Implementation of complete NLO radiative corrections to reaction $e^+e^- \to \mu^+\mu^-\gamma$ in PHOKHARA MC generator

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Collaboration

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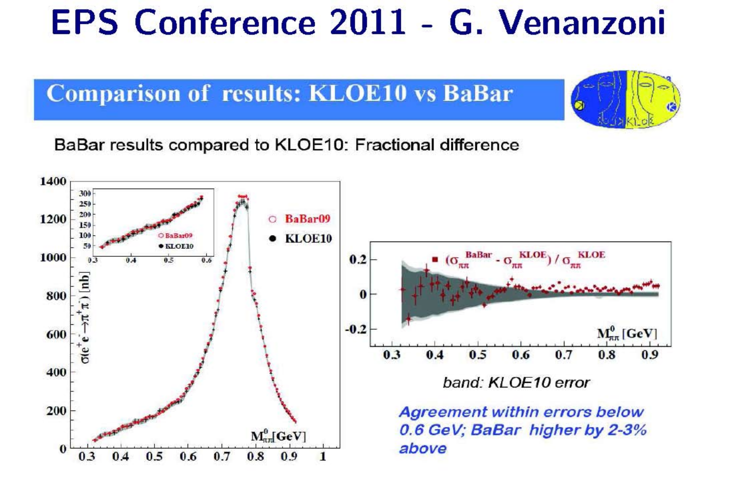
Motivation:Measure of the cross section for the process $e^+e^- \to \pi^+\pi^-$



Motivation

- The anomalous magnetic moment of the muon a_μ^{had} is mainly determined by low energy hadrons.

$$a_\mu^{had} = \frac{1}{4\pi^3}\int_{m_\Pi^2}^{\infty} dsK(s)\sigma_{had} \tag{1}$$

- The hadronic part of the fine structure constant.

$$\Delta\alpha^{had}(M_z) = -\frac{\alpha M_z^2}{3\pi} Re \int_{m_\Pi^2}^{\infty} ds \frac{R(s)}{s(s-M_z^2-i\epsilon)} \tag{2}$$

Aim of the work

- calculation of the full corrections for the process $e^+e^- \to \mu^+\mu^-\gamma$ with PHOKHARA MC generator;
- discussion of the numerical results for energies and with cuts near the real one for KLOE and BaBar experiments;
- comparison complete calculations with results from previous version of PHOKHARA;
- additional tests of agreement between two independent applications for calculating virtual corrections;

PHOKHARA9.0 Monte Carlo generator simulates processes with photon radiation $e^+e^- \to \textit{hadrons}(\textit{muons})$. The one-loop corrections for the process $e^+e^- \to \mu^+\mu^-\gamma$ can be presented in the following form:

$$
\begin{aligned}
\sigma(e^+e^- \to \mathrm{muons} + \mathrm{photons}) &= \\
\sigma(e^+e^- \to \mathrm{muons} + \mathrm{one\ hard\ photon}) & \\
+\sigma(e^+e^- \to \mathrm{muons} + \mathrm{two\ hard\ photons}) & \qquad (3)
\end{aligned}
$$

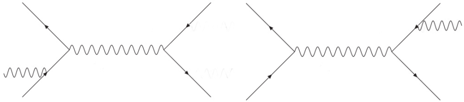
$\sigma(e^+e^- \to \mathrm{muons} + \mathrm{one\ hard\ photon})$

The contribution with emission of one real, hard photon:

$$\sigma_{1ph} = \sigma(e^+e^- \to \mathrm{muons} + \mathrm{one\ hard\ photon}) \tag{4}$$

consists of:

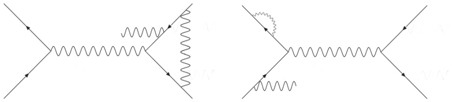
- σ_B Born contribution with emission of one real, hard photon



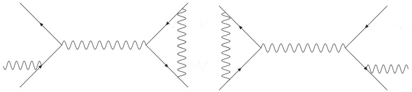
- σ_s the soft part with emission of one hard and one soft photon

- σ_V - virtual corrections that can be divided into 3 gauge invariant groups - boxes, pentaboxes, triangles. All this corrections interfere with Born diagrams

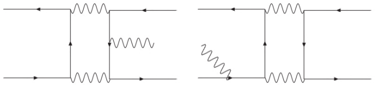
Samples of box diagrams



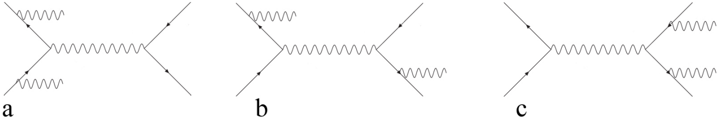
Samples of triangle diagrams



Samples of pentabox diagrams



$\sigma(e^+e^- \to \text{muons} + \text{two hard photons})$



Samples of contributions with two real photon emission for the process $e^+e^- \to \mu^+\mu^-\gamma$

What's new in PHOKHARA for $e^+e^- \to \mu^+\mu^-\gamma$?

- FSR emission of two hard photon, full amplitude for processes with one ISR photon and one FSR, interferences between all diagrams with emission of two photons;
- full soft part when one photon is soft and one is hard;
- all pentagons, box diagrams with radiation from final state and all interferences between box diagrams and Born;
- full corrections with triangles.

Tests in PHOKHARA for $e^+e^- \to \mu^+\mu^-\gamma$

- gauge invariance tests;
- test for the agreement between trace and helicity method for amplitudes with hard emission of two photons: for 2 FSR photon and 1 FSR and 1 ISR;
- tests for soft part for analytical formula and numerical integral;
- tests for soft part for agreement between quad and double precision for analytical formula;
- tests for cut independence of the sum of hard and soft part ;
- tests for agreement between two independent subroutines for all virtual corrections;

Virtual corrections

Two independent versions of codes for pentagons and box diagrams were used for tests and preliminary calculations:

- quad precision version (F. Campanario, G. Rodrigo);
- double precision version PJFry (V. Yundin, T. Riemann);

Two independent versions of subroutines for triangle diagrams were checked:

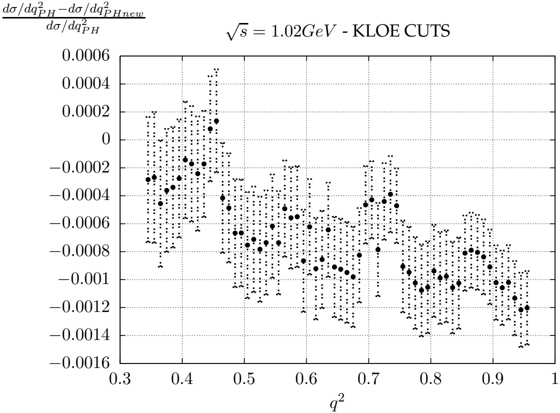
- extended version of PHOKHARA subroutine for triangle diagrams (double precision);
- double precision version PJFry (V. Yundin);

They were compared in points obtained from MC PHOKHARA generator. Also two version of PHOKHARA were created - double precision with PJFry and quad with the second version of virtual routines.

Example: Tests for differential cross sections $d\sigma_{1ph}$

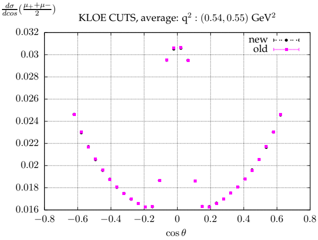
DIFFERENTIAL CROSS SECTION $d\sigma_{1ph}$				
	1.02 GeV	10.56 GeV	KLOE	BaBar
$\lvert\Delta_v\rvert$	number of events			
$< 10^{-14}$	2586	0	3130	0
$(10^{-14}, 10^{-13} >$	23179	18	28037	9
$(10^{-13}, 10^{-12} >$	231973	172	280415	99
$(10^{-12}, 10^{-11} >$	2245557	1732	2715161	882
$(10^{-11}, 10^{-10} >$	6280800	17514	6661834	9040
$(10^{-10}, 10^{-9} >$	1014394	176716	246579	92877
$(10^{-9}, 10^{-8} >$	175617	2194903	49486	3689968
$(10^{-8}, 10^{-7} >$	22226	5545970	10006	6206948
$(10^{-7}, 10^{-6} >$	2253	1564665	3229	151
$(10^{-6}, 10^{-5} >$	628	383219	988	18
$(10^{-5}, 10^{-4} >$	266	72470	340	8
$(10^{-4}, 0.001 >$	489	33022	795	0
$(0.001, 0.01 >$	31	8555	0	0
$(0.01, 0.1 >$	1	919	0	0
> 0.1	0	125	0	0

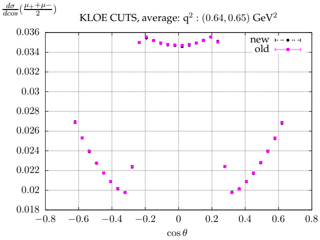
$\Delta = \frac{quad - double}{quad}$ - "double" means double precision version of PHOKHARA and "quad" means quad precision version of PHOKHARA. $d\sigma_{1ph} = d\sigma(e^+e^- \rightarrow \mathrm{muons + one\ hard\ photon})$

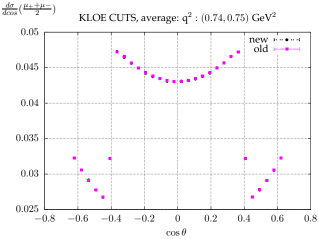


$\sigma_{PHnew} = 3.1144(2)$ nb
$\sigma_{PH} = 3.11710(3)$ nb
$\frac{\sigma_{PH} - \sigma_{PHnew}}{\sigma_{PH}} = 0.086(5)$ %

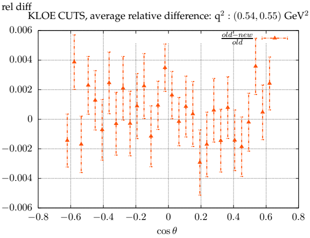
Angular distribution

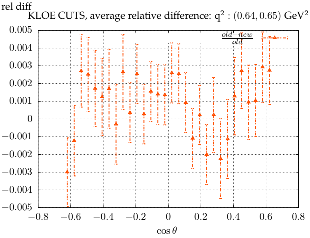


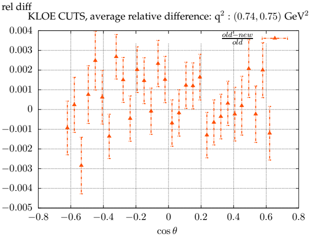




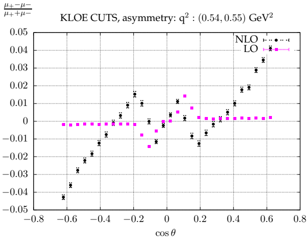
Angular distribution - relative differences

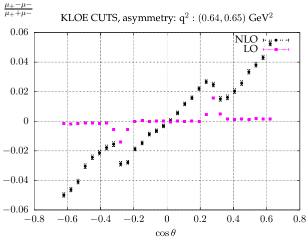


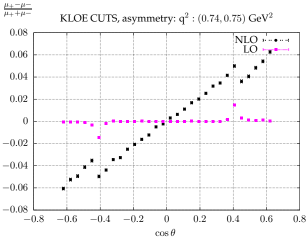


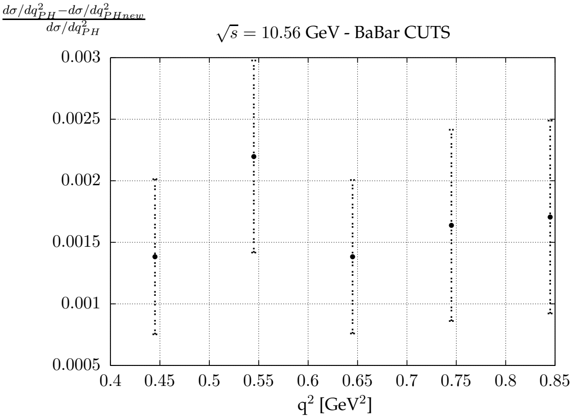


Asymmetry


$\frac{\mu_+-\mu_-}{\mu_++\mu_-}$
KLOE CUTS, asymmetry: q^2 : (0.54, 0.55) GeV^2
NLO
LO
cos θ

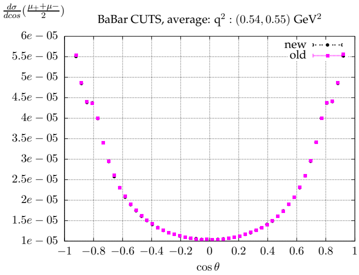

$\frac{\mu_+-\mu_-}{\mu_++\mu_-}$
KLOE CUTS, asymmetry: q^2 : (0.64, 0.65) GeV^2
NLO
LO
cos θ

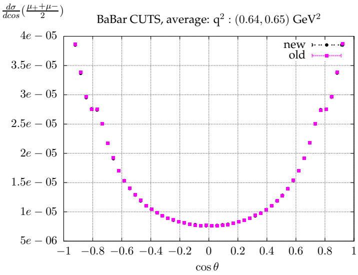

$\frac{\mu_+-\mu_-}{\mu_++\mu_-}$
KLOE CUTS, asymmetry: q^2 : (0.74, 0.75) GeV^2
NLO
LO
cos θ



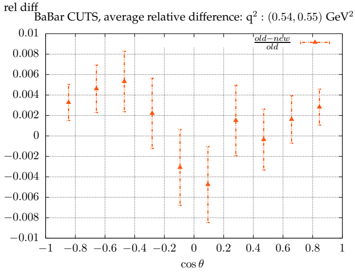
$$\frac{\sigma_{PH} - \sigma_{PHnew}}{\sigma_{PH}} = 0.17(3)\ \%$$

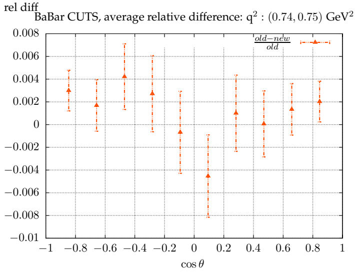
Angular distribution



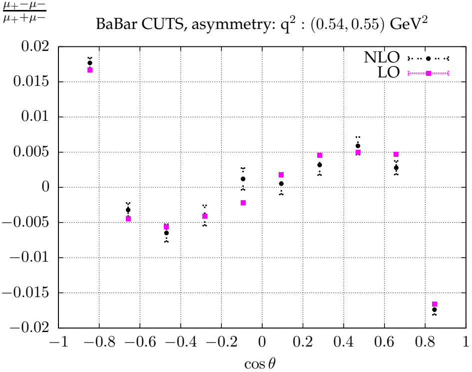


Angular distribution - relative differences





Asymmetry

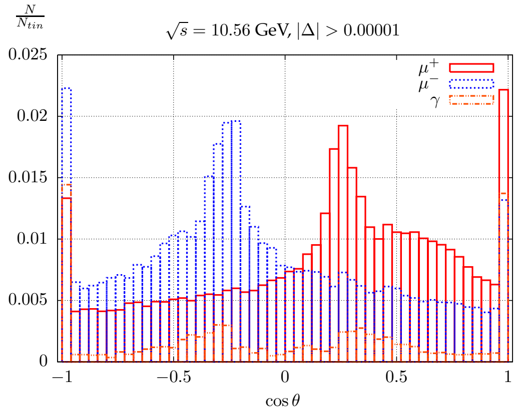


For BaBar cuts asymmetry is dominated by LO part of corrections. The experimental cuts and energy suppress additional influence of NLO corrections

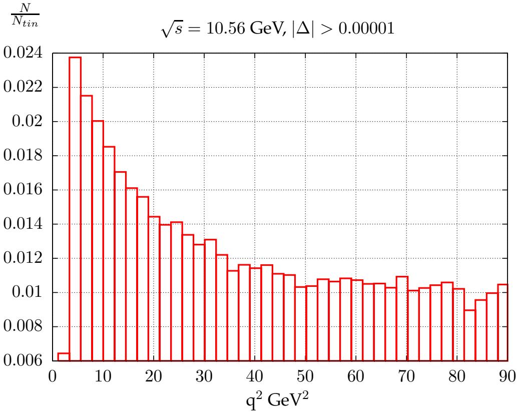
Conclusions

- new version of PHOKHARA9.0 MC generator contains full one loop corrections to reaction $e^+e^- \to \mu^+\mu^-\gamma$;
- the distributed version of generator contains PJFry routine;
- additional corrections inside PHOKHARA MC generator change results for BaBar and KLOE at the level of few ‰- for KLOE cross section 0.086(5) % and for BaBar 0.17(3) %;

Additional slides



Angular distribution for points with $|\Delta| > 0.00001$ and $\sqrt{s}$ =10.56 GeV. Y -axis shows ratio of the number of points with $|\Delta| > 0.00001$ N in the $\cos\theta$ interval to the total number of points N_{tin} in this same range



q^2 distribution for points with $|\Delta| > 0.00001$ and $\sqrt{s}$ =10.56 GeV. Y -axis shows ratio of the number of points with $|\Delta_v| > 0.00001$ N in the q^2 interval to the total number of points N_{tin} in this same range

CUTS

KLOE

- $\sqrt{s} = 1.02$GeV
- tracks between 50° and 130°
- missing photon angle$< 15^\circ (> 165^\circ)$
- $80 < m_{trk} < 115$ MeV

CUTS

BaBar

- $\sqrt{s} = 10.56$GeV
- tracks between 20° and 160°
- 3 GeV minimal photon energy/missing energy
- $|q_1| > 1$GeV (antimuon) and $|q_2| > 1$GeV (muon)