# Some remarks on non-planar diagrams

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As a consequence, they must be treated differently, both on computational and theoretical side:

- they demand different methods for analytical computations, e.g. to get as least dimensional Mellin–Barnes representations as possible,
- the non-planar ones can not yet be involved in some new constructions, e.g. twistor methods for calculating scattering amplitudes in $\mathcal{N} = 4\ SYM$.

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AMBRE is a Mathematica package that calculates M–B representations [J. Gluza, K. Kajda, T. Riemann and V. Yundin, Eur. Phys. J. C71 (2011)]

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The choice of the method should be made automatic, with the only input as given external and internal momenta of a diagram $G$, e.g.

$$k_1,\ k_1 - p_1,\ k_1 - p_1 - p_2,\ k_2,\ k_2 - p_4,\ k_2 - p_3 - p_4,\ k_1 + k_2.$$

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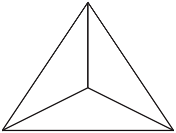
Do they determine uniquely the (non)planarity of $G$?

# Definitions

A graph is planar if it can be drawn on the sphere (plane) without intersections.



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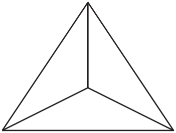

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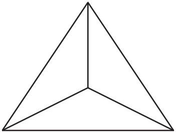

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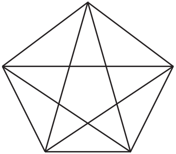
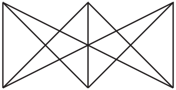


$K_5$



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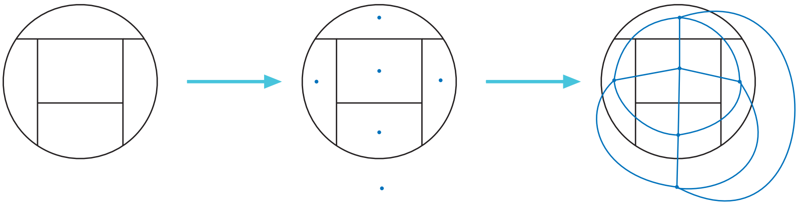

*Only planar graphs possess their duals.*

# Graph theory and Feynman diagrams

To say that a Feynman diagram $G$ is (non-)planar, one has to define the *adjoint* diagram $G^*$.

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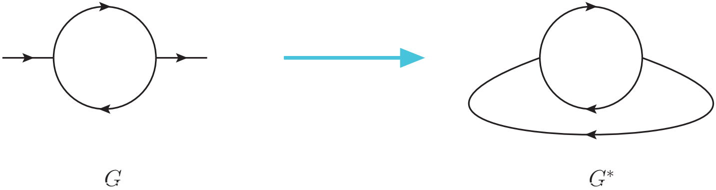
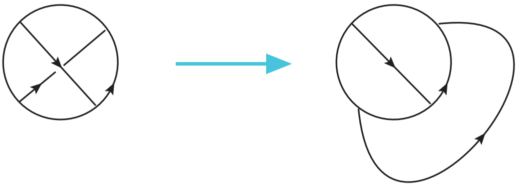



We say that a Feynman diagram $G$ is planar iff $G^*$ is planar.

Hence



is planar,

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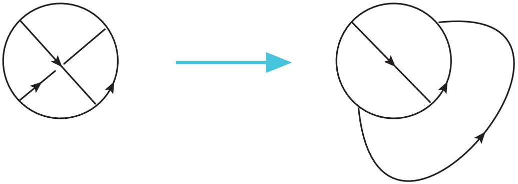
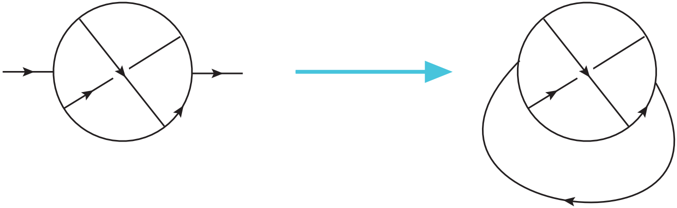


is planar,while



is not (it has $K_{3,3}$ as a subgraph).

# Planarity of Feynman diagrams

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However, there are *at least* 2 methods: combinatorial and geometrical one.

# Method I

Given a Feynman diagram with

- external momenta $p_1, ..., p_n$
- loop momenta $k_1, ..., k_m$ and Feynman parameters $x_1, ..., x_m$

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the Laplace matrix is

$$L_{ij} = \begin{cases} \sum_{s=1}^{m} x_s & \text{if } i = j,\ k_s \text{ is attached to } v_i,\ k_s \text{ is not a self-loop,} \\ -\sum_{s=1}^{m} x_s & \text{if } i \neq j,\ k_s \text{ connects } v_i, v_j. \end{cases}$$

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Diagonal elements of $L$ are computed by checking all possibilities of conserved momentum

(for external vertices) $\pm k_a \pm k_b = \pm p_e$ or $\pm k_a \pm k_b \pm k_c = \pm p_e$,

(for internal vertices) $\pm k_a \pm k_b = \pm k_c$ or $\pm k_a \pm k_b \pm k_c = \pm k_d$,

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Then introduce another form of Laplace matrix by $x_k \to 1$

$$L_{ij} = \begin{cases} \deg(v_i) & \text{if } i = j, \\ -1 & \text{if } i \neq j \text{ and } v_i, v_j \text{ are adjacent,} \end{cases}$$

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$L$ can be written as

$$L = D - A, \tag{1}$$

where $D$ is a degree matrix and $A$ is adjacency matrix given by

$$A_{ij} = \begin{cases} 1 & \text{if } i \neq j \text{ and } v_i, v_j \text{ are adjacent,} \\ 0 & \text{otherwise.} \end{cases}$$

The final part of the algorithm is to create the adjoint $G^*$.

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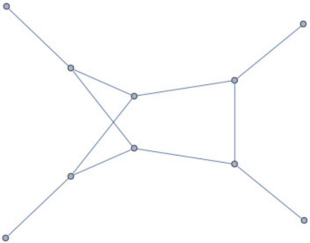
Eventually, given $A^*$, a `Mathematica` package `Combinatorica` yields an answer for the question of planarity of a Feynman diagram $G$.

```
<< PlanarityTest_1.2.m
by E. Dubovyk ver: 1.2
created: April 2013
last executed: 25.06.2013 at 15:37

(*non-planar dbox*)

PTest[{{k1, k2}, {p1, p2, p3, -p1 - p2 - p3}},
  {PR[k1 + k2 + p1 + p2 + p3, 0, n1] PR[k1 + k2, 0, n2] PR[k1, 0, n3] PR[k1 + p1, 0, n4] PR[k1 + p1 + p2, 0, n5] PR[k2 + p3,
The Diagram:
```



with the Laplacian matrix

$$\begin{pmatrix} x[1]+x[4] & 0 & 0 & -x[4] & -x[1] & 0 \\ 0 & x[2]+x[6] & 0 & 0 & -x[2] & -x[6] \\ 0 & 0 & x[3]+x[7] & 0 & -x[3] & -x[7] \\ -x[4] & 0 & 0 & x[4]+x[5] & 0 & -x[5] \\ -x[1] & -x[2] & -x[3] & 0 & x[1]+x[2]+x[3] & 0 \\ 0 & -x[6] & -x[7] & -x[5] & 0 & x[5]+x[6]+x[7] \end{pmatrix}$$

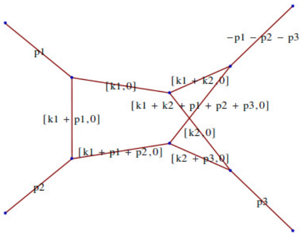
is non-planar.

```
(*planar dbox*)

PTestx[{{k1, k2}, {p1, p2, p3, -p1 - p2 - p3}},
```

# It is also possible to label the edges with propagators

```
In[7]:= DrawDiagram[{{k1, k2}, {p1, p2, p3, -p1 - p2 - p3}},
    {PR[k1 + k2 + p1 + p2 + p3, 0, n1] PR[k1 + k2, 0, n2] PR[k1, 0, n3] PR[k1 + p1, 0, n4] PR[k1 + p1 + p2, 0, n5]
     PR[k2 + p3, 0, n6] PR[k2, 0, n7]}];
```



# Method II

[Arkani-Hamed et al. 2010]:

planarity $\leftrightarrow$ (dual) conformal symmetry

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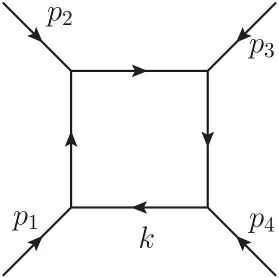
Example: simple one-loop planar box



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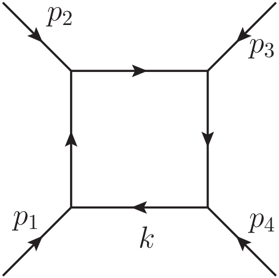
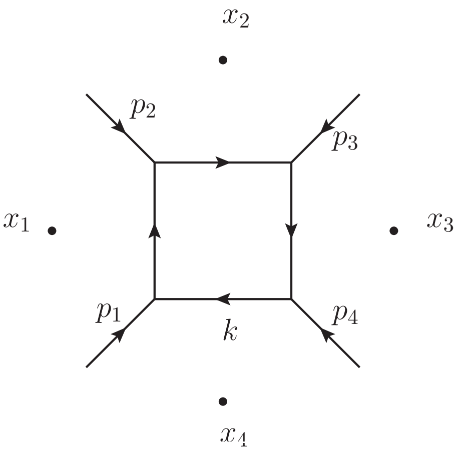


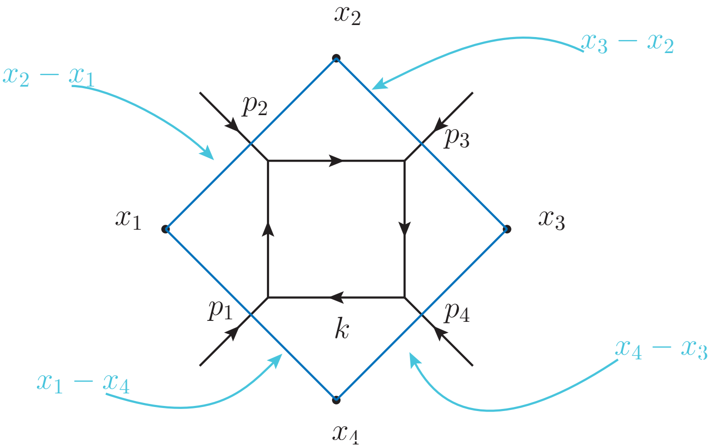

$$I = \int \mathrm{d}^4 k \frac{N}{k^2 \left(k+p_1\right)^2 \left(k+p_1+p_2\right)^2 \left(k+p_1+p_2+p_3\right)^2}.$$

Let introduce *dual variables* with incoming external momenta $p_1,\ldots,p_n$ and some propagators. Let

$$\begin{aligned} p_1 &= x_1 - x_4, \\ p_2 &= x_2 - x_1, \\ p_3 &= x_3 - x_2 \\ p_4 &= x_4 - x_3. \end{aligned}$$



Note that the lines connecting dual variables cross exactly given momenta



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$$\downarrow$$

$$I = \int \mathrm{d}^4 k \frac{N}{k^2 \left(k + x_1 - x_4\right)^2 \left(k + x_2 - x_4\right)^2 \left(k + x_3 - x_4\right)^2}.$$

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What are dual variables for the loop momentum $k$?

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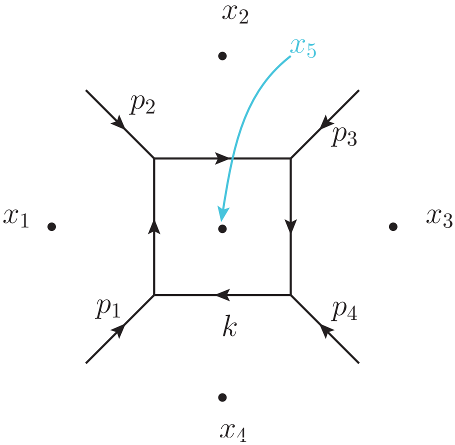
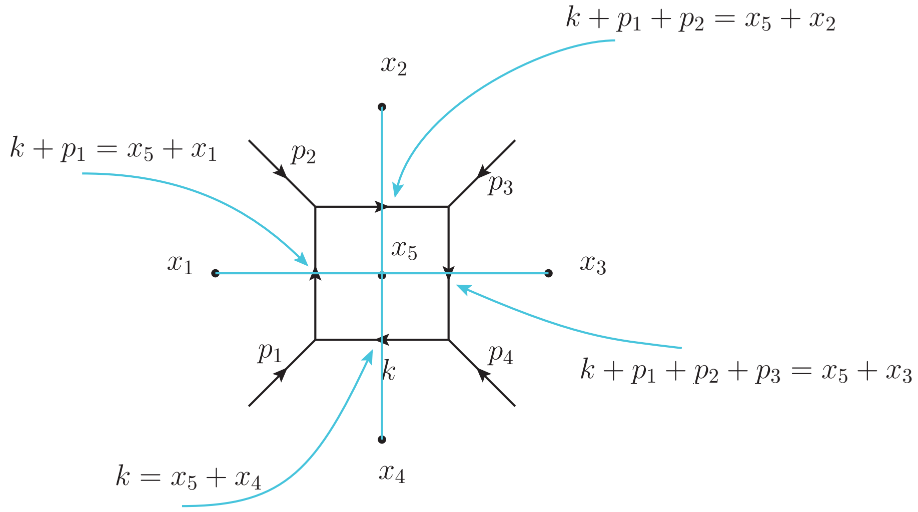
What are dual variables for the loop momentum $k$?

Since any momentum leads to the one new dual variable, let $x_5$ be introduced.

Note that the choice




gives unique recipe for $k$, that is $k = x_5 + x_4$.




$$x_5 + x_4 = x_5 + x_1 - p_1 = x_5 + x_2 - p_1 - p_2 = x_5 + x_3 - p_1 - p_2 - p_3 = k.$$

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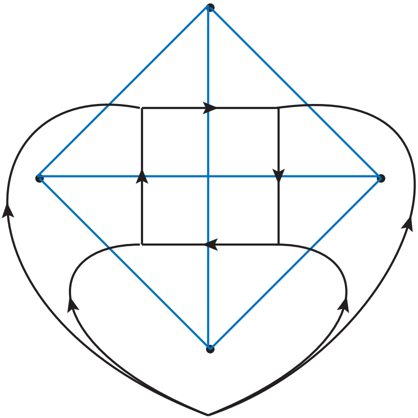

$$I = \int \mathrm{d}^4 x_5 \frac{N}{(x_5 + x_1)^2 (x_5 + x_2)^2 (x_5 + x_3)^2 (x_5 + x_4)^2}$$

Moreover, it is (dual) conformally invariant.

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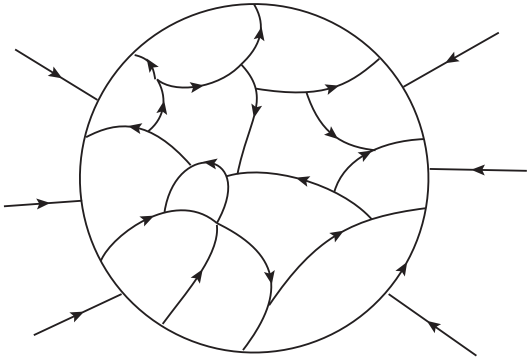

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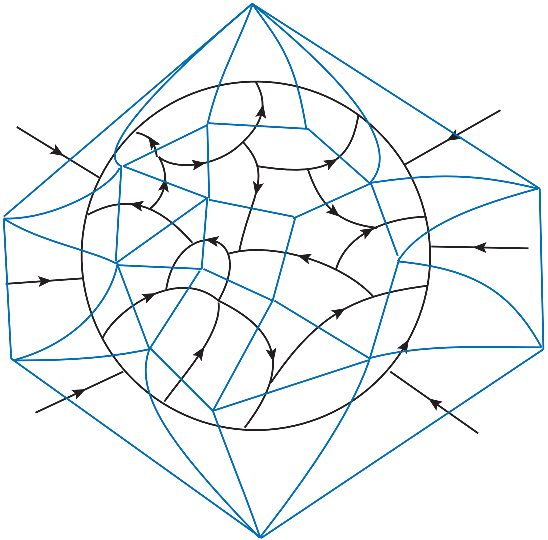

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What happens in the non-planar cases?

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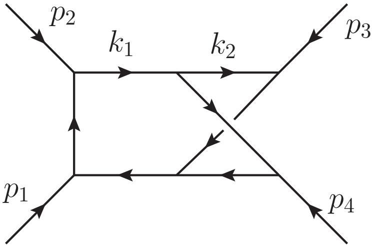
What happens in the non-planar cases?

Example — non-planar double box:



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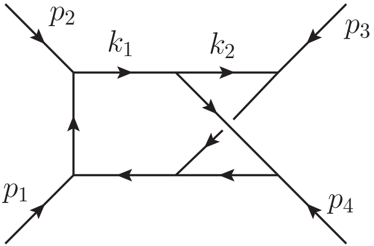
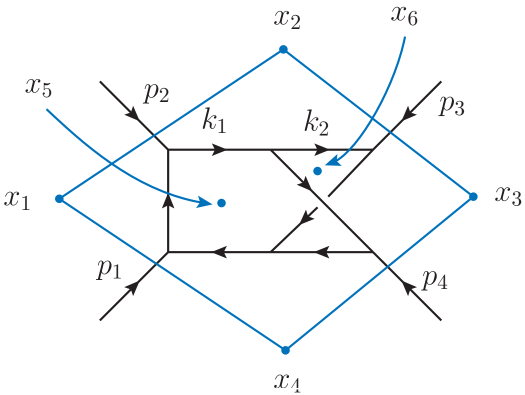
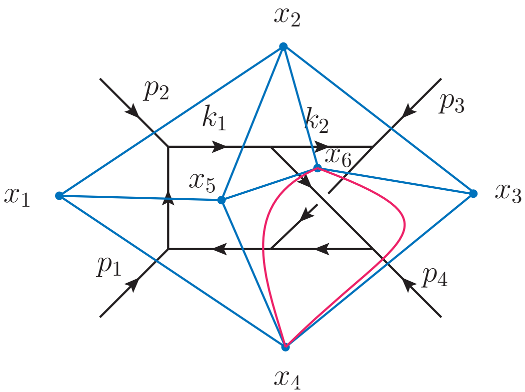



$$I = \int \frac{N \mathrm{d}^4 k_1 \mathrm{d}^4 k_2}{k_1^2 \left(k_1 - p_2\right)^2 \left(k_1 - p_1 - p_2\right)^2 k_2^2 \left(k_2 + p_3\right)^2 \left(k_1 - k_2 + p_4\right)^2 \left(k_1 - k_2\right)^2}$$

External dual variables are identical as the previous ones, while internal faces intersect each other:



One can not construct a dual diagram:



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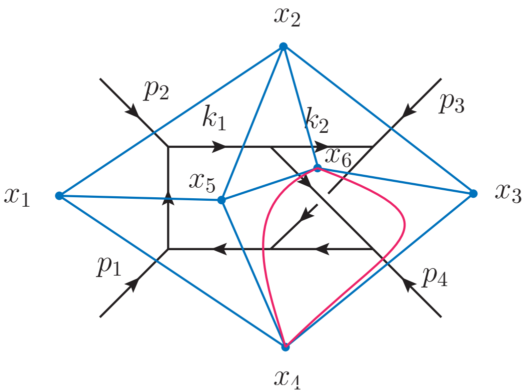


which is not a surprise from a graph-theoretical point of view.

Any choice gives a similar result

$$I = \int \frac{N \mathrm{d}^4 x_5 \mathrm{d}^4 x_6}{(x_5 + x_1)^2 (x_5 + x_2)^2 (x_5 + x_4)^2 (x_6 + x_2)^2 (x_6 + x_3)^2 (x_5 - x_6)^2} \times \frac{1}{(x_5 - x_6 + x_4 - x_3)^2},$$

which is not (dual) conformally invariant.

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planar diagrams $\longrightarrow$ single line-crossings $\longrightarrow x_i \pm x_j$

Any choice gives a similar result

$$I = \int \frac{N \mathrm{d}^4 x_5 \mathrm{d}^4 x_6}{(x_5+x_1)^2 (x_5+x_2)^2 (x_5+x_4)^2 (x_6+x_2)^2 (x_6+x_3)^2 (x_5-x_6)^2} \times \frac{1}{(x_5-x_6+x_4-x_3)^2},$$

which is not (dual) conformally invariant.

planar diagrams $\longrightarrow$ single line-crossings $\longrightarrow$ $x_i \pm x_j$

while

non-planar diagrams $\rightarrow$ double line-crossings $\rightarrow$ $x_i \pm x_j \pm x_k \pm x_n$

### Conclusion

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The method is general and applicable to all loop orders and for any vertex valences.

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*Remark*: since sometimes external legs could be permuted, which would lead to wrong relations $p_i = x_i - x_{i-1}$, it is even better to treat external momenta as loop momenta from the beginning.

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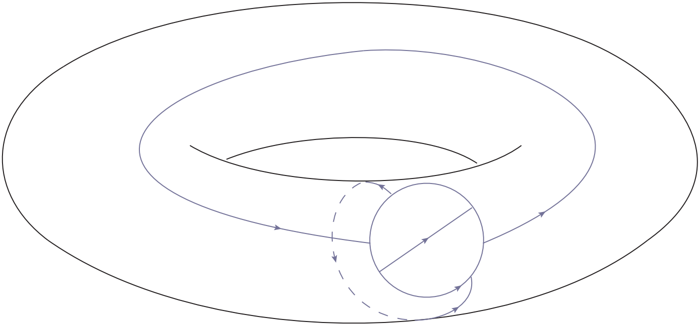

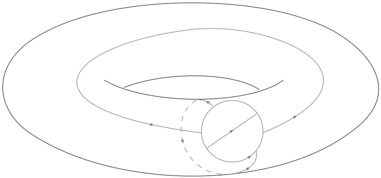

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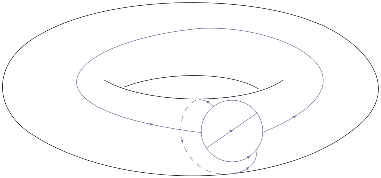

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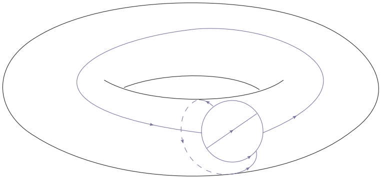

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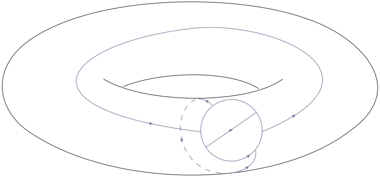

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While it is possible to find a dual, it is not the dual in the sense of correspondence $x_i - x_j \rightarrow k_i \rightarrow$ no conformal symmetry.

# Conclusions

- we found two ways of testing (non-)planarity of Feynman diagrams only upon given momenta,
- it allows to fully automatize the procedures in `AMBRE`, that differ on (non-)planar diagrams,
- the second method uses the fact, that non-planar diagrams break conformal symmetry,
- while it is the cornerstone of e.g. twistor methods in $SYM$ theories, the simple embedding on the higher-genus surface does not give the conformal invariant analogue,
- we should think about other methods of applying twistor techniques to non-planar diagrams.

# Thank you!

The work is supported by the scholarship project ŚWIDER