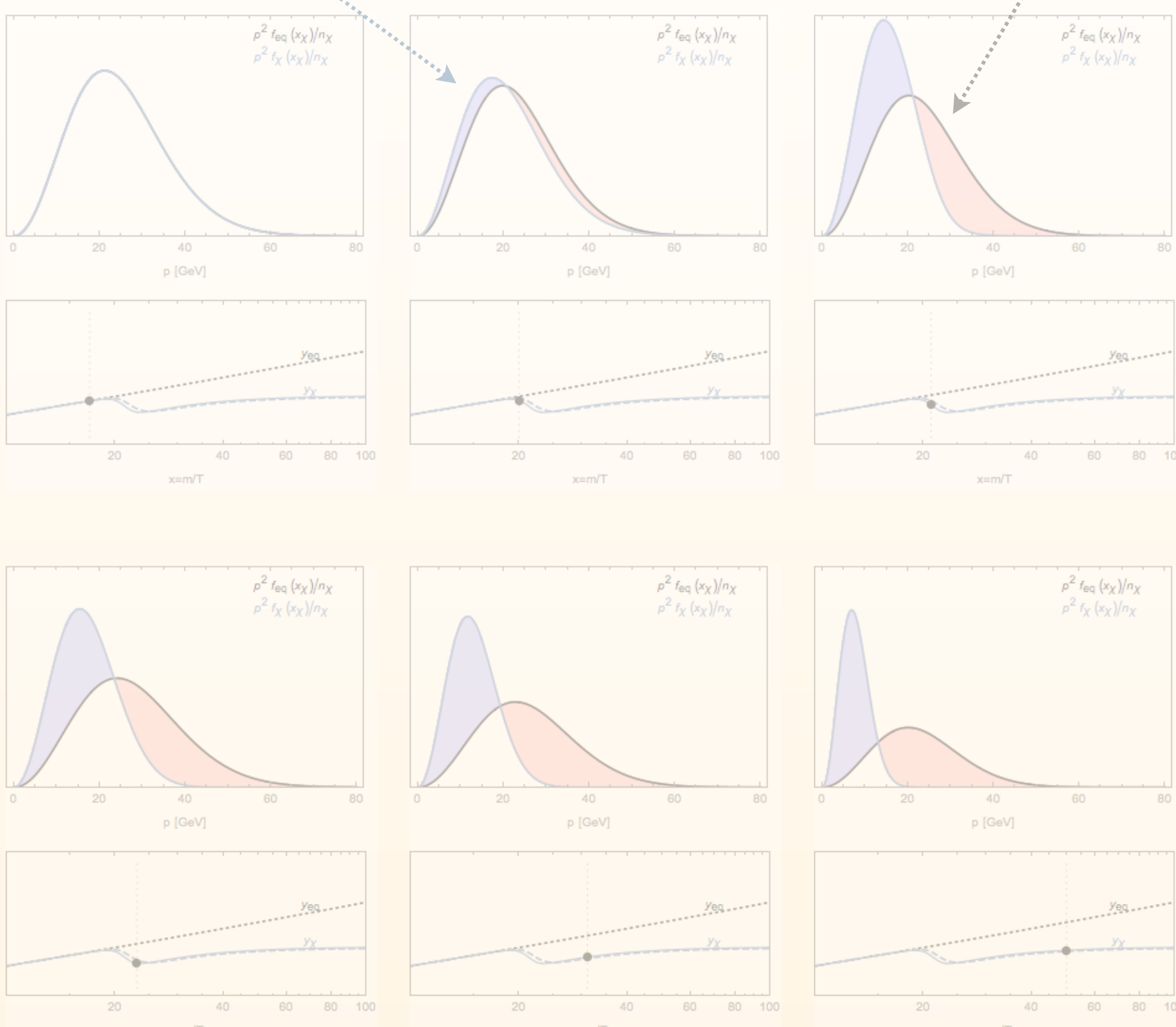

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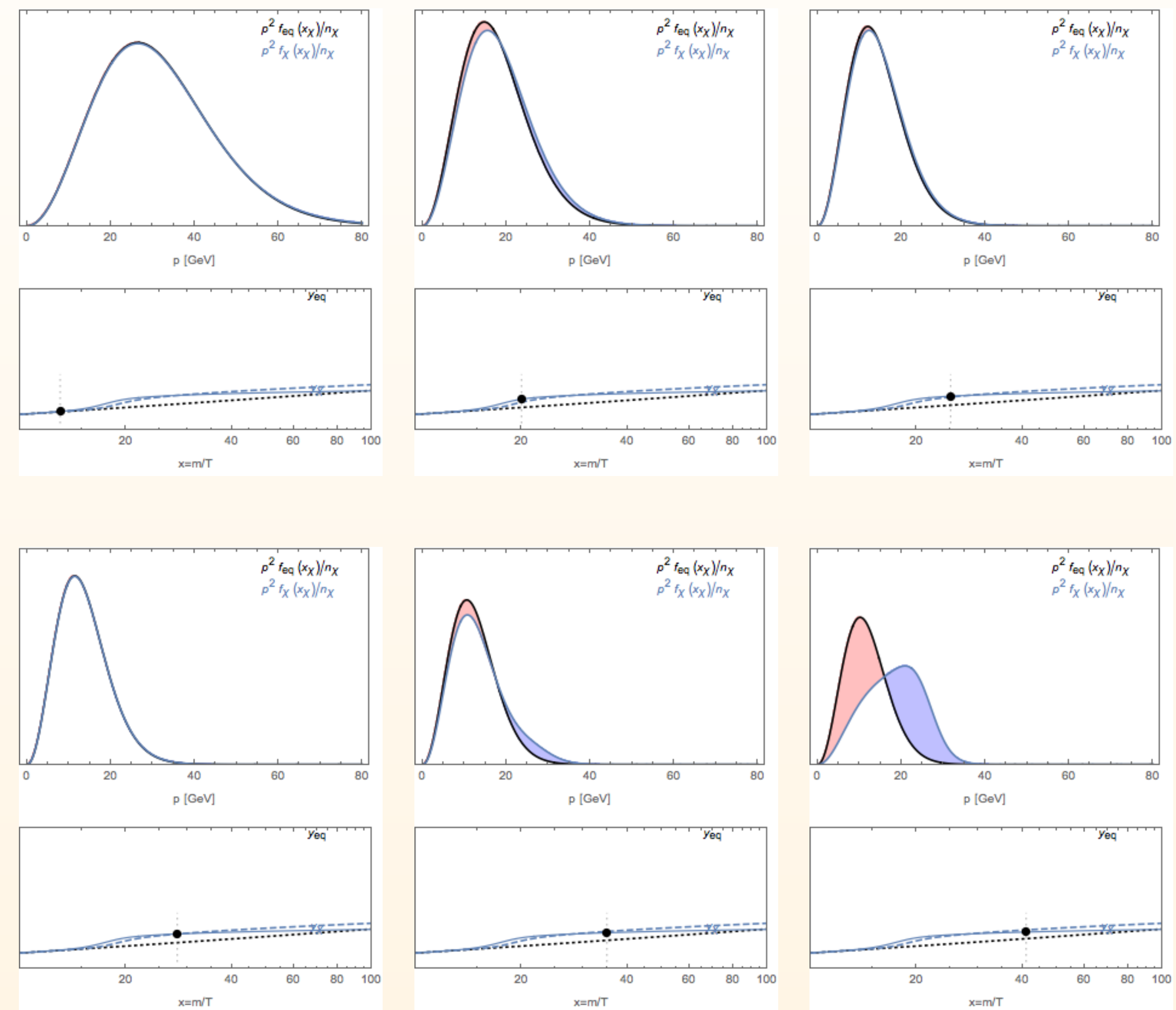

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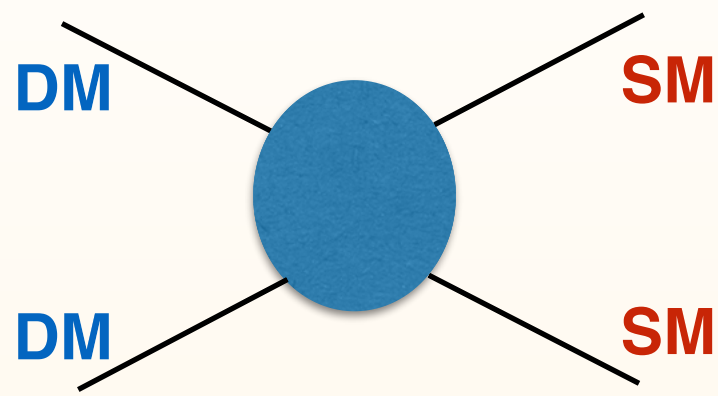
Chapter III:
Multi-component Dark Matter

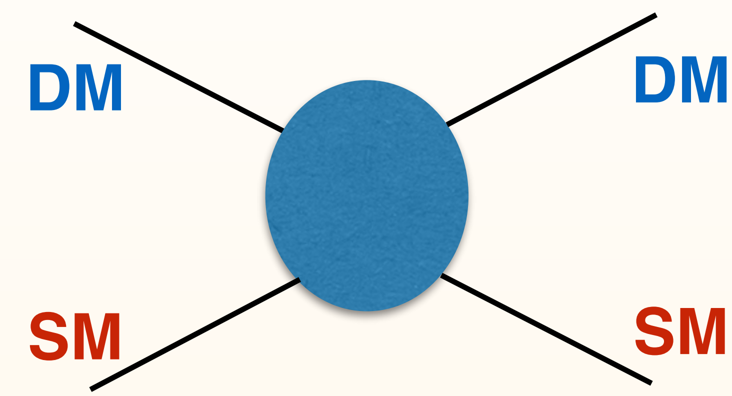
What if a non-minimal scenario?

In a minimal WIMP case <u>only two</u> types of processes are relevant:



annihilation

crossing sym.



(elastic) scattering

drives number density evolution

scatterings <u>typically</u> more frequent
(keeping the distribution to be in local thermal eq.)

Schmid, Schwarz, Widern '99; Green, Hofmann, Schwarz

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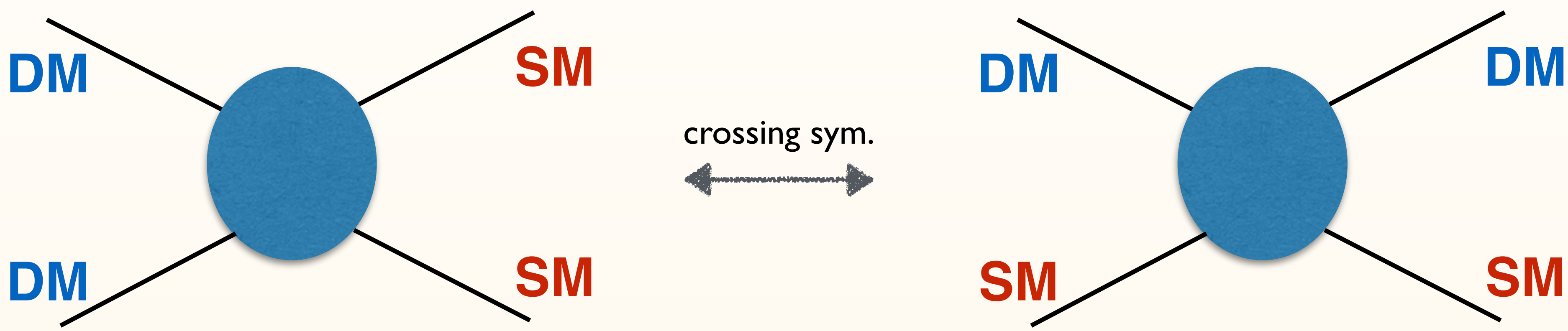


annihilation

drives number density evolution

(elastic) scattering

scatterings <u>typically</u> more frequent
(keeping the distribution to be in local thermal eq.)

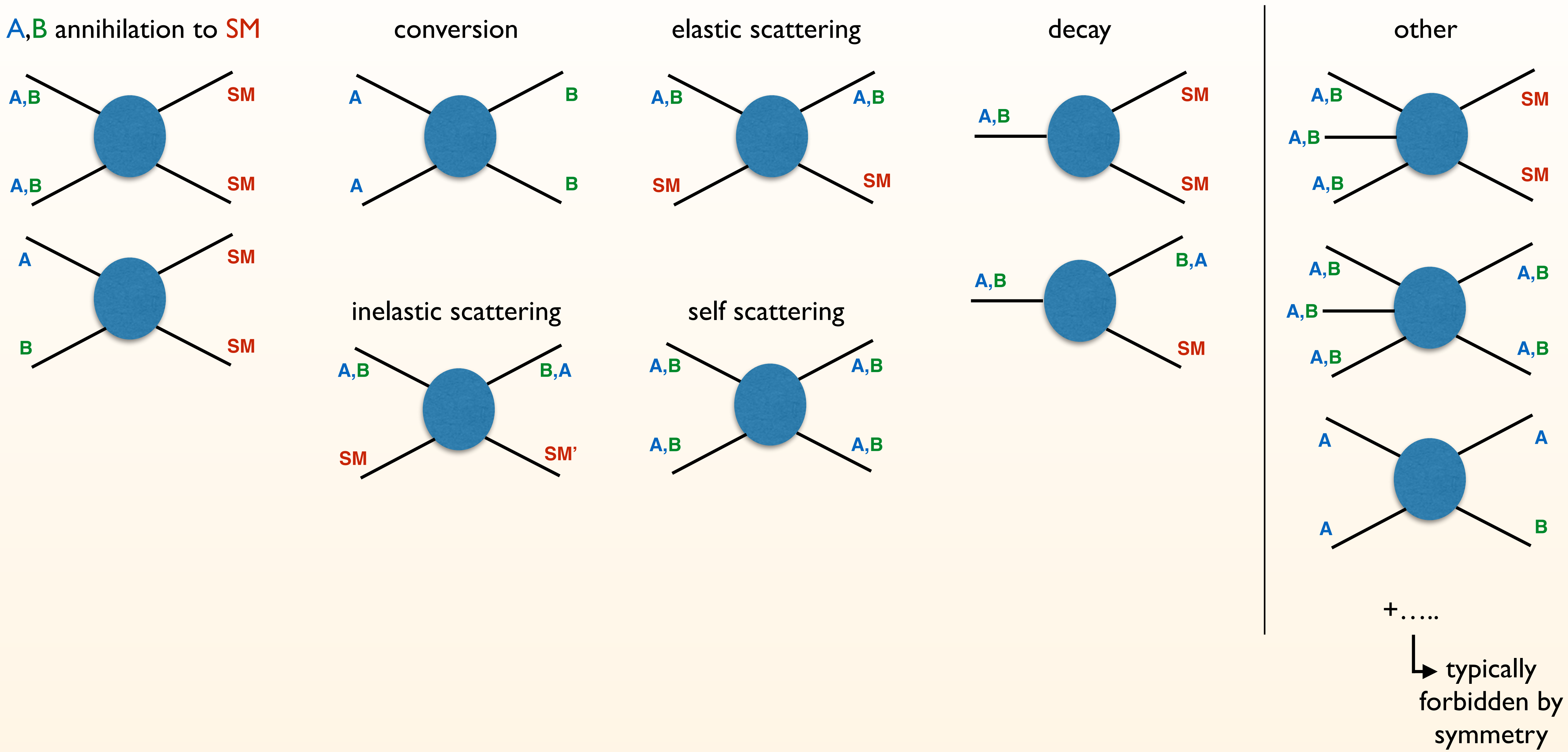
Schmid, Schwarz, Widern '99; Green, Hofmann, Schwarz

Recall: in *standard* thermal relic density calculation:

Critical assumption:
kinetic equilibrium at chemical decoupling

$$f_\chi \sim a(\mu) f_\chi^{\rm eq}$$

WHAT IF A NON-MINIMAL SCENARIO?

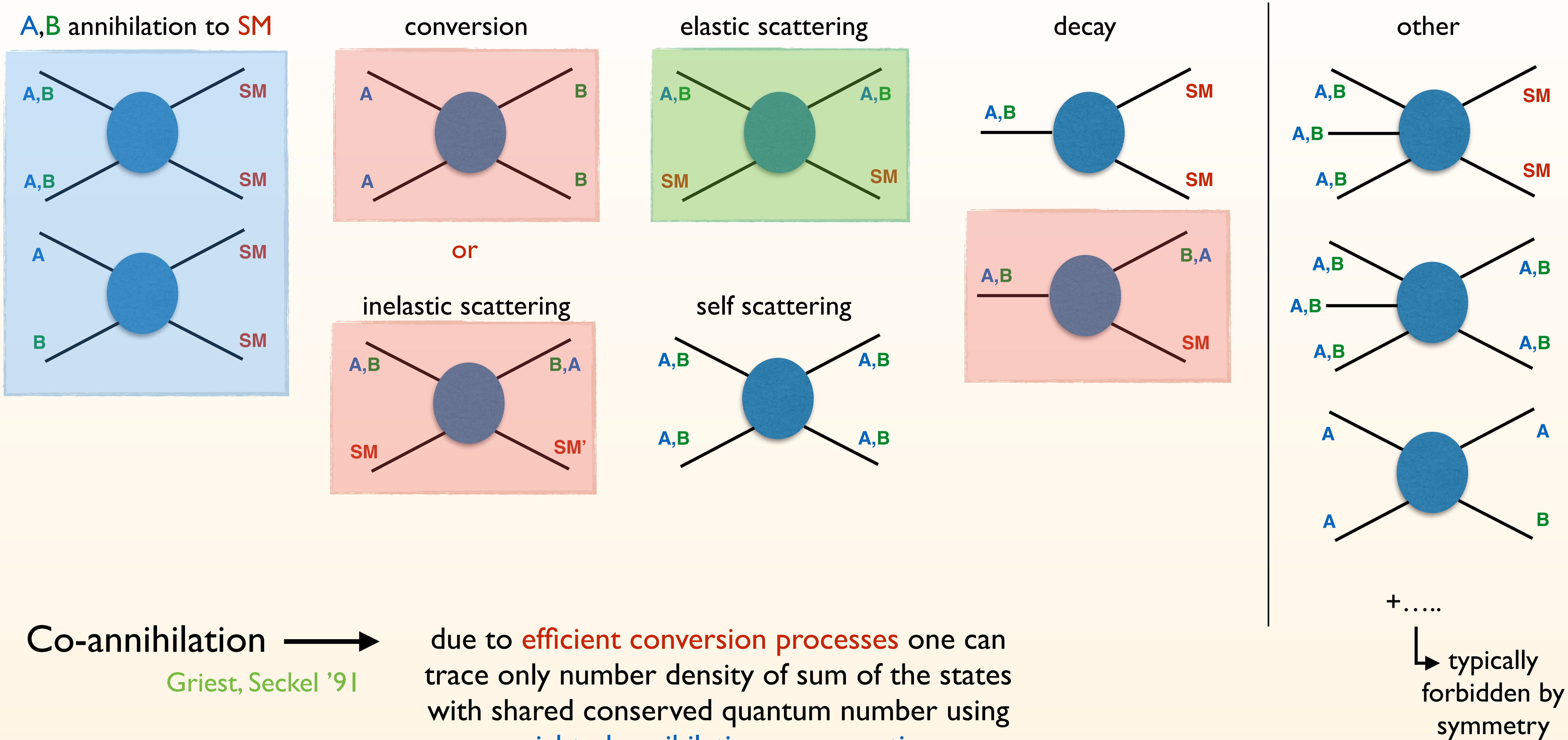


what one calculates

„defines" the mechanism
(necessary for it to work)

assumed in calculation (but not necessary)

WHAT IF A NON-MINIMAL SCENARIO?



Co-annihilation ⟶ due to efficient conversion processes one can trace only number density of sum of the states with shared conserved quantum number using weighted annihilation cross section

Griest, Seckel '91

what one calculates

„defines" the mechanism (necessary for it to work)

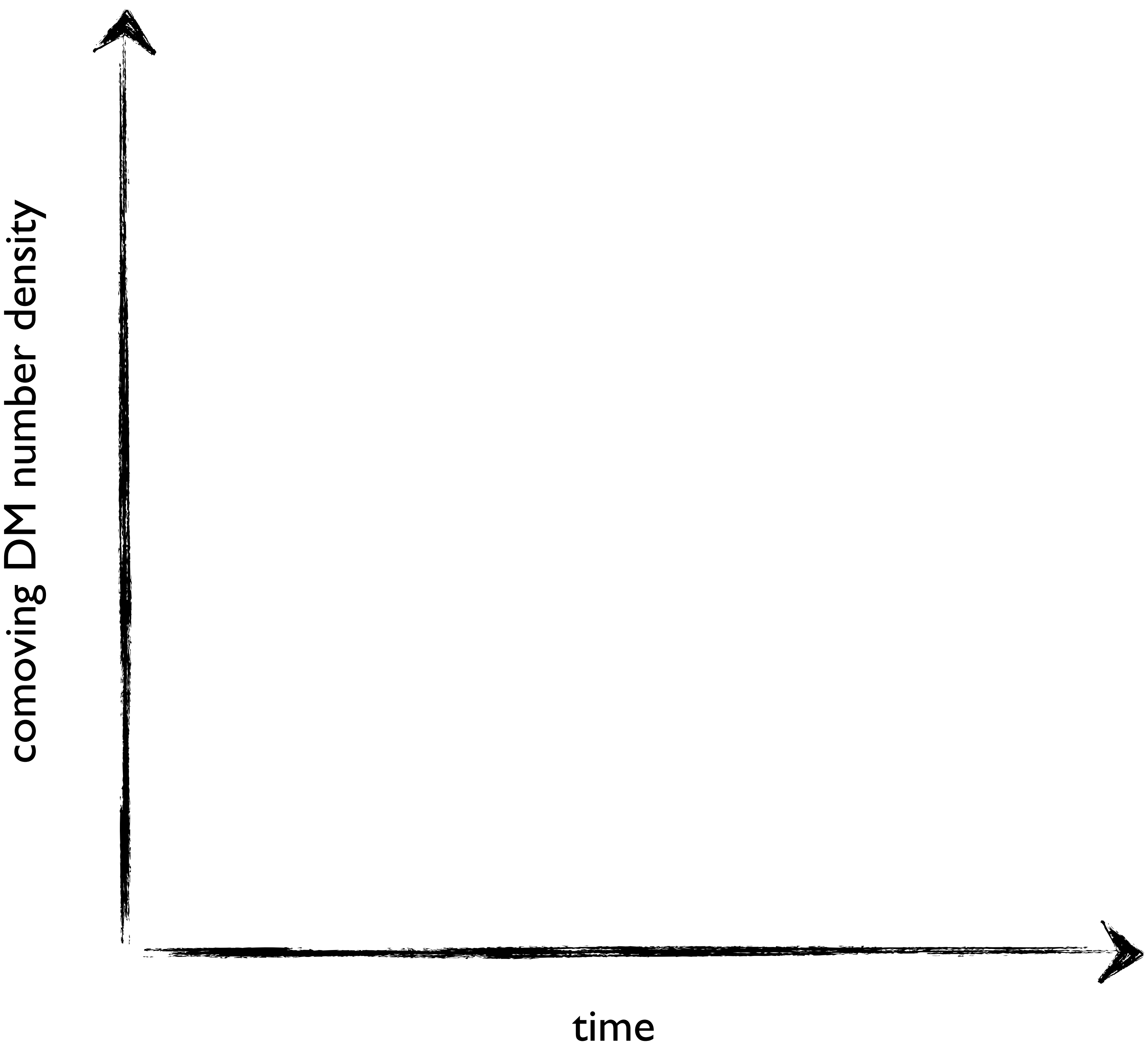
assumed in calculation (but not necessary)

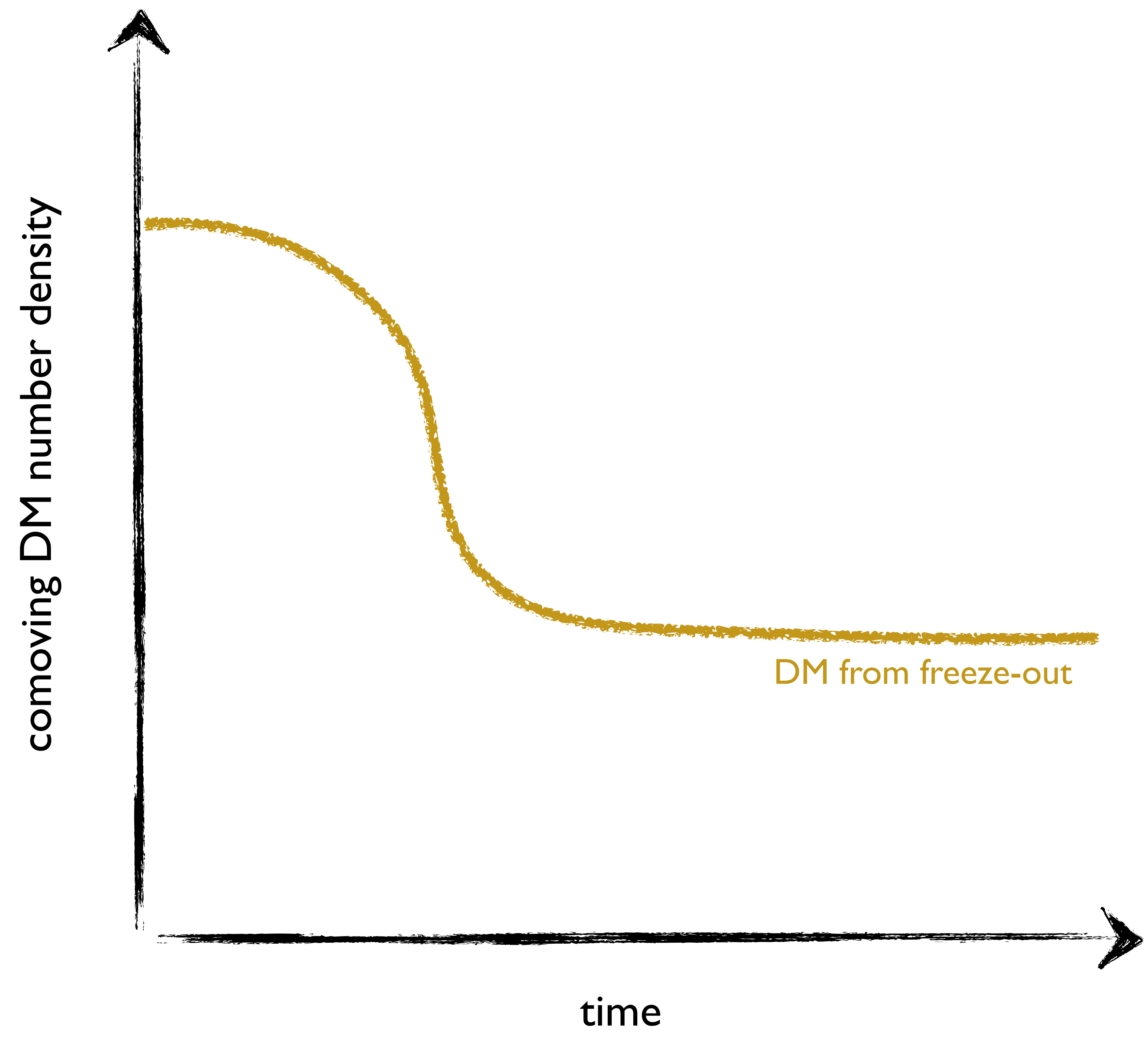
Example D:
When additional influx of DM arrives

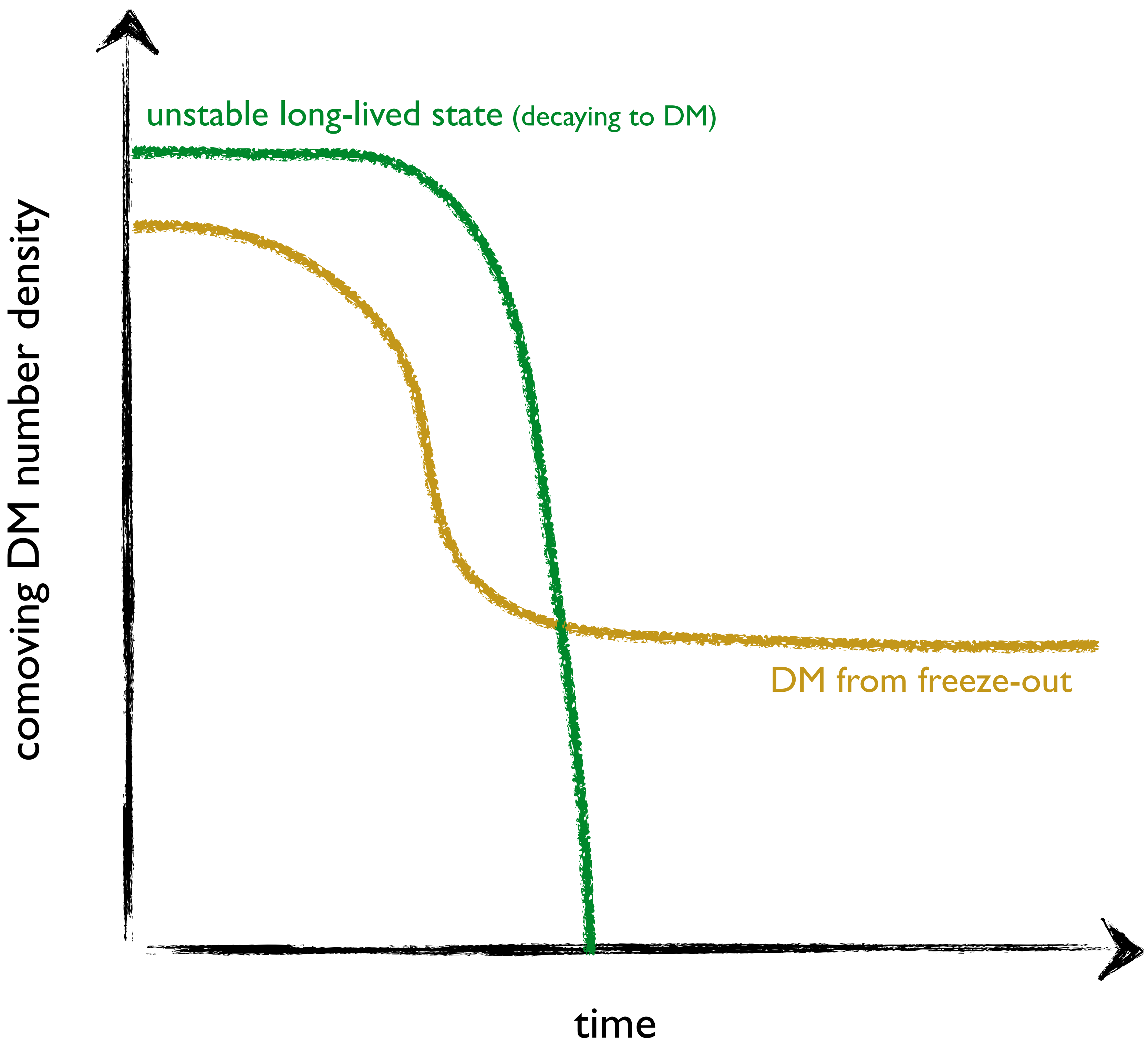
D) Multi-component dark sectors

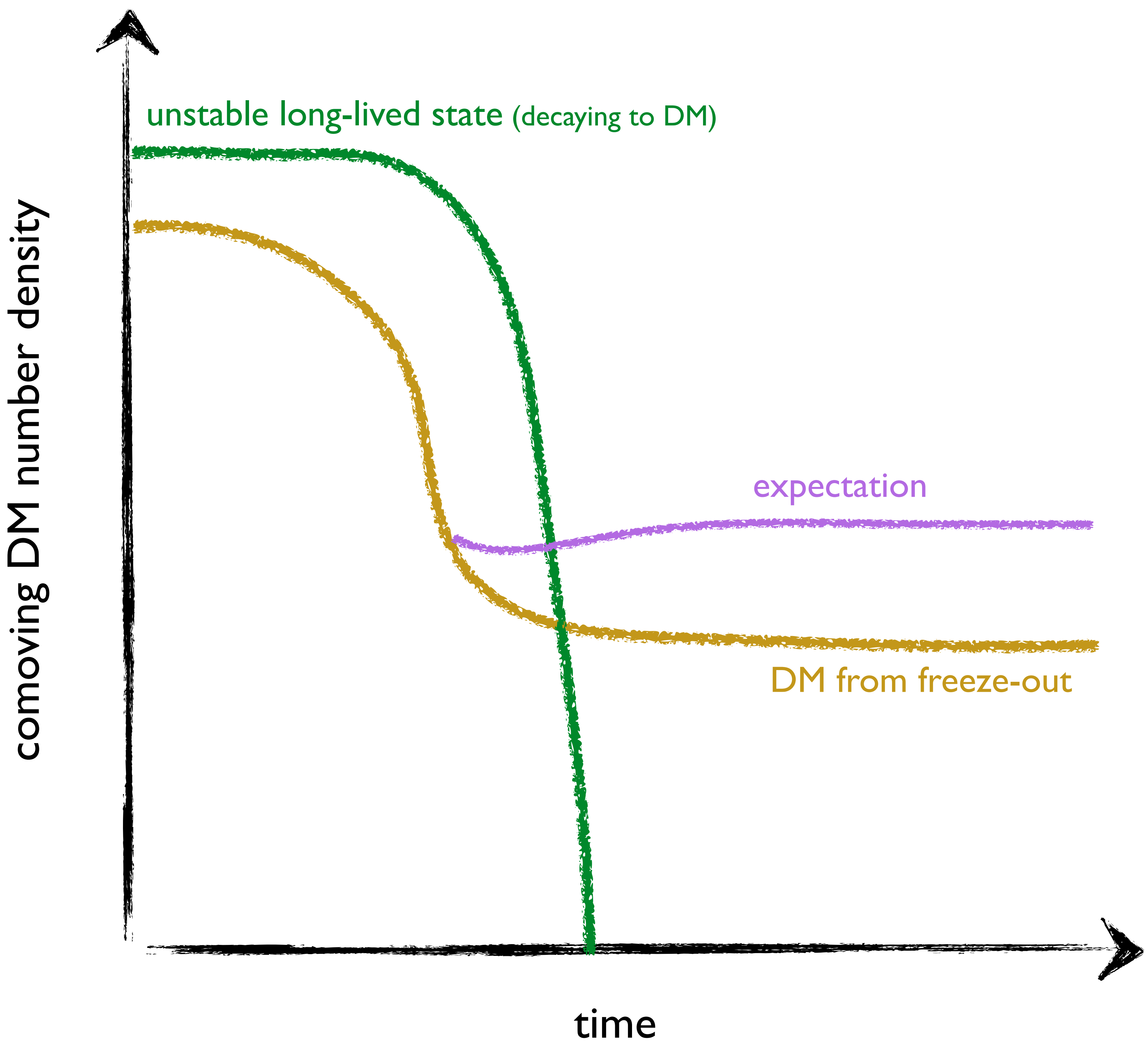
Sudden injection of more DM particles distorts $f_\chi(p)$
(e.g. from a decay or annihilation of other states)

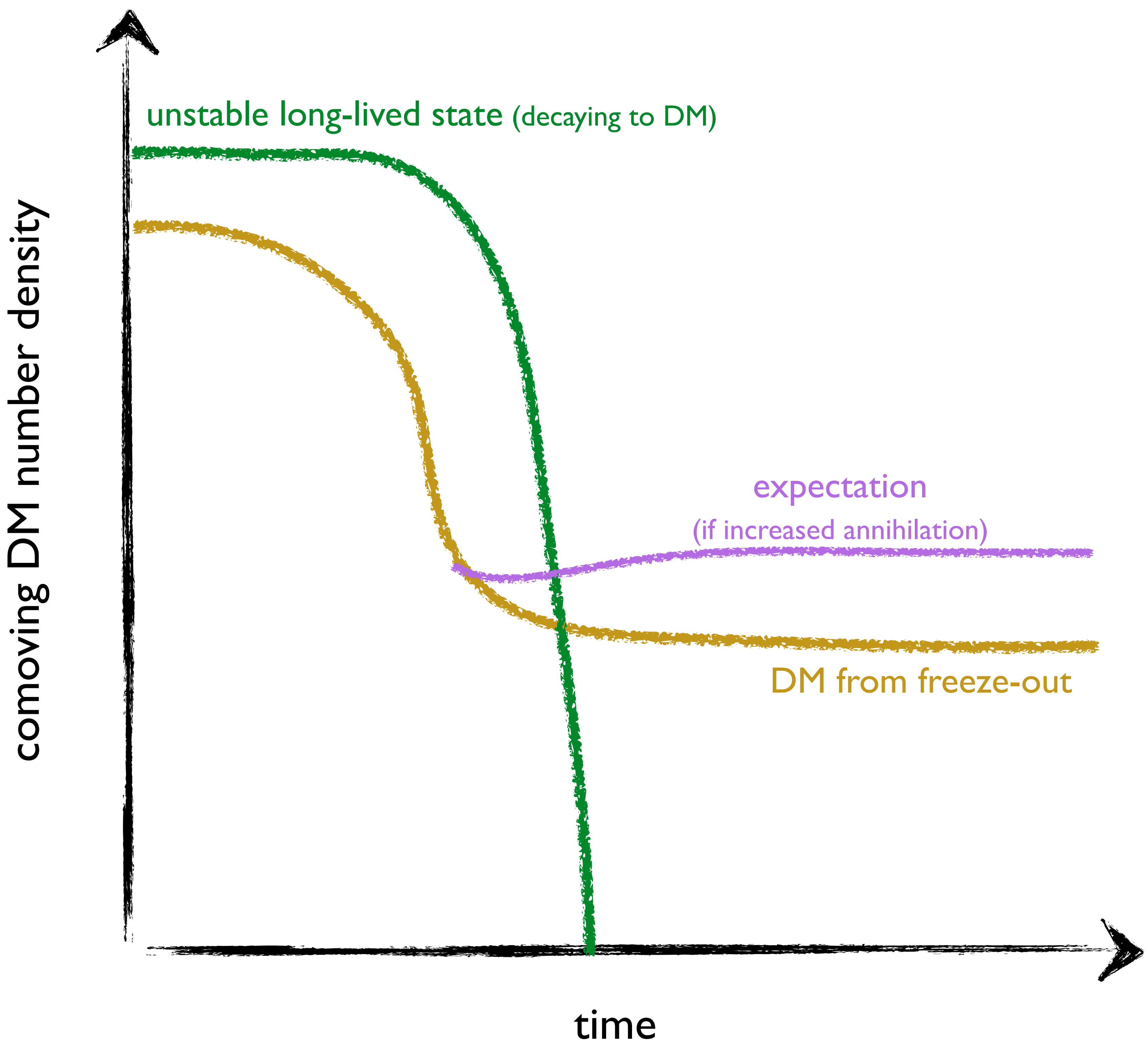
- this can modify the annihilation rate (if still active)
- how does the thermalization due to elastic scatterings happen?

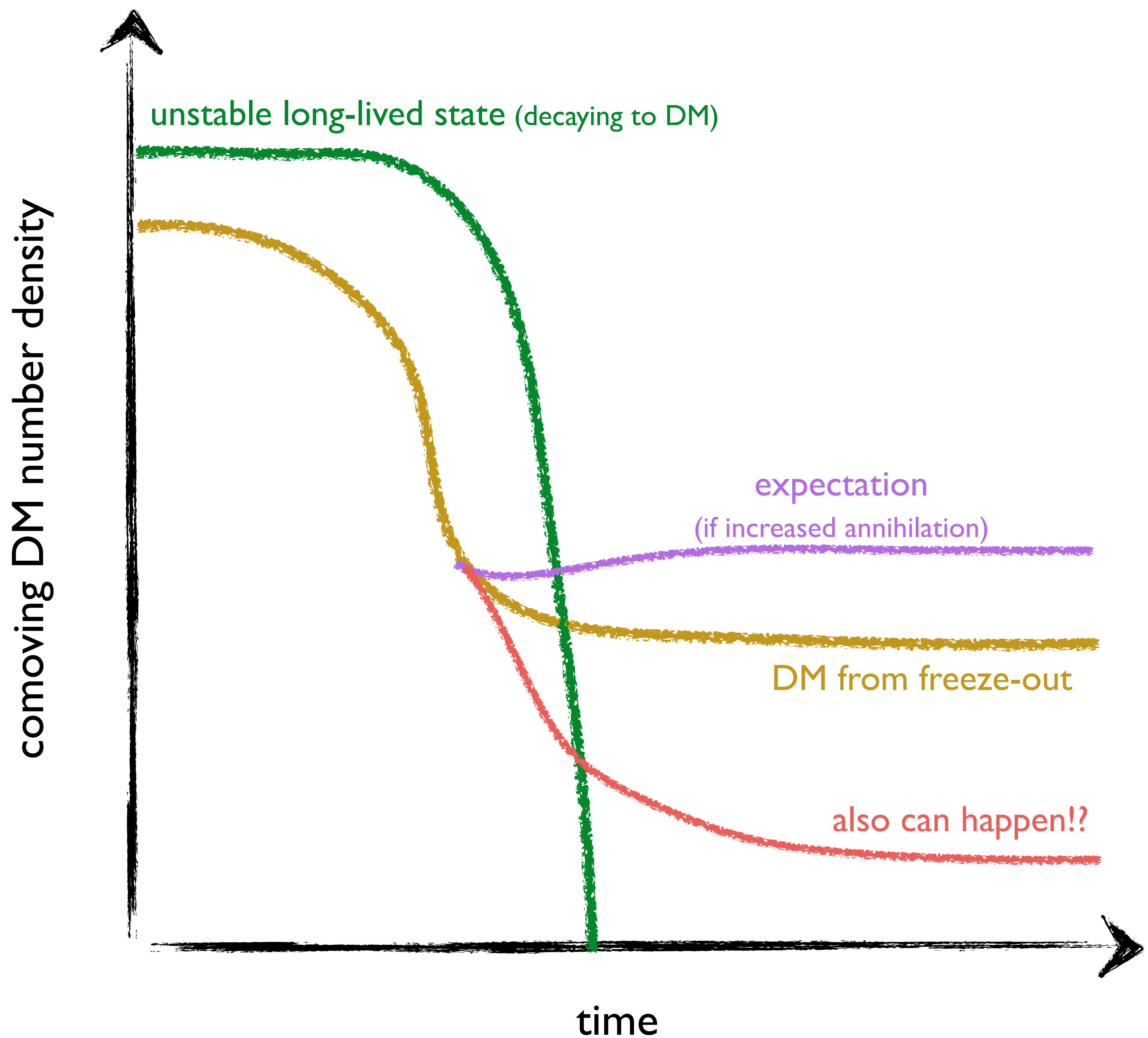

comoving DM number density
time


comoving DM number density
DM from freeze-out
time


unstable long-lived state (decaying to DM)
comoving DM number density
DM from freeze-out
time


unstable long-lived state (decaying to DM)
expectation
DM from freeze-out
comoving DM number density
time


unstable long-lived state (decaying to DM)
expectation
(if increased annihilation)
DM from freeze-out
comoving DM number density
time

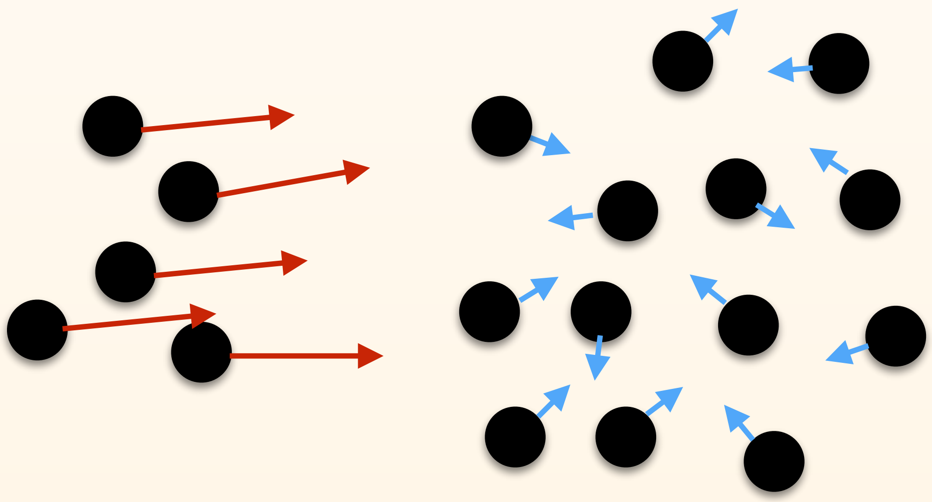

unstable long-lived state (decaying to DM)
expectation
(if increased annihilation)
DM from freeze-out
also can happen!?
comoving DM number density
time

1) DM produced via:

- 1st component from thermal freeze-out
- 2nd component from a decay $\phi \to \bar{\chi}\chi$

2) DM annihilation has a threshold

e.g. $\chi\bar{\chi} \to f\bar{f}$ with $m_\chi \lesssim m_f$



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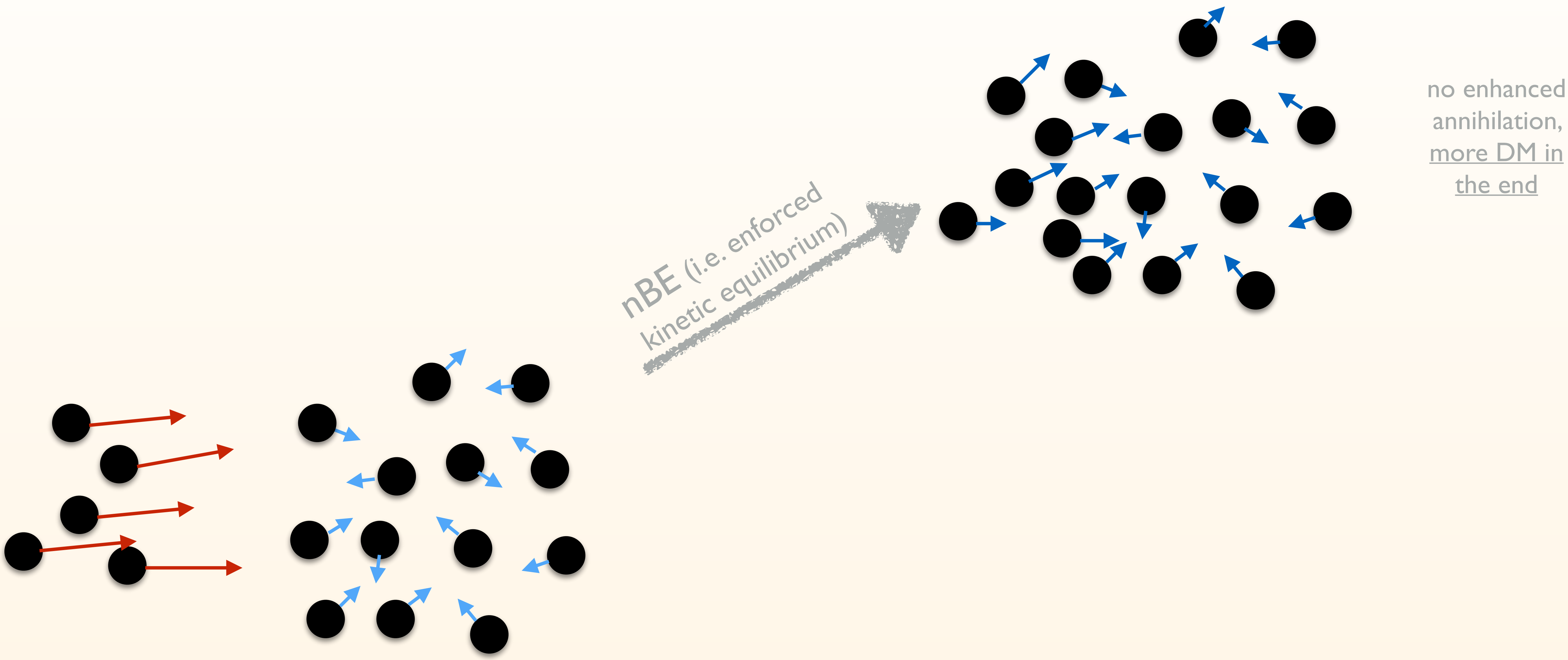

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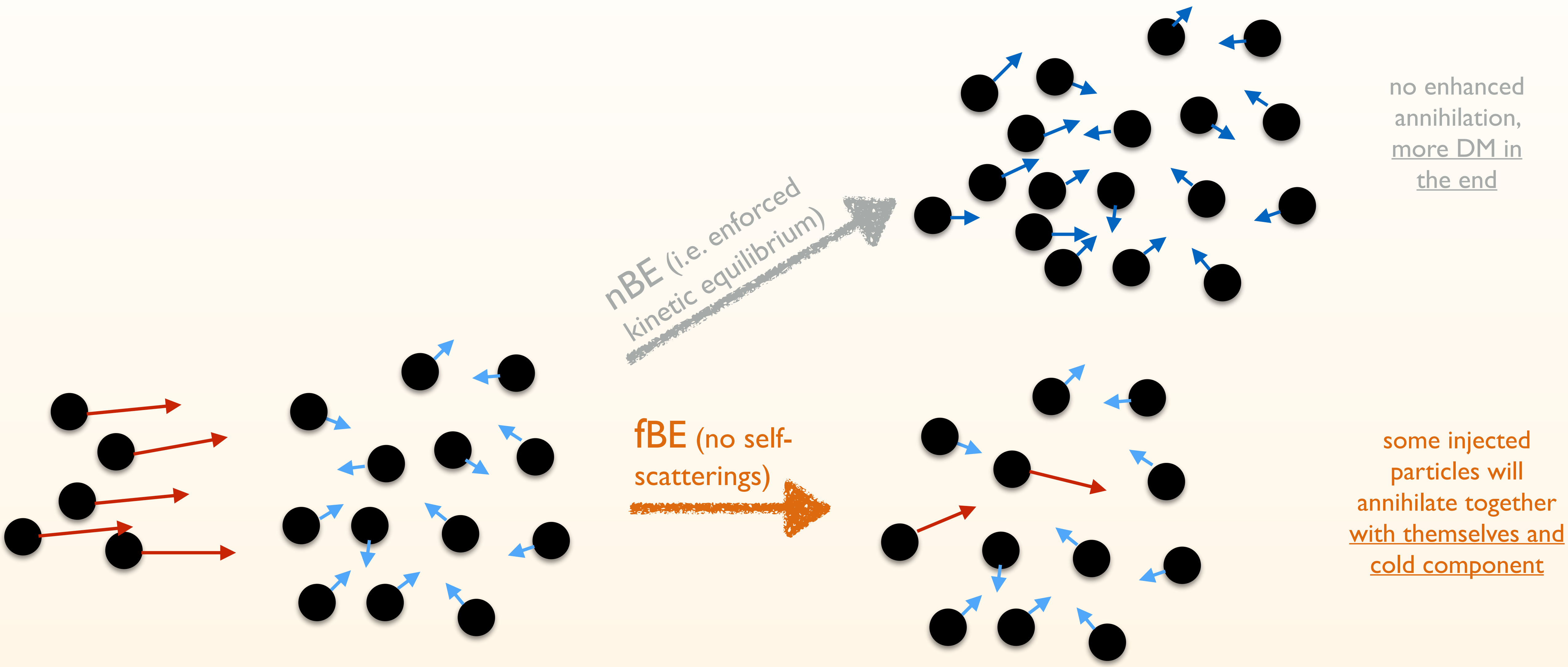

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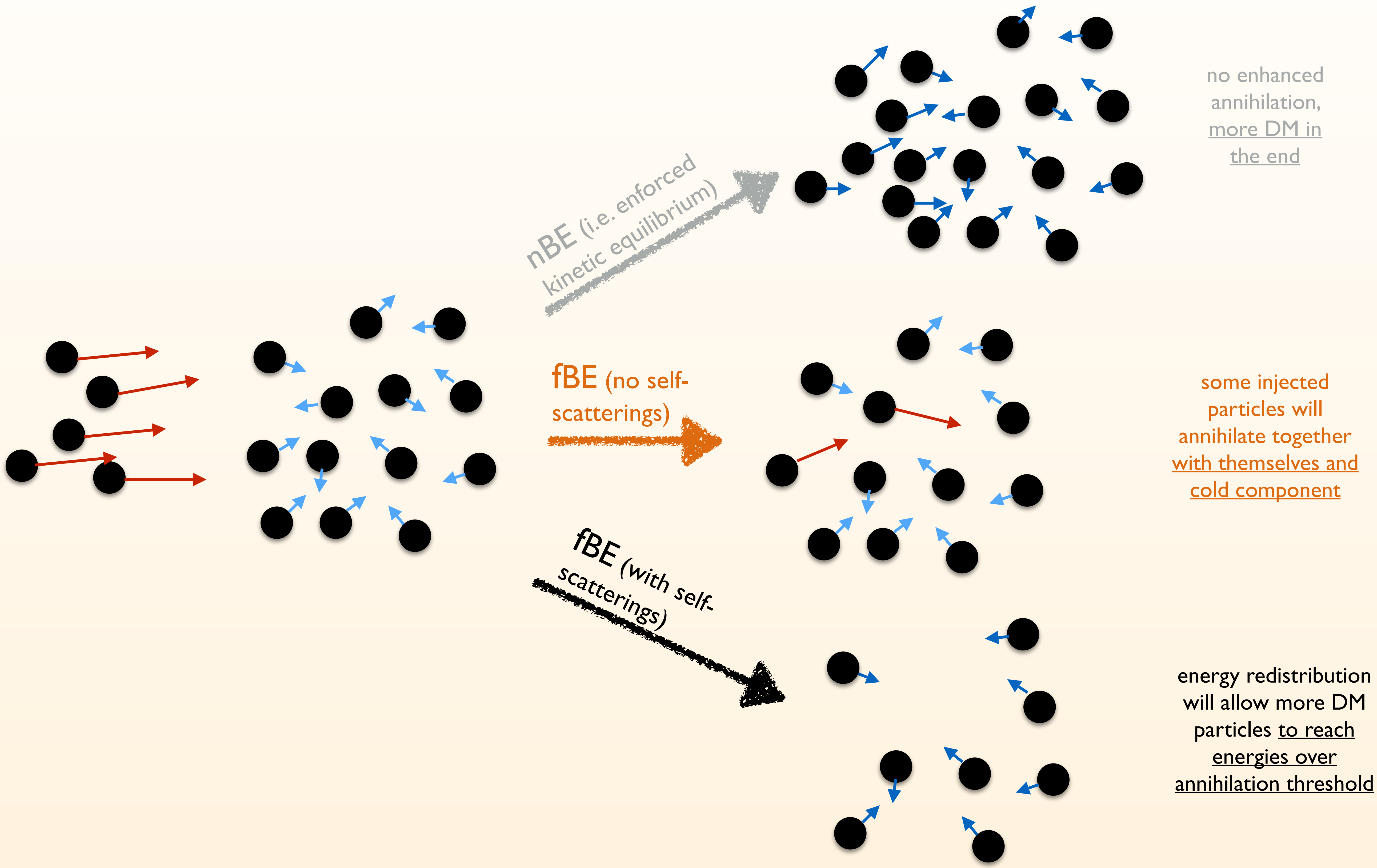

AH, Laletin 2204.07078

EXAMPLE EVOLUTION

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- 2nd component from a decay $\phi \to \bar{\chi}\chi$

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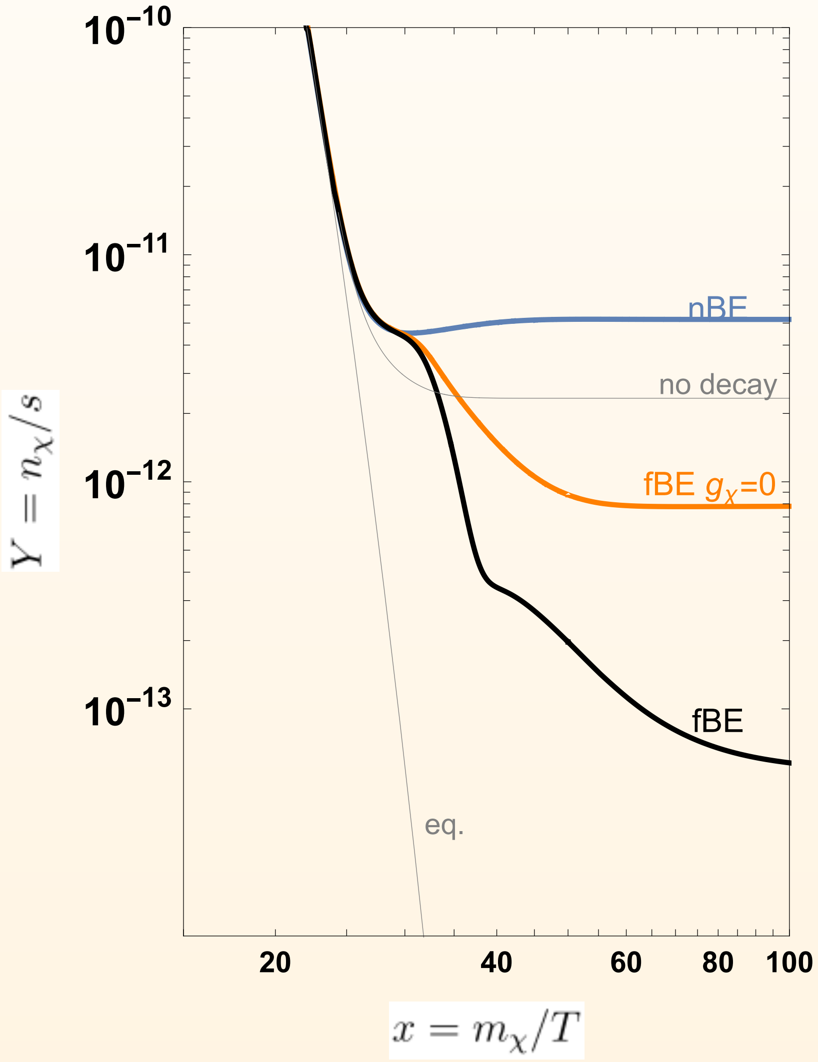
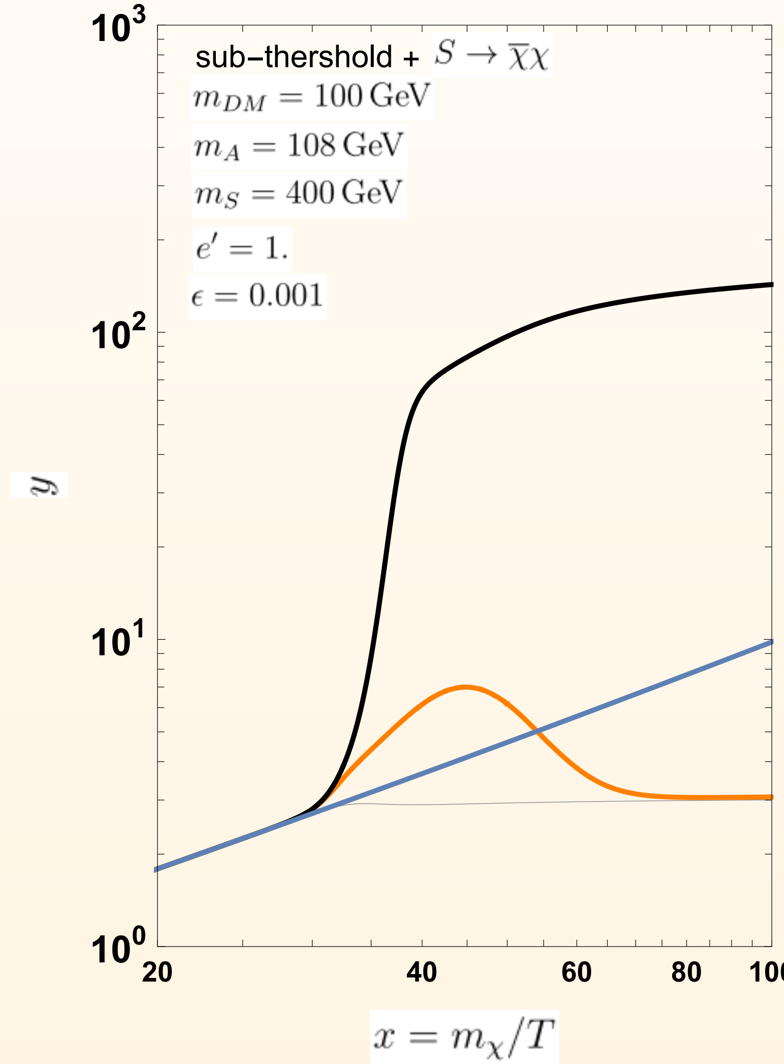
e.g. $\chi\bar{\chi} \to f\bar{f}$ with $m_\chi \lesssim m_f$

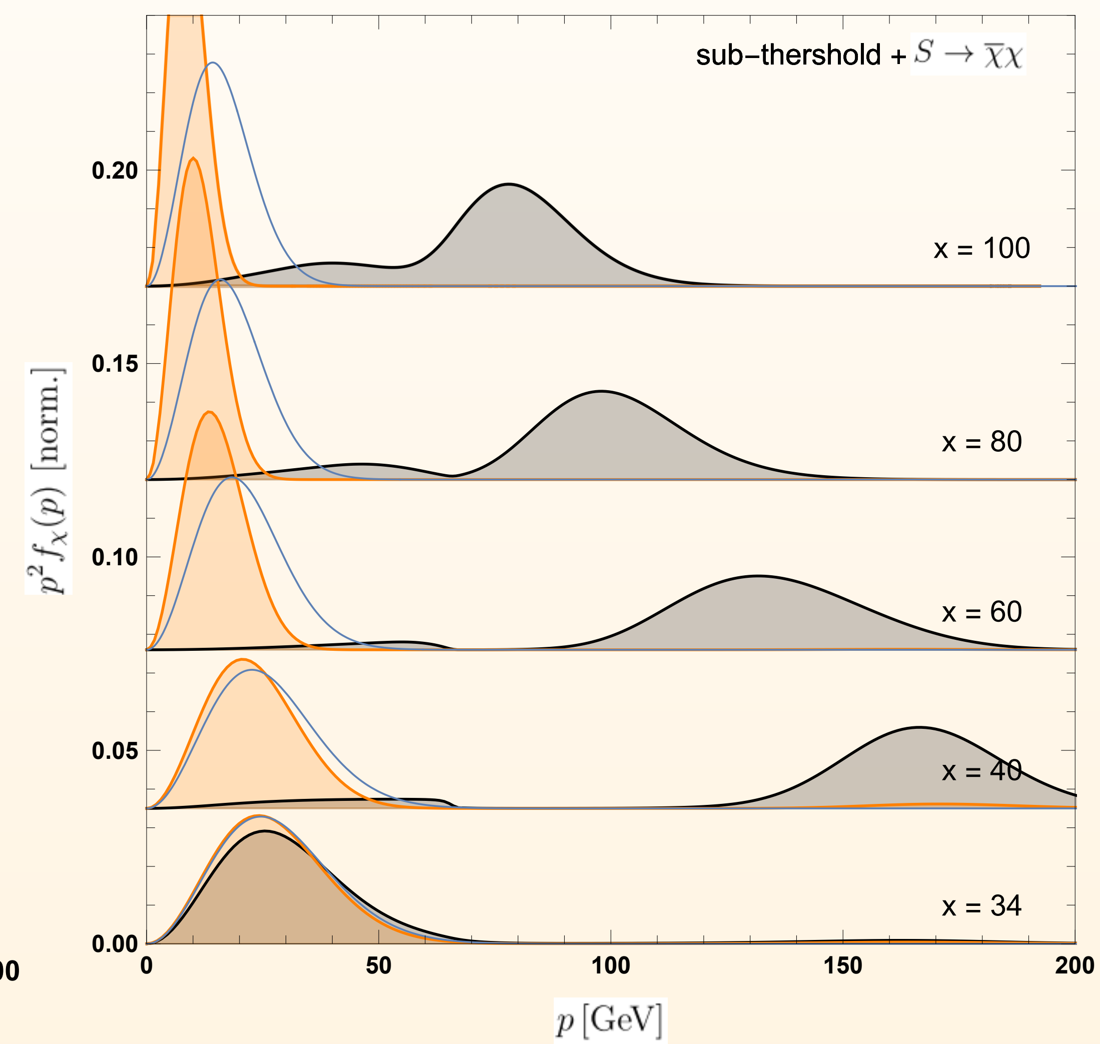
$Y \sim$ number density



$y \sim$ temperature



$p^2 f(p) \sim$ momentum distribution



TAKEAWAY MESSAGE

When computing relic density of dark matter one needs carefully to check if the standard treatment is sufficient for the case at hand

"Everything should be made as simple as possible, but no simpler."

attributed to* **Albert Einstein**

*The published quote reads:
"It can scarcely be denied that the supreme goal of all theory is to make the irreducible basic elements as simple and as few as possible without having to surrender the adequate representation of a single datum of experience."
„On the Method of Theoretical Physics" ,The Herbert Spencer Lecture, delivered at Oxford (10 June 1933); also published in *Philosophy of Science*, Vol. 1, No. 2 (April 1934), pp. 163-169., p. 165