MATTER TO THE DEEPEST

1-6 September 2013

QED radiative corrections for $\gamma\gamma$-production of hadrons in e^+e^- scattering

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Outline

- ✓ Introduction
- ✓ Rad.corrs
- ✓ Summary

Some definitions for this talk

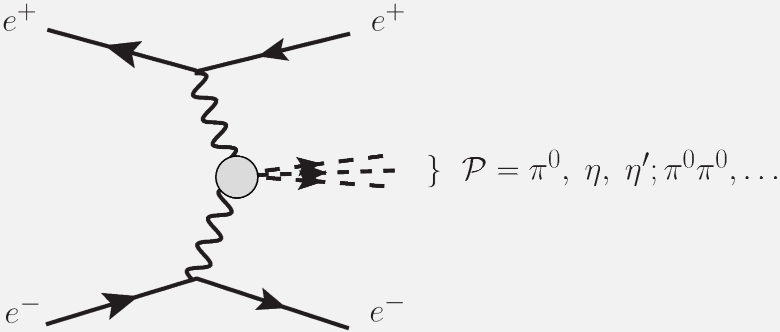
$e^+e^- \to e^+e^-\gamma^*\gamma^* \to e^+e^-\mathcal{P}$

Gamma-gamma production of hadrons —
the exclusive particle production via the *'photon-photon fusion'* sub-process of the e^+e^- scattering.

Agenda:

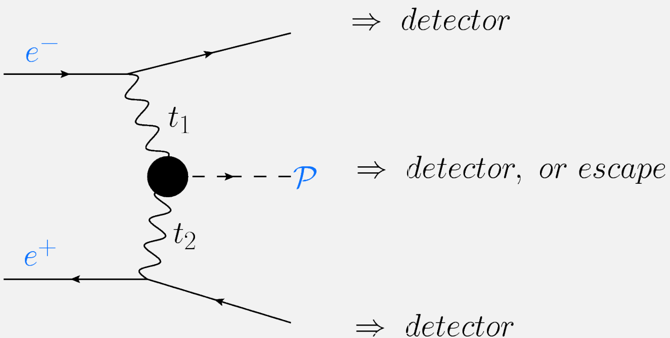
- focus on the *low s* physics case:
  $\sqrt{s}$ up to few GeV
- carry the QED calculations *at NLO*

Gamma-gamma production of hadrons



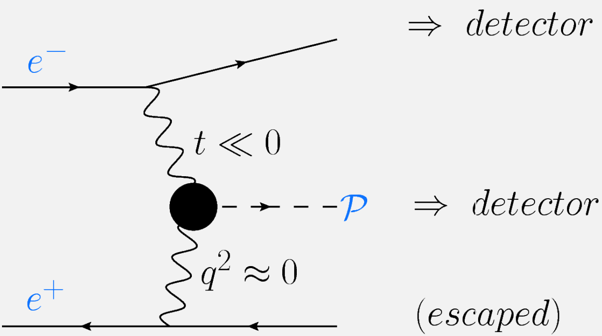
- the photons are being exchanged in *t*-channel ($e^+e^- \to e^+e^-\gamma^*\gamma^*$ part)
- one, two, at most three particles produced ($\gamma^*\gamma^* \to \mathcal{P}$ part)

Gamma-gamma experiment. Double-tag



- By detecting (tagging) both leptons, we get access to both variables: t_1 and t_2

Gamma-gamma experiment. Single-tag



- By tagging only one lepton, we measure only one invariant, $Q^2 = -t$

Motivation

Ongoing/planned data analyses:

- KLOE-2: $\sqrt{s} \approx M_\phi$
  - $\mathcal{P} = \pi^0$

    [D.Babusci et al., Eur.Phys.J. C72 (2012) 1917]
  - $\mathcal{P} = \pi^0\pi^0$, η

    [I.Prado Longhi in *MesonNet 2013 mini-proceedings* (arXiv:1308.2575)]
- BES-III: $\sqrt{s} \approx \psi(2S)$
  - $\mathcal{P} = \pi^0$, η, η', $\pi^0\pi^0$

    [C.Redmer in *MesonNet 2013 mini-proceedings* (arXiv:1308.2575)]

Our aim:

- to help our colleagues working on very challenging and exciting experimental data

Physics context

Two-photon transition form factors

- Shape, slope

  [Czyż, Ivashyn, Korchin, Shekhovtsova, Phys.Rev. D85 (2012) 094010]

- Asymptotics

  [Dorokhov, JETP Lett. 92 (2010) 707]

  [Bakulev, Mikhailov, Pimikov, Stefanis, Phys.Rev. D86 (2012) 031501, D87 (2013) 094025]

- Impact on muon $g - 2$

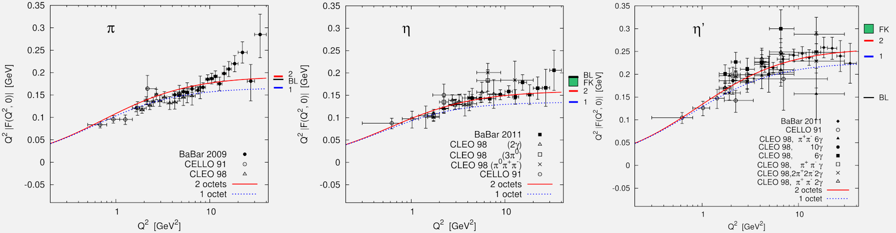
  [Jegerlehner, Nyffeler, Phys.Rept. 477 (2009) 1]

  [Babusci et al., Eur.Phys.J. C72 (2012) 1917]

  [Zhevlakov, Radzhabov, Dorokhov, Russ.Phys.J. 53 (2010) 625]

Transition form factors

π^0 η η'

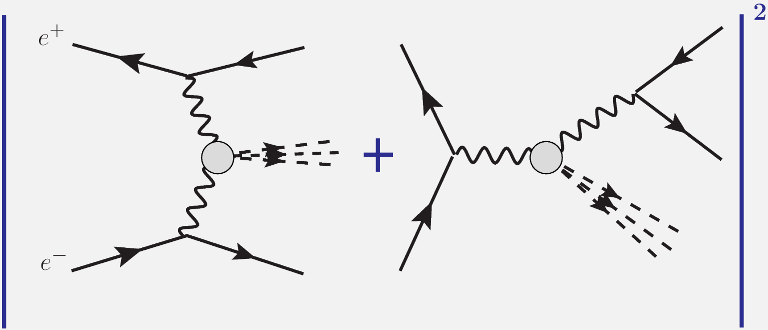


- Form factors are calculated in resonance effective chiral theory and implemented in EKHARA

[Czyż, Ivashyn, Korchin, Shekhovtsova, Phys.Rev. D85 (2012) 094010]

`http://prac.us.edu.pl/~ekhara`

Tree level (LO)



- Monte Carlo generator EKHARA

  [Comput.Phys.Commun., 182 (2011) 1338]

  `http://prac.us.edu.pl/~ekhara`
- Now we need to implement rad.corrs

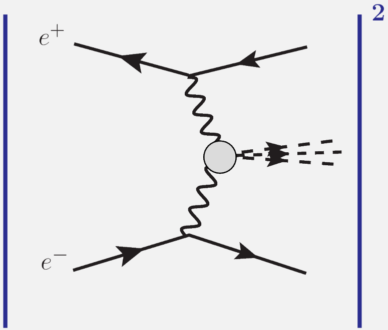
Types of rad.corrs

Leading corrections

- virtual correction to the vertex
- soft photon emission
- hard photon emission
- self energy / vacuum polarization
- box diagrams (extra photon exchange)

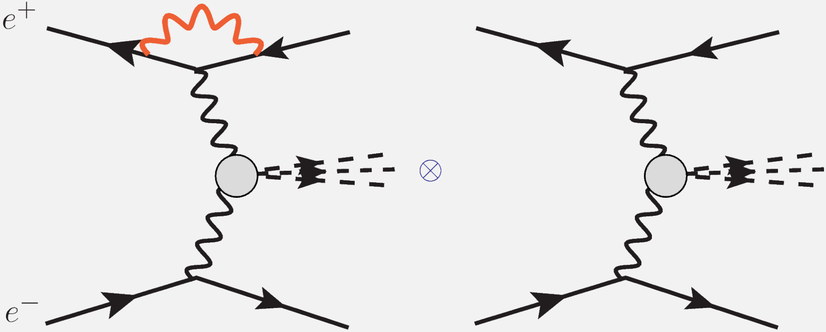
Soft photon emission

$(\cdots) \times$



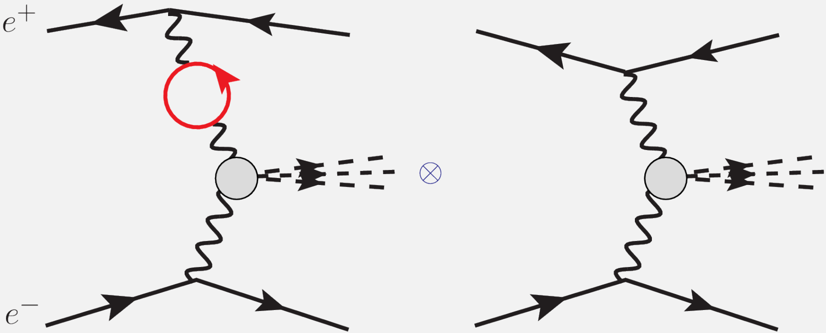
- Soft is what we never observe
- Contains infra-red divergent part
  (to be cancelled by virtual corrections)
- M_0 — separation of hard and soft photon

Virtual. Positron line



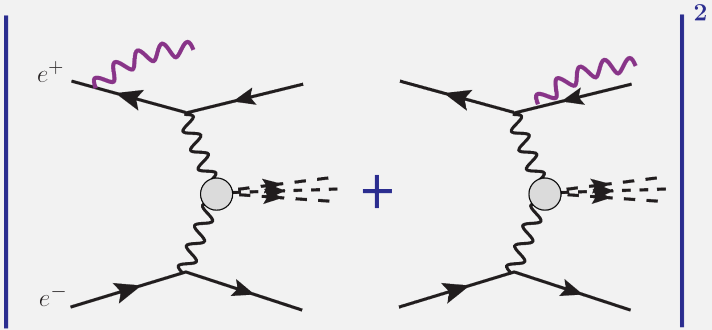
- IR-divergent part cancels by soft corrs
- similar correction for the electron line

Virtual. Vacuum polarization



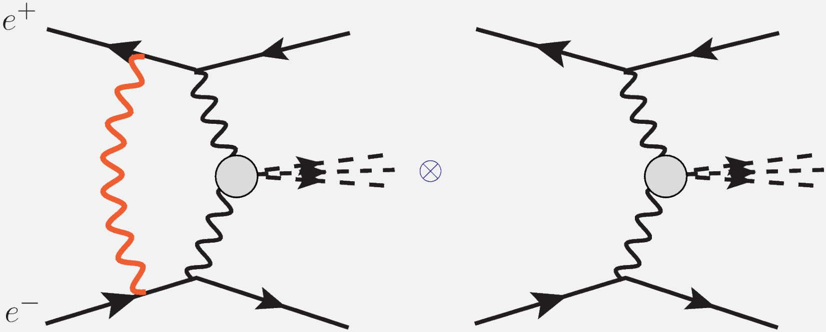
- e^-, μ^-, τ^- loops are there
- hadronic vac.pol. is also there
- similar correction for the electron line

Hard photon emission. Positron line



- M_0 — separation of hard and soft photon (matching)
- +1 particle in the final state
- similar correction for the electron line

Pentabox contribution



"contributions of the five-point functions will always be negligible or irrelevant"

[van Neerven, Vermaseren, Phys.Lett. B142 (1984) 80]

Standard approach to rad.corrs

- Approximations for "Soft+Virtual"
  - small t (leading terms in Q^2/m_e^2)
  - large t (leading terms in m_e^2/Q^2)

- Intergrated hard photon emission
  - such a way that $|T_{Hard}|^2 \Rightarrow \delta_H \times |T_{LO}|^2$

$$\frac{d\sigma}{dQ^2} \Rightarrow \left(1 + \delta(Q^2)\right) \times \frac{d\sigma_0}{dQ^2}$$

or

$$\sigma \Rightarrow \left(1 + \delta(Q^2)\right) \times \sigma_0$$

Example: approx. rad.corrs

Ong, Kessler (1988)

[Ong, Kessler, Phys.Rev. D38 (1988) 2280]

[Ong, Carimalo, Kessler, Phys.Lett. B142 (1984) 429]

- Used in data analyses by BaBar

  [Aubert et al., Phys.Rev. D80 (2009) 052002]

  [del Amo Sanchez at al., Phys.Rev. D84 (2011) 052001]

  [Lees et al., Phys.Rev. D81 (2010) 052010]

- In a modified form, it is implemented in GGRESRC

  [Druzhinin, Kardapoltsev, Tayursky, arXiv:1010.5969]

Ong, Kessler. Final formula

Rad.corr for single-tag case

$$\delta = \delta_{V+S} + \delta_{HI} + \delta_{HF}$$
$$= -\frac{\alpha}{\pi}\left(\left(\ln\frac{1}{r_{max}} - \frac{17}{12}\right)(L-1) + \frac{25}{36}\right)$$

✓ depends only on Q^2 (via L) and r_{max}

- $L \equiv \ln\frac{Q^2}{m^2}$
- r_{max} is maximal energy of undetected ISR hard photon normalized to the beam energy, typically: $r_{max} \ll 1$

Example: exact, but integrated

[Landrø, Mork, Olsen, Phys.Rev. D36 (1987) 44]

- Start with the *exact* formulae for the rad.corrs
- Integrate the hard-photon spectrum

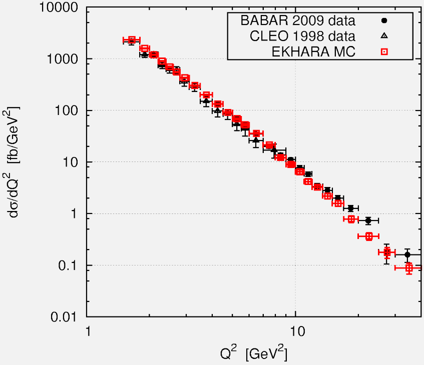
- Total rad.corr is then given only by the *vac.pol.* contributions
  (the rest gets fully cancelled)

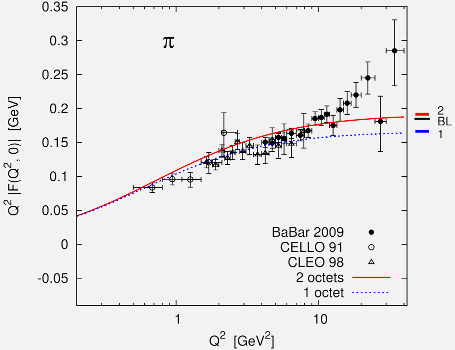
Final formula

$\delta = \delta_{vac.pol.}$

Why integrated results are bad

- intergrated form is good for analytical exercises
- for MC one needs unintegrated expressions



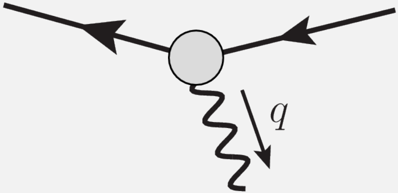


- differential cross section spans more than five orders of magnitude
- integrated rad.corr with small uncertainty can in fact be dramatically biased in the tail

Our approach to rad.corrs

- use exact QED formulae
- no analytic integration of hard photon spectrum
- implement in MC
- make it numerically efficient

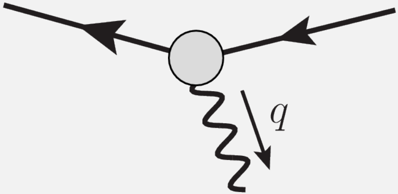
Vertex corrections



$$\Gamma^\mu = \frac{\alpha}{\pi}\left(\gamma^\mu\ F_1(q^2)\ -\ \frac{i}{2\ m}\sigma^{\mu\nu}q_\nu\ F_2(q^2)\right)$$

$$\sigma^{\mu\nu} \equiv \frac{i}{2}\left[\gamma^\mu,\ \gamma^\nu\right]$$

Vertex corrections. Exact formula

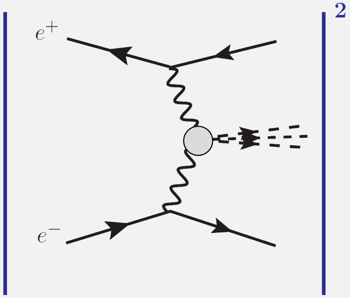


[Barbieri, Mignaco, Remiddi, Nuovo Cim. A11 (1972) 824]

$$F_1(t) = -\log\frac{\lambda}{\theta}\left(1+\frac{1+\theta^2}{1-\theta^2}\log\theta\right) - 1 - \frac{3\theta^2+2\theta+3}{4(1-\theta^2)}\log\theta$$
$$+\frac{1+\theta^2}{1-\theta^2}\left(-\frac{1}{4}\log^2\theta+\frac{1}{2}\zeta(2)+Li_2(-\theta)+\log\theta\log(1+\theta)\right)$$
$$F_2(t) = \frac{-\theta}{1-\theta^2}\log\theta \qquad \theta = -\frac{1-\sqrt{1-4m^2/t}}{1+\sqrt{1-4m^2/t}}$$

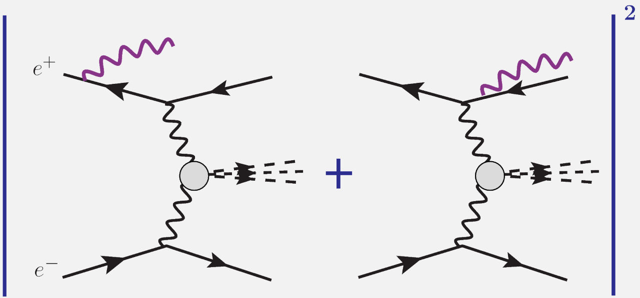
- λ is the infra-red regulator (“fictious photon mass”)

Soft photon emission. Formulae

$(\cdots) \times$ 

- The code from [Carloni Calame, Czyż, Gluza, Gunia et al., JHEP (2011)]
- Can be applied in any reference frame
- Contains infra-red regulator λ
- M_0 — separation between soft and hard photon

Hard Photon emission. Formulae



- $+1$ particle in the final state
- Misprint/sanity check:
  - ✓ helicity amplitudes
  - ✓ trace method

Work in progress

- Check independence of result from
  - ✓ IR regulator λ
  - ✓ Soft-Hard matching scale M_0

- Developing efficient mappings
- To compare with GGRESRC

Summary

- Exact formulae for QED leading rad.corrs are applied for $e^+e^- \to e^+e^-\gamma^*\gamma^* \to e^+e^-\mathcal{P}$
- For neutral hadrons — same rad.corrs
- Implementing in EKHARA MC generator
- Expect this to assist the ongoing data analyses (BES-III, KLOE-2, …)