Reduction at NLO and beyond I

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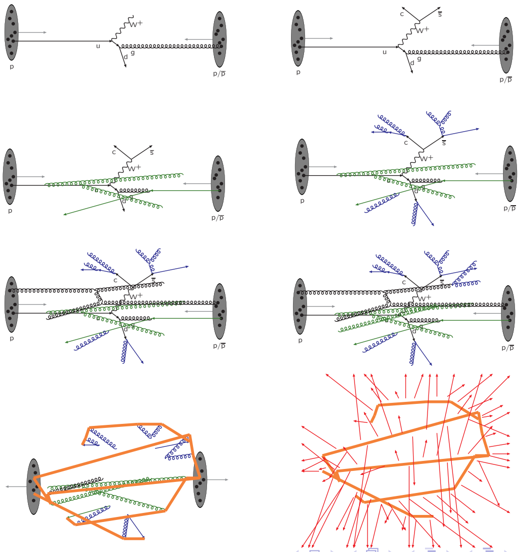
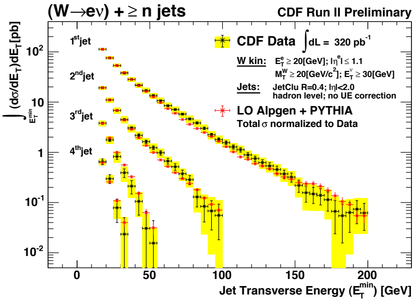
The big picture

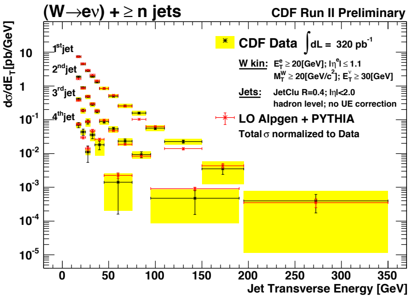
The evolution of the scattering process



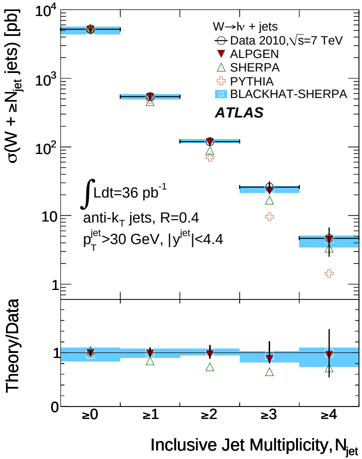
QCD VS EXPERIMENTS

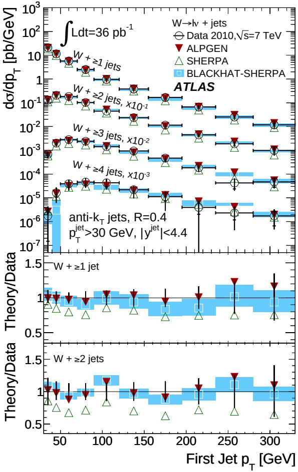
QCD quantitative description of data



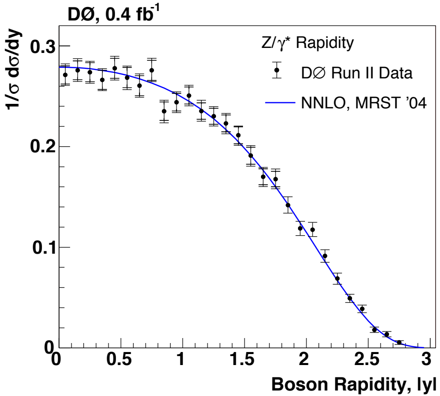


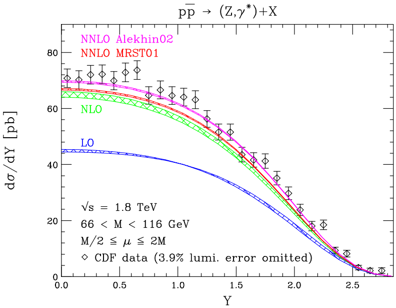
QCD quantitative description of data





(N)NLO needed in order to properly interpret the data at the LHC

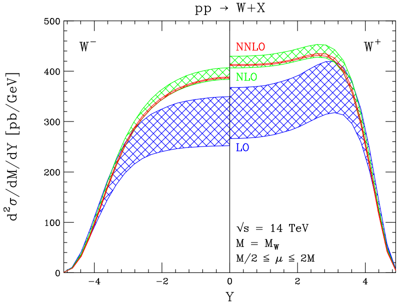




QCD beyond LO

(N)NLO corrections: impressive impact on theoretical uncertainties and differential shapes

C. Anastasiou, L. J. Dixon, K. Melnikov and F. Petriello



What do we need for an NLO calculation ?

$$p_1, p_2 \to p_3, \ldots, p_{m+2}$$

$$\begin{aligned}\sigma_{NLO} &= \int_m d\Phi_m |M_m^{(0)}|^2 J_m(\Phi) \\ &+ \int_m d\Phi_m 2Re(M_m^{(0)*} M_m^{(1)}(\epsilon_{UV}, \epsilon_{IR})) J_m(\Phi) \\ &+ \int_{m+1} d\Phi_{m+1} |M_{m+1}^{(0)}|^2 J_{m+1}(\Phi)\end{aligned}$$

$J_m(\Phi)$ jet function: Infrared safeness $J_{m+1} \to J_m$

What do we need for an NLO calculation ?

$$p_1, p_2 \to p_3, \ldots, p_{m+2}$$

$$\begin{aligned}\sigma_{NLO} &= \int_m d\Phi_m^{D=4}\left(|M_m^{(0)}|^2 + 2Re(M_m^{(0)*} M_m^{(CT)}(\epsilon_{UV}))\right) J_m(\Phi) \\ &+ \int_m d\Phi_m^{D=4} 2Re(M_m^{(0)*} M_m^{(1)}(\epsilon_{UV}, \epsilon_{IR})) J_m(\Phi) \\ &+ \int_{m+1} d\Phi_{m+1}^{D=4-2\epsilon_{IR}} |M_{m+1}^{(0)}|^2 J_{m+1}(\Phi)\end{aligned}$$

IR and UV divergencies, Four-Dimensional-Helicity scheme; scale dependence μ_R

Perturbative QCD at NLO

What do we need for an NLO calculation ?

$$p_1, p_2 \to p_3, \ldots, p_{m+2}$$

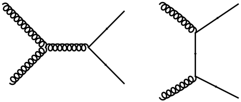
$$\begin{aligned}
\sigma_{NLO} &= \int_m d\Phi_m J_m(\Phi) \\
&+ \int_m d\Phi_m 2Re(M_m^{(0)*} M_m^{(1)}(\epsilon_{UV}, \epsilon_{IR})) J_m(\Phi) \\
&+ \int_{m+1} d\Phi_{m+1} |M_{m+1}^{(0)}|^2 J_{m+1}(\Phi)
\end{aligned}$$

QCD factorization$-\mu_F$ Collinear counter-terms when PDF are involved

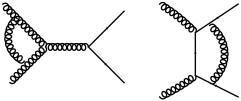
Perturbative QCD at NLO

What do we need for an NLO calculation ?

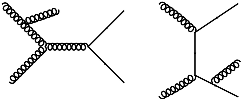
$M_m^{(0)}$



$M_m^{(1)}$



$M_{m+1}^{(0)}$



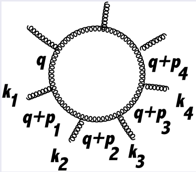
NLO TROUBLES

Problems arising in NLO calculations

- Large Number of Feynman diagrams
- Reduction to Scalar Integrals (or sets of known integrals)
- Numerical Instabilities (inverse Gram determinants, spurious phase-space singularities)
- Extraction of soft and collinear singularities (we need virtual and real corrections)

One-loop amplitudes

Any m-point one-loop amplitude can be written as



$$\int d^D q\, A(\bar{q}) = \int d^D q\, \frac{\bar{N}(\bar{q})}{\bar{D}_0 \bar{D}_1 \cdots \bar{D}_{m-1}}$$

A bar denotes objects living in $n = 4 + \epsilon$ dimensions

$$\bar{D}_i = (\bar{q} + p_i)^2 - m_i^2$$

$$\bar{q}^2 = q^2 + \tilde{q}^2$$

$$\bar{D}_i = D_i + \tilde{q}^2$$

The old "master" formula

$$
\begin{aligned}
\int A \;=\; & \sum_{i_0<i_1<i_2<i_3}^{m-1} d(i_0 i_1 i_2 i_3) D_0(i_0 i_1 i_2 i_3) \\
+ & \sum_{i_0<i_1<i_2}^{m-1} c(i_0 i_1 i_2) C_0(i_0 i_1 i_2) \\
+ & \sum_{i_0<i_1}^{m-1} b(i_0 i_1) B_0(i_0 i_1) \\
+ & \sum_{i_0}^{m-1} a(i_0) A_0(i_0) \\
+ & \text{ rational terms}
\end{aligned}
$$

D_0, C_0, B_0, A_0, scalar one-loop integrals: 't Hooft and Veltman
`QCDLOOP` Ellis & Zanderighi ; `OneLOop` A. van Hameren

Passarino-Veltman

G. Passarino and M. J. G. Veltman, "One Loop Corrections For E+ E- Annihilation Into Mu+ Mu- In The Weinberg Model," Nucl. Phys. B **160** (1979) 151.

For a generic one-loop Feynman graph

$$A(\bar{q}) = \frac{N(q)}{\bar{D}_0 \bar{D}_1 \cdots \bar{D}_{m-1}}$$

Decompose the numerator

$$N(q) \to q^{\mu_1} \ldots q^{\mu_m} \to g^{\mu_1 \mu_2} p_i^{\mu_3} \ldots$$

Tensor integrals

$$T^N_{\mu_1 \ldots \mu_P}(p_1, \ldots, p_{N-1}, m_0, \ldots, m_{N-1}) = \frac{(2\pi\mu)^{4-D}}{i\pi^2} \int d^D q \frac{q_{\mu_1} \cdots q_{\mu_P}}{D_0 D_1 \cdots D_{N-1}}$$

Contracting with external momenta and/or metric tensors on both sides

$$q p_k = \frac{1}{2}[D_k - D_0 - f_k], \qquad f_k = p_k^2 - m_k^2 + m_0^2.$$

$$\begin{aligned}
R^{N,k}_{\mu_1\ldots\mu_{P-1}} &= T^N_{\mu_1\ldots\mu_P} p_k^{\mu_P} \\
&= \frac{1}{2}\frac{(2\pi\mu)^{4-D}}{i\pi^2}\int d^Dq \left[\frac{q_{\mu_1}\cdots q_{\mu_{P-1}}}{D_0\ldots D_{k-1}D_{k+1}\ldots D_{N-1}}\right. \\
&\qquad \left. -\frac{q_{\mu_1}\cdots q_{\mu_{P-1}}}{D_1\ldots D_{N-1}} - f_k\frac{q_{\mu_1}\cdots q_{\mu_{P-1}}}{D_0\ldots D_{N-1}}\right] \\
&= \tfrac{1}{2}\left[T^{N-1}_{\mu_1\ldots\mu_{P-1}}(k) - T^{N-1}_{\mu_1\ldots\mu_{P-1}}(0) - f_k T^N_{\mu_1\ldots\mu_{P-1}}\right],
\end{aligned}$$

$$g^{\mu\nu} q_\mu q_\nu = q^2 = D_0 + m_0^2$$

$$\begin{aligned}
R^{N,00}_{\mu_1\ldots\mu_{P-2}} &= T^N_{\mu_1\ldots\mu_P} g^{\mu_{P-1}\mu_P} \\
&= \frac{(2\pi\mu)^{4-D}}{i\pi^2}\int d^Dq \left[\frac{q_{\mu_1}\cdots q_{\mu_{P-2}}}{D_1\ldots D_N} + m_0^2\frac{q_{\mu_1}\cdots q_{\mu_{P-2}}}{D_0\ldots D_N}\right] \\
&= \left[T^{N-1}_{\mu_1\ldots\mu_{P-2}}(0) + m_0^2 T^N_{\mu_1\ldots\mu_{P-2}}\right].
\end{aligned}$$

Passarino-Veltman

$$T^N_{\mu_1\ldots\mu_P}(p_1,\ldots,p_{N-1},m_0,\ldots,m_{N-1}) = \sum_{i_1,\ldots,i_P=0}^{N-1} T^N_{i_1\ldots i_P} p_{i_1\mu_1}\cdots p_{i_P\mu_P}.$$

$$\begin{aligned}
D_\mu &= \sum_{i=1}^{3} p_{i\mu} D_i, \\
D_{\mu\nu} &= g_{\mu\nu} D_{00} + \sum_{i,j=1}^{3} p_{i\mu} p_{j\nu} D_{ij}, \\
D_{\mu\nu\rho} &= \sum_{i=1}^{3} (g_{\mu\nu} p_{i\rho} + g_{\nu\rho} p_{i\mu} + g_{\mu\rho} p_{i\nu}) D_{00i} + \sum_{i,j,k=1}^{3} p_{i\mu} p_{j\nu} p_{k\rho} D_{ijk}, \\
D_{\mu\nu\rho\sigma} &= (g_{\mu\nu} g_{\rho\sigma} + g_{\mu\rho} g_{\nu\sigma} + g_{\mu\sigma} g_{\nu\rho}) D_{0000} \\
&\quad + \sum_{i,j=1}^{3} (g_{\mu\nu} p_{i\rho} p_{j\sigma} + g_{\nu\rho} p_{i\mu} p_{j\sigma} + g_{\mu\rho} p_{i\nu} p_{j\sigma} \\
&\qquad + g_{\mu\sigma} p_{i\nu} p_{j\rho} + g_{\nu\sigma} p_{i\mu} p_{j\rho} + g_{\rho\sigma} p_{i\mu} p_{j\nu}) D_{00ij} \\
&\quad + \sum_{i,j,k,l=1}^{3} p_{i\mu} p_{j\nu} p_{k\rho} p_{l\sigma} D_{ijkl}.
\end{aligned}$$

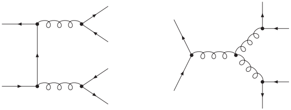
Recursive equations for the tensor coefficient functions

The process $\mathrm{pp} \to \mathrm{t\bar{t}b\bar{b}}$ in NLO QCD

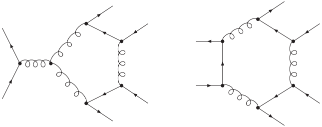
Bredenstein, Denner, S.D., Pozzorini '08,'09;   Bevilacqua, Czakon, Papadopoulos, Pittau, Worek '09

$q\bar{q} \to \mathrm{t\bar{t}b\bar{b}}$

LO: 7 tree graphs

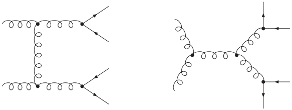


NLO: $\mathcal{O}(200)$ 1-loop diagrams
(24 pentagons, 8 hexagons)

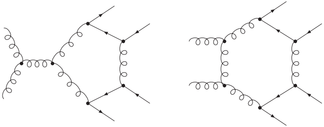


$\mathrm{gg} \to \mathrm{t\bar{t}b\bar{b}}$

LO: 36 tree graphs



NLO: $\mathcal{O}(\gtrsim 1000)$ 1-loop diagrams
(> 100 pentagons, 40 hexagons)



$2 \to 4$ processes define present "NLO multi-leg frontier".

Our Feynman-diagrammatic approach for virtual 1-loop corrections

$$\mathcal{M}_{1-\text{loop}} = \sum_{\text{(sub)diagrams }\Gamma} \mathcal{M}_\Gamma$$

generated with FEYNARTS (Küblbeck et al. '90; Hahn '01)

$$\mathcal{M}_\Gamma = \sum_n \underbrace{C^{(\Gamma)}}_{\text{colour factor}} \underbrace{F_n^{(\Gamma)}} \underbrace{\hat{\mathcal{M}}_n}_{\text{spin structures like } [\bar{u}_t(k_t)\not{\varepsilon}_{g_1}(k_{g_1})v_{\bar{t}}(k_{\bar{t}})](\varepsilon_{g_2}(k_g)\cdot k_t)\dots}$$

$\uparrow$ invariant functions containing 1-loop tensor integrals $T^{\mu\nu\rho\dots}$

$$T^{\mu\nu\rho\dots} = (p_k^\mu p_l^\nu p_m^\rho \dots) T_{kl\dots} + (g^{\mu\nu} p_m^\rho \dots) T_{00m\dots} + \dots$$

$T_{kl\dots}$ = linear combination of scalar 1-loop integrals A_0, B_0, C_0, D_0

– recursively calculable à la Passarino/Veltman '79 for regular points

– specially designed methods for rescuing cases with small Gram dets. Denner, S.D. '05

– 5-/6-point integrals reduced to 4-point integrals Denner, S.D. '02,'05

Features:

– advantage: get all colour/spin channels in one stroke

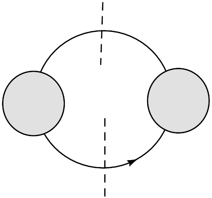
$\hookrightarrow$ speed: $\mathcal{M}_{1-\text{loop}}^{q\bar{q}/gg\to t\bar{t}b\bar{b}}$ in $\mathcal{O}(0.2\text{sec/event})$ very fast !

– lengthy algebra $\to$ automation (MATHEMATICA)

– two independent calculations, one using features of FORMCALC (Hahn)

Unitarity

Gluing trees to make loops ?



Started in 90's, mainly QCD, amplitude level (analytical results)
Z. Bern, L. J. Dixon, D. C. Dunbar and D. A. Kosower, [arXiv:hep-ph/9403226].
Gluing tree amplitudes plus colinear limits → extract coefficients

Gluing trees to make loops ?

$$\mathcal{C} * \int A = \sum_{i_0<i_1<i_2<i_3}^{m-1} d(i_0 i_1 i_2 i_3)\mathcal{C} * D_0(i_0 i_1 i_2 i_3) + \sum_{i_0<i_1<i_2}^{m-1} c(i_0 i_1 i_2)\mathcal{C} * C_0(i_0 i_1 i_2) + \sum_{i_0<i_1}^{m-1} b(i_0 i_1)\mathcal{C} * B_0(i_0 i_1)$$

Unitarity

Applying the unitarity method:

From the known behavior of scalar functions

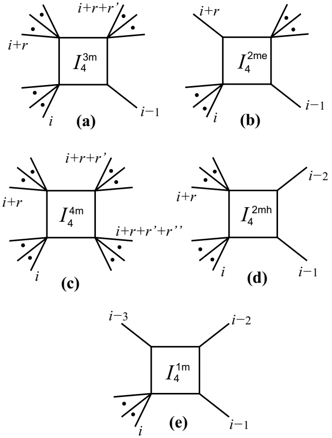
	Integral	Unique Function
a	$I_4^{0\mathrm{m}}(s,t)$	$\ln(-s)\ln(-t)$
b	$I_3^{1\mathrm{m}}(s)$	$\ln(-s)^2$
c	$I_3^{1\mathrm{m}}(t)$	$\ln(-t)^2$
d	$I_2(s)$	$\ln(-s)$
e	$I_2(t)$	$\ln(-t)$

Table 1: The set of integral functions that may appear in a cut-constructible massless four-point amplitude, together with the independent logarithms.

To calculate/guess the rational coefficients + collinear limits

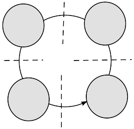
$$\int d\mu A^{\mathrm{tree}}(\ell_1, i, ..., j, \ell_2) A^{\mathrm{tree}}(-\ell_2, j+1, ..., i-1, -\ell_1) =$$
$$\sum \left(\widehat{b}\Delta I^{1m} + \widehat{c}\Delta I^{2m\ e} + \widehat{d}\Delta I^{2m\ h} + \widehat{g}\Delta I^{3m} + \widehat{f}\Delta I^{4m} \right).$$

Cut-construct-ability for arbitrary number of legs (to be compared with PV)



Quadruple cuts

R. Britto, F. Cachazo and B. Feng, [arXiv:hep-th/0412103].
Quadruple cut with complex momenta $\rightarrow d(i_0 i_1 i_2 i_3)$



The amazing formula

$$\widehat{f} = \frac{1}{|\mathcal{S}|} \sum_{\mathcal{S},J} n_J (A_1^{\mathrm{tree}} A_2^{\mathrm{tree}} A_3^{\mathrm{tree}} A_4^{\mathrm{tree}}),$$

Familiarize yourself in using complex-valued momenta

The old "master" formula

$$\int A = \sum_{i_0<i_1<i_2<i_3}^{m-1} d(i_0 i_1 i_2 i_3) D_0(i_0 i_1 i_2 i_3)$$
$$+ \sum_{i_0<i_1<i_2}^{m-1} c(i_0 i_1 i_2) C_0(i_0 i_1 i_2)$$
$$+ \sum_{i_0<i_1}^{m-1} b(i_0 i_1) B_0(i_0 i_1)$$
$$+ \sum_{i_0}^{m-1} a(i_0) A_0(i_0)$$
$$+ \text{rational terms}$$

D_0, C_0, B_0, A_0, scalar one-loop integrals: 't Hooft and Veltman
`QCDLOOP` Ellis & Zanderighi ; `OneLOop` A. van Hameren

The old "master" formula

$$\int \frac{N(q)}{\bar{D}_0 \bar{D}_1 \cdots \bar{D}_{m-1}} = \sum_{i_0<i_1<i_2<i_3}^{m-1} d(i_0 i_1 i_2 i_3) \int \frac{1}{\bar{D}_{i0}\bar{D}_{i1}\bar{D}_{i2}\bar{D}_{i2}}$$
$$+ \sum_{i_0<i_1<i_2}^{m-1} c(i_0 i_1 i_2) \int \frac{1}{\bar{D}_{i0}\bar{D}_{i1}\bar{D}_{i2}}$$
$$+ \sum_{i_0<i_1}^{m-1} b(i_0 i_1) \int \frac{1}{\bar{D}_{i0}\bar{D}_{i1}}$$
$$+ \sum_{i_0}^{m-1} a(i_0) \int \frac{1}{\bar{D}_{i0}}$$
$$+ \text{ rational terms}$$

The new "master" formula

$$
\begin{aligned}
\frac{N(q)}{\bar{D}_0 \bar{D}_1 \cdots \bar{D}_{m-1}} &= \sum_{i_0<i_1<i_2<i_3}^{m-1} \frac{d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3)}{\bar{D}_{i0} \bar{D}_{i1} \bar{D}_{i2} \bar{D}_{i2}} \\
&+ \sum_{i_0<i_1<i_2}^{m-1} \frac{c(i_0 i_1 i_2) + \tilde{c}(q; i_0 i_1 i_2)}{\bar{D}_{i0} \bar{D}_{i1} \bar{D}_{i2}} \\
&+ \sum_{i_0<i_1}^{m-1} \frac{b(i_0 i_1) + \tilde{b}(q; i_0 i_1)}{\bar{D}_{i0} \bar{D}_{i1}} \\
&+ \sum_{i_0}^{m-1} \frac{a(i_0) + \tilde{a}(q; i_0)}{\bar{D}_{i0}} \\
&+ \text{rational terms}
\end{aligned}
$$

OPP "MASTER" FORMULA - I

General expression for the 4-dim $N(q)$ at the integrand level in terms of D_i

$$
\begin{aligned}
N(q) &= \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3) \right] \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} D_i \\
&+ \sum_{i_0<i_1<i_2}^{m-1} \left[c(i_0 i_1 i_2) + \tilde{c}(q; i_0 i_1 i_2) \right] \prod_{i \neq i_0, i_1, i_2}^{m-1} D_i \\
&+ \sum_{i_0<i_1}^{m-1} \left[b(i_0 i_1) + \tilde{b}(q; i_0 i_1) \right] \prod_{i \neq i_0, i_1}^{m-1} D_i \\
&+ \sum_{i_0}^{m-1} \left[a(i_0) + \tilde{a}(q; i_0) \right] \prod_{i \neq i_0}^{m-1} D_i
\end{aligned}
$$

OPP "MASTER" FORMULA - II

$$
\begin{aligned}
N(q) &= \sum_{i_0<i_1<i_2<i_3}^{m-1}\left[d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3)\right] \prod_{i\neq i_0, i_1, i_2, i_3}^{m-1} D_i + \sum_{i_0<i_1<i_2}^{m-1}\left[c(i_0 i_1 i_2) + \tilde{c}(q; i_0 i_1 i_2)\right] \prod_{i\neq i_0, i_1, i_2}^{m-1} D_i \\
&+ \sum_{i_0<i_1}^{m-1}\left[b(i_0 i_1) + \tilde{b}(q; i_0 i_1)\right] \prod_{i\neq i_0, i_1}^{m-1} D_i + \sum_{i_0}^{m-1}\left[a(i_0) + \tilde{a}(q; i_0)\right] \prod_{i\neq i_0}^{m-1} D_i
\end{aligned}
$$

- The quantities $d(i_0 i_1 i_2 i_3)$ are the coefficients of 4-point functions with denominators labeled by i_0, i_1, i_2, and i_3.
- $c(i_0 i_1 i_2)$, $b(i_0 i_1)$, $a(i_0)$ are the coefficients of all possible 3-point, 2-point and 1-point functions, respectively.

What is the explicit expression of the spurious term?

OPP "MASTER" FORMULA - II

$$
\begin{aligned}
N(q) &= \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3)\right] \prod_{i\neq i_0,i_1,i_2,i_3}^{m-1} D_i + \sum_{i_0<i_1<i_2}^{m-1} \left[c(i_0 i_1 i_2) + \tilde{c}(q; i_0 i_1 i_2)\right] \prod_{i\neq i_0,i_1,i_2}^{m-1} D_i \\
&+ \sum_{i_0<i_1}^{m-1} \left[b(i_0 i_1) + \tilde{b}(q; i_0 i_1)\right] \prod_{i\neq i_0,i_1}^{m-1} D_i + \sum_{i_0}^{m-1} \left[a(i_0) + \tilde{a}(q; i_0)\right] \prod_{i\neq i_0}^{m-1} D_i
\end{aligned}
$$

The quantities $\tilde{d}$, $\tilde{c}$, $\tilde{b}$, $\tilde{a}$ are the "spurious" terms

- They still depend on q (integration momentum)
- They should vanish upon integration

What is the explicit expression of the spurious term?

Spurious Terms - I

Following F. del Aguila and R. Pittau, arXiv:hep-ph/0404120

- Express any q in $N(q)$ as

$$q^\mu = -p_0^\mu + \sum_{i=1}^{4} G_i\, \ell_i^\mu \;,\; {\ell_i}^2 = 0$$

$$k_1 = \ell_1 + \alpha_1 \ell_2\,, \quad k_2 = \ell_2 + \alpha_2 \ell_1\,,\; k_i = p_i - p_0$$
$${\ell_3}^\mu =< \ell_1|\gamma^\mu|\ell_2]\,,\; {\ell_4}^\mu =< \ell_2|\gamma^\mu|\ell_1]$$

- The coefficients G_i either reconstruct denominators D_i
  or vanish upon integration

→ They give rise to d, c, b, a coefficients
→ They form the spurious $\tilde{d}$, $\tilde{c}$, $\tilde{b}$, $\tilde{a}$ coefficients

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Spurious Terms - II

- $\tilde{d}(q)$ term (only 1)

$$\tilde{d}(q) = \tilde{d}\ T(q)\,,$$

where $\tilde{d}$ is a constant (does not depend on q)

$$T(q) \equiv Tr[(\not{q} + \not{p}_0)\not{\ell}_1\not{\ell}_2\not{k}_3\gamma_5]$$

- $\tilde{c}(q)$ terms (they are 6)

$$\tilde{c}(q) = \sum_{j=1}^{j_{max}} \{\tilde{c}_{1j}[(q+p_0)\cdot\ell_3]^j + \tilde{c}_{2j}[(q+p_0)\cdot\ell_4]^j\}$$

In the renormalizable gauge, $j_{max} = 3$

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A simple example

$$\int \frac{1}{D_0 D_1 D_2 D_3 D_4}$$

$$1 = \sum \left[d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3) \right] D_{i_4}$$

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A SIMPLE EXAMPLE

To be compared with standard formula (Denner)

$$\begin{vmatrix} T_0^5 & -T_0^4(0) & -T_0^4(1) & -T_0^4(2) & -T_0^4(3) & -T_0^4(4) \\ 1 & Y_{00} & Y_{01} & Y_{02} & Y_{03} & Y_{04} \\ 1 & Y_{10} & Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ 1 & Y_{20} & Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ 1 & Y_{30} & Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ 1 & Y_{40} & Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{vmatrix} = 0, \qquad (4.52)$$

which can be solved for T_0^5 if the determinant of the matrix Y_{ij}, $i, j = 0, \ldots, 4$ is nonzero. Note that in the integral $T_0^4(0)$ the momenta have not been shifted. In particular (4.52) yields the scalar five-point function T_0^5 in terms of five scalar four-point functions.

$$Y_{ij} = m_i^2 + m_j^2 - (p_i - p_j)^2.$$

A next to simple example

$$\int \frac{1}{D_0 D_1 D_2 D_3 \ldots D_{m-1}}$$

$$1 = \sum \left[d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3) \right] D_{i_4} D_{i_5} \ldots D_{i_{m-1}}$$

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$$d(i_0 i_1 i_2 i_3) = \frac{1}{2} \left(\prod_{j \neq i_0, i_1, i_2, i_3} \frac{1}{D_j(q^+)} + \prod_{j \neq i_0, i_1, i_2, i_3} \frac{1}{D_j(q^-)} \right)$$

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General strategy

Now we know the form of the spurious terms:

$$
\begin{aligned}
N(q) &= \sum_{i_0<i_1<i_2<i_3}^{m-1}\left[d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3)\right]\prod_{i\neq i_0,i_1,i_2,i_3}^{m-1} D_i + \sum_{i_0<i_1<i_2}^{m-1}\left[c(i_0 i_1 i_2) + \tilde{c}(q; i_0 i_1 i_2)\right]\prod_{i\neq i_0,i_1,i_2}^{m-1} D_i \\
&+ \sum_{i_0<i_1}^{m-1}\left[b(i_0 i_1) + \tilde{b}(q; i_0 i_1)\right]\prod_{i\neq i_0,i_1}^{m-1} D_i + \sum_{i_0}^{m-1}\left[a(i_0) + \tilde{a}(q; i_0)\right]\prod_{i\neq i_0}^{m-1} D_i
\end{aligned}
$$

Our calculation is now reduced to an algebraic problem

Extract all the coefficients by evaluating N(q) for a set of values of the integration momentum q

There is a very good set of such points: Use values of q for which a set of denominators D_i vanish $\rightarrow$ The system becomes "triangular": solve first for 4-point functions, then 3-point functions and so on

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$$N(q) = \sum_{i_0<i_1<i_2<i_3}^{m-1}\left[d(i_0i_1i_2i_3) + \tilde{d}(q;i_0i_1i_2i_3)\right]\prod_{i\neq i_0,i_1,i_2,i_3}^{m-1} D_i + \sum_{i_0<i_1<i_2}^{m-1}\left[c(i_0i_1i_2) + \tilde{c}(q;i_0i_1i_2)\right]\prod_{i\neq i_0,i_1,i_2}^{m-1} D_i$$
$$+ \sum_{i_0<i_1}^{m-1}\left[b(i_0i_1) + \tilde{b}(q;i_0i_1)\right]\prod_{i\neq i_0,i_1}^{m-1} D_i + \sum_{i_0}^{m-1}\left[a(i_0) + \tilde{a}(q;i_0)\right]\prod_{i\neq i_0}^{m-1} D_i$$

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Example

$$N(q) = d + \tilde{d}(q) + \sum_{i=0}^{3} [c(i) + \tilde{c}(q;i)] D_i + \sum_{i_0<i_1}^{3} \left[b(i_0 i_1) + \tilde{b}(q; i_0 i_1) \right] D_{i_0} D_{i_1}$$

$$+ \sum_{i_0=0}^{3} [a(i_0) + \tilde{a}(q; i_0)] D_{i\neq i_0} D_{j\neq i_0} D_{k\neq i_0}$$

We look for a q of the form $q^\mu = -p_0^\mu + x_i \ell_i^\mu$ such that

$$D_0 = D_1 = D_2 = D_3 = 0$$

$\rightarrow$ we get a system of equations in x_i that has two solutions $q_0^\pm$

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$$+ \sum_{i_0=0}^{3} [a(i_0) + \tilde{a}(q; i_0)] D_{i\neq i_0} D_{j\neq i_0} D_{k\neq i_0}$$

Our "master formula" for $q = q_0^{\pm}$ is:

$$N(q_0^{\pm}) = [d + \tilde{d}\, T(q_0^{\pm})]$$

$\rightarrow$ solve to extract the coefficients d and $\tilde{d}$

Example

$$
\begin{aligned}
N(q) - d - \tilde{d}(q) &= \sum_{i=0}^{3} \left[c(i) + \tilde{c}(q; i) \right] D_i + \sum_{i_0 < i_1}^{3} \left[b(i_0 i_1) + \tilde{b}(q; i_0 i_1) \right] D_{i_0} D_{i_1} \\
&+ \sum_{i_0=0}^{3} \left[a(i_0) + \tilde{a}(q; i_0) \right] D_{i \neq i_0} D_{j \neq i_0} D_{k \neq i_0}
\end{aligned}
$$

Then we can move to the extraction of c coefficients using

$$N'(q) = N(q) - d - \tilde{d}\, T(q)$$

and setting to zero three denominators (ex: $D_1 = 0,\ D_2 = 0,\ D_3 = 0$)

Example

$$N(q) - d - \tilde{d}(q) \quad = \quad [c(0) + \tilde{c}(q;0)]\, D_0$$

We have infinite values of q for which

$$D_1 = D_2 = D_3 = 0 \quad \text{and} \quad D_0 \neq 0$$

$\rightarrow$ Here we need 7 of them to determine $c(0)$ and $\tilde{c}(q;0)$

Rational Terms - I

R_1: the rational terms from the reduction itself

- Let's go back to the integrand

$$A(\bar{q}) = \frac{N(q)}{\bar{D}_0 \bar{D}_1 \cdots \bar{D}_{m-1}}$$

- Insert the expression for $N(q)$ $\to$ we know all the coefficients

$$N(q) = \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d + \tilde{d}(q)\right] \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} D_i + \sum_{i_0<i_1<i_2}^{m-1} \left[c + \tilde{c}(q)\right] \prod_{i \neq i_0, i_1, i_2}^{m-1} D_i + \cdots$$

- Finally rewrite all denominators using

$$\frac{D_i}{\bar{D}_i} = \bar{Z}_i \,, \quad \text{with} \quad \bar{Z}_i \equiv \left(1 - \frac{\tilde{q}^2}{\bar{D}_i}\right)$$

Rational Terms - I

$$
\begin{aligned}
A(\bar{q}) &= \sum_{i_0<i_1<i_2<i_3}^{m-1} \frac{d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3)}{\bar{D}_{i_0}\bar{D}_{i_1}\bar{D}_{i_2}\bar{D}_{i_3}} \prod_{i\neq i_0,i_1,i_2,i_3}^{m-1} \bar{Z}_i \\
&+ \sum_{i_0<i_1<i_2}^{m-1} \frac{c(i_0 i_1 i_2) + \tilde{c}(q; i_0 i_1 i_2)}{\bar{D}_{i_0}\bar{D}_{i_1}\bar{D}_{i_2}} \prod_{i\neq i_0,i_1,i_2}^{m-1} \bar{Z}_i \\
&+ \sum_{i_0<i_1}^{m-1} \frac{b(i_0 i_1) + \tilde{b}(q; i_0 i_1)}{\bar{D}_{i_0}\bar{D}_{i_1}} \prod_{i\neq i_0,i_1}^{m-1} \bar{Z}_i \\
&+ \sum_{i_0}^{m-1} \frac{a(i_0) + \tilde{a}(q; i_0)}{\bar{D}_{i_0}} \prod_{i\neq i_0}^{m-1} \bar{Z}_i
\end{aligned}
$$

The rational part is produced, after integrating over $d^n q$, by the $\tilde{q}^2$ dependence in $\bar{Z}_i$

$$\bar{Z}_i \equiv \left(1 - \frac{\tilde{q}^2}{\bar{D}_i}\right)$$

Rational Terms - I

The "Extra Integrals" are of the form

$$I^{(n;2\ell)}_{s;\mu_1\cdots\mu_r} \equiv \int d^n q\, \tilde{q}^{2\ell} \frac{q_{\mu_1}\cdots q_{\mu_r}}{\bar{D}(k_0)\cdots\bar{D}(k_s)} \,,$$

where

$$\bar{D}(k_i) \equiv (\bar{q}+k_i)^2 - m_i^2 \,, k_i = p_i - p_0$$

These integrals:

- **have dimensionality** $\mathcal{D} = 2(1+\ell-s)+r$
- **contribute only when** $\mathcal{D} \geq 0$**, otherwise are of** $\mathcal{O}(\epsilon)$

Rational Terms - II

Expand in D-dimensions ?

$$\bar{D}_i = D_i + \tilde{q}^2$$

$$\begin{aligned} N(q) &= \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d(i_0 i_1 i_2 i_3; \tilde{q}^2) + \tilde{d}(q; i_0 i_1 i_2 i_3; \tilde{q}^2) \right] \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} \bar{D}_i \\ &+ \sum_{i_0<i_1<i_2}^{m-1} \left[c(i_0 i_1 i_2; \tilde{q}^2) + \tilde{c}(q; i_0 i_1 i_2; \tilde{q}^2) \right] \prod_{i \neq i_0, i_1, i_2}^{m-1} \bar{D}_i \\ &+ \sum_{i_0<i_1}^{m-1} \left[b(i_0 i_1; \tilde{q}^2) + \tilde{b}(q; i_0 i_1; \tilde{q}^2) \right] \prod_{i \neq i_0, i_1}^{m-1} \bar{D}_i \\ &+ \sum_{i_0}^{m-1} \left[a(i_0; \tilde{q}^2) + \tilde{a}(q; i_0; \tilde{q}^2) \right] \prod_{i \neq i_0}^{m-1} \bar{D}_i + \tilde{P}(q) \prod_{i}^{m-1} \bar{D}_i \end{aligned}$$

Rational Terms - II

Expand in D-dimensions ?

$$
\begin{aligned}
N(q) &= \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d(i_0 i_1 i_2 i_3;\tilde{q}^2) + \tilde{d}(q;i_0 i_1 i_2 i_3;\tilde{q}^2)\right] \prod_{i\neq i_0,i_1,i_2,i_3}^{m-1} \bar{D}_i \\
&+ \sum_{i_0<i_1<i_2}^{m-1} \left[c(i_0 i_1 i_2;\tilde{q}^2) + \tilde{c}(q;i_0 i_1 i_2;\tilde{q}^2)\right] \prod_{i\neq i_0,i_1,i_2}^{m-1} \bar{D}_i \\
&+ \sum_{i_0<i_1}^{m-1} \left[b(i_0 i_1;\tilde{q}^2) + \tilde{b}(q;i_0 i_1;\tilde{q}^2)\right] \prod_{i\neq i_0,i_1}^{m-1} \bar{D}_i \\
&+ \sum_{i_0}^{m-1} \left[a(i_0;\tilde{q}^2) + \tilde{a}(q;i_0;\tilde{q}^2)\right] \prod_{i\neq i_0}^{m-1} \bar{D}_i + \tilde{P}(q) \prod_{i}^{m-1} \bar{D}_i
\end{aligned}
$$

$$m_i^2 \to m_i^2 - \tilde{q}^2$$

Rational Terms - II

Expand in D-dimensions ?

$$
\begin{aligned}
N(q) &= \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d(i_0 i_1 i_2 i_3;\tilde{q}^2) + \tilde{d}(q;i_0 i_1 i_2 i_3;\tilde{q}^2)\right] \prod_{i\neq i_0,i_1,i_2,i_3}^{m-1} \bar{D}_i \\
&+ \sum_{i_0<i_1<i_2}^{m-1} \left[c(i_0 i_1 i_2;\tilde{q}^2) + \tilde{c}(q;i_0 i_1 i_2;\tilde{q}^2)\right] \prod_{i\neq i_0,i_1,i_2}^{m-1} \bar{D}_i \\
&+ \sum_{i_0<i_1}^{m-1} \left[b(i_0 i_1;\tilde{q}^2) + \tilde{b}(q;i_0 i_1;\tilde{q}^2)\right] \prod_{i\neq i_0,i_1}^{m-1} \bar{D}_i \\
&+ \sum_{i_0}^{m-1} \left[a(i_0;\tilde{q}^2) + \tilde{a}(q;i_0;\tilde{q}^2)\right] \prod_{i\neq i_0}^{m-1} \bar{D}_i + \tilde{P}(q) \prod_{i}^{m-1} \bar{D}_i
\end{aligned}
$$

$$m_i^2 \rightarrow m_i^2 - \tilde{q}^2$$

Rational Terms - II

Polynomial dependence on $\tilde{q}^2$

$$b(ij;\tilde{q}^2) = b(ij) + \tilde{q}^2 b^{(2)}(ij)\,, \quad c(ijk;\tilde{q}^2) = c(ijk) + \tilde{q}^2 c^{(2)}(ijk)\,.$$

$$\int d^n\bar{q}\frac{\tilde{q}^2}{\bar{D}_i\bar{D}_j} = -\frac{i\pi^2}{2}\left[m_i^2 + m_j^2 - \frac{(p_i-p_j)^2}{3}\right] + \mathcal{O}(\epsilon)\,,$$

$$\int d^n\bar{q}\frac{\tilde{q}^2}{\bar{D}_i\bar{D}_j\bar{D}_k} = -\frac{i\pi^2}{2} + \mathcal{O}(\epsilon)\,, \quad \int d^n\bar{q}\frac{\tilde{q}^4}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_l} = -\frac{i\pi^2}{6} + \mathcal{O}(\epsilon)\,.$$

Rational Terms - II

Polynomial dependence on $\tilde{q}^2$

$$b(ij;\tilde{q}^2) = b(ij) + \tilde{q}^2 b^{(2)}(ij)\,, \quad c(ijk;\tilde{q}^2) = c(ijk) + \tilde{q}^2 c^{(2)}(ijk)\,.$$

$$\int d^n\bar{q}\,\frac{\tilde{q}^2}{\bar{D}_i\bar{D}_j} = -\frac{i\pi^2}{2}\left[m_i^2 + m_j^2 - \frac{(p_i-p_j)^2}{3}\right] + \mathcal{O}(\epsilon)\,,$$

$$\int d^n\bar{q}\,\frac{\tilde{q}^2}{\bar{D}_i\bar{D}_j\bar{D}_k} = -\frac{i\pi^2}{2} + \mathcal{O}(\epsilon)\,, \quad \int d^n\bar{q}\,\frac{\tilde{q}^4}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_l} = -\frac{i\pi^2}{6} + \mathcal{O}(\epsilon)\,.$$

Rational Terms - II

Furthermore, by defining

$$\mathcal{D}^{(m)}(q,\tilde{q}^2) \equiv \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d(i_0 i_1 i_2 i_3; \tilde{q}^2) + \tilde{d}(q; i_0 i_1 i_2 i_3; \tilde{q}^2) \right] \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} \bar{D}_i \,,$$

the following expansion holds

$$\mathcal{D}^{(m)}(q,\tilde{q}^2) = \sum_{j=2}^{m} \tilde{q}^{(2j-4)} d^{(2j-4)}(q)\,,$$

where the last coefficient is independent on q

$$d^{(2m-4)}(q) = d^{(2m-4)}\,.$$

Rational Terms - II

In practice, once the 4-dimensional coefficients have been determined, one can redo the fits for different values of $\tilde{q}^2$, in order to determine $b^{(2)}(ij)$, $c^{(2)}(ijk)$ and $d^{(2m-4)}$.

$$
\begin{aligned}
\mathrm{R}_1 &= -\frac{i}{96\pi^2} d^{(2m-4)} - \frac{i}{32\pi^2} \sum_{i_0<i_1<i_2}^{m-1} c^{(2)}(i_0 i_1 i_2) \\
&- \frac{i}{32\pi^2} \sum_{i_0<i_1}^{m-1} b^{(2)}(i_0 i_1) \left(m_{i_0}^2 + m_{i_1}^2 - \frac{(p_{i_0} - p_{i_1})^2}{3} \right) .
\end{aligned}
$$

G. Ossola, C. G. Papadopoulos and R. Pittau,arXiv:0802.1876 [hep-ph]

Rational Terms - R_2

A different source of Rational Terms, called R_2, can also be generated from the ϵ-dimensional part of $N(q)$

$$\bar{N}(\bar{q}) = N(q) + \tilde{N}(\tilde{q}^2, \epsilon; q)$$

$$\mathrm{R}_2 \equiv \frac{1}{(2\pi)^4} \int d^n\,\bar{q}\,\frac{\tilde{N}(\tilde{q}^2, \epsilon; q)}{\bar{D}_0 \bar{D}_1 \cdots \bar{D}_{m-1}} \equiv \frac{1}{(2\pi)^4} \int d^n\,\bar{q}\,\mathcal{R}_2$$

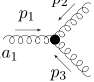
$$\begin{aligned} \bar{q} &= q + \tilde{q}\,, \\ \bar{\gamma}_{\bar{\mu}} &= \gamma_\mu + \tilde{\gamma}_{\tilde{\mu}}\,, \\ \bar{g}^{\bar{\mu}\bar{\nu}} &= g^{\mu\nu} + \tilde{g}^{\tilde{\mu}\tilde{\nu}}\,. \end{aligned}$$

New vertices/particles or GKMZ-approach

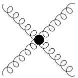
HELAC R2 TERMS

Contribution from $d-$dimensional parts in numerators:

$$= \frac{ig^2 N_{col}}{48\pi^2} \delta_{a_1 a_2} \left[\frac{p^2}{2} g_{\mu_1\mu_2} + \lambda_{HV} \left(g_{\mu_1\mu_2} p^2 - p_{\mu_1} p_{\mu_2} \right) + \frac{N_f}{N_{col}} (p^2 - 6\, m_q^2)\, g_{\mu_1\mu_2} \right]$$

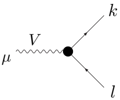


$$= -\frac{g^3 N_{col}}{48\pi^2} \left(\frac{7}{4} + \lambda_{HV} + 2\frac{N_f}{N_{col}} \right) f^{a_1 a_2 a_3}\, V_{\mu_1\mu_2\mu_3}(p_1, p_2, p_3)$$

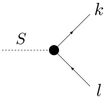


$$= -\frac{ig^4 N_{col}}{96\pi^2} \sum_{P(234)} \Bigg\{ \Bigg[\frac{\delta_{a_1a_2}\delta_{a_3a_4} + \delta_{a_1a_3}\delta_{a_4a_2} + \delta_{a_1a_4}\delta_{a_2a_3}}{N_{col}} + 4\, Tr(t^{a_1}t^{a_3}t^{a_2}t^{a_4} + t^{a_1}t^{a_4}t^{a_2}t^{a_3})\,(3+\lambda_{HV}) - Tr(\{t^{a_1}t^{a_2}\}\{t^{a_3}t^{a_4}\})\,(5+2\lambda_{HV}) \Bigg] g_{\mu_1\mu_2} g_{\mu_3\mu_4}$$
$$+ 12\frac{N_f}{N_{col}} Tr(t^{a_1}t^{a_2}t^{a_3}t^{a_4}) \left(\frac{5}{3} g_{\mu_1\mu_3} g_{\mu_2\mu_4} - g_{\mu_1\mu_2} g_{\mu_3\mu_4} - g_{\mu_2\mu_3} g_{\mu_1\mu_4} \right) \Bigg\}$$

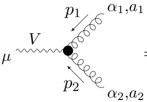
HELAC R2 terms



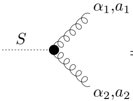
$$= -\frac{g^2}{16\pi^2}\frac{N_{col}^2-1}{2N_{col}}\,\delta_{kl}\gamma_\mu\left(v+a\gamma_5\right)\left(1+\lambda_{HV}\right)$$



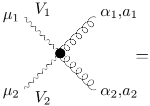
$$= -\frac{g^2}{8\pi^2}\frac{N_{col}^2-1}{2N_{col}}\,\delta_{kl}\left(c+d\gamma_5\right)\left(1+\lambda_{HV}\right)$$



$$= a\,\frac{ig^2}{12\pi^2}\,\delta_{a_1a_2}\epsilon_{\mu\alpha_1\alpha_2\beta}\left(p_1-p_2\right)^\beta$$



$$= c\,\frac{g^2}{8\pi^2}\,\delta_{a_1a_2}g_{\alpha_1\alpha_2}\,m_q$$



$$= -\frac{ig^2}{24\pi^2}\,\delta_{a_1a_2}(v_1v_2+a_1a_2)\left(g_{\mu_1\mu_2}g_{\alpha_1\alpha_2}+g_{\mu_1\alpha_1}g_{\mu_2\alpha_2}+g_{\mu_1\alpha_2}g_{\mu_2\alpha_1}\right)$$

Cuts in $D-$dimensions with particles in D_s dimensions

Giele, Kunszt, Melnikov, Zanderighi

$$\sum_{i=1}^{D_s-2} e_\mu^{(i)}(l) e_\nu^{(i)}(l) = -g_{\mu\nu}^{(D_s)} + \frac{l_\mu b_\nu + b_\mu l_\nu}{l \cdot b}, \qquad \sum_{i=1}^{2^{(D_s-2)/2}} u^{(i)}(l)\overline{u}^{(i)}(l) = \not{l} + m = \sum_{\mu=1}^{D} l_\mu \gamma^\mu + m \, .$$

$$\mathcal{N}^{(D_s)}(l) = \mathcal{N}_0(l) + (D_s - 4)\mathcal{N}_1(l).$$

$$\mathcal{N}_0(l) = \frac{(D_2-4)\mathcal{N}^{(D_1)}(l) - (D_1-4)\mathcal{N}^{(D_2)}(l)}{D_2 - D_1},$$

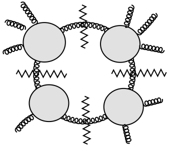
$$\mathcal{N}_1(l) = \frac{\mathcal{N}^{(D_1)}(l) - \mathcal{N}^{(D_2)}(l)}{D_2 - D_1}.$$

$$\frac{\mathcal{N}^{(D_s)}(l)}{d_1 d_2 \cdots d_N} = \sum_{[i_1|i_5]} \frac{\overline{e}^{(D_s)}_{i_1 i_2 i_3 i_4 i_5}(l)}{d_{i_1} d_{i_2} d_{i_3} d_{i_4} d_{i_5}} + \sum_{[i_1|i_4]} \frac{\overline{d}^{(D_s)}_{i_1 i_2 i_3 i_4}(l)}{d_{i_1} d_{i_2} d_{i_3} d_{i_4}} + \sum_{[i_1|i_3]} \frac{\overline{c}^{(D_s)}_{i_1 i_2 i_3}(l)}{d_{i_1} d_{i_2} d_{i_3}} + \sum_{[i_1|i_2]} \frac{\overline{b}^{(D_s)}_{i_1 i_2}(l)}{d_{i_1} d_{i_2}} + \sum_{[i_1|i_1]} \frac{\overline{a}^{(D_s)}_{i_1}(l)}{d_{i_1}} .$$

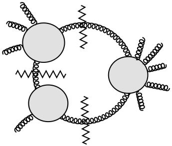
$$A_N^{[1]} = \sum_{[i_1|i_4]} d^{(0,0)}_{i_1 i_2 i_3 i_4} \, I^{(4-2\epsilon)}_{i_1 i_2 i_3 i_4} + \sum_{[i_1|i_3]} c^{(0,0)}_{i_1 i_2 i_3} \, I^{(4-2\epsilon)}_{i_1 i_2 i_3} + \sum_{[i_1|i_2]} b^{(0,0)}_{i_1 i_2} \, I^{(4-2\epsilon)}_{i_1 i_2}$$
$$- \sum_{[i_1|i_4]} \frac{d^{(4,0)}_{i_1 i_2 i_3 i_4}}{6} + \sum_{[i_1|i_3]} \frac{c^{(2,0)}_{i_1 i_2 i_3}}{2} - \sum_{[i_1|i_2]} \frac{(q_{i_1} - q_{i_2})^2}{6} b^{(2,0)}_{i_1 i_2} + \mathcal{O}(\epsilon) \, .$$

Constructing the one-loop amplitudes

The unitarity approach, by `Blackhat` and `Rocket` collaborations, with primitive amplitudes



$\rightarrow d$



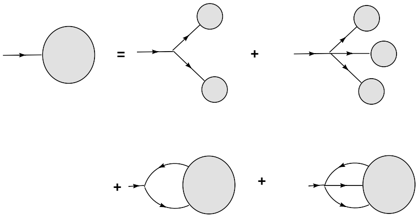
$-\sum d \rightarrow c$

Certain repetition in blobs, but unique cut coefficient

Constructing the one-loop amplitudes

The HELAC-1LOOP approach

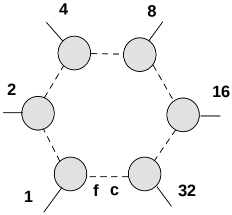
Dyson-Schwinger equations - reduced to Berends-Giele for ordered amplitudes



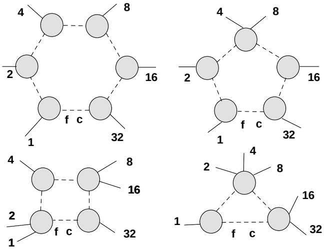
First line: tree-order generating

Constructing the one-loop amplitudes

Numerator functions computed by HELAC and reduced by CuTtools



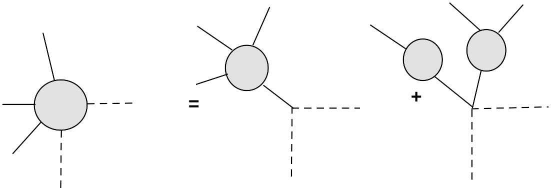
$\rightarrow d, c, b, a, R_1$



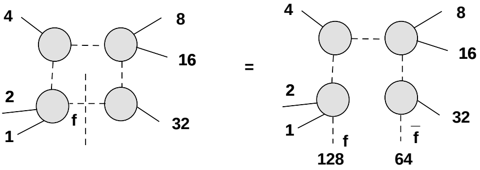
$\rightarrow d, c, b, a, R_1$

Constructing the one-loop amplitudes

All blobs are tree-order currents, independent of loop-momentum

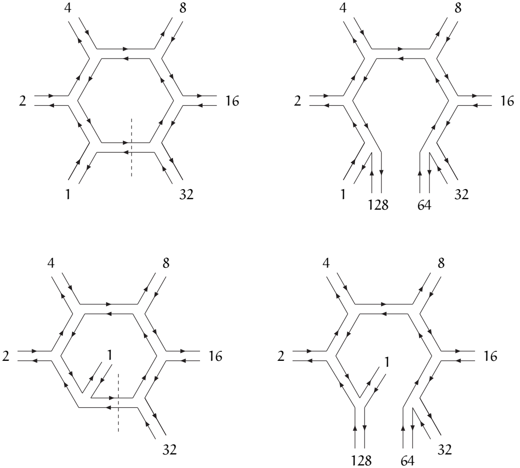


The $n \to n+2$ construction



Color treatment

HELAC is using color-connection representation of amplitudes + color-flow Feynman rules (Kanaki & Papadopoulos) - valid also at one loop

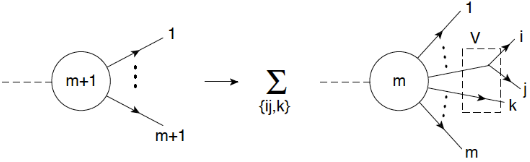


Real corrections

Real corrections: $D \to 4$ dimensions (Catani & Seymour)

$$\int_{m+1} d\sigma^R + \int_m d\sigma^V$$

$$\sigma^{NLO} = \int_{m+1} \left[\left(d\sigma^R\right)_{\epsilon=0} - \left(d\sigma^A\right)_{\epsilon=0} \right] + \int_m \left[d\sigma^V + \int_1 d\sigma^A \right]_{\epsilon=0}$$

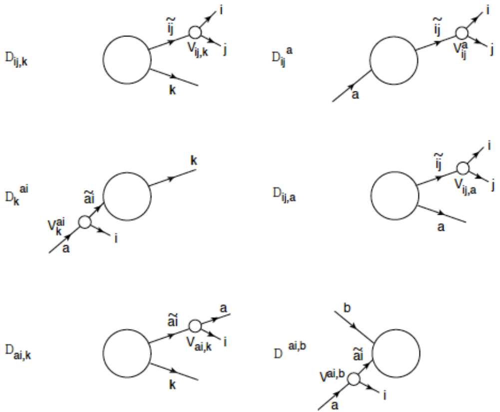


$$\tilde{p}_k^\mu = \frac{1}{1-y_{ij,k}} p_k^\mu \quad , \quad \tilde{p}_{ij}^\mu = p_i^\mu + p_j^\mu - \frac{y_{ij,k}}{1-y_{ij,k}} p_k^\mu$$

$$d\phi(p_i, p_j, p_k; Q) = \frac{d^d p_i}{(2\pi)^{d-1}} \delta_+(p_i^2) \frac{d^d p_j}{(2\pi)^{d-1}} \delta_+(p_j^2) \frac{d^d p_k}{(2\pi)^{d-1}} \delta_+(p_k^2) \; (2\pi)^d \delta^{(d)}(Q - p_i - p_j - p_k)$$

$$d\phi(p_i, p_j, p_k; Q) = d\phi(\tilde{p}_{ij}, \tilde{p}_k; Q) \; [dp_i(\tilde{p}_{ij}, \tilde{p}_k)]$$

Dipoles in real life



Real corrections

Dipoles in real life: the formulae

$$
\begin{aligned}
d\sigma^A &= \mathcal{N}_{in} \sum_{\{m+1\}} d\phi_{m+1}(p_1, ..., p_{m+1}; Q) \, \frac{1}{S_{\{m+1\}}} \\
&\cdot \sum_{\substack{\text{pairs} \\ \text{i,j}}} \sum_{k \neq i,j} \mathcal{D}_{ij,k}(p_1, ..., p_{m+1}) \, F_J^{(m)}(p_1, ..\tilde{p}_{ij}, \tilde{p}_k, .., p_{m+1})
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{D}_{ij,k} \, (p_1, ..., p_{m+1}) &= -\frac{1}{2p_i \cdot p_j} \\
\cdot \quad {}_m\!< 1, .., \widetilde{ij}, .., \tilde{k}, .., m+1| \, & \frac{\boldsymbol{T}_k \cdot \boldsymbol{T}_{ij}}{\boldsymbol{T}_{ij}^2} \, \boldsymbol{V}_{ij,k} \, |1, .., \widetilde{ij}, .., \tilde{k}, .., m+1 >_m
\end{aligned}
$$

$$
\begin{aligned}
d\sigma^R - d\sigma^A &= \mathcal{N}_{in} \sum_{\{m+1\}} d\phi_{m+1}(p_1, ..., p_{m+1}; Q) \, \frac{1}{S_{\{m+1\}}} \\
&\cdot \Big\{ |\mathcal{M}_{m+1}(p_1, ..., p_{m+1})|^2 \, F_J^{(m+1)}(p_1, ..., p_{m+1}) \\
&- \sum_{\substack{\text{pairs} \\ \text{i,j}}} \sum_{k \neq i,j} \mathcal{D}_{ij,k}(p_1, ..., p_{m+1}) \, F_J^{(m)}(p_1, ..\tilde{p}_{ij}, \tilde{p}_k, .., p_{m+1}) \Big\}
\end{aligned}
$$

$$
\begin{aligned}
&\int_{m+1} d\sigma^A = -\int_m \mathcal{N}_{in} \sum_{\{m\}} d\phi_m(p_1, .., p_m; Q) \, \frac{1}{S_{\{m\}}} \, F_J^{(m)}(p_1, ..., p_m) \\
&\cdot \sum_i \sum_{k \neq i} |\mathcal{M}_m^{i,k}(p_1, ..., p_m)|^2 \, \frac{\alpha_S}{2\pi} \frac{1}{\Gamma(1-\epsilon)} \left(\frac{4\pi\mu^2}{2p_i \cdot p_k} \right)^\epsilon \frac{1}{\boldsymbol{T}_i^2} \, \mathcal{V}_i(\epsilon) \ ,
\end{aligned}
$$

Institute of Nuclear Physics "Demokritos" Bergische Universitat Wuppertal Institute of Nuclear Physics PAN RWTH Aachen University

Content

HELAC-NLO & Associated Tools

Projects

HELAC-PHEGAS - A generator for all parton level processes in the Standard Model

HELAC-DIPOLES - Dipole formalism for the arbitrary helicity eigenstates of the external partons

HELAC-1LOOP - A program for numerical evaluation of QCD virtual corrections to scattering amplitudes

ONELOOP - A program for the evaluation of one-loop scalar functions

CUTTOOLS - A program implementing the OPP reduction method to compute one-loop amplitudes

PARNI - A program for importance sampling and density estimation

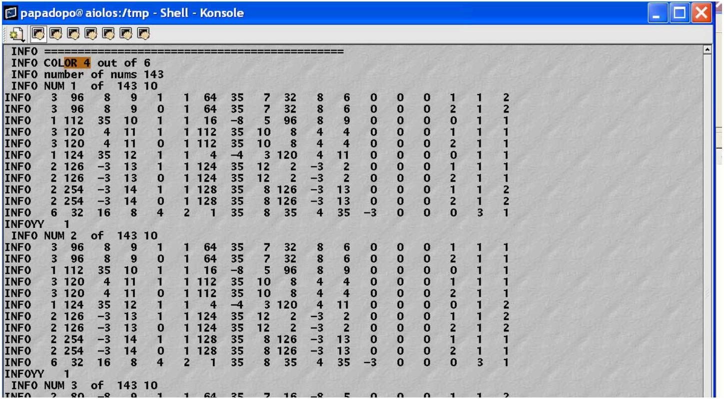
KALEU - A general-purpose parton-level phase space generator

HELAC-ONIA - An automatic matrix element generator for heavy quarkonium physics

:: top ::

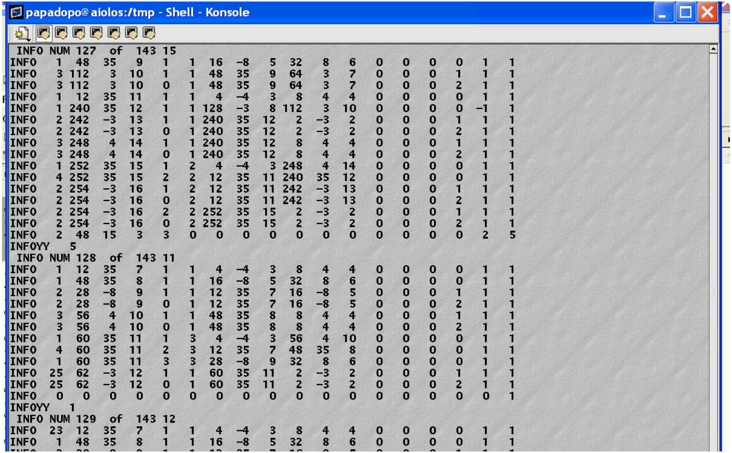
```
 kmax,ncc,nmax 6 24 6 16
 INFO ==============================================
 INFO COLOR 1 out of 6
INFO    1  12  35   7   1   1   4  -4   3   8   4   4   0   0   0   0   1   2
INFO    1  48  35   8   1   1  16  -8   5  32   8   6   0   0   0   0   1   2
INFO    2  14  -3   9   1   1  12  35   7   2  -3   2   0   0   0   1   1   2
INFO    2  14  -3   9   0   1  12  35   7   2  -3   2   0   0   0   2   1   2
INFO    2  28  -8  10   1   1  12  35   7  16  -8   5   0   0   0   1   1   2
INFO    2  28  -8  10   0   1  12  35   7  16  -8   5   0   0   0   2   1   2
INFO    3  44   8  11   1   1  12  35   7  32   8   6   0   0   0   1   1   2
INFO    3  44   8  11   0   1  12  35   7  32   8   6   0   0   0   2   1   2
INFO    2  50  -3  12   1   1  48  35   8   2  -3   2   0   0   0   1   1   2
INFO    2  50  -3  12   0   1  48  35   8   2  -3   2   0   0   0   2   1   2
INFO    2  52  -4  13   1   1  48  35   8   4  -4   3   0   0   0   1   1   2
INFO    2  52  -4  13   0   1  48  35   8   4  -4   3   0   0   0   2   1   2
INFO    3  56   4  14   1   1  48  35   8   8   4   4   0   0   0   1   1   2
INFO    3  56   4  14   0   1  48  35   8   8   4   4   0   0   0   2   1   2
INFO    1  60  35  15   1   4   4  -4   3  56   4  14   0   0   0   0   1   2
INFO    1  60  35  15   2   4  16  -8   5  44   8  11   0   0   0   0   1   2
INFO    1  60  35  15   3   4  28  -8  10  32   8   6   0   0   0   0   1   2
INFO    1  60  35  15   4   4  52  -4  13   8   4   4   0   0   0   0   1   2
INFO    2  62  -3  16   1   3  12  35   7  50  -3  12   0   0   0   1   1   2
INFO    2  62  -3  16   0   3  12  35   7  50  -3  12   0   0   0   2   1   2
INFO    2  62  -3  16   2   3  48  35   8  14  -3   9   0   0   0   1   1   2
INFO    2  62  -3  16   0   3  48  35   8  14  -3   9   0   0   0   2   1   2
INFO    2  62  -3  16   3   3  60  35  15   2  -3   2   0   0   0   1   1   2
INFO    2  62  -3  16   0   3  60  35  15   2  -3   2   0   0   0   2   1   2
 INFO ==============================================
 INFO COLOR 2 out of 6
INFO    1  12  35   7   1   1   4  -4   3   8   4   4   0   0   0   0   1   1
INFO    1  48  35   8   1   1  16  -8   5  32   8   6   0   0   0   0   1   1
```

HELAC 1-LOOP



papadopo@aiolos:/tmp - Shell - Konsole
INFO ==
INFO COLOR 4 out of 6
INFO number of nums 143
INFO NUM 1 of 143 10
INFO 3 96 8 9 1 1 64 35 7 32 8 6 0 0 0 1 1 2
INFO 3 96 8 9 0 1 64 35 7 32 8 6 0 0 0 2 1 2
INFO 1 112 35 10 1 1 16 -8 5 96 8 9 0 0 0 0 1 1
INFO 3 120 4 11 1 1 112 35 10 8 4 4 0 0 0 1 1 1
INFO 3 120 4 11 0 1 112 35 10 8 4 4 0 0 0 2 1 1
INFO 1 124 35 12 1 1 4 -4 3 120 4 11 0 0 0 0 1 1
INFO 2 126 -3 13 1 1 124 35 12 2 -3 2 0 0 0 1 1 1
INFO 2 126 -3 13 0 1 124 35 12 2 -3 2 0 0 0 2 1 1
INFO 2 254 -3 14 1 1 128 35 8 126 -3 13 0 0 0 1 1 2
INFO 2 254 -3 14 0 1 128 35 8 126 -3 13 0 0 0 2 1 2
INFO 6 32 16 8 4 2 1 35 8 35 4 35 -3 0 0 0 3 1
INFOYY 1
INFO NUM 2 of 143 10
INFO 3 96 8 9 1 1 64 35 7 32 8 6 0 0 0 1 1 1
INFO 3 96 8 9 0 1 64 35 7 32 8 6 0 0 0 2 1 1
INFO 1 112 35 10 1 1 16 -8 5 96 8 9 0 0 0 0 1 1
INFO 3 120 4 11 1 1 112 35 10 8 4 4 0 0 0 1 1 1
INFO 3 120 4 11 0 1 112 35 10 8 4 4 0 0 0 2 1 1
INFO 1 124 35 12 1 1 4 -4 3 120 4 11 0 0 0 0 1 2
INFO 2 126 -3 13 1 1 124 35 12 2 -3 2 0 0 0 1 1 2
INFO 2 126 -3 13 0 1 124 35 12 2 -3 2 0 0 0 2 1 2
INFO 2 254 -3 14 1 1 128 35 8 126 -3 13 0 0 0 1 1 1
INFO 2 254 -3 14 0 1 128 35 8 126 -3 13 0 0 0 2 1 1
INFO 6 32 16 8 4 2 1 35 8 35 4 35 -3 0 0 0 3 1
INFOYY 1
INFO NUM 3 of 143 10

HELAC 1-LOOP



papadopo@aiolos:/tmp - Shell - Konsole

```
 INFO NUM 127  of  143 15
INFO   1  48  35   9   1   1  16  -8   5  32   8   6   0   0   0   0   1   1
INFO   3 112   3  10   1   1  48  35   9  64   3   7   0   0   0   1   1   1
INFO   3 112   3  10   0   1  48  35   9  64   3   7   0   0   0   2   1   1
INFO   1  12  35  11   1   1   4  -4   3   8   4   4   0   0   0   0   1   1
INFO   1 240  35  12   1   1 128  -3   8 112   3  10   0   0   0   0  -1   1
INFO   2 242  -3  13   1   1 240  35  12   2  -3   2   0   0   0   1   1   1
INFO   2 242  -3  13   0   1 240  35  12   2  -3   2   0   0   0   2   1   1
INFO   3 248   4  14   1   1 240  35  12   8   4   4   0   0   0   1   1   1
INFO   3 248   4  14   0   1 240  35  12   8   4   4   0   0   0   2   1   1
INFO   1 252  35  15   1   2   4  -4   3 248   4  14   0   0   0   0   1   1
INFO   4 252  35  15   2   2  12  35  11 240  35  12   0   0   0   0   1   1
INFO   2 254  -3  16   1   2  12  35  11 242  -3  13   0   0   0   1   1   1
INFO   2 254  -3  16   0   2  12  35  11 242  -3  13   0   0   0   2   1   1
INFO   2 254  -3  16   2   2 252  35  15   2  -3   2   0   0   0   1   1   1
INFO   2 254  -3  16   0   2 252  35  15   2  -3   2   0   0   0   2   1   1
INFO   2  48  15   3   3   0   0   0   0   0   0   0   0   0   0   0   2   5
INFOYY   5
 INFO NUM 128  of  143 11
INFO   1  12  35   7   1   1   4  -4   3   8   4   4   0   0   0   0   1   1
INFO   1  48  35   8   1   1  16  -8   5  32   8   6   0   0   0   0   1   1
INFO   2  28  -8   9   1   1  12  35   7  16  -8   5   0   0   0   1   1   1
INFO   2  28  -8   9   0   1  12  35   7  16  -8   5   0   0   0   2   1   1
INFO   3  56   4  10   1   1  48  35   8   8   4   4   0   0   0   1   1   1
INFO   3  56   4  10   0   1  48  35   8   8   4   4   0   0   0   2   1   1
INFO   1  60  35  11   1   3   4  -4   3  56   4  10   0   0   0   0   1   1
INFO   4  60  35  11   2   3  12  35   7  48  35   8   0   0   0   0   1   1
INFO   1  60  35  11   3   3  28  -8   9  32   8   6   0   0   0   0   1   1
INFO  25  62  -3  12   1   1  60  35  11   2  -3   2   0   0   0   1   1   1
INFO  25  62  -3  12   0   1  60  35  11   2  -3   2   0   0   0   2   1   1
INFO   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   1
INFOYY   1
 INFO NUM 129  of  143 12
INFO  23  12  35   7   1   1   4  -4   3   8   4   4   0   0   0   0   1   1
INFO   1  48  35   8   1   1  16  -8   5  32   8   6   0   0   0   0   1   1
```

HELAC-1L

A. van Hameren, C. G. Papadopoulos and R. Pittau, JHEP **0909** (2009) 106 [arXiv:0903.4665 [hep-ph]].

Process		**6** D_i	**5** D_i	**4** D_i	**3** D_i	**2** D_i	R_2	**CT**	**Total**
$gg \to t\bar{t}b\bar{b}$	non-planar	18	120	268	220	112	51	6	795
	planar	13	32	35	40	48	25	2	195
$gg \to t\bar{t}gg$	non-planar	168	576	480	224	56	50	14	1568
	planar	2	14	40	60	60	34	3	213

HELAC-1L

$pp \to t\bar{t}b\bar{b}$			
$u\bar{u} \to t\bar{t}b\bar{b}$			
	ϵ^{-2}	ϵ^{-1}	ϵ^{0}
HELAC-1L	-2.347908989000179E-07	-2.082520105681483E-07	3.909384299635230E-07
$I(\epsilon)$	-2.347908989000243E-07	-2.082520105665445E-07	
$gg \to t\bar{t}b\bar{b}$			
HELAC-1L	-1.435108168334016E-06	-2.085070773763073E-06	3.616343483497464E-06
$I(\epsilon)$	-1.435108168334035E-06	-2.085070773651439E-06	

	p_x	p_y	p_z	E
$u(g)$	0	0	250	250
$\bar{u}(g)$	0	0	-250	250
t	12.99421901255723	-9.591511769543683	75.05543670827210	190.1845561691092
$\bar{t}$	53.73271578143694	-0.2854146459513714	17.68101382654795	182.9642163285034
b	-41.57664370692741	3.895531135098977	-91.94931862397770	100.9874727883170
$\bar{b}$	-25.15029108706678	5.981395280396083	-0.7871319108423604	25.86375471407044

HELAC-1L

$pp \to VVb\bar{b}$ and $pp \to VV+$ 2 jets			
$u\bar{u} \to W^+W^-b\bar{b}$			
	ϵ^{-2}	ϵ^{-1}	ϵ^{0}
HELAC-1L	-2.493916939359002E-07	-4.885901774740355E-07	1.592538533368835E-07
$I(\epsilon)$	-2.493916939359001E-07	-4.885901774752593E-07	
$gg \to W^+W^-b\bar{b}$			
HELAC-1L	-2.686310592221201E-07	-6.078682316434646E-07	-2.431624440346638E-07
$I(\epsilon)$	-2.686310592221206E-07	-6.078682340168020E-07	

	p_x	p_y	p_z	E
$u(g)$	0	0	250	250
$\bar{u}(g)$	0	0	-250	250
W^+	22.40377113462118	-16.53704884550758	129.4056091248114	154.8819879118765
W^-	92.64238702192333	-0.4920930146078141	30.48443210132545	126.4095336206695
b	-71.68369328357026	6.716416578342183	-158.5329205583824	174.1159068988160
$\bar{b}$	-43.36246487297426	10.31272528177322	-1.357120667754454	44.59257156863792

HELAC-1L

$pp \to V+$ 3 jets			
$u\bar{d} \to W^+ggg$			
	ϵ^{-2}	ϵ^{-1}	ϵ^{0}
HELAC-1L	-1.995636628164684E-05	-5.935610843551600E-05	-5.323285370666314E-05
$I(\epsilon)$	-1.995636628164686E-05	-5.935610843566534E-05	
$u\bar{u} \to Zggg$			
HELAC-1L	-7.148261887172997E-06	-2.142170009323704E-05	-1.906378375774021E-05
$I(\epsilon)$	-7.148261887172976E-06	-2.142170009540120E-05	

	p_x	p_y	p_z	E
u	0	0	250	250
$\bar{d}$	0	0	-250	250
W^+	23.90724239064912	-17.64681636854432	138.0897548661186	162.5391101447744
g	98.85942812363483	-0.5251163702879512	32.53017998659339	104.0753327455388
g	-76.49423931754684	7.167141557113385	-169.1717405928078	185.8004692730082
g	-46.27243119673712	11.00479118171890	-1.448194259904179	47.58508783667868

HELAC-1L

$pp \to t\bar{t}+$ 2 jets			
$u\bar{u} \to t\bar{t}gg$			
	ϵ^{-2}	ϵ^{-1}	ϵ^{0}
HELAC-1L	-6.127108113312741E-05	-1.874963444741646E-04	-3.305349683690902E-04
$I(\epsilon)$	-6.127108113312702E-05	-1.874963445081074E-04	
$gg \to t\bar{t}gg$			
HELAC-1L	-3.838786514961561E-04	-9.761168899507888E-04	-5.225385984750410E-04
$I(\epsilon)$	-3.838786514961539E-04	-9.761168898436521E-04	

	p_x	p_y	p_z	E
$u(g)$	0	0	250	250
$\bar{u}(g)$	0	0	-250	250
t	12.99421901255723	-9.591511769543683	75.05543670827210	190.1845561691092
$\bar{t}$	53.73271578143694	-0.2854146459513714	17.68101382654795	182.9642163285034
g	-41.57664370692741	3.895531135098977	-91.94931862397770	100.9874727883170
g	-25.15029108706678	5.981395280396083	-0.7871319108423604	25.86375471407044

HELAC-1L

$pp \to b\bar{b}b\bar{b}$			
$u\bar{u} \to b\bar{b}b\bar{b}$			
	ϵ^{-2}	ϵ^{-1}	ϵ^{0}
HELAC-1L	-9.205269484951069E-08	-2.404679886692200E-07	-2.553568662778129E-07
$I(\epsilon)$	-9.205269484951025E-08	-2.404679886707971E-07	
$gg \to b\bar{b}b\bar{b}$			
HELAC-1L	-2.318436429821683E-05	-6.958360737366907E-05	-7.564212339279291E-05
$I(\epsilon)$	-2.318436429821662E-05	-6.958360737341511E-05	

	p_x	p_y	p_z	E
$u(g)$	0	0	250	250
$\bar{u}(g)$	0	0	-250	250
b	24.97040523056789	-18.43157602837212	144.2306511496888	147.5321146846735
$\bar{b}$	103.2557390255471	-0.5484684659584054	33.97680766420219	108.7035966213640
b	-79.89596300367462	7.485866671764871	-176.6948628845280	194.0630765341365
$\bar{b}$	-48.33018125244035	11.49417782256567	-1.512595929362970	49.70121215982584

HELAC-DIPOLES

$\mathcal{E}_0$ - massless emitter, $\mathcal{S}_0$ - massless spectator, $\mathcal{E}_M$ - massive emitter, $\mathcal{S}_M$ - massive spectator, $\mathcal{E}_I$ - initial state emitter, $\mathcal{E}_F$ - final state emitter, $\mathcal{S}_I$ - initial state spectator, $\mathcal{S}_F$ - final state spectator, ✓ - check, ■ - does not occur.

	$\mathcal{E}_0/\mathcal{S}_0$	$\mathcal{E}_0/\mathcal{S}_M$	$\mathcal{E}_M/\mathcal{S}_0$	$\mathcal{E}_M/\mathcal{S}_M$
$\mathcal{E}_I/\mathcal{S}_I$				
$g \to gg$	✓	■	■	■
$g \to q\bar{q}$	✓	■	■	■
$q \to qg$	✓	■	■	■
$q \to gq$	✓	■	■	■
$\mathcal{E}_I/\mathcal{S}_F$				
$g \to gg$	✓	✓	■	■
$g \to q\bar{q}$	✓	✓	■	■
$q \to qg$	✓	✓	■	■
$q \to gq$	✓	✓	■	■

	$\mathcal{E}_0/\mathcal{S}_0$	$\mathcal{E}_0/\mathcal{S}_M$	$\mathcal{E}_M/\mathcal{S}_0$	$\mathcal{E}_M/\mathcal{S}_M$
$\mathcal{E}_F/\mathcal{S}_I$				
$g \to gg$	✓	■	■	■
$g \to q\bar{q}$	✓	■	✓	■
$q \to qg$	✓	■	✓	■
$q \to gq$	✓	■	✓	■
$\mathcal{E}_F/\mathcal{S}_F$				
$g \to gg$	✓	✓	■	■
$g \to q\bar{q}$	✓	✓	✓	✓
$q \to qg$	✓	✓	✓	✓
$q \to gq$	✓	✓	✓	✓

Table 1: *Independent dipole splitting formulae, which need to be tested in order to ensure the correctness of the code. In the splitting description, e.g. $g \to gg$, the left hand side particle always denotes the virtual state.*

HELAC-DIPOLES

PROCESS	REAL EMISSION + DIPOLES [msec]	REAL EMISSION [msec]	NR OF DIPOLES
$gg \to ggg$	3.8	1.0	27
$gg \to gggg$	8.5	2.6	56
$gg \to ggggg$	300	42	100
$u\bar{d} \to W^+gggg$	9.3	2.4	56
$gg \to t\bar{t}b\bar{b}g$	12	2.9	55

Table 2: *The CPU time needed to evaluate the real emission matrix element together with all of the dipole subtraction terms per phase-space point (this corresponds to $\alpha_{max} = 1$). All numbers have been obtained on an Intel 2.53 GHz Core 2 Duo processor with the Intel Fortran compiler using the -fast option.*

HELAC-DIPOLES

- Arbitrary processes QCD+EW

HELAC-DIPOLES

- Arbitrary processes QCD+EW
- Massive and massless external states

HELAC-DIPOLES

- Arbitrary processes QCD+EW
- Massive and massless external states
- Helicity (& color) sampling for partons

HELAC-DIPOLES

- Arbitrary processes QCD+EW
- Massive and massless external states
- Helicity (& color) sampling for partons
- Random helicities for non-partons

HELAC-DIPOLES

- Arbitrary processes QCD+EW
- Massive and massless external states
- Helicity (& color) sampling for partons
- Random helicities for non-partons
- Restrictions on PS α_{max}

HELAC-DIPOLES

- Arbitrary processes QCD+EW
- Massive and massless external states
- Helicity (& color) sampling for partons
- Random helicities for non-partons
- Restrictions on PS α_{max}

```
--------------------------------------------------
Dipole Subtraction Configuration
--------------------------------------------------
only real emission:                F
only last particle soft/collinear:        F
only divergent dipoles:            T
random polarization for non-partons:      T
sign mode (0-both,1-positive,2-negative):          0
helicity sum (0-fast,1-slow,2-flat MC):          1
events with polarization sum=              100000
events for sampling optimization=            20000
event increment for sampling update=           10000
alphaMinCut=                       1.000000000000000E-006
alphaMaxII=                       1.00000000000000
alphaMaxIF=                        1.00000000000000
alphaMaxFI=                        1.00000000000000
alphaMaxFF=                         1.00000000000000
kappa=                        0.000000000000000E+000
jet veto included:               F
pt of vetoing jet=                 50.0000000000000
color sampling: F
--------------------------------------------------
--------------------------------------------------
Number of Dipoles:          55
Number of Processes:          7
--------------------------------------------------
--------------------------------------------------
```

HOW HELAC-NLO WORKS-VIRTUAL

Generate $w = 1$ events (Les Houches format) using HELAC at tree order.
Information included: LH + color assignment, helicity. Optimization!

HOW HELAC-NLO WORKS-VIRTUAL

Generate $w = 1$ events (Les Houches format) using HELAC at tree order. Information included: LH + color assignment, helicity. Optimization!


```
<event>
   6   81  1.000000E+00  1.726000E+02  7.546772E-03  1.180000E-01
    21  -1   0   0  103  101  0.000000000000000E+00  0.000000000000000E+00  4.885658920243087E+02  4.885658920243087E+02  0.000000000000000E+00  0.000000E+00  9.0000E+00
    21  -1   0   0  104  102  0.000000000000000E+00  0.000000000000000E+00 -4.885658920243087E+02  4.885658920243087E+02  0.000000000000000E+00  0.000000E+00  9.0000E+00
     6   1   1   2  103    0  1.648551153938704E+02 -2.128833463956879E+01  1.563411288268662E+01  2.401366022681282E+02  1.726000000000000E+02  0.000000E+00  9.0000E+00
    -6   1   1   2    0  102 -6.677109609683933E+01  6.109017946596872E+01 -3.256227583127494E+02  3.794882475549945E+02  1.726000000000000E+02  0.000000E+00  9.0000E+00
     5   1   1   2  104    0  4.725480269309031E+00  2.281431584259000E+01  1.753945210216305E+02  1.769351891952198E+02  0.000000000000000E+00  0.000000E+00  9.0000E+00
    -5   1   1   2    0  101 -1.028094995663402E+02 -6.261616066898994E+01  1.345941244084323E+02  1.805717450302747E+02  0.000000000000000E+00  0.000000E+00  9.0000E+00
#   9.193930413382987E-08      4      3      4     14     13
#   0.000000000000000E+00  0.000000000000000E+00  0.000000000000000E+00  0.000000000000000E+00  7.869627745360847E-01 -1.175027485420859E+00 -7.869627745360847E-01
-1.175027485420859E+00
#   0.000000000000000E+00  0.000000000000000E+00  0.000000000000000E+00  0.000000000000000E+00  1.376454726499085E+00 -3.246111302748730E-01 -1.376454726499085E+00
-3.246111302748730E-01
#   1.863667555432868E+01 -3.562491121497572E+00 -9.077135267012881E+00  6.153194387677511E+00 -1.970622777463714E+01 -1.717507312227297E+00 -6.433090024792207E+00
-6.899515402964241E+00
#  -6.580432295368123E+00 -2.321633716694498E-01  2.264652765353805E+01  1.423921666814779E+01 -2.316151832172334E+01  1.257559440674843E+01  4.439749203374159E+00
6.683084353093276E+00
#  -5.059138841333641E-01 -1.133454593457765E+00  1.833599061114253E+01 -4.015116252979888E+00  1.833599061114253E+01  4.015116252979888E+00  5.059138841333641E-01
-1.133454593457765E+00
#   2.672004287479594E+00  1.755067698199695E+01  2.615285048432793E+00 -6.256029470621641E+00 -2.615285048432793E+00 -6.256029470621641E+00  2.672004287479594E+00
-1.755067698199695E+01
# pdf  3.605966723564206E-02  1.350916463377768E-01
</event>
```

HOW HELAC-NLO WORKS-VIRTUAL

Do this sum by MC (sample a configuration $\{i\} = 1, 2, 3$ $\{j\} = 1, 2, 3$)

$$\sum_{\{i\},\{j\}} |\mathcal{M}^{i_1,i_2,\dots,i_k}_{j_1,j_2,\dots,j_k}|^2$$

Express in terms of color connections A_σ

$$\mathcal{M}^{i_1,i_2,\dots,i_k}_{j_1,j_2,\dots,j_k} = \sum_\sigma \delta_{i_{\sigma_1},j_1}\delta_{i_{\sigma_2},j_2}\dots\delta_{i_{\sigma_k},j_k} A_\sigma$$

Very significant reduction in CPU-time

Process	n_{conn}	$\langle n_{conn}\rangle_{MC}$	Ratio
$gg \to b\bar{b}\, W^+W^-$	6	1.74	3.5
$gg \to t\bar{t}b\bar{b}$	24	3.04	7.9
$gg \to t\bar{t}gg$	120	6.27	19.1

HOW HELAC-NLO WORKS-VIRTUAL

Generate $w = 1$ events (Les Houches format) using HELAC at tree order. Information included: LH + color assignment, helicity. Optimization!

Calculate using `HELAC-1L` virtual part for each $w = 1$ event. Produce a new LH file including virtual corrections. Includes UV renormlization

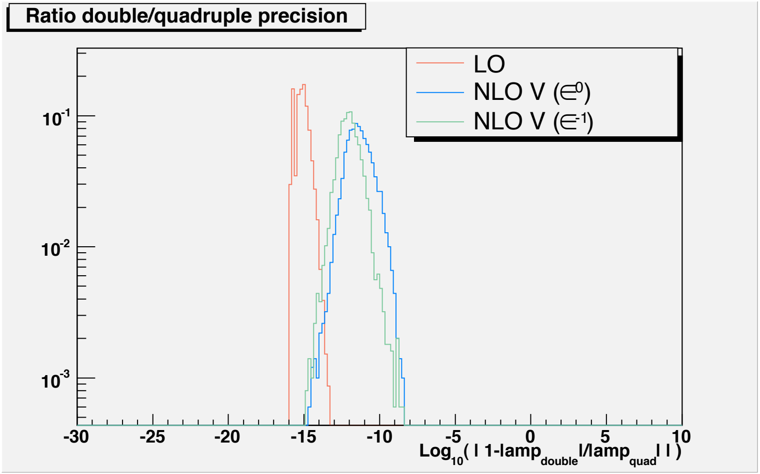
HOW HELAC-NLO WORKS-VIRTUAL

Generate $w = 1$ events (Les Houches format) using HELAC at tree order. Information included: LH + color assignment, helicity. Optimization!

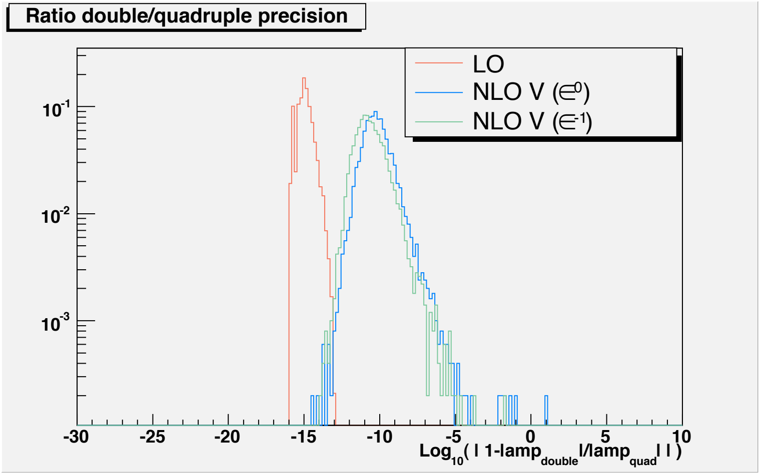
Calculate using `HELAC-1L` virtual part for each $w = 1$ event. Produce a new LH file including virtual corrections. Includes UV renormlization

The final LH file can now be used to produce any kinematical distribution !

HOW HELAC-NLO WORKS - STABILITY



HOW HELAC-NLO WORKS - STABILITY



HOW HELAC-NLO WORKS-REAL

`HELAC-DIPOLES`

Generate CS Dipoles and calculate $R - D$, jet-algorithm, histograms

HOW HELAC-NLO WORKS-REAL

`HELAC-DIPOLES`

Generate CS Dipoles and calculate $R - D$, jet-algorithm, histograms

Calculate I operator contributions, histograms

HOW HELAC-NLO WORKS-REAL

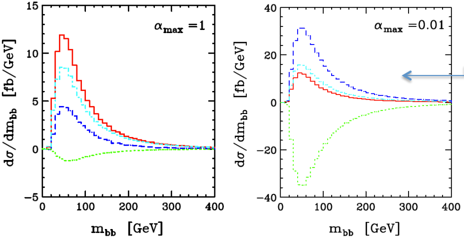
HELAC-DIPOLES

Generate CS Dipoles and calculate $R - D$, jet-algorithm, histograms

Calculate I operator contributions, histograms

Calculate KP operator contributions, histograms

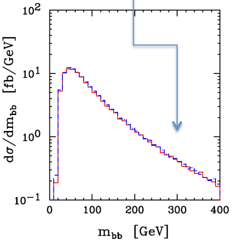
Real Emission



Subtracted real emission
K + P operators
I operators
Full result

Cutoff independence !!!

- Phase space restriction on the dipoles phase space $\alpha_{\mathbf{max}} \in (\mathbf{0}, \mathbf{1}]$
- Less dipole subtraction terms per event
- Increased numerical stability
- Reduced missed binning problem
- Large cancellations between subtracted real radiation and integrated dipoles



Bevilacqua, Czakon, Papadopoulos, Pittau, Worek '09

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$pp \to VVV$

T. Binoth, G. Ossola, C. G. Papadopoulos and R. Pittau, JHEP **0806** (2008) 082 [arXiv:0804.0350 [hep-ph]].

Process	scale μ	Born cross section [fb]	NLO cross section [fb]
ZZZ	$3M_Z$	9.7(1)	15.3(1)
WZZ	$2M_Z + M_W$	20.2(1)	40.4(2)
WWZ	$M_Z + 2M_W$	96.8(6)	181.7(8)
WWW	$3M_W$	82.5(5)	146.2(6)

Table 1: Cross section for the four processes, corresponding to the distributions in Fig 4. Different values of the factorization(renormalization) scale are used for the different processes.

$pp \to VVV$

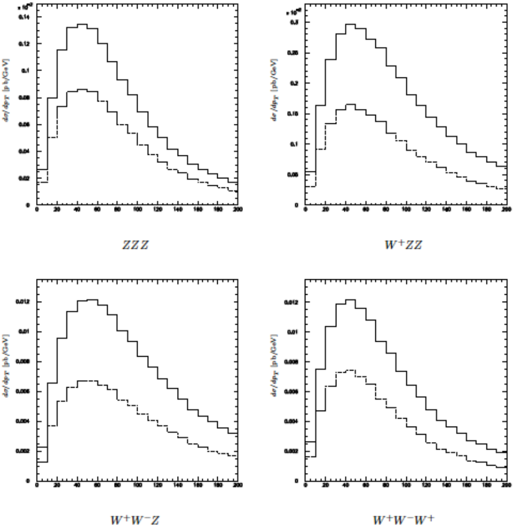


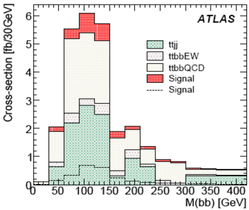
Figure 4: Transverse momentum distribution, as defined in the text, for the four processes $pp \to VVV$: NLO (solid line) compared with the LO contribution (dashed line).

Motivations for ttbb and ttjj

- $\mathbf{pp \to t\bar{t}H}$ potential discovery channel
  - $\mathbf{H \to b\bar{b}}$
  - $\mathbf{m_H \leq 135\ GeV}$
- top & bottom Yukawa coupling
- Large QCD backgrounds: $\mathbf{t\bar{t}b\bar{b}}$ & $\mathbf{t\bar{t}jj}$

- *Problem 1:* combinatorial background of b-jets:
  - bb pair can be chosen incorrectly, lack of distinctive kinematic feature of Higgs decay jets
- *Problem 2:* b-tagging efficiency:
  - two b-jets for Higgs candidate can arise from mistagged QCD light jets
- *Goal:* Backgrounds need to be controlled

ATLAS TDR, CERN-OPEN-2008-020



S/B ~ 1/9

Summary table	Significance loose/tight	Luminosity
ATLAS (Lepton+jets)	2.2	30 fb^{-1}
CMS (Lepton+jets)	2.5/1.9	60 fb^{-1}
CMS(Combined)	3.9/3.3	60 fb^{-1}

G. Aad, J. Steggemann, ATLAS & CMS @ TOP 2008

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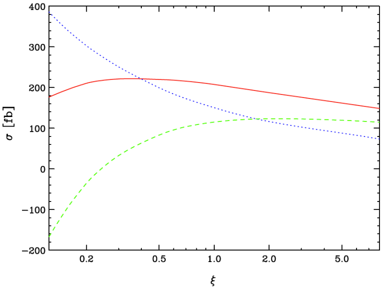
Pp -> ttH -> ttbb @ LHC

- Scale dependence and integrated cross sections



$m_H = 130$ GeV

$$\sigma_{LO} = (150.375 \pm 0.077)\ \text{fb}$$

$$\sigma_{NLO} = (207.268 \pm 0.150)\ \text{fb}$$

$$\sigma_{NLO}^{veto} = (114.880 \pm 0.152)\ \text{fb}$$

Scale dependence reduced:
33% @ LO down to **10% @ NLO**
28% @ NLO with **jet veto** of 50 GeV

K factor of **K = 1.38** (**K = 0.76**)
NLO QCD Corrections **38%** (**24%**)

Bevilacqua, Czakon, Garzelli, Hameren, Papadopoulos, Pittau, Worek '10 (Les Houches 2009)

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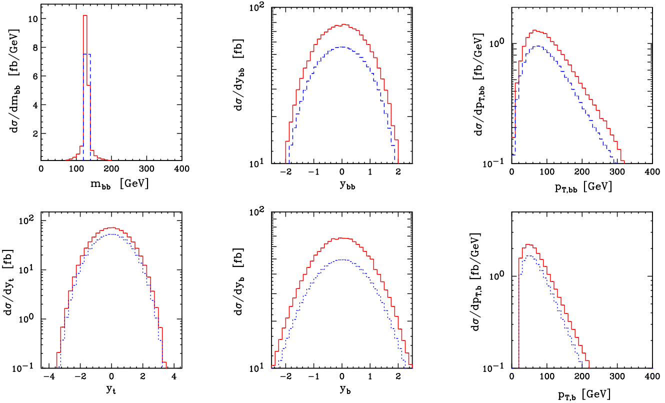
pp -> ttH -> ttbb @ LHC

- Differential cross section, bb pair, single bottom & top kinematics, *LO* & *NLO*



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Pp -> ttbb @ LHC

- Integrated cross sections and scale dependence, *Permille level agreement !*

Process	$\sigma^{LO}_{[23,\ 24]}$ [fb]	σ^{LO} [fb]	$\sigma^{NLO}_{[23,\ 24]}$ [fb]	$\sigma^{NLO}_{\alpha_{max}=1}$ [fb]	$\sigma^{NLO}_{\alpha_{max}=0.01}$ [fb]
$q\bar{q} \to t\bar{t}b\bar{b}$	85.522(26)	85.489(46)	87.698(56)	87.545(91)	87.581(134)
$pp \to t\bar{t}b\bar{b}$	1488.8(1.2)	1489.2(0.9)	2638(6)	2642(3)	2636(3)

$\xi \cdot m_t$	$1/8 \cdot m_t$	$1/2 \cdot m_t$	$1 \cdot m_t$	$2 \cdot m_t$	$8 \cdot m_t$
σ^{LO} [fb]	8885(36)	2526(10)	1489.2(0.9)	923.4(3.8)	388.8(1.4)
σ^{NLO} [fb]	4213(65)	3498(11)	2636(3)	1933.0(3.8)	1044.7(1.7)

$$\sigma_{\mathbf{LO}} = (\mathbf{1489.2 \pm 0.9})\ \mathbf{fb}$$

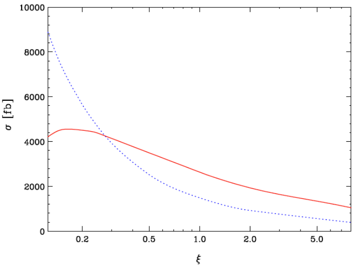
$$\sigma_{\mathbf{NLO}} = (\mathbf{2636 \pm 3})\ \mathbf{fb}$$

Bevilacqua, Czakon, Papadopoulos, Pittau, Worek '09
Bredenstein, Denner, Dittmaier, Pozzorini '08, '09

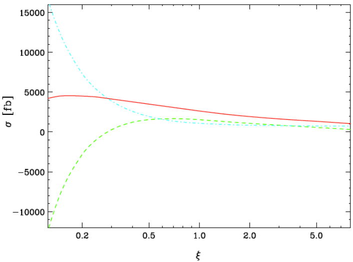
Scale dependence reduced:
70% @ LO down to **33% @ NLO**
K factor of **K = 1.77**
for quarks initial states only **K = 1.03**
With jet veto of 50 GeV **K = 1.20**

Pp -> ttbb @ LHC

- Scale dependence graphically



Varying scale up or down by a factor two changes cross section by **70% @ LO** and by **33% @ NLO**



Scale dependence at NLO decomposed into contribution of *Virtual Corrections* & *Real Radiation*

Bevilacqua, Czakon, Papadopoulos, Pittau, Worek '09

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Pp -> ttbb @ LHC

- Differential cross sections

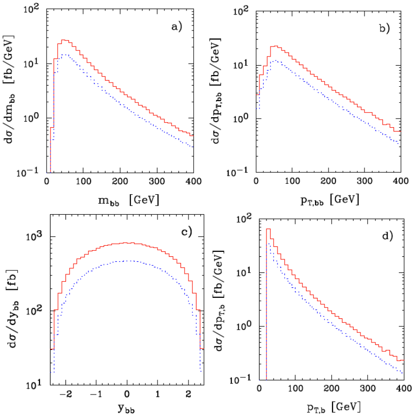
- b-jet pair kinematics
  - Invariant mass
  - Transverse momentum
  - Rapidity distribution

- single b-jet kinematics
  - Transverse momentum

LO & NLO

- Relatively small variation compared to the size but shape change important

Bevilacqua, Czakon, Papadopoulos, Pittau, Worek '09



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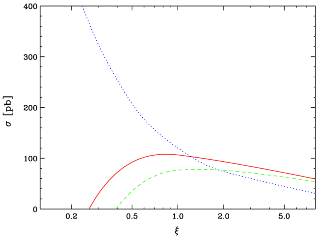
Pp -> ttjj @ LHC

- Scale dependence & integrated cross sections

Bevilacqua, Czakon, Papadopoulos, Worek '10

Process	$\sigma^{\rm LO}$ [pb]	Contribution
$pp \to t\bar{t}jj$	120.17(8)	100%
$qg \to t\bar{t}qg$	56.59(5)	47.1%
$gg \to t\bar{t}gg$	52.70(6)	43.8%
$qq' \to t\bar{t}qq'$, $q\bar{q} \to t\bar{t}q'\bar{q}'$	7.475(8)	6.2%
$gg \to t\bar{t}q\bar{q}$	1.981(3)	1.6%
$q\bar{q} \to t\bar{t}gg$	1.429(1)	1.2%



Scale dependence reduced:
72% @ LO down to **13% @ NLO**
54% @ NLO with **jet veto** of 50 GeV

$$\sigma_{\rm LO} = (120.17 \pm 0.08)\ {\rm pb}$$

$$\sigma_{\rm NLO} = (106.94 \pm 0.17)\ {\rm pb}$$

$$\sigma_{\rm NLO}^{\rm veto} = (76.58 \pm 0.17)\ {\rm pb}$$

K factor of **K = 0.89** (**K = 0.64**)
Negative shift of **11%** (**36%**)

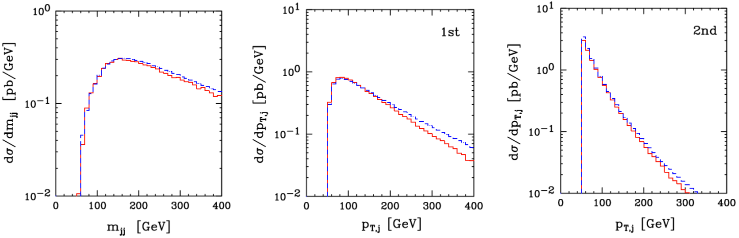
pp -> ttjj @ LHC

- Differential cross section

LO & NLO

- m_{jj} size of the corrections transmitted to distributions for low p_T, shapes change for hight p_T

- *p_T of 1st hardest & 2nd hardest jet* (ordered in p_T) altered shapes up to **39% & 28%** in tails



Bevilacqua, Czakon, Papadopoulos, Worek '10

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pp or $p\bar{p} \to \ell_1^+ \nu_{\ell_1} \ell_2^- \bar{\nu}_{\ell_2} b\bar{b}$

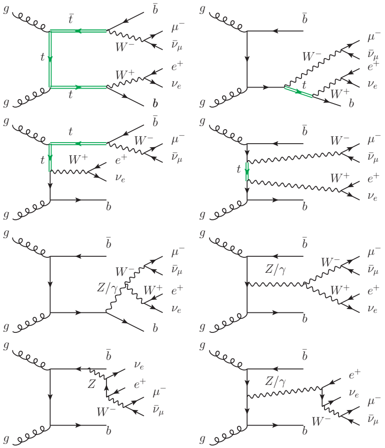


Figure 1: *Representative Feynman diagrams contributing to the leading order process $gg \to e^+\nu_e\mu^-\bar{\nu}_\mu b\bar{b}$ at $\mathcal{O}(\alpha_s^2\alpha^4)$, with different off-shell intermediate states: double-, single-, and non-resonant top quark contributions.*

pp or $p\bar{p} \rightarrow \ell_1^+ \nu_{\ell_1} \ell_2^- \bar{\nu}_{\ell_2} b\bar{b}$

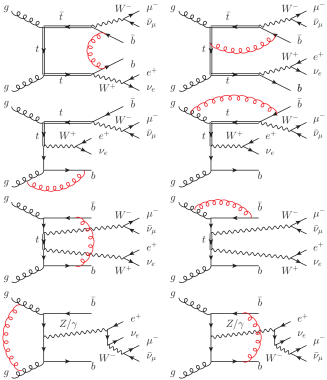


Figure 2: *Representative Feynman diagrams contributing to the virtual corrections to the partonic subprocess* $gg \rightarrow e^+\nu_e\mu^-\bar{\nu}_\mu b\bar{b}$ *at* $\mathcal{O}(\alpha_s^3\alpha^4)$.

pp or $p\bar{p} \rightarrow \ell_1^+ \nu_{\ell_1} \ell_2^- \bar{\nu}_{\ell_2} b\bar{b}$

Real radiation sub-processes

$$gg \rightarrow e^+\nu_e\mu^-\bar{\nu}_\mu b\bar{b}g$$

$$qg \rightarrow e^+\nu_e\mu^-\bar{\nu}_\mu b\bar{b}q$$

$$gq \rightarrow e^+\nu_e\mu^-\bar{\nu}_\mu b\bar{b}q$$

$$q\bar{q} \rightarrow e^+\nu_e\mu^-\bar{\nu}_\mu b\bar{b}g$$

pp or $p\bar{p} \to \ell_1^+ \nu_{\ell_1} \ell_2^- \bar{\nu}_{\ell_2} b\bar{b}$

Algorithm	$\sigma_{\rm LO}$ [fb]	$\sigma_{\rm NLO}^{\alpha_{\rm max}=1}$ [fb]	$\sigma_{\rm NLO}^{\alpha_{\rm max}=0.01}$ [fb]
anti-k_T	34.922 ± 0.014	35.705 ± 0.047	35.697 ± 0.049
k_T	34.922 ± 0.014	35.727 ± 0.047	35.723 ± 0.049
C/A	34.922 ± 0.014	35.724 ± 0.047	35.746 ± 0.050

Table 1: *Integrated cross section at LO and NLO for* $p\bar{p} \to e^+\nu_e\mu^-\bar{\nu}_\mu b\bar{b} + X$ *production at the TeVatron run II with* $\sqrt{s} = 1.96$ TeV, *for three different jet algorithms, the anti-*k_T, k_T *and the Cambridge/Aachen jet algorithm. The two NLO results refer to different values of the dipole phase space cutoff* α_{max}. *The scale choice is* $\mu_R = \mu_F = m_t$.

pp or $p\bar{p} \to \ell_1^+ \nu_{\ell_1} \ell_2^- \bar{\nu}_{\ell_2} b\bar{b}$

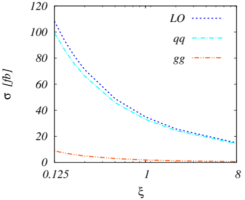


Figure 3: *Scale dependence of the LO cross section with the individual contributions of the partonic channels (left panel) and scale dependence of the LO and NLO cross sections (right panel) for the* $p\bar{p} \to e^+\nu_e\mu^-\bar{\nu}_\mu b\bar{b} + X$ *process at the TeVatron run II with* $\sqrt{s} = 1.96$ *TeV, where renormalization and factorization scales are set to the common value* $\mu = \mu_R = \mu_F = \xi m_t$.

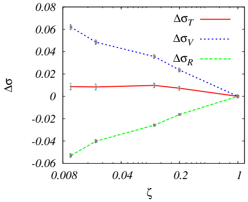


Figure 4: *Dependence of the NLO cross section, $\sigma_{\rm T}$, (red solid line) and the individual contributions, the real emission part, $\sigma_{\rm R}$, (green dashed line) and the LO plus virtual part, $\sigma_{\rm V}$, (blue dotted line), on the rescaling parameter ζ defined as $\Gamma_{rescaled} = \zeta\Gamma_t$ for the $p\bar{p} \to e^+\nu_e\mu^-\bar{\nu}_\mu b\bar{b} + X$ process at the TeVatron run II with $\sqrt{s} = 1.96$ TeV. $\Delta\sigma$ is defined as follows: $\Delta\sigma_i(\zeta) = (\sigma_i(\zeta) - \sigma_i(\zeta = 1))/\sigma_{\rm T}(\zeta = 1)$ with $i = V, R, T$.*

pp or $p\bar{p} \to \ell_1^+ \nu_{\ell_1} \ell_2^- \bar{\nu}_{\ell_2} b\bar{b}$

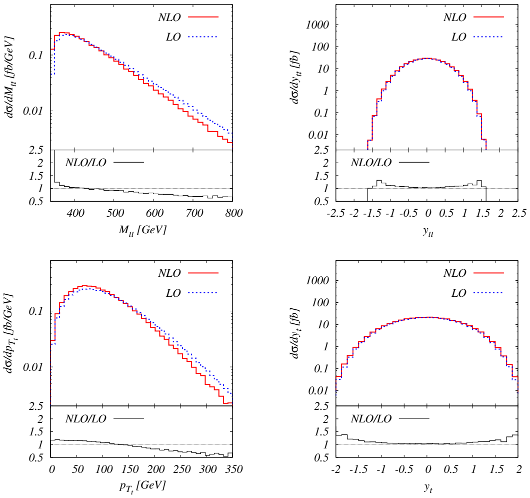


Figure 6: *Differential cross section distributions as a function of the invariant mass* $m_{t\bar{t}}$

pp or $p\bar{p} \to \ell_1^+ \nu_{\ell_1} \ell_2^- \bar{\nu}_{\ell_2} b\bar{b}$

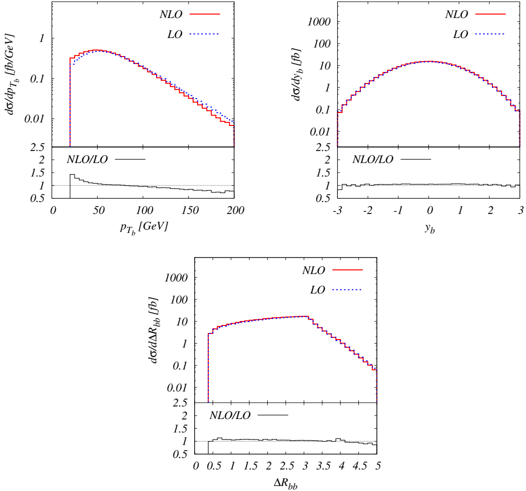


Figure 7: *Differential cross section distributions as a function of the averaged transverse*

Asymmetries

$$A^t_{FB} = 0.051 \pm 0.0013$$

O. Antunano, J. H. Kuhn and G. Rodrigo, Phys. Rev. **D77** (2008) 014003, [arXiv:0709.1652 [hep-ph]].

$$A^b_{FB} = 0.033 \pm 0.0013 \quad A^\ell_{FB} = 0.033 \pm 0.0013$$

W. Bernreuther, A. Brandenburg, Z. G. Si and P. Uwer, Nucl. Phys. **B690**(2004) 81, [hep-ph/0403035].

Working with PS matching: POWHEG

- Interface HELAC-NLO with POWHEG

  P. Nason, JHEP 0411 (2004) 040, hep-ph/0409146; S. Frixione, P. Nason and C. Oleari, JHEP 0711 (2007) 070, arXiv:0709.2092; S. Alioli, P. Nason, C. Oleari and E. Re, JHEP 1006 (2010) 043, arXiv:1002.2581

- POWHEG plays the role of the driver (phase-space) and HELAC-NLO of the provider for all needed ingredients, namely all matrix elements
- Complexity of the process $pp \to t\bar{t}$+jet

$qg \to t\bar{t}q$	$gq \to t\bar{t}q$	$\bar{q}g \to t\bar{t}\bar{q}$	$g\bar{q} \to t\bar{t}\bar{q}$
$gg \to t\bar{t}g$	$q\bar{q} \to t\bar{t}g$	$\bar{q}q \to t\bar{t}g$	

Table 1: Flavour structures of the Born processes, q = u, d, c, s, b.

$qg \to t\bar{t}qg$	$qq \to t\bar{t}qq$	$q\bar{q} \to t\bar{t}q\bar{q}$
$gq \to t\bar{t}qg$	$\bar{q}\bar{q} \to t\bar{t}\bar{q}\bar{q}$	$\bar{q}q \to t\bar{t}q\bar{q}$
$\bar{q}g \to t\bar{t}\bar{q}g$	$q\bar{q} \to t\bar{t}gg$	$q\bar{q} \to t\bar{t}q'\bar{q}'$
$g\bar{q} \to t\bar{t}\bar{q}g$	$\bar{q}q \to t\bar{t}gg$	$\bar{q}q \to t\bar{t}q'\bar{q}'$
$qq' \to t\bar{t}qq'$	$q\bar{q}' \to t\bar{t}q\bar{q}'$	$gg \to t\bar{t}gg$
$\bar{q}q' \to t\bar{t}\bar{q}q'$	$\bar{q}\bar{q}' \to t\bar{t}\bar{q}\bar{q}'$	$gg \to t\bar{t}q\bar{q}$

Table 2: Flavour structures of the real-emission processes, q, q' = u, d, c, s, b.

Working with PS matching: POWHEG

- Agreement with MADGRAPH : $\sigma^{LO} = 631.6 \pm 1.1$ $\sigma^{LO} = 630.5 \pm 0.8$
- Agreement with Melnikov and Schulze(2010), Nucl. Phys. B840 (2010)129–159.
- Technical cuts independence

$p_\perp^{\rm t.c.}$[GeV]	$\sigma^{\rm LO}$ [pb]	$\sigma^{\rm NLO}$ [pb]
20	1.583	1.773 ± 0.003
5	1.583	1.780 ± 0.006
1	1.583	1.780 ± 0.010

Table 3: Dependence of the NLO cross section on the technical cut $p_\perp^{\rm t.c.}$.

- Results

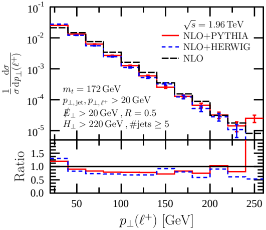


Figure 1: Transverse momentum distribution of the antilepton.

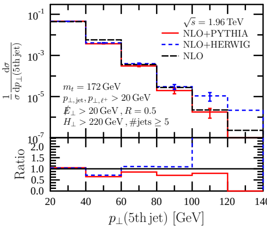


Figure 3: Transverse momentum distribution of the 5th hardest jet.

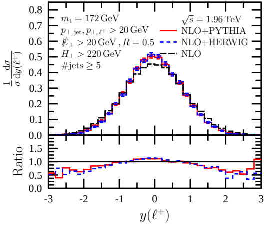


Figure 2: Rapidity distribution of the antilepton.

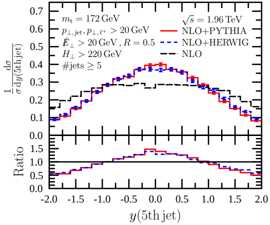


Figure 4: Rapidity distribution of the 5th hardest jet.

Working with PS matching: POWHEG

- More results

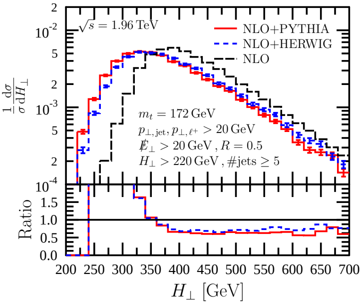


Figure 5: Distribution of the scalar sum of transverse momenta.

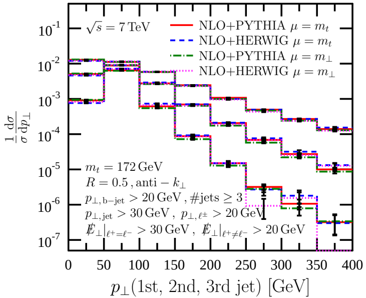


Figure 6: Transverse momentum distributions of the first, second and third hardest jet.

The Les Houches Wish List (2010)

2010

process wanted at NLO	background to
1. $pp \to VV +$ jet	$t\bar{t}H$, new physics Dittmaier, Kallweit, Uwer; Campbell, Ellis, Zanderighi
2. $pp \to H + 2$ jets	H in VBF Campbell, Ellis, Zanderighi; Ciccolini, Denner Dittmaier
3. $pp \to t\bar{t}b\bar{b}$	$t\bar{t}H$ Bredenstein, Denner Dittmaier, Pozzorini; Bevilacqua, Czakon, Papadopoulos, Pittau, Worek
4. $pp \to t\bar{t} + 2$ jets	$t\bar{t}H$ Bevilacqua, Czakon, Papadopoulos, Worek
5. $pp \to VVb\bar{b}$	VBF $\to H \to VV$, $t\bar{t}H$, new physics
6. $pp \to VV + 2$ jets	VBF $\to H \to VV$ VBF: Bozzi, Jäger, Oleari, Zeppenfeld
7. $pp \to V + 3$ jets	new physics Berger Bern, Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower, Maitre; Ellis, Melnikov, Zanderighi
8. $pp \to VVV$	SUSY trilepton Lazopoulos, Melnikov, Petriello; Hankele, Zeppenfeld; Binoth, Ossola, Papadopoulos, Pittau
9. $pp \to b\bar{b}b\bar{b}$	Higgs, new physics GOLEM

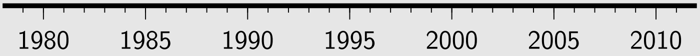
Feynman diagram methods

now joined by

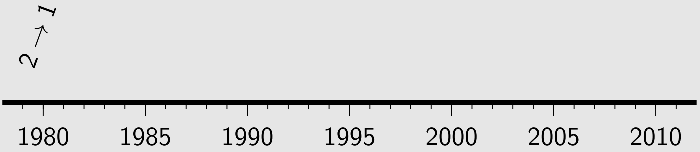
unitarity based methods

L. Dixon CERN HO10

The NLO revolution

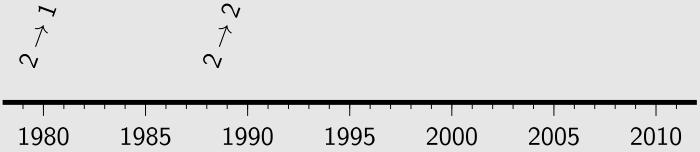


The NLO revolution



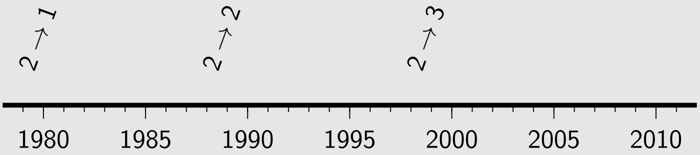
1979: NLO Drell-Yan [Altarelli, Ellis & Martinelli]

The NLO revolution



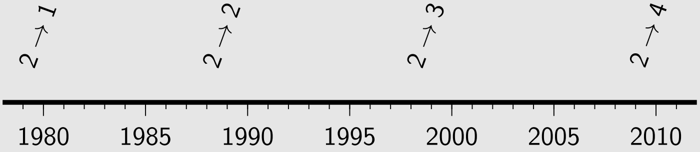
1987: NLO high-p_t photoproduction [Aurenche et al]
1988: NLO $b\bar{b}$, $t\bar{t}$ [Nason et al]
1993: dijets, Vj [JETRAD, Giele, Glover & Kosower]

The NLO revolution

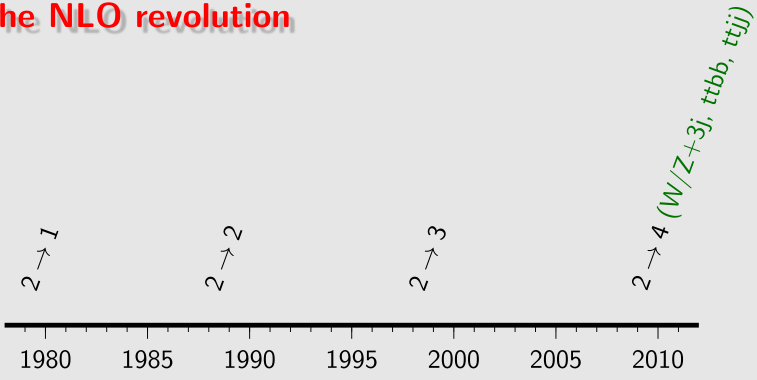


1998: NLO $Wb\bar{b}$ [MCFM: Ellis & Veseli]
2000: NLO $Zb\bar{b}$ [MCFM: Campbell & Ellis]
2001: NLO $3j$ [NLOJet++: Nagy]
...
2007: NLO $t\bar{t}j$ [Dittmaier, Uwer & Weinzierl '07]
...

The NLO revolution

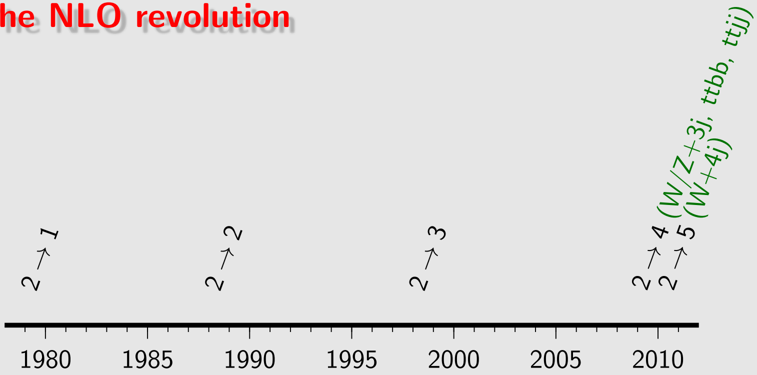


The NLO revolution



2009: NLO W+3j [Rocket: Ellis, Melnikov & Zanderighi] [unitarity]
2009: NLO W+3j [BlackHat: Berger et al] [unitarity]
2009: NLO $t\bar{t}b\bar{b}$ [Bredenstein et al] [traditional]
2009: NLO $t\bar{t}b\bar{b}$ [HELAC-NLO: Bevilacqua et al] [unitarity]
2009: NLO $q\bar{q} \to b\bar{b}b\bar{b}$ [Golem: Binoth et al] [traditional]
2010: NLO $t\bar{t}jj$ [HELAC-NLO: Bevilacqua et al] [unitarity]
2010: NLO Z+3j [BlackHat: Berger et al] [unitarity]

The NLO revolution



2010: NLO $W{+}4j$ [BlackHat: Berger et al, preliminary] [unitarity]

NLO SOFTWARE

Automatizing NLO calculations

- PV and in-house codes, based on FORMCALC, FEYNARTS, LOOPTOOLS and GOLEM
- BLACKHAT+SHERPA collaboration: QCD+EWK bosons, massless color partons; CS-dipole
- Rockect QCD processes, basically gluons + in-house real radiation corrections - MCFM
- HELAC-NLO: CuTtools, HELAC-1LOOP, HELAC-DIPOLES, OneLOop, PHEGAS, KALEU: all NLO-QCD
- Newcomers: GoSam, OpenLoops, RECOLA, MadLoop, MadFKS, aMC@NLO, ...