One Loop External tensor integral Contractions - the OLEC library

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Outline

Definitions

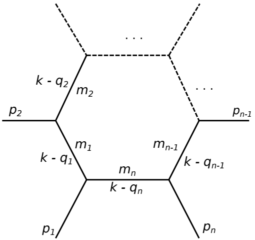
based on talk by T.Riemann “Tensor reduction for higher rank one-loop integrals”

n-point tensor integrals of rank R: (n,R)-integrals

$$I_n^{\mu_1\dots\mu_R} = \int \frac{dk^d}{i\pi^{d/2}} \frac{\prod_{r=1}^{R} k^{\mu_r}}{\prod_{j=1}^{n} c_j^{\nu_j}}$$

$d = 4 - 2\varepsilon$ and denominators c_j have indices ν_j and

$$c_j = (k - q_j)^2 - m_j^2 + i\varepsilon$$



tensor integrals due to, e.g.:

- fermion propagators
- three-gauge boson couplings

Calculation methods

In general tensor integrals are reduced to basic one-loop scalar integrals:
1-point – A_0; 2-point – B_0; 3-point – C_0; 4-point – D_0;
The scalar integrals were calculated originally by 't Hooft and Veltman, Nucl. Phys. '79

Passarino-Veltman algorithm:
contraction of (n,R)-integral with external momenta and metric tensor to cancel propagators $\rightarrow$ invertion of the resulting system of linear equations $\rightarrow$ result consists of $(n-1)$-point and $(R-1)$-rank integrals

Problem – appearance of inverse Gram determinants, which may become small $\rightarrow$ numerical instabilities.

Numerical libraries I

For the 4-dimensional scalar integrals with up to four external legs:

QCDLoop/FF package – all finite and divergent scalar 4-dimensional integrals with real masses in dimensional regularization.
http://qcdloop.fnal.gov/

OneLOop – all finite and divergent scalar 4-dimensional integrals with real or complex masses in dimensional regularization.
http://helac-phegas.web.cern.ch/helac-phegas/OneLOop.html

Numerical libraries II

For the tensor integrals:

LoopTools/FF – tensor integrals up to rank 4 with up to 5 legs. No soft-collinear case. No treatment of small four point Gram determinants.
http://www.feynarts.de/looptools/

Golem95C – tensor integrals up to rank 6 with up to 6 legs. Numerical integration for small Gram determinants in massless case. Massive is potentially unstable for small four point Gram determinants. Complex internal masses.
https://golem.hepforge.org/95/

PJFry - tensor integrals up to rank 5 with up to 5 legs. Correct treatment of small Gram determinants.
https://github.com/Vayu/PJFry/

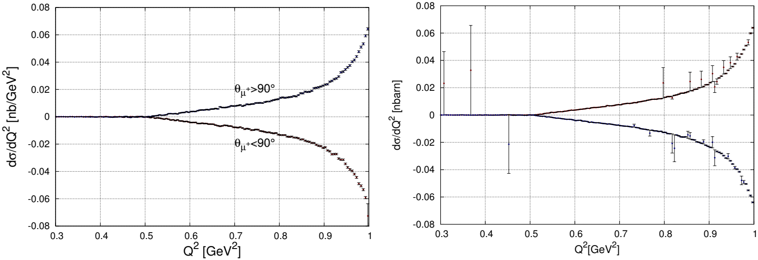


Figure 1: Muon pair distributions including 5-point functions at KLOE.

$2,5 \cdot 10^6$-(left graph) and 10^9-(right graph) events have been generated. Looptools and FF packages have been used.

[J. Gluza, M. Gunia, T. Riemann, M. Worek: [arXiv:1201.0968]]

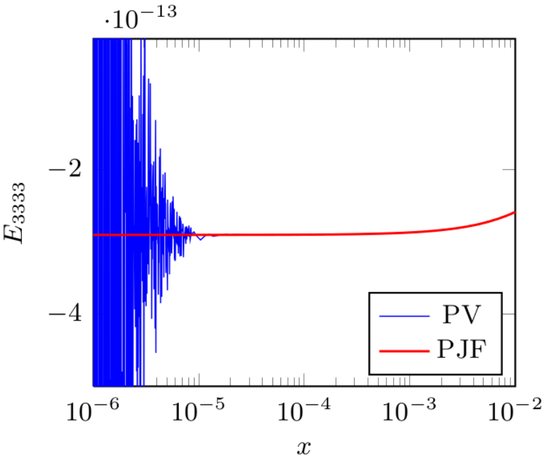


Figure 2: Pentagon form-factors with simple Passarino-Veltman reduction and PJFry code in small Gram region.
[V. Yundin, PhD thesis]

	double [s]	quad [s]	$\frac{quad}{double}$
$\sqrt{s} = 1.02$ GeV	7.76	170	22
$\sqrt{s} = 10.56$ GeV	7.92	171	22
KLOE	1.53	57.29	5
BaBar	2.30	135	59

Table 1: Time of calculations for two tested versions of PHOKHARA generator (double – PJFry, quad – Rodrigo/Campanario).
[Private comunication, M. Gunia PhD thesis.]

Our motivation for OLEC library

- numerical stability
- speed
- alternative approach for crosschecks

http://prac.us.edu.pl/ gluza/olec/

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OLEC - One Loop External Contractions

Jochem Fleischer, Janusz Gluza, Marek Gluza, Tord Riemann

The project is based on the approach described by J. Fleischer and T. Riemann in Phys.Lett. B701 (2011) 646, further developed in Phys.Rev. D83 (2011) 073004 and Phys.Lett. B707 (2012) 375.

The present version 0.9 includes contractions of chords (i.e. appropriately organized external momenta) with one-loop tensor five point functions of up to rank 3. It includes the mathematica package OLECv0.9.m supplied by Contracts_examples.nb and the c++ package OLECv0.9.tgz.
It is advise to look first at the mathematica examples and arXiv:1211.3921 [hep-ph] (basic introduction and further references).

The main package is OLECv0.9.tgz. For calculation of basic scalar one-loop integrals, it can be linked with both LoopTools (LT) and OneLoop (OL).
See README and Makefiles for details.

- **The *Mathematica* package OLECv0.9.m, version 0.9.**
  Contracts_examples.nb shows how to use it. The results are compared with the previous package hexagon.m and LoopTools. For using with OneLoop, we prepared mathlink.tgz which should be downloaded and compiled.

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The present version includes contractions of chords (i.e. appropriately organized external momenta) with one-loop tensor five point functions of up to rank 3. It is advise to look first at arXiv:1211.3921 [hep-ph] (basic introduction and further references).

The main package is written in C++ (Improved Fortran version will be available in nearest future). For calculation of basic scalar one-loop integrals, it can be linked with both LoopTools (LT) and OneLoop (OL).

Some numerical results:

```
=====================================================
CE1_LT(1)=(   0.948042733312E+005,   0.102830199036E+006)
CE1   (1)=(   0.948042733312E+005,   0.102830199036E+006)
CE1_LT(2)=(   0.886783019590E+005,   0.219969623798E+006)
CE1   (2)=(   0.886783019590E+005,   0.219969623798E+006)
CE1_LT(3)=(   0.898106174204E+005,   0.203976367496E+006)
CE1   (3)=(   0.898106174204E+005,   0.203976367496E+006)
CE1_LT(4)=(  -0.553336532378E+005,   0.960635750549E+005)
CE1   (4)=(  -0.553336532378E+005,   0.960635750549E+005)
CE1_LT(5)=(  -0.000000000000E+000,  -0.000000000000E+000)
CE1   (5)=(   0.000000000000E+000,   0.000000000000E+000)
```

Figure 3: Rank 1 contractions with external momenta.
CE1 - OLEC, CE1_LT - LoopTools decomposition.

Reduction and small Gram determinants

a brief overview

5-point tensor recursion:
Any (5,R) integral can be expressed by a (5,R-1) and (4,R-1)
[T.Diakonidis, J.Fleischer, T.Riemann, B.Tausk: Phys. Lett. B683 (2010)]
[J.Fleischer, T.Riemann: Phys. Rev.D83 (2011)]

$$I_5^{\mu_1\ldots\mu_{R-1}\mu} = I_5^{\mu_1\ldots\mu_{R-1}} Q_0^{\mu} - \sum_{s=1}^{5} I_4^{\mu_1\ldots\mu_{R-1},s} Q_s^{\mu}$$

with auxiliary vectors an inverse Gram determinants

$$Q_s^{\mu} = \sum_{i=1}^{5} q_i^{\mu} \frac{\binom{s}{i}_5}{\binom{}{}_5}$$

contraction:

$$q_{i_1\mu_1}\ldots q_{i_R\mu_R} I_5^{\mu_1\ldots\mu_R} = \int \frac{dk^d}{i\pi^{d/2}} \frac{\prod_{r=1}^{R}(q_{i_r}k)}{\prod_{j=1}^{5} c_j^{\nu_j}}$$

One may arrange a one-loop calculation (e.g., crose sections Born$\times$1-loop) such that all the 1-loop integrals appear only in such contractions.

$R = 1$:

$$I_5^{\mu} = I_5 Q_0^{\mu} - \sum_{s=1}^{5} I_4^s Q_s^{\mu} \to (q_i \cdot I_5) = I_5 (q_i \cdot Q_0) - \sum_{s=1}^{5} I_4^s (q_i \cdot Q_s)$$

$R = 2$:

$$I_5^{\mu\nu} = I_5^{\mu} Q_0^{\nu} - \sum_{s=1}^{5} I_4^{\mu,s} Q_s^{\nu} \to (q_i q_j I_5) = (q_i \cdot I_5)(q_j \cdot Q_0) - \sum_{s=1}^{5} (q_i \cdot I_4^s)(q_j \cdot Q_s)$$

$R = 3$:

$$\ldots \to (q_i q_j q_k I_5) = (q_i q_j I_5)(q_k \cdot Q_0) - \ldots$$

Finally using dimensionally recurence relations [Ref.] and algebra of signed minors [Ref.] one can express any contracted integral through scalar function (D_0, C_0, B_0, A_0) and (if nessesary) integrals in shifted dimensions. integrals in shifted dimensions allows us to avoid small Gram determinant cases.

Example: R=2 contraction

$$(q_i q_j I_5) = (q_i \cdot I_5)(q_j \cdot Q_0) - \sum_{s=1}^{5} (q_i \cdot I_4^s)(q_j \cdot Q_s)$$

$$(q_i \cdot I_4^s) = -\frac{1}{2}\left\{ (Y_{i5} - Y_{55}) I_4^s + \sum_{t=1}^{5} (\delta_{at} - \delta_{5t}) I_3^{st} + (\delta_{as} - \delta_{5s}) R^s \right\}$$

$$R^s = \frac{1}{\binom{s}{s}_5} \left[\binom{s}{0}_5 I_4^s - \sum_{t=1}^{5} \binom{s}{t}_5 I_3^{st} \right]$$

$$\binom{s}{s}_5 \to 0 \Longrightarrow R^s = \frac{1}{\binom{0s}{0s}_5} \left[\binom{s}{0}_5 I_4^{[d+],s} - \sum_{t=1}^{5} \binom{0s}{0t}_5 I_3^{st} \right]$$

integrals in shifted dimensions $I_n^{[d+]^l}$: $[d+]^l = 4 - 2\varepsilon + 2l$;
integrals in $[d+]^l$-dimensions can be reduced to integrals with lower dimensions

$$I_4^{[d+]^l} = \frac{1}{d+2l-4}\frac{1}{\binom{s}{s}_5}\left[\ldots\right]^{[d+]^{(l-1)}}$$

in case of small 4-point Gram determinats - another approach:

$$I_4^{[d+]^l} = \sum_{j=1}^{\infty} a_j^L r^j \left[Z_{4F}^{(L+j)} - b_j^L Z_{4D}^{(L+j)}\right]$$

$$Z_4^{[d+]^l} = \frac{1}{\binom{0}{0}}\sum_{t=1}^{4}\binom{t}{0} I_3^{[d+]^l,t}$$

with small parameter $r = \frac{()}{\binom{0}{0}}$.

Or, as an alternative way, integrals in $[d+]^l$-dimensions can be calculated using hypergeometric representation:
[J. Fleischer, F. Jegerlehner, O.V. Tarasov: Nucl.Phys. B672 (2003)]

Conclusions and Outlook I

Current status of the project (version 1.0)

- new cashe system
- improved matrix algebra
- tests of different optimization levels
- new examples

Conclusions and Outlook II

To Do Next:

- rank 4 and 5 contractions
- small Gram determinats
  - evaluation using expantion in small parameter (PJFry approach)
  - hypergeometric representation
- application to real processes, like $e^+e^- \to \mu^+\mu^-\gamma$ in PHOKHARA generator, LHC, low energy physics
- 6- and 7-point functions
- ...