Advances in Lattice QCD

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UNIWERSYTET JAGIELLOŃSKI W KRAKOWIE

5th Symposium of the Division for Physics of Fundamental Interactions of the Polish Physical Society

Lattice Quantum Field Theory

Lattice regularization

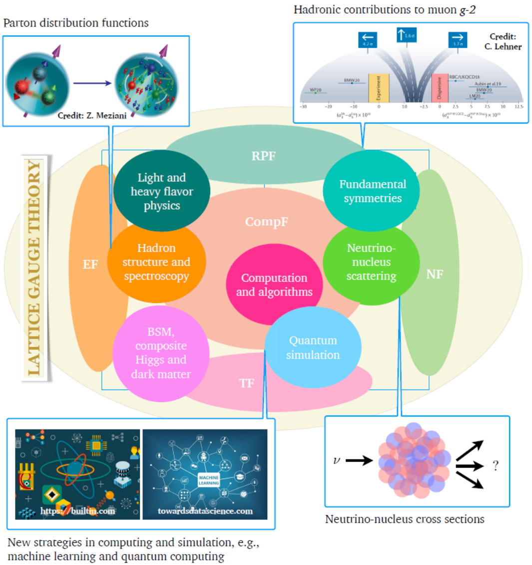
Discretize your theory with a minimal lattice spacing a (UV) and a finite volume (usually torus) (IR). Evaluate numerically the finite path integral to estimate quantum expectation values.

Recovery of relevant continuum (Euclidean) physics

- statistical limit
- infinite volume limit
- physical quark masses limit
- continuum (renormalization) limit
- other required limits (large hadron's momentum)
- ...

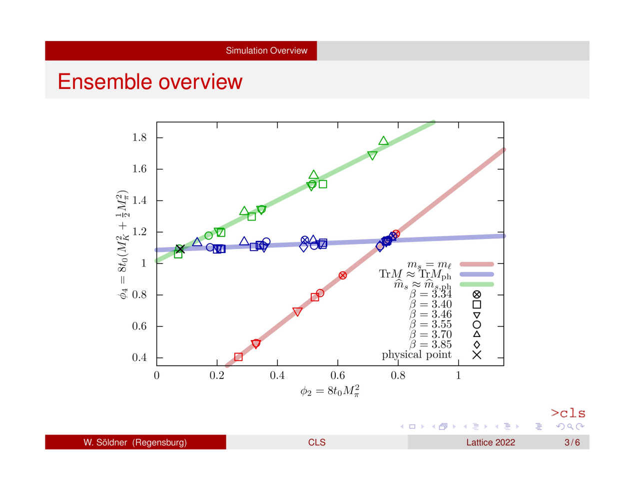
All this takes a lot of human and computer time!
$\Rightarrow$ but allows to keep all systematic uncertainties under full control!

Lattice Quantum Field Theory: where are we now?



Taken from Snowmass 2022 report: https://arxiv.org/pdf/2209.10758.pdf

Coordinated Lattice Simulations effort



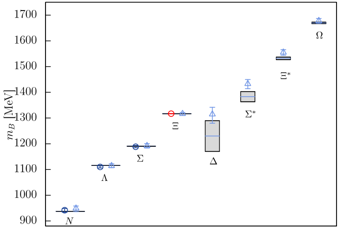
W. Söldner (U. Regensburg) Lattice2022 presentation

Coordinated Lattice Simulations effort

Low lying baryon spectrum

○ Octet baryon spectrum from BChPT, finite a and FV fits (determine $\sqrt{8t_{0,ph}}$) Agreement with QCD "expt." masses within 1% overall uncertainty.

△ Octet and decuplet masses from SSE BChPT, finite a and FV fits.



Unstable decuplet baryons: grey bands indicate the expt. Breit-Wigner width. Proper treatment via the Lüscher formalism required.

S. Collins (U. Regensburg) Lattice2022 presentation

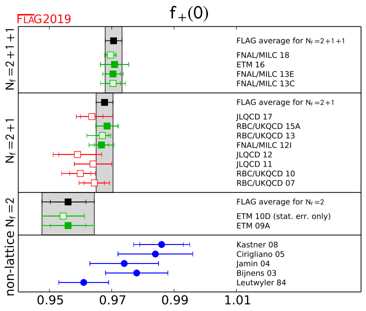
Phenomenology

FLAG (Flavor Averaging Group) reports and decay constants

$$f_+(0) = \langle K|Q^{\bar{u}s}|\pi\rangle, \qquad f_\pi = \langle \pi|Q|0\rangle, \qquad f_K = \langle K|Q|0\rangle$$

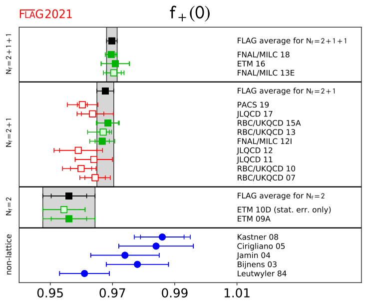
Progress

2019



$f_+(0) = 0.9706(27)$

2021



$f_+(0) = 0.9698(17)$

QCD+QED$_C$

QED on the lattice: theoretically challenging!

- Gauss law does not allow charged states on a periodic torus
- massless photon $\Rightarrow$ long range interactions

C^* boundary conditions

QED with C-parity boundary conditions (QED$_C$): the matter fields are multiplied by the charge conjugation at the edge of the lattice in the spatial directions. This provides a consistent QFT. Charged interpolating operators can be constructed in a gauge-invariant way $\Rightarrow$ correlation functions of charged operators can be estimated.

QCD+QED$_C$

C^* boundary conditions

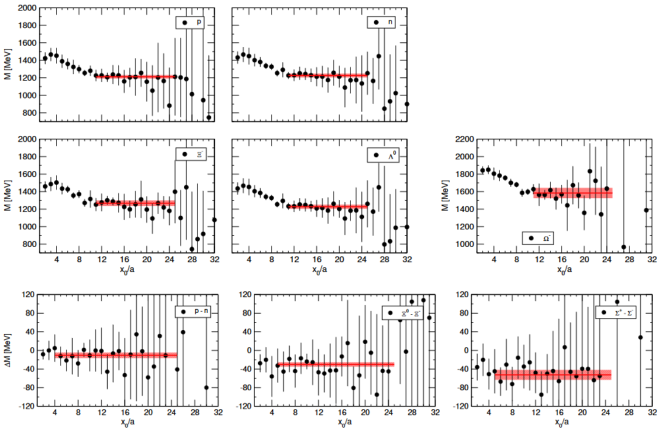


Figure 7: Baryon effective masses and effective ϕ observables for the ensemble A360a50b324+RW2, with the selected plateaux and the fits to a constant. Values in MeV are obtained by using the reference value $(8t_0)^{1/2} = 0.415$ fm.

Taken from RC* collaboration, https://arxiv.org/pdf/2209.13183.pdf

x-dependence of structure functions

Twist-2 PDF

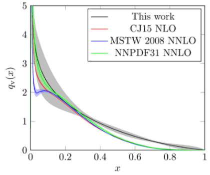
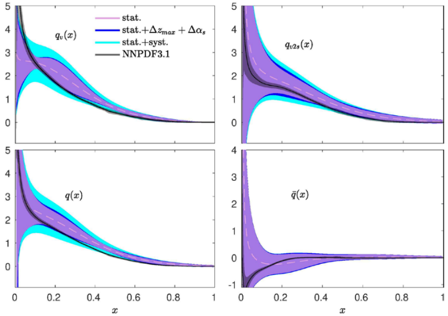


Figure 1: Recent results for the nucleon's unpolarized isovector PDFs. Left: q_v [39]. Right: q_v (upper left), q_{v2s} (upper right), q (lower left), $\bar{q}$ (lower right) [36]. Plots reprinted based on the arXiv distribution license.



Taken from K.Cichy, https://arxiv.org/pdf/2111.04552.pdf

x-dependence of structure functions

Further exploratory studies

- flavor-singlet quark PDFs
- gluon PDFs
- twist-3 quark PDFs
- generalized PDFs (GPD)
- transverse momentum dependent PDFs (TMD)

Working principle

How to simulate these field theories?

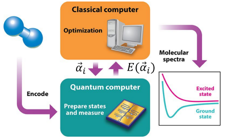
Example: hybrid quantum-classical algorithms

Key concept

Classical computer: main computation

Quantum computer: classically hard/intractable part

Advantages: even for small quantum hardware!



Sim et al. (2018)

[1] Peruzzo et al. (2014)

Variational Quantum Eigensolver (VQE) [1]

Goal

Find ground state and excited states of Hamiltonian $\mathcal{H}$

Variational approach

Minimize $E(\vec{\alpha}) = \langle\psi(\vec{\alpha})|\mathcal{H}|\psi(\vec{\alpha})\rangle$ w.r.t. parameters $\vec{\alpha}$

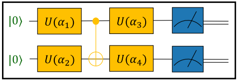
Classical computer

Given $E(\vec{\alpha}_i)$, find optimized parameters $\vec{\alpha}_{i+1}$

Quantum device

Given $\vec{\alpha}_i$, prepare $|\psi(\vec{\alpha})\rangle$ and measure $E(\vec{\alpha}_i)$

$|\psi(\vec{\alpha})\rangle =$



13

Taken from L.Flucke, Lattice2022 plenary talk

Quantum computing

Revolution is happening now!

More details: parallel talks @ this conference

Quantum Electrodynamics in 1+1D

1+1D Schwinger model with chemical potential
Studying multi-flavor chemical potential with VQE
→ talk by Stefan Kühn tomorrow at 2:20pm

1+1D Schwinger model with θ-term
Comparing quantum algorithms for state preparation
→ talk by Alexei Bazavov tomorrow at 3:20pm

1+1D Schwinger model with Wilson fermions
Comparing Wilson and staggered fermions
→ talk by Takis Angelides tomorrow at 5:30pm

1+1D Schwinger model at finite temperature
Mapping out $T-\mu$ phase diagram with VQE
→ talk by Akio Tomiya on Thursday at 12:30pm

Discrete and higher-dimensional gauge theories

1+1D D_n gauge theory
Preparing ground state & evolution on quantum annealer
→ talk by Michael Fromm on Wednesday at 2pm

1+1D Z_2 gauge theory
Developing quantum algorithms for thermal states
→ talk by Connor Powers tomorrow at 3:20pm

2+1D U(1) gauge theory
Finding a resource-efficient implementation
→ talk by Christopher Kane tomorrow at 2pm

16

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Revolution is happening now!

More details: parallel talks @ this conference

SU(N) lattice gauge theories

1+1D SU(2) gauge theory

Implementing real-time evolution on IBM-Q hardware

→ talk by Emanuele Mendicelli tomorrow at 4:30pm

Comparing Schwinger-boson & loop-string hadron form.

→ talk by Jesse Stryker on Thursday at 11:50am

SU(2) gauge theories

Developing efficient digitization via finite subgroups

→ talk by Timo Jakobs tomorrow at 2:40pm

Defining canonical momenta for discretized gauge fields

→ talk by Carsten Urbach tomorrow at 3pm

SU(3) gauge theories

Developing efficient digitization & improved Hamiltonians

→ talk by Henry Lamm tomorrow at 5:50pm

Lattice field theories

1+1D Ising model

Studying NISQ algorithms for open quantum systems

→ talk by Bharath Sambasivam tomorrow at 6:10pm

1+1D extended O(2) model

Studying discrete approximations of continuous groups

→ talk by Leon Hostetler on Wednesday at 3:40pm

1+1D Wess-Zumino model

Studying dynamical breaking of supersymmetry

→ talk by Christopher Culver tomorrow at 2:20pm

17

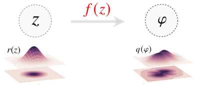
Taken from L.Flucke, Lattice2022 plenary talk

Another revolution is happening now!

Fields via flow models

Example application: Embarrassingly parallel direct sampling

Flow model as an approximate trivialising map



"**Base**" distribution: Efficient to sample e.g., Haar-uniform

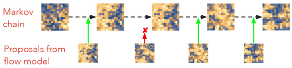
"**Model**" distribution $q(\phi) \approx \frac{1}{Z}e^{-S(\phi)}$

- Independent samples of the base distribution map to independent samples of the model distribution

- **Train the model:** Gradient descent to minimise "loss function" with minimum at $q(\phi) = \frac{1}{Z}e^{-S(\phi)}$

$$L(q) = \underbrace{\int d\phi\, q(\phi)}_{\text{Estimate stochastically by sampling from the model, i.e., "self training"}}[\log q(\phi) + S(\phi)]$$

- **Guarantee exactness:** Reweight or form a Markov chain with Metropolis-Hastings accept/reject step



Taken from P.Shanahan, Lattice2022 parallel talk

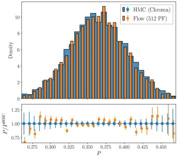
Another revolution is happening now!

Flow models for QCD in 4D

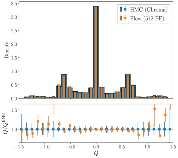
Initial QCD demonstration [this talk +upcoming manuscripts on scaling and 4D]

- Direct combination of published results on gauge-equivariant flows and pseudofermions [Boyda et al., 2008.05456, Abbott et al., 2207.08945]
- Illustration at straightforward parameters $V=4^4$, $N_f=2$, $\beta=1$, $\kappa=0.1$
- Observables from flow ensemble in precise agreement with HMC at high statistics (65k samples)
- Development and scaling of QCD-specific architectures in full swing — stay tuned!

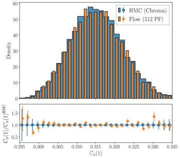
Plaquette



Topological charge (Wilson flowed)



Pion correlation function (t=1)



Taken from P.Shanahan, Lattice2022 parallel talk

Conclusions: Recent advances? Too many of them...

A VERY subjective subset:

- improving algorithms and computational strategies $\Rightarrow$ increasing precision: permille precision on stable light hadron masses
- such precision requires control of corrections which so far were neglected $\Rightarrow$ QCD+QED
- improved precision proliferates on other quantities $\Rightarrow$ structure functions
- ever increasing computational cost $\Rightarrow$ will machine learning or quantum computers help?