

# Advances in small-x physics

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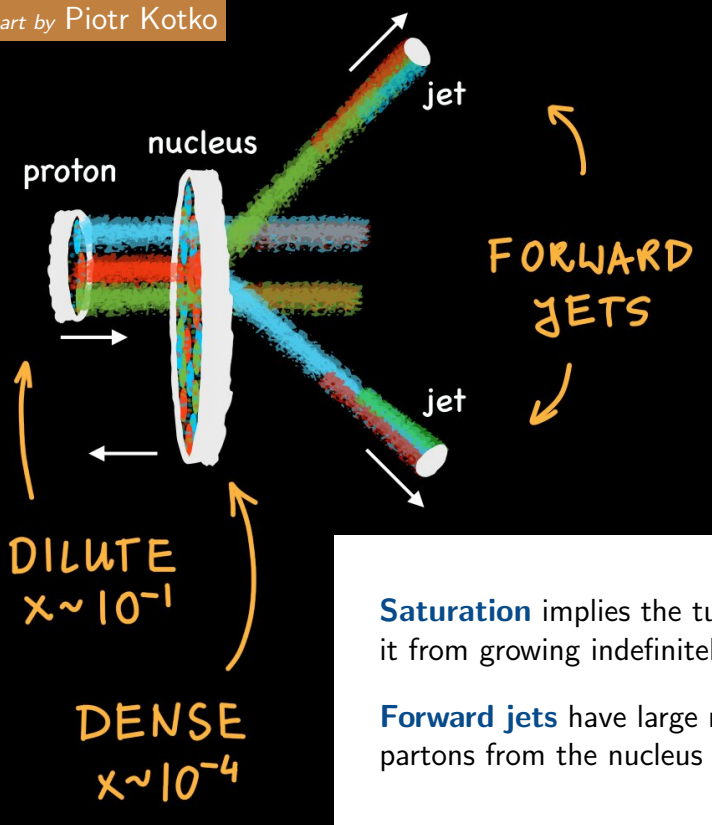
*on*

21-10-2022

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# QCD evolution, dilute vs. dense, forward jets

art by Piotr Kotko



A **dilute** system carries a few **high- $x$**  partons contributing to the hard scattering.

A **dense** system carries many **low- $x$**  partons.

At high density, gluons are imagined to undergo recombination, and to saturate.

This is modeled with non-linear evolution equations, involving explicit **non-vanishing  $k_T$** .

**Saturation** implies the turnover of the gluon density, stopping it from growing indefinitely for small  $x$ .

**Forward jets** have large rapidities, and trigger events in which partons from the nucleus have small  $x$ .

The CGC is an effective field theory for high energy QCD.

introduction from  
Morreale, Salazar 2021

Partons carrying large hadron momentum fraction  $x$  are treated as static color sources  $\rho$ .

Their color charge distribution is non-perturbative and is dictated by a gauge invariant weight functional  $W_{x_0}[\rho]$ . The sources generate a current  $J^{\mu,a}$ .

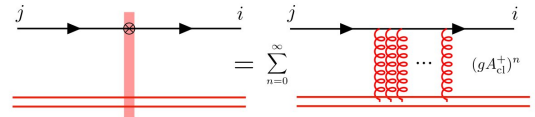
The partons carrying small  $x$  are treated as a dynamical classical field  $A^{\mu,a}$ .

Sources and fields are related by the Yang-Mills equations  $[D_\mu, F_{\mu\nu}] = J_\nu$ .

The expectation value  $\langle \mathcal{O} \rangle_{x_0}$  of an observable  $\mathcal{O}$  is calculated as the path integral  $\mathcal{O}[\rho]$  in the presence of sources from  $W_{x_0}[\rho]$ , averaged over all possible configurations  $\rho$ .

The interaction of a highly energetic color charged particle with the classical field  $A$  in the eikonal approximation is encoded in the light-like Wilson lines

$$U(x_T) = \text{Pexp} \left\{ ig \int_{-\infty}^{\infty} dx^+ A^{-,a}(x^+, x_T) t^a \right\}$$

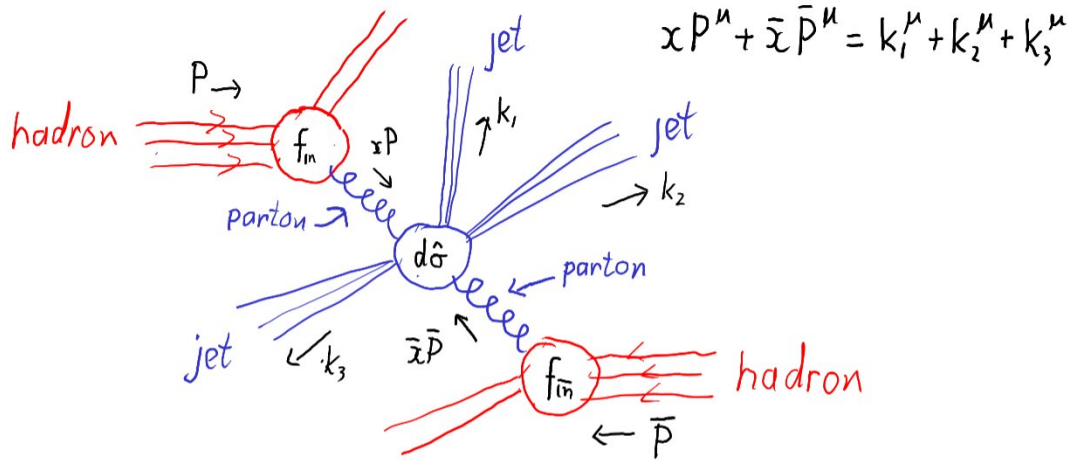


Balitsky, Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner

Evolution in  $x$  of  $W_x[\rho]$  implies an infinite hierarchy (known as the **B-JIMWLK** hierarchy) of non-linear coupled equations dictating the evolution of  $n$ -point Wilson line correlators.

Cross section calculations involve particle wave functions and Wilson line correlators.

# Collinear factorization in QCD



$$xP^\mu + \bar{x}\bar{P}^\mu = k_1^\mu + k_2^\mu + k_3^\mu$$

factorization scale

$$d\sigma = \int_0^1 dx f_{in}(x, \mu) \int_0^1 d\bar{x} f_{in}(\bar{x}, \mu) d\hat{\sigma}(x, \bar{x}, \{k_i\}_{i=1}^n, \mu)$$

$\nwarrow$  universal PDFs, fit to data       $\nearrow$  parton-level cross section calculable perturbatively

# ITMD Factorization

For forward dijet production  
in dilute-dense hadronic collisions

Generalized TMD factorization (Dominguez, Marquet, Xiao, Yuan 2011)

$$d\sigma_{AB \rightarrow X} = \int dk_T^2 \int d\chi_A \sum_i \int d\chi_B \sum_b \Phi_{gb}^{(i)}(\chi_A, k_T, \mu) f_{b/B}(\chi_B, \mu) d\hat{\sigma}_{gb \rightarrow X}^{(i)}(\chi_A, \chi_B, \mu)$$

For  $\chi_A \ll 1$  and  $P_T \gg k_T \sim Q_s$  (jets almost back-to-back).

TMD gluon distributions  $\Phi_{gb}^{(i)}(\chi_A, k_T, \mu)$  satisfy non-linear evolution equations.

Partonic cross section  $d\hat{\sigma}_{gb}^{(i)}$  is on-shell, but depends on color-structure  $i$ .

Improved TMD factorization (Kotko, Kutak, Marquet, Petreska, Sapeta, AvH 2015)

$$d\sigma_{AB \rightarrow X} = \int dk_T^2 \int d\chi_A \sum_i \int d\chi_B \sum_b \Phi_{gb}^{(i)}(\chi_A, k_T, \mu) f_{b/B}(\chi_B, \mu) d\hat{\sigma}_{gb \rightarrow X}^{(i)}(\chi_A, \chi_B, \mathbf{k}_T, \mu)$$

Originally a model interpolating between High Energy Factorization and Generalized TMD factorization:  $P_T \gtrsim k_T \gtrsim Q_s$ .

Partonic cross section  $d\hat{\sigma}_{gb}^{(i)}$  is **off-shell** and depends on color-structure  $i$ .

ITMD formalism is obtained from the CGC formalism, by including so-called kinematic twist corrections (Antinluk, Boussarie, Kotko 2019).

# ITMD for 2 jets (or ITMD\* for more)

Schematic hybrid (non-ITMD) factorization formula

$$d\sigma = \sum_{y=g,u,d,\dots} \int dx_1 d^2k_T \int dx_2 d\Phi_{g^*y \rightarrow n} \frac{1}{\text{flux}_{gy}} \mathcal{F}_g(x_1, k_T, \mu) f_y(x_2, \mu) \sum_{\text{color}} \left| \mathcal{M}_{g^*y \rightarrow n}^{(\text{color})} \right|^2$$

$$\mathcal{F}_g \sum_{\text{color}} \left| \mathcal{M}^{(\text{color})} \right|^2 = \mathcal{F}_g \sum_{i_1, i_2, \dots, i_{n+2}} \sum_{j_1, j_2, \dots, j_{n+2}} \left( \tilde{\mathcal{M}}_{j_1 j_2 \dots j_{n+2}}^{i_1 i_2 \dots i_{n+2}} \right)^* \left( \tilde{\mathcal{M}}_{j_1 j_2 \dots j_{n+2}}^{i_1 i_2 \dots i_{n+2}} \right)$$

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with (Bomhof, Mulders, Pijlman 2006; Bury, Kotko, Kutak 2018)

$$\begin{aligned} & (N_c^2 - 1) \sum_{i_1, \dots, i_n} \sum_{j_1, \dots, j_{n+2}} \sum_{\bar{i}_1, \dots, \bar{i}_{n+2}} \sum_{\bar{j}_1, \dots, \bar{j}_{n+2}} \left( \tilde{\mathcal{M}}_{j_1 j_2 \dots j_{n+2}}^{i_1 i_2 \dots i_{n+2}} \right)^* \left( \tilde{\mathcal{M}}_{\bar{j}_1 \bar{j}_2 \dots \bar{j}_{n+2}}^{\bar{i}_1 \bar{i}_2 \dots \bar{i}_{n+2}} \right) \\ & \times 2 \int \frac{d^4 \xi}{(2\pi)^3 P^+} \delta(\xi_+) e^{ik \cdot \xi} \left\langle P \left| \left( \hat{F}^+(\xi) \right)_{i_1}^{j_1} \left( \hat{F}^+(0) \right)_{\bar{i}_1}^{\bar{j}_1} \left( u^{[\lambda_2]} \right)_{i_2 \bar{i}_2} \left( u^{[\lambda_2] \dagger} \right)^{\bar{j}_2 \bar{j}_2} \dots \right. \right. \\ & \left. \left. \dots \left( u^{[\lambda_{n+2}]} \right)_{i_{n+2} \bar{i}_{n+2}} \left( u^{[\lambda_{n+2} \dagger]} \right)^{\bar{j}_{n+2} \bar{j}_{n+2}} \right| P \right\rangle \end{aligned}$$

where  $P$  is the light-like momentum of the hadron (with  $P^- = 0$ ), and  $k^\mu = xP^\mu + k_T^\mu$ ,

where  $\hat{F}$  is the field strength,

and  $U^\pm$  is a Wilson line from 0 to  $\xi$ , via a “staple-like detour” to  $\pm\infty$  depending on the type and state (initial/final) of parton.

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where  $P$  is

where  $\hat{F}$  is  
and  $u^\pm$  is  
type and st

$$\tilde{\mathcal{M}}_{j_1 j_2 \dots j_{n+2}}^{i_1 i_2 \dots i_{n+2}} = \sum_{\sigma \in S_{n+2}} \delta_{j_{\sigma(1)}}^{i_1} \delta_{j_{\sigma(2)}}^{i_2} \dots \delta_{j_{\sigma(n+2)}}^{i_{n+2}} \mathcal{A}_\sigma$$

$p^\mu + k_T^\mu$ ,

ling on the



# ITMD for 2 jets (or ITMD\* for more)

Schematic hybrid (non-ITMD) factorization formula

$$d\sigma = \sum_{y=g,u,d,\dots} \int dx_1 d^2k_T \int dx_2 d\Phi_{g^*y \rightarrow n} \frac{1}{\text{flux}_{gy}} \mathcal{F}_g(x_1, k_T, \mu) f_y(x_2, \mu) \sum_{\text{color}} \left| \mathcal{M}_{g^*y \rightarrow n}^{(\text{color})} \right|^2$$

ITMD\* formula: replace

$$\mathcal{F}_g \sum_{\text{color}} \left| \mathcal{M}^{(\text{color})} \right|^2 = \mathcal{F}_g \sum_{\sigma \in S_{n+2}} \sum_{\tau \in S_{n+2}} \mathcal{A}_\sigma^* \mathcal{C}_{\sigma\tau} \mathcal{A}_\tau \quad , \quad \mathcal{C}_{\sigma\tau} = N_c^{\lambda(\sigma,\tau)}$$

with “TMD-valued color matrix”

$$(N_c^2 - 1) \sum_{\sigma \in S_{n+2}} \sum_{\tau \in S_{n+2}} \mathcal{A}_\sigma^* \tilde{\mathcal{C}}_{\sigma\tau}(x, |k_T|) \mathcal{A}_\tau \quad , \quad \tilde{\mathcal{C}}_{\sigma\tau}(x, |k_T|) = N_c^{\bar{\lambda}(\sigma,\tau)} \tilde{\mathcal{F}}_{\sigma\tau}(x, |k_T|)$$

where each function  $\tilde{\mathcal{F}}_{\sigma\tau}$  is one of 10 functions

$$\begin{aligned} & \mathcal{F}_{qg}^{(1)} \quad , \quad \mathcal{F}_{qg}^{(2)} \quad , \quad \mathcal{F}_{qg}^{(3)} \\ & \mathcal{F}_{gg}^{(1)} \quad , \quad \mathcal{F}_{gg}^{(2)} \quad , \quad \mathcal{F}_{gg}^{(3)} \quad , \quad \mathcal{F}_{gg}^{(4)} \quad , \quad \mathcal{F}_{gg}^{(5)} \quad , \quad \mathcal{F}_{gg}^{(6)} \quad , \quad \mathcal{F}_{gg}^{(7)} \end{aligned}$$

# ITMD for 2 jets (or ITMD\* for more)

$$\mathcal{F}_{qg}^{(1)}(x, k_T) = \left\langle \text{Tr} \left[ \hat{F}^{i+}(\xi) u^{[-]\dagger} \hat{F}^{i+}(0) u^{[+]} \right] \right\rangle, \quad \langle \dots \rangle = 2 \int \frac{d^4 \xi \delta(\xi_+)}{(2\pi)^3 P^+} e^{ik \cdot \xi} \langle P | \dots | P \rangle$$

$$\mathcal{F}_{qg}^{(2)}(x, k_T) = \left\langle \frac{\text{Tr} [u^{[\square]}]}{N_c} \text{Tr} \left[ \hat{F}^{i+}(\xi) u^{[+]\dagger} \hat{F}^{i+}(0) u^{[+]} \right] \right\rangle$$

$$\mathcal{F}_{qg}^{(3)}(x, k_T) = \left\langle \text{Tr} \left[ \hat{F}^{i+}(\xi) u^{[+]\dagger} \hat{F}^{i+}(0) u^{[\square]} u^{[+]} \right] \right\rangle$$

$$\mathcal{F}_{gg}^{(1)}(x, k_T) = \left\langle \frac{\text{Tr} [u^{[\square]\dagger}]}{N_c} \text{Tr} \left[ \hat{F}^{i+}(\xi) u^{[-]\dagger} \hat{F}^{i+}(0) u^{[+]} \right] \right\rangle$$

$$\mathcal{F}_{gg}^{(2)}(x, k_T) = \frac{1}{N_c} \left\langle \text{Tr} \left[ \hat{F}^{i+}(\xi) u^{[\square]\dagger} \right] \text{Tr} \left[ \hat{F}^{i+}(0) u^{[\square]} \right] \right\rangle$$

$$\mathcal{F}_{gg}^{(3)}(x, k_T) = \left\langle \text{Tr} \left[ \hat{F}^{i+}(\xi) u^{[+]\dagger} \hat{F}^{i+}(0) u^{[+]} \right] \right\rangle$$

$$\mathcal{F}_{gg}^{(4)}(x, k_T) = \left\langle \text{Tr} \left[ \hat{F}^{i+}(\xi) u^{[-]\dagger} \hat{F}^{i+}(0) u^{[-]} \right] \right\rangle$$

$$\mathcal{F}_{gg}^{(5)}(x, k_T) = \left\langle \text{Tr} \left[ \hat{F}^{i+}(\xi) u^{[\square]\dagger} u^{[+]\dagger} \hat{F}^{i+}(0) u^{[\square]} u^{[+]} \right] \right\rangle$$

$$\mathcal{F}_{gg}^{(6)}(x, k_T) = \left\langle \frac{\text{Tr} [u^{[\square]}]}{N_c} \frac{\text{Tr} [u^{[\square]\dagger}]}{N_c} \text{Tr} \left[ \hat{F}^{i+}(\xi) u^{[+]\dagger} \hat{F}^{i+}(0) u^{[+]} \right] \right\rangle$$

$$\mathcal{F}_{gg}^{(7)}(x, k_T) = \left\langle \frac{\text{Tr} [u^{[\square]}]}{N_c} \text{Tr} \left[ \hat{F}^{i+}(\xi) u^{[\square]\dagger} u^{[+]\dagger} \hat{F}^{i+}(0) u^{[+]} \right] \right\rangle$$

For 2 jets one only needs  $\mathcal{F}_{qg}^{(1)}, \mathcal{F}_{qg}^{(2)}, \mathcal{F}_{gg}^{(1)}, \mathcal{F}_{gg}^{(2)}, \mathcal{F}_{gg}^{(6)}$ , and the latter four can be obtained from the first in the mean-field and  $1/N_c$  approximation.

$$\phi(x, k^2) = \phi^{(0)}(x, k^2)$$

linear BFKL with **kinematic constraint**

$$+ \frac{\alpha_s(k^2) N_c}{\pi} \int_x^1 \frac{dz}{z} \int_{k_0^2}^{\infty} \frac{dl^2}{l^2} \left\{ \frac{l^2 \phi\left(\frac{x}{z}, l^2\right) \theta\left(\frac{k^2}{z} - l^2\right) - k^2 \phi\left(\frac{x}{z}, k^2\right)}{|l^2 - k^2|} + \frac{k^2 \phi\left(\frac{x}{z}, k^2\right)}{\sqrt{|4l^4 + k^4|}} \right\}$$

$$+ \frac{\alpha_s(k^2)}{2\pi k^2} \int_x^1 dz \left( P_{gg}(z) - \frac{2N_c}{z} \right) \int_{k_0^2}^{k^2} dl^2 \phi\left(\frac{x}{z}, l^2\right) + \frac{\alpha_s(k^2)}{2\pi} \int_x^1 dz P_{gq}(z) \Sigma\left(\frac{x}{z}, k^2\right)$$

$$- \frac{2\alpha_s^2(k^2)}{\mathbf{R}^2} \left[ \left( \int_{k^2}^{\infty} \frac{dl^2}{l^2} \phi(x, l^2) \right)^2 + \phi(x, k^2) \int_{k^2}^{\infty} \frac{dl^2}{l^2} \ln\left(\frac{l^2}{k^2}\right) \phi(x, l^2) \right]$$

non-linear term from triple-pomeron vertex, with  $\mathbf{R}_A = \mathbf{R} A^{1/3}$

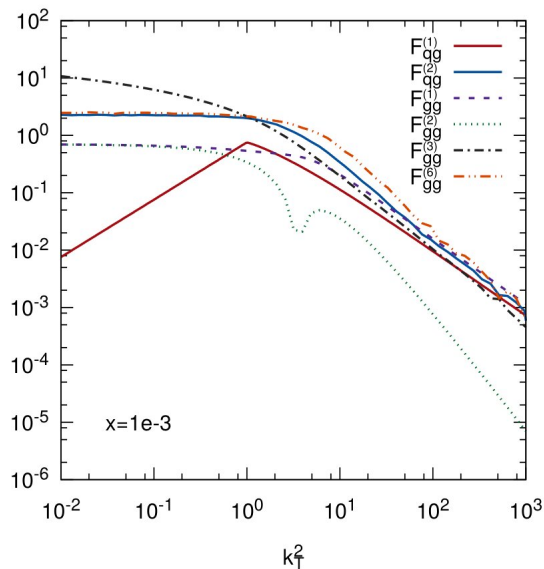
DGLAP corrections

Kutak, Sapeta 2012:

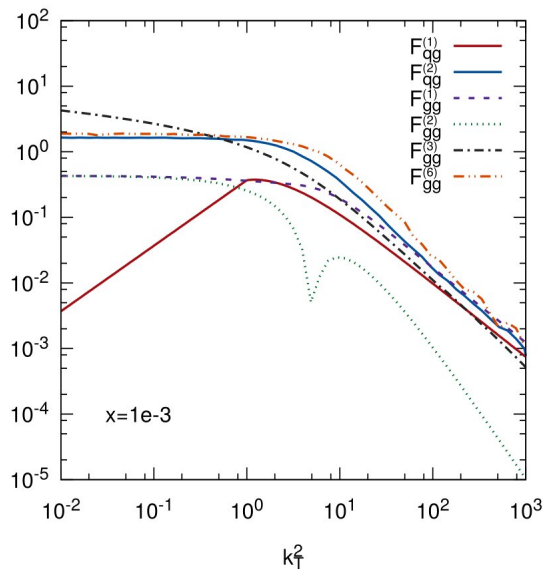
$$\text{Starting distribution } \phi^{(0)}(x, k^2) = \frac{\alpha_s(k^2)}{2\pi k^2} \int_x^1 dz P_{gg}(z) \frac{x}{z} g\left(\frac{x}{z}\right), \quad xg(x) = N(1-x)^\beta(1-Dx)$$

fitted to combined HERA  $F_2$  data, and with  $\phi(x, k^2 < 1) = k^2 \phi(x, 1)$ .

KS gluon TMDs in proton



KS gluon TMDs in lead

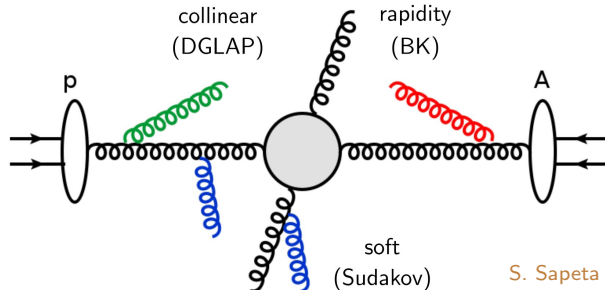


Dependence of  $\mathcal{F}_{qg}^{(1)}$  on  $k_T$  below 1GeV approximated by power-like fall-off. For higher values of  $|k_T|$  it is a solution to the BK equation.

TMDs decrease as  $1/|k_T|$  for increasing  $|k_T|$ , except  $\mathcal{F}_{gg}^{(2)}$ , which decreases faster (even becomes negative, absolute value shown here).

# Sudakov resummation for dijets

Having hard jets in the final state, large logarithms associated with the hard scale have to be resummed. This resummation can be accounted for by inclusion of the Sudakov factor.



Within the small- $x$  saturation formalism, Sudakov effects are most conveniently included in  $b$ -space, via an “initial-state luminosity” (Mueller, Xiao, Yuan 2013)

$$\mathcal{L}_{g^*/B}^{ag \rightarrow cd}(x_p, x, k_T, \mu) = \int db_T b_T J_0(b_T k_T) e^{-S_{\text{Sud}}^{ag \rightarrow cd}(\mu, b_T)} \\ \times f_{a/p}(x_p, \mu_b) \int dk'_T k'_T J_0(b_T k'_T) \mathcal{F}_{g^*/B}(x, k'_T)$$

with  $\mu_b = 2e^{-\gamma_E}/b_*$ ,  $b_* = b_T/\sqrt{1 + b_T^2/b_{\text{max}}^2}$ . The scale choice  $\mu_b$  eliminates threshold logarithms, but “breaks” factorization between initial-state variables, which complicates the Monte Carlo approach, or requires expensive 4-dim luminosity grids.

# Sudakov resummation for dijets

The Sudakov receives perturbative and non-perturbative contributions for each channel

$$S_{\text{Sud}}^{ab \rightarrow cd}(\mu, b_T) = \sum_{i=a,b,c,d} S_p^i(\mu, b_T) + \sum_{i=a,c,d} S_{\text{np}}^i(\mu, b_T)$$

Perturbative part (Mueller, Xiao, Yuan 2013)

$$S_p^i(Q, b_T) = \frac{\alpha_s}{2\pi} \int_{\mu_b^2}^{Q^2} \frac{d\mu^2}{\mu^2} \left[ A^i \ln \frac{Q^2}{\mu^2} - B^i \right]$$

$$\{A, B\}^{qg \rightarrow qg} = \{2(C_A + C_F), 3C_F + 2C_A\beta_0\} \quad , \quad \{A, B\}^{gg \rightarrow gg} = \{4C_A, 6C_A\beta_0\}$$

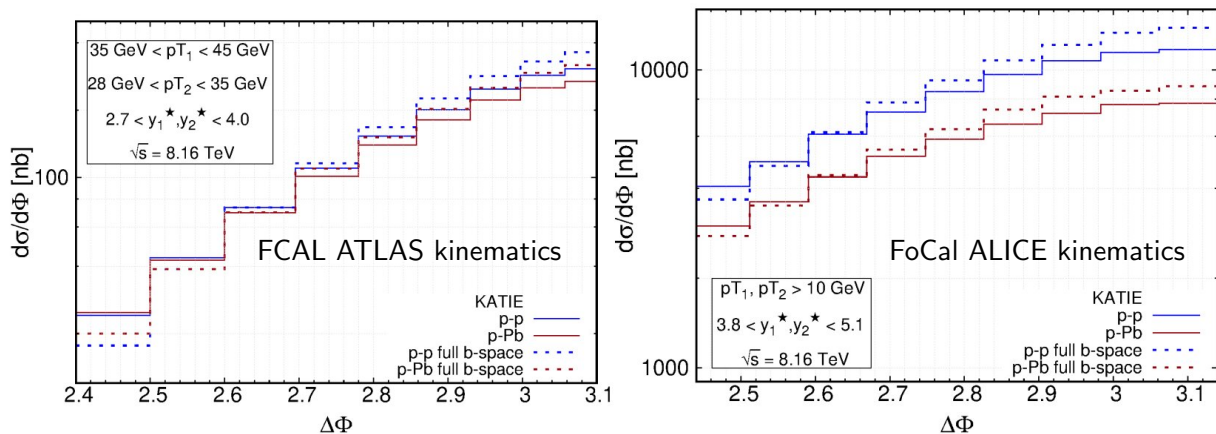
$$b_{\text{max}} = 0.5 \text{GeV}^{-1}$$

Non-perturbative contribution for small- $x$  gluon already in TMD and should be omitted in our application (Staśto, Wei, Xiao, Yuan 2018).

# Dijet azimuthal correlations

in p-p and p-Pb collisions at forward LHC calorimeters

Abdullah Al-Mashad, AvH, Kakkad,  
Kotko, Kutak, van Mechelen, Sapeta  
2022



Predictions for the azimuthal angle  $\Delta\Phi$  between the two hardest jets in **p-p** and **p-Pb** collisions, in the kinematics of FCAL (ATLAS) and the planned FoCal (ALICE) detectors.

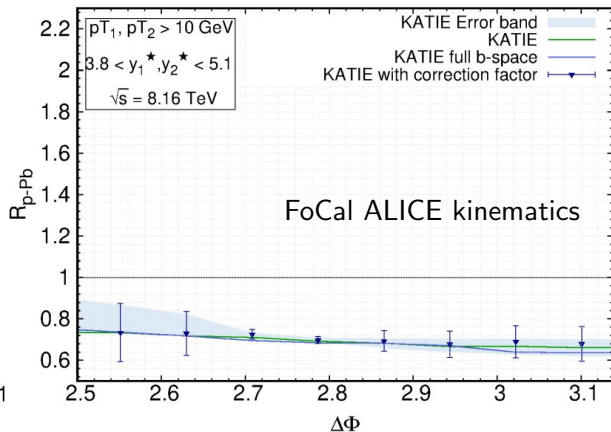
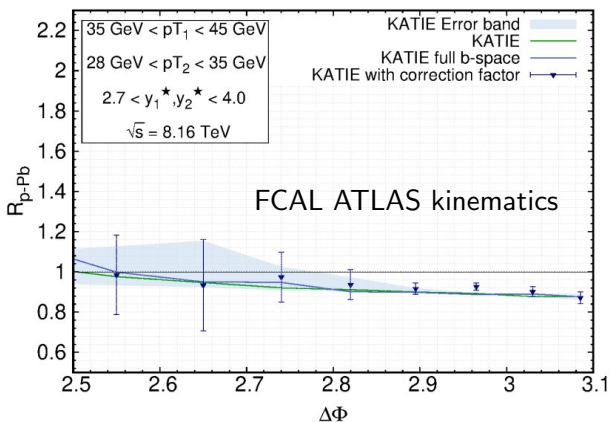
Saturation manifests itself as suppression of **p-Pb** compared to **p-p**, especially near  $\Delta\Phi = \pi$ , but Sudakov factors have the same effect.

Difference between “threshold-log correct” (dashed) and “factorized” (solid) calculation is within accuracy of LO parton-level calculation.

# Dijet azimuthal correlations

in p-p and p-Pb collisions at forward LHC calorimeters

Abdullah Al-Mashad, AvH, Kakkad,  
Kotko, Kutak, van Mechelen, Sapeta  
2022



Predictions for the nuclear modification ratio  $R_{p-pB} = (d\sigma^{p+pB}/d\Delta\Phi)/(d\sigma^{p+p}/d\Delta\Phi)$  as function of the azimuthal angle  $\Delta\Phi$  between the two hardest jets p-p and p-Pb collisions.

Points with error bars are corrected with final-state shower effects using Pythia, and represent uncertainty both from statistics and scale dependence.

Sudakov factors, feared to wash out saturation effects, appear to cancel and the latter stay manifest.