Progress in the parametrisation of the Neutrino sector

Thomas Gajdosik, Andrius Juodagalvis,

Darius Jurčiukonis, Tomas Sabonis

Vilnius University

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the νSM and the Grimus-Lavoura ansatz

what we understood ... what we could do

further plans – and how to teach the νSM

The Standard Model (SM) with the fermionic singlets: νSM

- is not a new idea: W. Grimus and H. Neufeld, Nucl. Phys. B **325** (1989) 18.

- includes the "original" SM
  - with the gauge symmetry $SU(3)_{\text{color}} \times SU(2)_{\text{weak}} \times U(1)_Y$
  - and the continuous global symmetries $U(1)_L \times U(1)_R \sim U(1)_V \times U(1)_A$

- adds n_R fermionic gauge singlets N_R with
  - kinetic terms $\mathcal{L}_{\text{kin}} = \bar{N}_R i \not{D} N_R = \bar{N}_R i \not{\partial} N_R$
  - Yukawa terms $\mathcal{L}_{\text{Y}} = Y_\nu \bar{N}_R \Psi_\ell \cdot \phi + h.c.$
    - ∗ with $n_h \geq 1$ Higgs doublets ϕ
  - and a Majorana mass term $\mathcal{L}_M = M_R (N_R^\top \mathbf{C}^{-1} N_R + h.c.)$

  ➥ breaks the chiral symmetry $U(1)_A$ explicitly: $U(1)_L \times U(1)_R \to U(1)_V$

- spontaneous symmetry breaking of the Higgs sector
  - generates the masses in the SM with the vev v
    - ∗ including the Dirac mass term $M_D = v Y_\nu$
  - N_R mixes with ν_L (neutral part of the lepton doublets)

  ⇒ small neutrino masses due to the seesaw mechanism

The Standard Model (SM) with the fermionic singlets: νSM

- the mixing gives a $(3+n_R)\times(3+n_R)$ symmetric mass matrix

$$M_\nu = \begin{pmatrix} M_L & M_D^\top \\ M_D & M_R \end{pmatrix}$$

  - M_L represents a Majorana mass term for the SM neutrinos
    - which violates the $SU(2)_{\text{weak}}\times U(1)_Y$ gauge symmetry

      $\Rightarrow$ does not exist at tree level: $M_L|_{\text{tree}} = 0$

- M_ν can be diagonalised $\quad W^\top M_\nu W = \text{diag}(m_{\nu_1},\ldots,m_{\nu_{3+n_R}}) = \text{diag}(M_\ell, M_h)$

  with the Grimus-Lavoura ansatz of a unitary W

$$W = \begin{pmatrix} \sqrt{1-BB^\dagger} & B \\ -B^\dagger & \sqrt{1-B^\dagger B} \end{pmatrix}$$

  and an arbitrary complex $3\times n_R$ matrix B

  - W. Grimus and L. Lavoura, JHEP **0011** (2000) 042 [arXiv:hep-ph/0008179].

The Standard Model (SM) with the fermionic singlets: νSM

In the Grimus-Lavoura approach

- analysing the size of the elements M_D, M_R, M_ℓ, and M_h
  - by finding the singular values of each . . .
- solving hierarchical equations for B and the masses M_ℓ and M_h

  $\Rightarrow$ "The Seesaw mechanism at arbitrary order:

  Disentangling the small scale from the large scale "
  - works for arbitrary n_R and arbitrary number of generations n_L
- gives masses and couplings from the νSM Lagrangian
- Our idea was to "reverse engineer" the approach
  - to get the νSM Lagrangian from
    * masses: the measured Δm^2 of the light neutrino spectrum
    * known mixing: the measured PMNS-matrix
    * and a minimal set of additional parameters

The Standard Model (SM) with the fermionic singlets: νSM

- direct reverse engineering for arbitrary n_R and n_L does not work
  - one has to reduce the problem to $n'_R = n'_L = \min(n_R, n_L)$
- standard scenario is $n_R = n_L = 3$ ... (number of generations in the SM)
  - the Casas-Ibarra parametrisation

$$M_D = iM_h^{1/2} \cdot O \cdot M_\ell^{1/2}$$

solves the leading order seesaw equation

$$M_\ell = -M_D^\top M_h^{-1} M_D$$

with an arbitrary (complex) orthogonal matrix O.

  * O has 3 complex angles $\sim$ 6 real parameters
- the case $n_R > 3$ introduces more "invisible" gauge singlets
  - could allow for neutrino-like states of intermediate mass: sterile neutrinos?
  - is left out of the discussion
- the cases $n_R < 3$ have less parameters

$\Rightarrow$ more restrictive predictions possible

The νSM with $n_L = 3$ and $n_R = 1$

taking $n_L = 3$ ... number of generations in the SM

$n_R = 1$ is the most restrictive case

- can be solved analytically

  $\Rightarrow$ optimal toy model for teaching

- gives a single massive light neutrino and two mass less ones

  $\Rightarrow$ ruled out at tree level by the measured mass differences $\Delta m^2_{\odot}$ and Δm^2_{atm}

- including a single Higgs doublet (like in the SM)
  - loop corrections to the masses are proportional to the masses themselves

  $\Rightarrow$ still ruled out by $\Delta m^2_{\odot}$ and Δm^2_{atm}

- including more Higgs doublets
  - "directions" of Yukawa couplings and of vevs can be different
    * in bare perturbation theory loop corrections can generate masses for neutrinos
  - realistic treatments have to include the full Higgs sector
    * possible problems with flavour changing rare decays: $b \to s\gamma$, etc.

The νSM toy model with $n_L = 3$ and $n_R = 1$

- Yukawa term $\times$ vev: $M_D = \frac{v}{\sqrt{2}} Y_\nu = (m_1, m_2, m_3)$ is a 1×3-matrix
  - using the SVD, it can be parametrised as $M_D = m_D \vec{a}^\top$
    - with the singular value m_D (with $m_D^2 = |m_1|^2 + |m_2|^2 + |m_3|^2$)
- M_ν at tree level is

$$M_\nu = \begin{pmatrix} M_L & M_D^\top \\ M_D & M_R \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & |m_1|e^{i\phi_1} \\ 0 & 0 & 0 & |m_2|e^{i\phi_2} \\ 0 & 0 & 0 & |m_3|e^{i\phi_3} \\ |m_1|e^{i\phi_1} & |m_2|e^{i\phi_2} & |m_3|e^{i\phi_3} & m_R \end{pmatrix}$$

- the diagonalisation of M_ν can be constructed from
  1. a phase absorption matrix $\hat{P} = \mathrm{diag}(e^{-i\phi_1}, e^{-i\phi_2}, e^{-i\phi_3}, 1)$
     - to make the entries of M_D real
  2. a rotation $R_a = R_1.R_2$
     - to simplify $\vec{a} \to (0,0,1)^\top$
  3. solving the $1+1$ dimensional seesaw
     - which gives the analytic relations: $m_D^2 = m_\ell m_h$ and $m_R = (m_h - m_\ell) \sim m_h$

Diagonalisation of M_ν

$$M_\nu = \begin{pmatrix} 0 & 0 & 0 & |m_1|e^{i\phi_1} \\ 0 & 0 & 0 & |m_2|e^{i\phi_2} \\ 0 & 0 & 0 & |m_3|e^{i\phi_3} \\ |m_1|e^{i\phi_1} & |m_2|e^{i\phi_2} & |m_3|e^{i\phi_3} & m_R \end{pmatrix}$$

1. the phase absorption
   – with $\hat{P} = \text{diag}(e^{-i\phi_1}, e^{-i\phi_2}, e^{-i\phi_3}, 1)$ gives

$$\hat{P}^\top M_\nu \hat{P} = \begin{pmatrix} 0 & 0 & 0 & |m_1| \\ 0 & 0 & 0 & |m_2| \\ 0 & 0 & 0 & |m_3| \\ |m_1| & |m_2| & |m_3| & m_R \end{pmatrix}$$

2.1. the rotation R_1 with

$$R_1 = \begin{pmatrix} \cos\alpha_1 & \sin\alpha_1 & 0 & 0 \\ -\sin\alpha_1 & \cos\alpha_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad \tan\alpha_1 = \frac{|m_1|}{|m_2|}$$

gives

$$R_1^\top \hat{P}^\top M_\nu \hat{P} R_1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{|m_1|^2 + |m_2|^2} \\ 0 & 0 & 0 & |m_3| \\ 0 & \sqrt{|m_1|^2 + |m_2|^2} & |m_3| & m_R \end{pmatrix}$$

2.2. the rotation R_2 with

$$R_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha_2 & \sin\alpha_2 & 0 \\ 0 & -\sin\alpha_2 & \cos\alpha_2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad \tan\alpha_2 = \frac{|m_2|}{|m_3|\cos\alpha_1} = \frac{\sqrt{|m_1|^2 + |m_2|^2}}{|m_3|}$$

gives

$$R_2^\top R_1^\top \hat{P}^\top M_\nu \hat{P} R_1 R_2$$

$$= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{|m_1|^2 + |m_2|^2 + |m_3|^2} \\ 0 & 0 & \sqrt{|m_1|^2 + |m_2|^2 + |m_3|^2} & m_R \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_D \\ 0 & 0 & m_D & m_R \end{pmatrix}$$

3.1. solving the $1+1$ dimensional real seesaw with the rotation R_3 with

$$R_3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\alpha_3 & \sin\alpha_3 \\ 0 & 0 & -\sin\alpha_3 & \cos\alpha_3 \end{pmatrix} \quad \text{and} \quad \tan^2\alpha_3 = \frac{|m_\ell|}{|m_h|}$$

gives the analytic relations: $m_D^2 = m_\ell m_h$ and $m_R = (m_h - m_\ell) \sim m_h$

3.1. (continued) R_3 gives

$$R_3^\top R_2^\top R_1^\top \hat{P}^\top M_\nu \hat{P} R_1 R_2 R_3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -m_\ell & 0 \\ 0 & 0 & 0 & m_h \end{pmatrix}$$

3.2. the minus sign at m_ℓ has to be absorbed by $\hat{P}_3 = \text{diag}(1, 1, i, 1)$:

$$\hat{P}_3^\top R_3^\top R_2^\top R_1^\top \hat{P}^\top M_\nu \hat{P} R_1 R_2 R_3 \hat{P}_3 = U^\top M_\nu U = \text{diag}(0, 0, m_\ell, m_h)$$

4. the diagonalisation matrix U is unitary as
   - the real orthogonal matrices R_i are unitary
   - and the diagonal phase matrices $\hat{P}$ and $\hat{P}_3$ are unitary, too

$$U = \begin{pmatrix} c_1 e^{-i\phi_1} & s_1 c_2 e^{-i\phi_1} & i s_1 s_2 c_3 e^{-i\phi_1} & s_1 s_2 s_3 e^{-i\phi_1} \\ s_1 e^{-i\phi_2} & c_1 c_2 e^{-i\phi_2} & i c_1 s_2 c_3 e^{-i\phi_2} & c_1 s_2 s_3 e^{-i\phi_2} \\ 0 & -s_2 e^{-i\phi_3} & i c_2 c_3 e^{-i\phi_3} & c_2 s_3 e^{-i\phi_3} \\ 0 & 0 & -i s_3 & c_3 \end{pmatrix}$$

- including one-loop effects (with only one Higgs doublet)
  - the Higgs contributes in the same flavour direction as the vev (Yukawa coupling)
    * only the already massive fermionic states can receive corrections

The νSM with $n_L = 3$ and $n_R = 1$

extension to include $n_H > 1$ Higgs doublets

- the Yukawa matrices are still $n_R \times n_L = 3 \times 1$
  - the Higgs fields with the same quantum numbers mix
    - orthogonal superpositions are equivalent

- we choose a parametrisation of the Higgs doublets such that:
  - only the first doublet has a vev, and is represented by $\vec{a}_1$
  - the other doublets are represented by $\vec{a}_2$
    - $\vec{a}_1$ and $\vec{a}_2$ are linearly independent

- Dirac mass term like in the $n_H = 1$ toy model: $M_D = m_D \vec{a}_1^\top$
  - tree-level diagonalisation is the same
  - loop corrections involve now also the other Higgses $(\vec{a}_2)$

  $\Rightarrow$ radiative mass generation becomes possible

The νSM with $n_L = 3$ and $n_R = 1$

extension to include $n_H > 1$ Higgs doublets (continued)

- the mass matrix at one loop can be written

$$M_\nu^{[1]} = \begin{pmatrix} 0 + \delta M_L & M_D^\top + \delta M_D^\top \\ M_D + \delta M_D & M_R + \delta M_R \end{pmatrix} \sim \begin{pmatrix} \delta M_L & M_D^\top \\ M_D & M_R \end{pmatrix}$$

  - with only δM_L being of relative importance

- $M_\nu^{[1]}$ has to be diagonalised, giving
  - one mass less neutrino
  - one radiatively generated massive neutrino
  - one seesaw neutrino
  - one heavy state

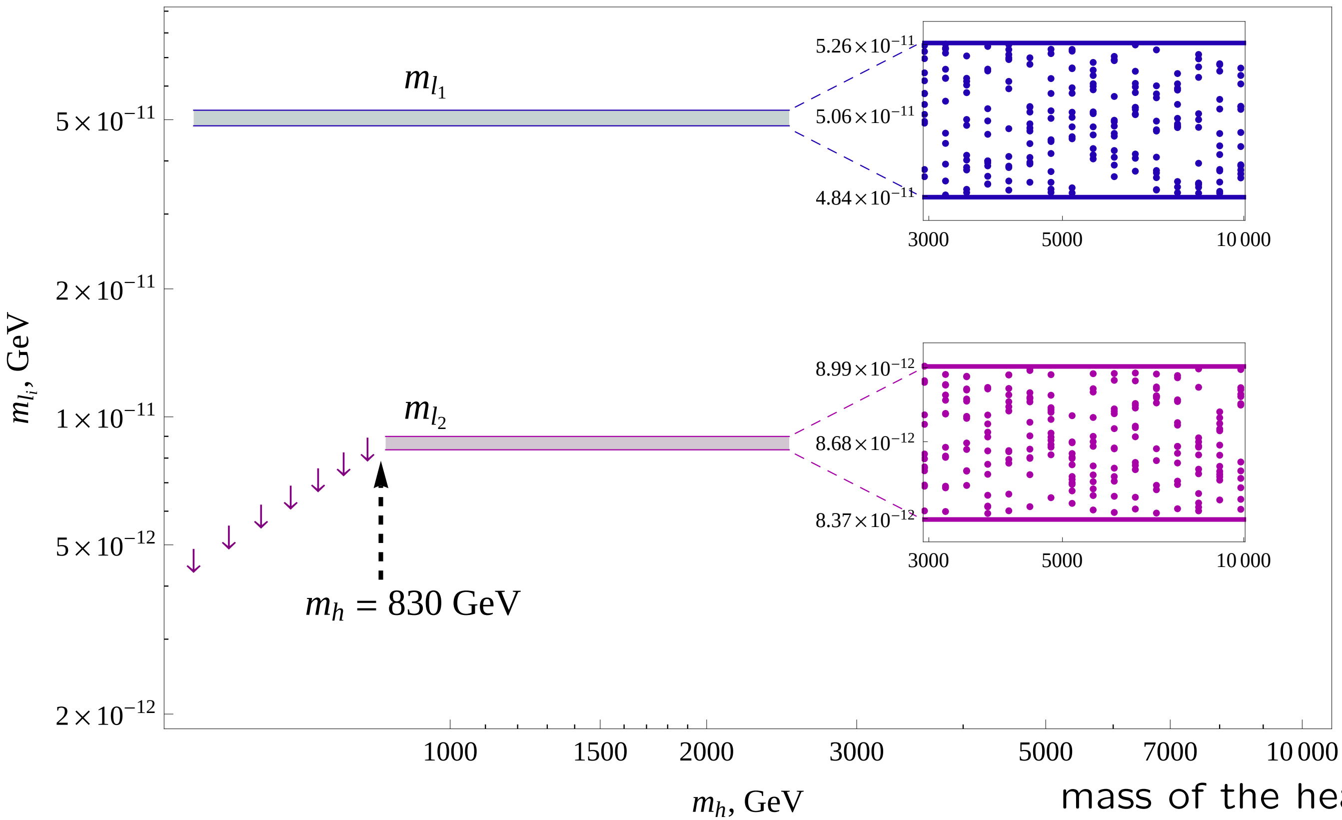
- for the numeric evaluation
  - the numeric diagonalisation is much faster than the analytic one

  $\Rightarrow$ we do not specify rotation angles, but Yukawa couplings
  - we do not deal with Higgs sector predictions
    - we take only the mass of the lightest neutral Higgs as 125 GeV
    - and the masses of the heavier neutral Higgses as input $>$ 200 GeV

The νSM with $n_L = 3$ and $n_R = 1$

including $n_H = 2$ Higgs doublets

- we require the loop corrected masses to fulfil $\Delta m^2_\odot$ and Δm^2_{atm}
  - with the lightest neutrino mass zero, the masses are given (with errors)
    - with heavy singlet mass bigger than 830 GeV we can fulfil our constraints
  - all other parameters a varied in the Monte Carlo study, done by Darius Jurčiukonis

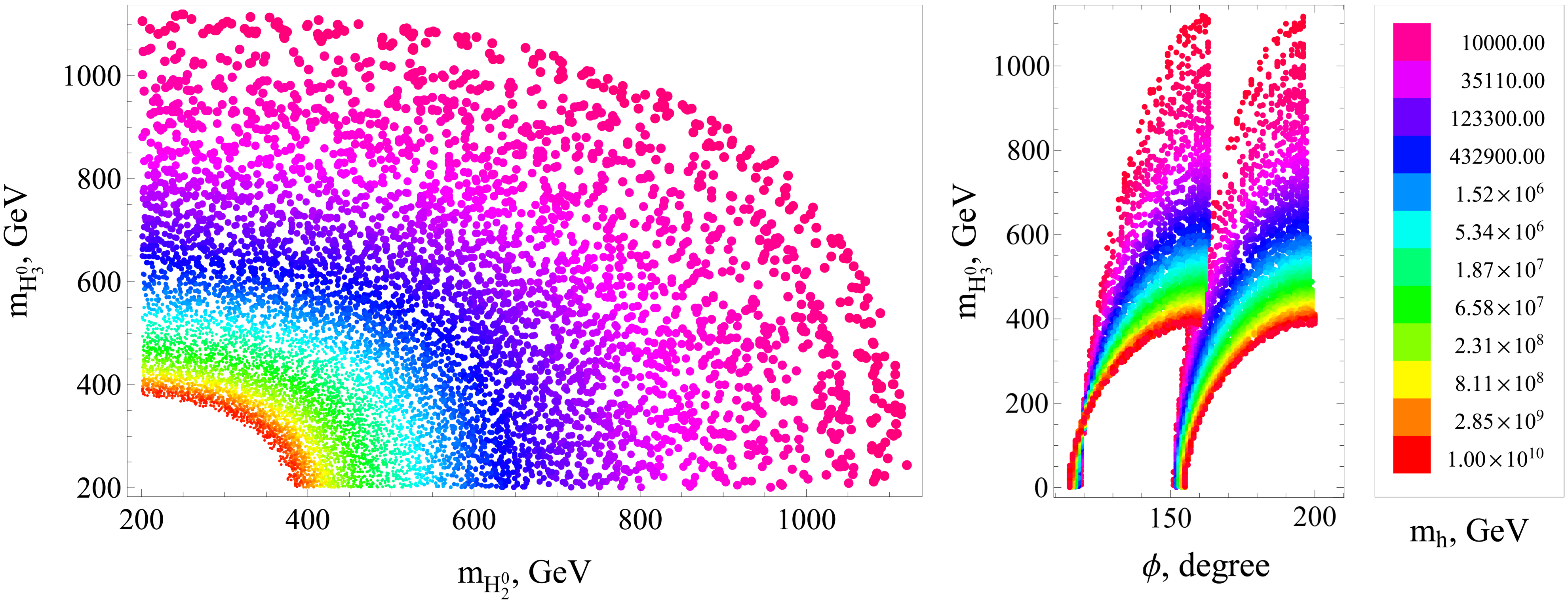


mass of the heavy singlet fermion

The νSM with $n_L = 3$ and $n_R = 1$

including $n_H = 2$ Higgs doublets

- we specifically take $\vec{a}_1^\top = (0, 0, 1)$ and $\vec{a}_2^\top = (0, 1, e^{i\phi})$
  - we name the heavy CP-even Higgs H_2^0 and the CP-odd Higgs H_3^0
  - every dot fulfils the criteria of $\Delta m_\odot^2$ and Δm_{atm}^2



The νSM with $n_L = 3$ and $n_R = 2$

taking $n_R = 2$ allows two mass differences at tree level

- like in the 3+1 case, the tree level mass matrix can be reduced
  - this time to an effective 2+2 case
    - * one needs two "rotations" to remove the two elements of the first line
    - * both rotations have to be determined simultaneously ...
    - * both rotations might need unitary matrices: $U_{13}^\top U_{12}^\top M_\nu U_{12} U_{13}$
    - * the principle discussion already in
      W. Grimus and H. Neufeld, Nucl. Phys. B **325** (1989) 18.

- remaining 2+2 matrix is suitable for Casas-Ibarra parametrisation
  - one has only a single complex angle as a free parameter
    - ⇒ six real parameters determine the couplings
    - * two masses are needed for the heavy singlets

- gives a single mass less neutrino and two massive light ones
  - can accommodate at tree level the measured mass differences $\Delta m^2_\odot$ and Δm^2_{atm}

The νSM with $n_L = 3$ and $n_R = 2$

renormalisation can change the masses

$\Rightarrow$ full treatment has to account for loop effects

- if only to define the tree level properly ...

- including a single Higgs doublet (like in the SM)
  - loop corrections to the masses are proportional to the masses themselves
  - one can still accommodate $\Delta m^2_\odot$ and Δm^2_{atm}
    - normal and inverted hierarchy

- including more Higgs doublets
  - can make all neutrinos massive through loop corrections
    - might be important for cosmological limits: $\sum m_\nu$
  - realistic treatments have to include the full Higgs sector
    - possible problems with flavour changing rare decays: $b \to s\gamma$, etc.

- a renormalisation treatment with on-shell renormalisation conditions is given in
A. A. Almasy, B. A. Kniehl and A. Sirlin, Nucl. Phys. B **818** (2009) 115 [arXiv:0902.3793 [hep-ph]].
  - a more explicit and a numeric treatment for this approach are still missing

The νSM with $n_L = 3$ and $n_R = 2$

Numeric diagonalisation is much faster than working with analytic angles

- for our numeric study we define
  - the neutrino masses $m_{\nu_1} > m_{\nu_2} > m_{\nu_3}$
  - the heavy singlet masses $m_{h_1} > m_{h_2}$
  - two Dirac masses $m^2_{D_i} = m_{\nu_i} m_{h_i}$
  - we parametrise the Yukawa couplings for n_H Higgs doublets as

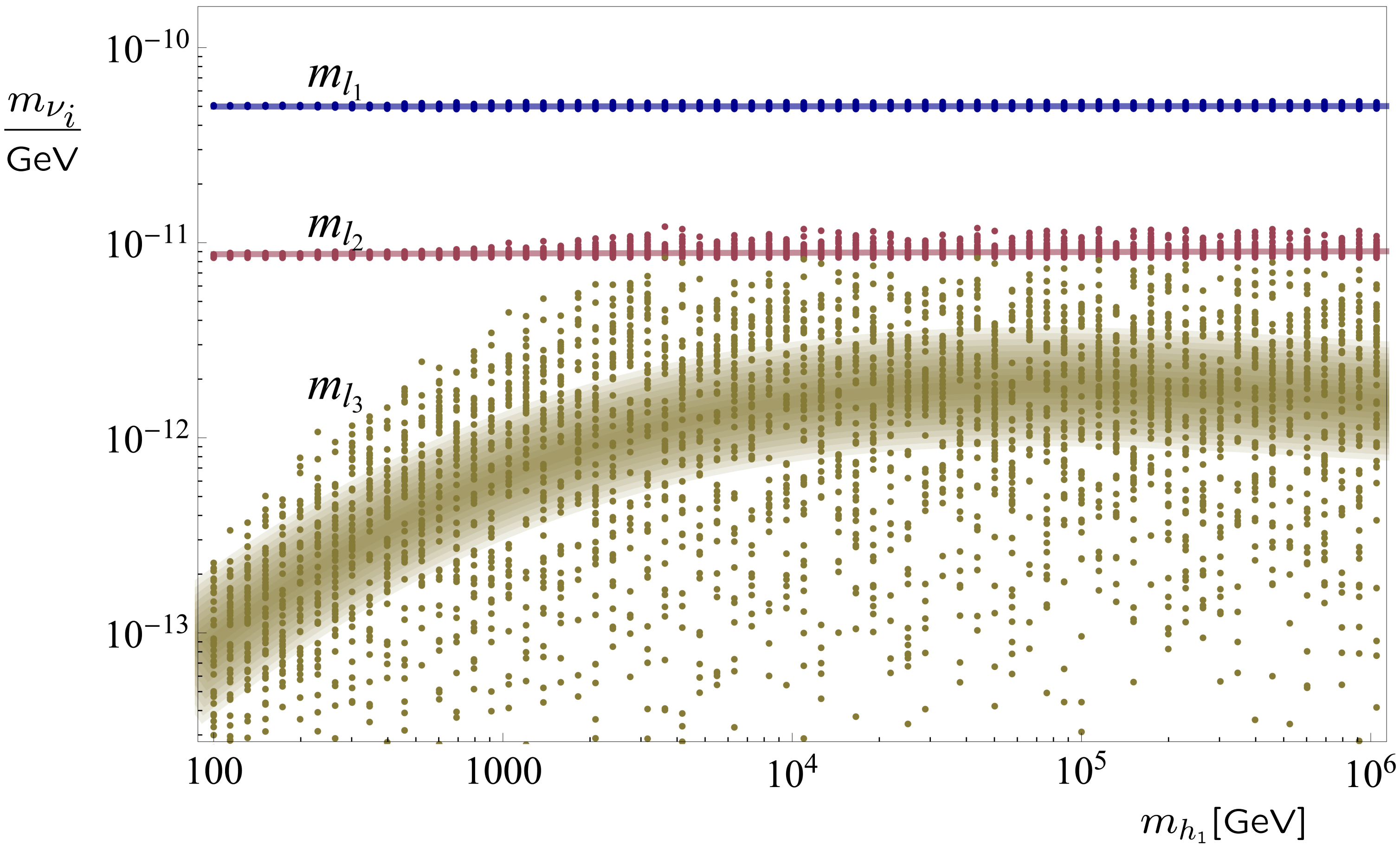
$$\Delta_k = \frac{\sqrt{2}}{v} \begin{pmatrix} m_{D_2} \vec{a}_k^\top \\ m_{D_1} \vec{b}_k^\top \end{pmatrix} \quad , \quad k = 1 \ldots n_H$$

- Monte Carlo parameter sets are generated for $\{m_{h_i}, \vec{a}_k, \vec{b}_k\}$
  - tree level masses of the light neutrinos are fixed by
    - $m_{\nu_3} = 0$
    - $\Delta m^2_\odot$ and Δm^2_{atm} – both hierarchies are allowed
  - the other tree-level model parameters are calculated
  - loop corrections are calculated and the mass matrix diagonalised again
    - like in our 3+1 Monte Carlo study
  - if the renormalised masses still fit $\Delta m^2_\odot$ and Δm^2_{atm}
    - the Monte Carlo parameter set is kept

The νSM with $n_L = 3$ and $n_R = 2$

Displaying the light neutrino masses in normal hierarchy

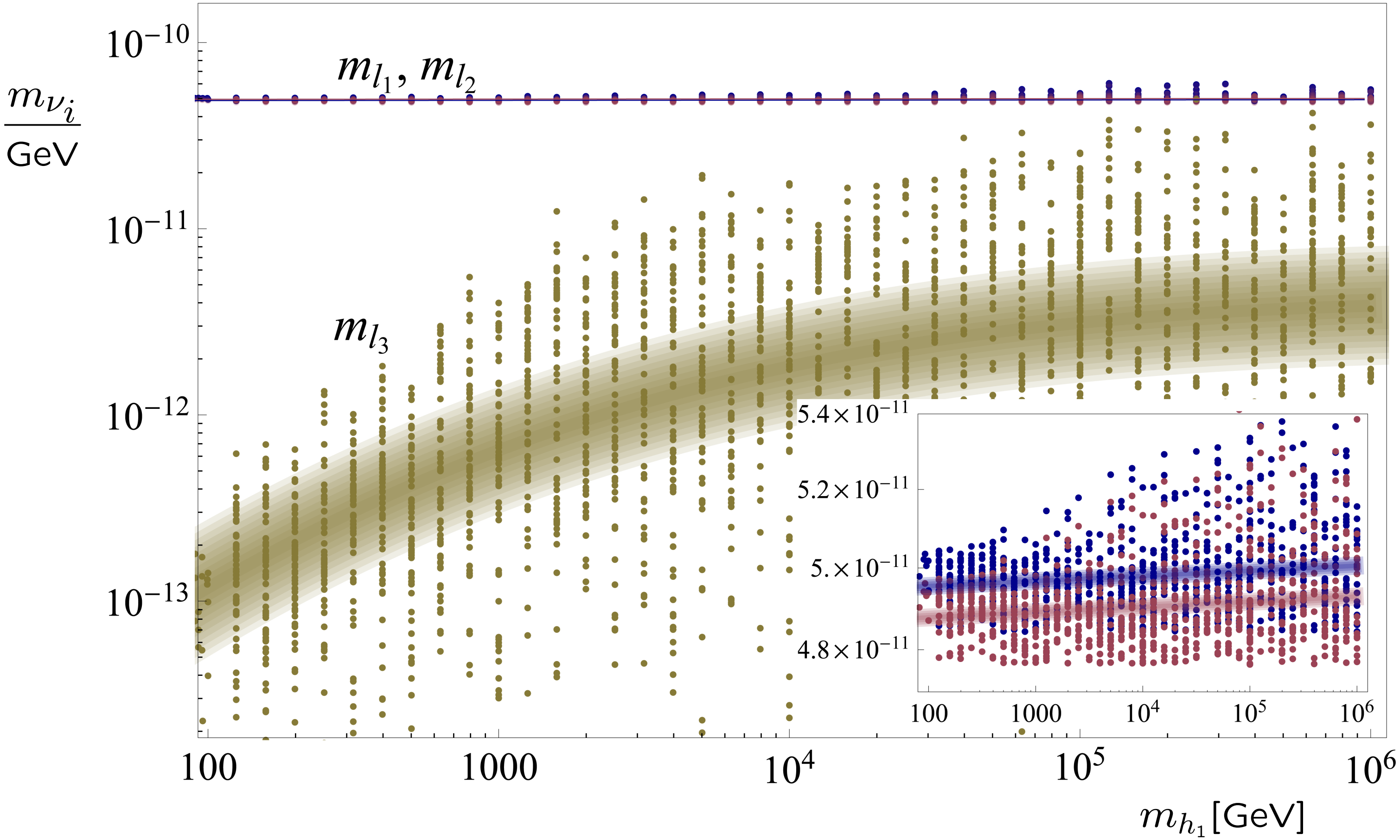
- depending on the heavier singlet mass
  - the lighter singlet is taken as $m_{h_2} = 100$ GeV



The νSM with $n_L = 3$ and $n_R = 2$

Displaying the light neutrino masses in inverted hierarchy

- depending on the heavier singlet mass
  - the lighter singlet is taken as $m_{h_2} = 100$ GeV



Our plans = work in progress

- make the νSM available for students

⇒ formulate the renormalisation of the chiral νSM in an easier way

- the route of our numerical analysis is too compact for students:

  W. Grimus and H. Neufeld, Nucl. Phys. B **325** (1989) 18
  W. Grimus and L. Lavoura, JHEP **0011** (2000) 042 [arXiv:hep-ph/0008179]
  W. Grimus and L. Lavoura, Phys. Lett. B **546** (2002) 86 [arXiv:hep-ph/0207229]
  W. Grimus and L. Lavoura, Eur. Phys. J. C **39** (2005) 219 [arXiv:hep-ph/0409231]
  D. A. Sierra and C. E. Yaguna, JHEP **1108** (2011) 013 [arXiv:1106.3587 [hep-ph]].

  * understanding the renormalised normal SM
  * understanding the notation ...

- the route of AKS is too general for students:

  A. A. Almasy, B. A. Kniehl and A. Sirlin, Nucl. Phys. B **818** (2009) 115 [arXiv:0902.3793 [hep-ph]] and references therein

  * understanding the full renormalisation program
  * specialising the general formulae to the 3+1 / 3+2 case

? it might become simpler when using only Weyl spinors

  * a Master student tries to understand the SM in terms of Weyl spinors (Vytautas Dudenas)

Our plans = work in progress

- Tomas Sabonis works on his thesis
  - will include the analytic derivations of the 3+1 and 3+2 cases

- we want to estimate the τ polarisation at the LHC
  (Master thesis of Adomas Jelinskas)
  - we need to learn much more about the τ
    * theory
    * Tauola

  $\Rightarrow$ again a task for a student

- far future:
  - fully renormalising the 3+1 case

Thank you

for discussion

and comments

and of course for the conference! ☺