

Higgs-boson induced reheating and ultraviolet freeze-in dark matter

Anna Socha

University of Warsaw

based on: A. Ahmed, B. Grządowski, AS [2111.06065](#) and [2207.11218](#)

Motivation

- Inflation is the most compelling phenomenon that explains the puzzles of the early Universe.

Motivation

- Inflation is the most compelling phenomenon that explains the puzzles of the early Universe.
- The relevance of the reheating dynamics is typically ignored with a minimal assumption, such as instantaneous reheating.

Motivation

- Inflation is the most compelling phenomenon that explains the puzzles of the early Universe.
- The relevance of the reheating dynamics is typically ignored with a minimal assumption, such as instantaneous reheating.
- Even in the case of non-instantaneous reheating models, it is usually assumed that the inflaton decay rate, Γ_ϕ , is constant.

Motivation

- Inflation is the most compelling phenomenon that explains the puzzles of the early Universe.
- The relevance of the reheating dynamics is typically ignored with a minimal assumption, such as instantaneous reheating.
- Even in the case of non-instantaneous reheating models, it is usually assumed that the inflaton decay rate, Γ_ϕ , is constant.
- However, these widely used assumptions could be violated in generic models of perturbative reheating, e.g., when the inflaton field has a non-trivial potential.

Motivation

- Inflation is the most compelling phenomenon that explains the puzzles of the early Universe.
- The relevance of the reheating dynamics is typically ignored with a minimal assumption, such as instantaneous reheating.
- Even in the case of non-instantaneous reheating models, it is usually assumed that the inflaton decay rate, Γ_ϕ , is constant.
- However, these widely used assumptions could be violated in generic models of perturbative reheating, e.g., when the inflaton field has a non-trivial potential.
- The understanding of the reheating era is essential for the **dark matter (DM) sector**, especially in the context of the **freeze-in DM production**.

Non-instantaneous reheating



Time-averaged Boltzmann equations

Expansion

$$\dot{\rho}_\phi + 3(1 + \bar{w})H\rho_\phi = -\langle\Gamma_\phi\rangle\rho_\phi$$

$$\dot{\rho}_\mathcal{R} + 4H\rho_\mathcal{R} = \langle\Gamma_\phi^\mathcal{R}\rangle\rho_\phi$$

$$\bar{w} \equiv \langle p_\phi \rangle / \langle \rho_\phi \rangle$$

$$H^2 = \frac{\rho_\phi + \rho_\mathcal{R}}{3M_{pl}^2}$$

Interactions

Time-dependent decay rate

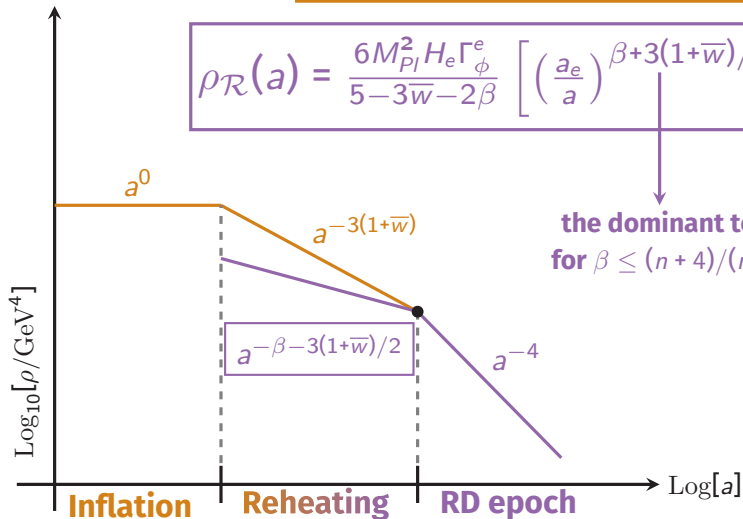
$$\langle\Gamma_\phi^\mathcal{R}\rangle \simeq \langle\Gamma_\phi\rangle = \Gamma_\phi^e \left(\frac{a_e}{a}\right)^\beta$$

constant parameter

Non-instantaneous reheating

$$\rho_\phi(a) \stackrel{H \gg \Gamma_\phi}{\simeq} 3M_{\text{Pl}}^2 H_e^2 \left(\frac{a_e}{a}\right)^{3(1+\bar{w})}$$

$$\rho_{\mathcal{R}}(a) = \frac{6M_{\text{Pl}}^2 H_e \Gamma_\phi^e}{5-3\bar{w}-2\beta} \left[\left(\frac{a_e}{a}\right)^{\beta+3(1+\bar{w})/2} - \left(\frac{a_e}{a}\right)^4 \right]$$



Example model

As an illustration, we consider

the α -attractor T-model:

$$\alpha = 1/6,$$

$$\Lambda = 3 \cdot 10^{-3} M_{\text{Pl}}$$

$$V(\phi) = \Lambda^4 \tanh^{2n} \left(\frac{|\phi|}{\sqrt{6\alpha} M_{\text{Pl}}} \right)$$

R. Kallosh [et al.](#), arXiv:1306.5220
R. Kallosh [et al.](#), arXiv:1311.0472

$$\simeq \begin{cases} \Lambda^4 & |\phi| \gg M_{\text{Pl}} \\ \Lambda^4 \left| \frac{\phi}{M_{\text{Pl}}} \right|^{2n} & |\phi| \ll M_{\text{Pl}} \end{cases},$$

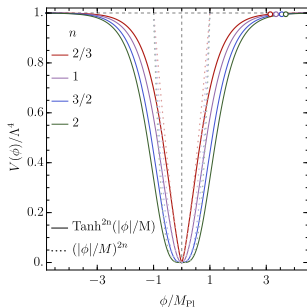
where $n > 0$, $\sqrt{6\alpha} \lesssim 10$, and $\Lambda \lesssim 1.6 \times 10^{16}$ GeV.

N. Aghanim [et al.](#), arXiv:1807.06209

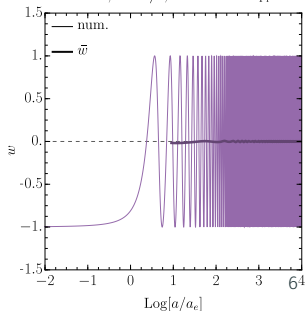
For power-law potentials

$$\bar{w} = \frac{n-1}{n+1}$$

$\frac{n-1}{n+1}$	$\xrightarrow{2/3}$	\rightarrow	$-1/5$	
	$\xrightarrow{1}$	\rightarrow	0	matter-like
	$\xrightarrow{3/2}$	\rightarrow	$1/5$	
	$\xrightarrow{2}$	\rightarrow	$1/3$	radiation-like



$n=1, \alpha=1/6, \Lambda=3 \cdot 10^{-3} M_{\text{Pl}}$



Higgs portal

$$g_{h\phi} M_{\text{Pl}} \phi |\mathbf{h}|^2$$

Y. Shtanov, et al., arXiv:9407247

homogeneous, classical
background field



coherently oscillating

$$\phi = \varphi(t) \cdot \mathcal{P}(t)$$

rapidly-oscillating

slowly-varying envelope

⇒ **Reheating** i.e., energy transfer between the inflaton and the SM sector

$$\frac{1}{V} \frac{dE_g}{dt} = \rho_\phi \Gamma_\phi = g_{h\phi}^2 M_{\text{Pl}}^2 \frac{\varphi^2(t)}{8\pi} \sum_{i=0}^3 \sum_{k=1}^{\infty} k\omega |\mathcal{P}_k|^2 \sqrt{1 - \left(\frac{2m_{h_i}}{k\omega}\right)^2}$$

⇒ **Higgs mass** induced by the oscillating inflaton background

$$m_{h_0}^2 = g_{h\phi} M_{\text{Pl}} \varphi \begin{cases} |\mathcal{P}|, & \mathcal{P}(t) > 0 \\ 2|\mathcal{P}|, & \mathcal{P}(t) < 0 \end{cases} \quad v_h = \begin{cases} 0, & \mathcal{P}(t) > 0 \\ \sqrt{|m_{h_0}^2|/(2\lambda_h)}, & \mathcal{P}(t) < 0 \end{cases}$$

Kinematic suppression

The inflaton decay rate can be written as

$$\langle \Gamma_\phi \rangle = \frac{g_{h\phi}^2}{32\pi} \frac{M_{\text{Pl}}^2}{m_\phi(a)} \gamma_h(a), \quad m_\phi(a) = V_{,\phi\phi}|_{\phi=\varphi}$$

effective mass

$$\gamma_h(a) \simeq \sum_{i=0}^3 \sum_{k=1}^{\infty} k |\mathcal{P}_k|^2 \left\langle \sqrt{1 - \left(\frac{2m_{h_i}(a)}{k\omega(a)} \right)^2} \right\rangle$$

$\omega \propto m_\phi$

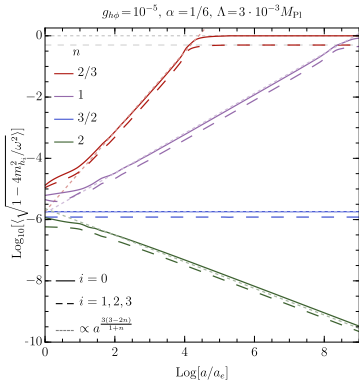
It turns out that

$$\gamma_h \propto \begin{cases} a^0, & m_{h_i} = 0 \\ a^{\frac{3(3-2n)}{1+n}}, & m_{h_i} \neq 0 \end{cases}$$

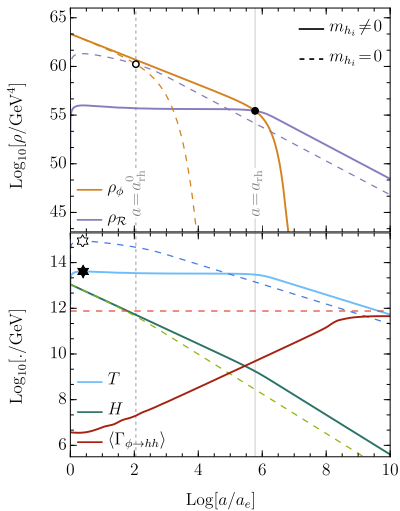
which, in turn, implies

$$\langle \Gamma_\phi \rangle \propto \begin{cases} a^{\frac{3(n-1)}{n+1}}, & m_{h_i} = 0 \\ a^{\frac{6-3n}{1+n}}, & m_{h_i} \neq 0 \end{cases}$$

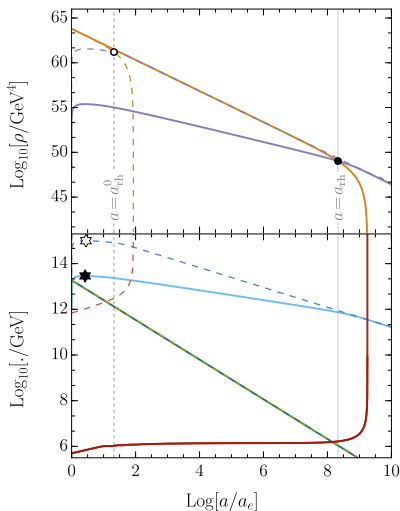
$$-\beta$$



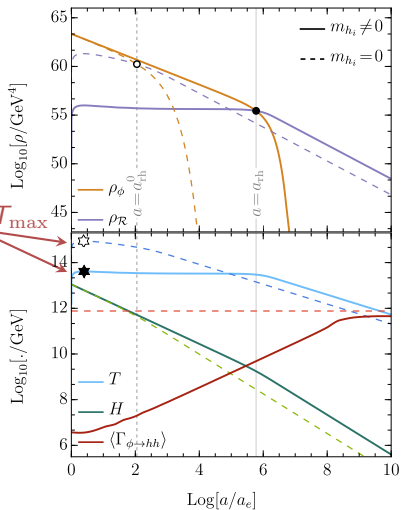
$$g_{h\phi} = 10^{-5}, n=1, \alpha=1/6, \Lambda=3 \cdot 10^{-3} M_{\text{Pl}}$$



$$g_{h\phi} = 10^{-5}, n=2, \alpha=1/6, \Lambda=3 \cdot 10^{-3} M_{\text{Pl}}$$



$$g_{h\phi} = 10^{-5}, n=1, \alpha=1/6, \Lambda=3 \cdot 10^{-3} M_{\text{Pl}}$$



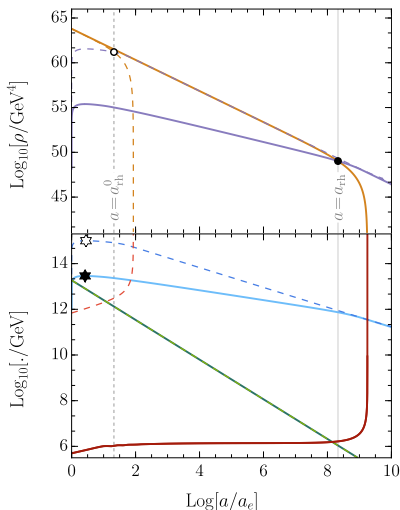
$$m_{h_i} = 0$$

$$\beta = 0, \rho_{\mathcal{R}} \propto a^{-3/2}$$

$$m_{h_i} \neq 0$$

$$\beta = -\frac{3}{2}, \rho_{\mathcal{R}} \propto a^0$$

$$g_{h\phi} = 10^{-5}, n=2, \alpha=1/6, \Lambda=3 \cdot 10^{-3} M_{\text{Pl}}$$



$$\beta = -1, \rho_{\mathcal{R}} \propto a^{-1}$$

$$\beta = 0, \rho_{\mathcal{R}} \propto a^{-2}$$

Spin-1

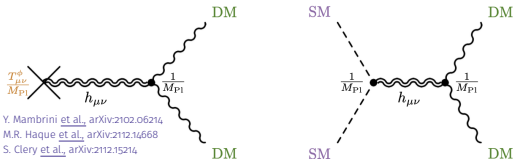
Dark Matter

Gravitational production

Boltzmann equation

$$\mathcal{L}_{\text{DM}} = -\frac{1}{4} X_{\mu\nu} X^{\mu\nu} + \frac{1}{2} m_X^2 X_\mu X^\mu + \mathcal{L}_{\text{int}}$$

$$\mathcal{L}_{\text{int}} = \frac{h^{\mu\nu}}{M_{\text{Pl}}} \left(T_\phi^{\mu\nu} + T_X^{\mu\nu} + T_{\text{SM}}^{\mu\nu} \right)$$



M. Garny [et al.](#), arXiv:1511.03278
Y. Tang [et al.](#), arXiv:1708.05138
M. Garny [et al.](#), arXiv:1709.09688

$$\frac{dN_X}{da} = \frac{a^2}{H} (\mathcal{S}_\phi + \mathcal{S}_{\text{SM}})$$

$$N_X \equiv n_X a^3$$

Gravitational production

Production from the inflaton

time dependent

$$\mathcal{S}_\phi = \frac{1}{8\pi} \frac{\rho_\phi^2}{M_{\text{Pl}}^4} \sum_{k=1}^{\infty} |\mathcal{P}_k^{2n}|^2 \left[\left(1 - \frac{2m_X^2}{(k\omega)^2}\right)^2 + \frac{8m_X^4}{(k\omega)^4} \right]$$

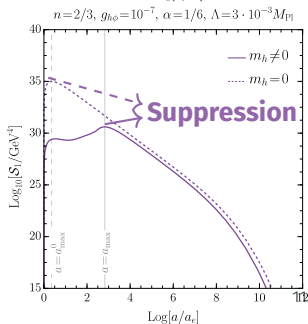
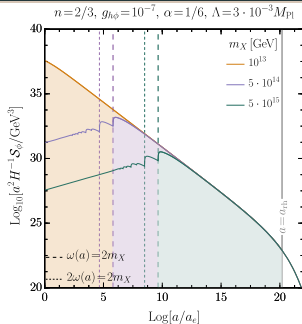
$$\times \sqrt{1 - \frac{4m_X^2}{(k\omega)^2}}$$

Gravitational freeze-in

$$\mathcal{S}_{\text{SM}} = [n_0 \langle \sigma|v \rangle_0 + n_{1/2} \langle \sigma|v \rangle_{1/2} + n_1 \langle \sigma|v \rangle_1] \bar{n}_X^2$$

$$\langle \sigma|v \rangle_1 \bar{n}_X^2 \propto \begin{cases} \frac{T^4}{\pi^5} \left(\frac{T}{M_{\text{Pl}}}\right)^4, & m_X \ll T, \\ \frac{1}{16\pi^4} \frac{m_X^5 T^3}{M_{\text{Pl}}^4} e^{-2m_X/T}, & m_X \gg T. \end{cases}$$

Thermally-averaged cross section for the annihilation of massless SM vectors

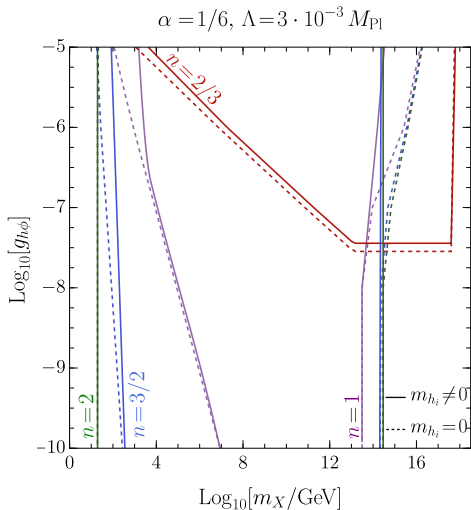


The DM relic abundance can be calculated as

$$\Omega_X^{\text{grav}} h^2 \simeq \frac{m_X}{\rho_c} \frac{N_X^{\text{grav}}(a_{\text{rh}})}{a_{\text{rh}}^3} \frac{s_0}{s(a_{\text{rh}})} h^2.$$

Successful DM model has to predict the correct amount of DM i.e.,

$$\Omega_X^{\text{grav}} h^2 = \Omega_X^{(\text{obs})} h^2 = 0.1198 \pm 0.0012$$



Heavy DM particles are produced by the freeze-in from the SM sector

The DM relic abundance can be calculated as

$$\Omega_X^{\text{grav}} h^2 \simeq \frac{m_X}{\rho_c} \frac{N_X^{\text{grav}}(a_{\text{rh}})}{a_{\text{rh}}^3} \frac{s_0}{s(a_{\text{rh}})} h^2.$$

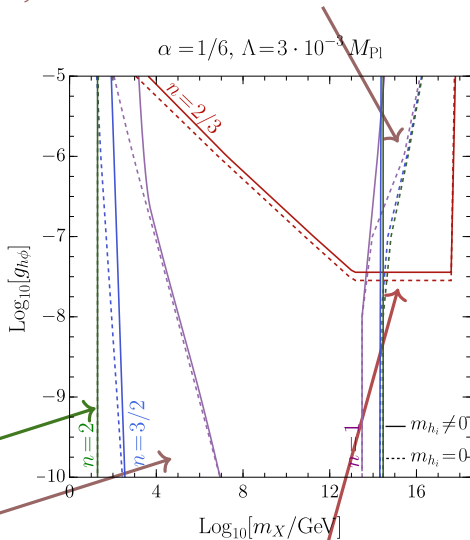
Successful DM model has to predict the correct amount of DM i.e.,

$$\Omega_X^{\text{grav}} h^2 = \Omega_X^{(\text{obs})} h^2 = 0.1198 \pm 0.0012$$

For the $n=2$ case,

$\Omega_X^{\text{grav}} h^2$ does not depend on $g_{h\phi}$

Light DM particles are dominantly produced from the inflaton



For the $n=2/3$ case,

$\Omega_X^{\text{grav}} h^2$ does not depend on m_X for heavy DM

Summary

We have demonstrated that the non-standard ($n \neq 1$) cosmologies and the kinematical suppression of radiation production significantly affect the thermal bath evolution and the DM production.

- In particular, we have shown that the duration of reheating and the evolution of the radiation energy density, $\rho_{\mathcal{R}}$, are sensitive to the inflaton potential shape.

Summary

We have demonstrated that the non-standard ($n \neq 1$) cosmologies and the kinematical suppression of radiation production significantly affect the thermal bath evolution and the DM production.

- In particular, we have shown that the duration of reheating and the evolution of the radiation energy density, $\rho_{\mathcal{R}}$, are sensitive to the inflaton potential shape.
- Moreover, we have discussed the role of kinematical suppression in the reheating dynamics. We have demonstrated that the non-zero mass of the Higgs boson leads to the elongation of the reheating period, changes the $\rho_{\mathcal{R}}(a)$ and $T(a)$ evolution, and conduces to the decrease of T_{\max} .

Summary

We have demonstrated that the non-standard ($n \neq 1$) cosmologies and the kinematical suppression of radiation production significantly affect the thermal bath evolution and the DM production.

- In particular, we have shown that the duration of reheating and the evolution of the radiation energy density, $\rho_{\mathcal{R}}$, are sensitive to the inflaton potential shape.
- Moreover, we have discussed the role of kinematical suppression in the reheating dynamics. We have demonstrated that the non-zero mass of the Higgs boson leads to the elongation of the reheating period, changes the $\rho_{\mathcal{R}}(a)$ and $T(a)$ evolution, and conduces to the decrease of T_{\max} .
- Finally, we have pointed out that the DM freeze-in production is very sensitive to the reheating dynamics.

Thank you for your attention!

Back-up slides

EoM for the inflaton

The equation of motion (EoM) for the ϕ field is:

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi}(\phi) = 0, \quad H^2 = \rho_{\phi}/(3M_{\text{Pl}})^2.$$

We can distinguish two types of solutions to the ϕ 's EoM:

1. Slow-roll $\ddot{\phi} \simeq 0$, $\epsilon_V, \eta_V \ll 1$

$$\epsilon_V = M_{\text{Pl}}^2/2(V_{,\phi}/V)^2$$

$$\eta_V = M_{\text{Pl}}^2(V_{,\phi\phi}/V)^2$$

$$\phi(t) \simeq \text{const.}$$

2. Oscillations

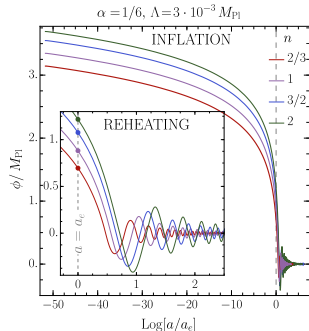
$$\phi(t) = \varphi(t) \cdot \mathcal{P}(t)$$

Slowly-varying
Envelope function

$$\rho_{\phi} = V(\varphi)$$

Rapidly-oscillating
quasi-periodic function

Y. Shtanov et al, arXiv:9407247



value of the scale factor
at the end of inflation ($\epsilon_V = 1$)

Analytical solutions to the inflaton EoM after the end of inflation

The EoM for the envelope function φ is given by:

$$\dot{\varphi} = -\frac{3}{n+1}H\varphi, \quad \Rightarrow \quad \varphi(a) = \varphi(a_e) \left(\frac{a}{a_e}\right)^{-\frac{3}{n+1}},$$

while \mathcal{P} satisfies the following approximate equation:

M. A. G. Garcia [et al., arXiv:2004.08404](#)
M. A. G. Garcia [et al., arXiv:2012.10756](#)

$$\dot{\mathcal{P}} \simeq \pm \sqrt{\frac{m_\phi^2(1 - |\mathcal{P}|^{2n})}{n(2n-1)}}, \quad m_\phi^2 = 2n(2n-1)\Lambda^2 \left(\frac{\Lambda}{\sqrt{6\alpha}M_{\text{Pl}}}\right)^2 \left(\frac{\rho_\phi}{\Lambda^4}\right)^{\frac{n-1}{n}}$$

The solution to the above equation can be written as

Inverse of the regularized incomplete beta function

$$\mathcal{P}(t) = \left[I_z^{-1} \left(\frac{1}{2n}, \frac{1}{2} \right) \right]^{\frac{1}{2n}}, \quad z = 1 - \frac{4}{T}(t - t_0),$$

where T denotes the period of the oscillations:

$$T = \frac{\sqrt{16\pi n(2n-1)} \Gamma\left(1 + \frac{1}{2n}\right)}{m_\phi \Gamma\left(\frac{1}{2} + \frac{1}{2n}\right)}$$

depend on time
for $n \neq 1$

Particles production in a classical inflaton background

The inflaton field can be regarded as a homogeneous, classical field that coherently oscillates in time.

For the interactions linear in the $\phi = \varphi \cdot \mathcal{P}$ term, the energy gain per unit volume per unit time due to the pair production of f particles with mass m can be calculated as

$$\frac{1}{V} \frac{dE_g}{dt} = \frac{\varphi^2(t)}{8\pi} \sum_{k=1}^{\infty} k\omega |\mathcal{P}_k|^2 \left| \mathcal{M}_{0 \rightarrow f}(k) \right|^2 \operatorname{Re} \left[\sqrt{1 - \frac{4m^2}{k^2\omega^2}} \right],$$

where

$$\mathcal{P}(t) = \sum_{k=-\infty}^{\infty} \mathcal{P}_k e^{-ik\omega t}, \quad \mathcal{P}_k = \frac{1}{\mathcal{T}(t_0)} \int_{t_0}^{t_0 + \mathcal{T}(t_0)} dt \mathcal{P}(t) e^{ik\omega t}.$$

The matrix element $\mathcal{M}_{0 \rightarrow f}$ accounts for the quantum process of two particles production out of the vacuum.

Particles production in a classical inflaton background

For the interactions proportional to the $\phi = \varphi \cdot \mathcal{P}$ term, the lowest-order non-vanishing S-matrix element takes the form

$$S_{if}^{(1)} = \sum_k \mathcal{P}_k \langle f | \int d^4x \varphi(t) e^{-ik\omega t} \mathcal{L}_{\text{int}}(x) | i \rangle$$

where

$$|i\rangle \equiv |0\rangle, \quad |f\rangle \equiv \hat{a}_f^\dagger \hat{a}_f^\dagger |0\rangle.$$

If the **envelope** $\varphi(t)$ varies on the time-scale much longer than the time-scale relevant for processes of particle creation, the S-matrix element can be written as

$$S_{if}^{(1)} = i\varphi(t) \sum_k \mathcal{P}_k \mathcal{M}_{0 \rightarrow f}(k) \times (2\pi)^4 \delta(k\omega - 2E_f) \delta^3(p_{f_1} + p_{f_2}).$$