

# Associated Higgs production at NLO with GoSam

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Max-Planck-Institut für Physik  
(Werner-Heisenberg-Institut)

Alexander von Humboldt  
Stiftung / Foundation



# Publications

Based on the work described in:

- ▶ S. Alioli, S. Badger, ..., HvD, ... et al., “*Update of the Binoth Les Houches Accord for a standard interface between Monte Carlo tools and one-loop programs*” arXiv: 1308.3462
- ▶ HvD, G. Luisoni, P. Mastrolia, E. Mirabella, G. Ossola and T. Peraro, arXiv:1308.3462 [hep-ph]. “*NLO QCD correction to Higgs boson production in association with a top quark pair and a jet*”, arXiv:1307.8437 [hep-ph].
- ▶ G. Cullen, HvD, N. Greiner, G. Luisoni, P. Mastrolia, E. Mirabella, G. Ossola., T. Peraro, and F. Tramontano, “*NLO QCD corrections to Higgs boson production plus three jets in gluon fusion*”, arXiv:1307.4737 [hep-ph].
- ▶ S. Heinemeyer, C. Mariotti, ..., HvD, ... et al., “*Handbook of LHC Higgs Cross Sections: 3. Higgs Properties*”, arXiv:1307.1347 [hep-ph]
- ▶ HvD, N. Greiner, G. Luisoni, P. Mastrolia, E. Mirabella, G. Ossola., T. Peraro, J.F. von Soden-Fraunhofen, and F. Tramontano, “*NLO QCD corrections to the production of Higgs plus two jets at the LHC*,” Phys. Lett. B **721**, 74 (2013).
- ▶ P. Mastrolia, E. Mirabella, G. Ossola., T. Peraro, HvD, “*The Integrand Reduction of One- and Two-Loop Scattering Amplitudes*” PoS LL2012 028 (2012).

# Outline

Motivation

GoSam

Interfaces with external MCs

Scattering amplitudes at one-loop

Determining the parametric form of the numerator

Extended rank numerator

Higgs + 2 jets in GF

Higgs + 3 jets in GF

Higgs in association with top pair and jet

Higgs plus 2 and 3 jets in VBF

Conclusions and Outlook

# Motivation

# Motivation

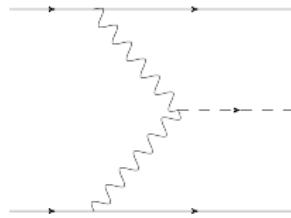
- ▶ Boson discovered by Atlas and CMS → Higgs?

# Motivation

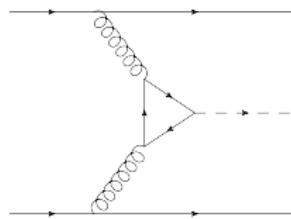
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- ▶ Need to determine properties:  
spin, CP properties, couplings

# Motivation

- ▶ Boson discovered by Atlas and CMS → Higgs?
- ▶ Need to determine properties:  
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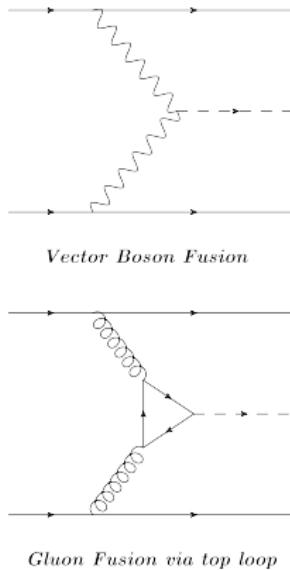
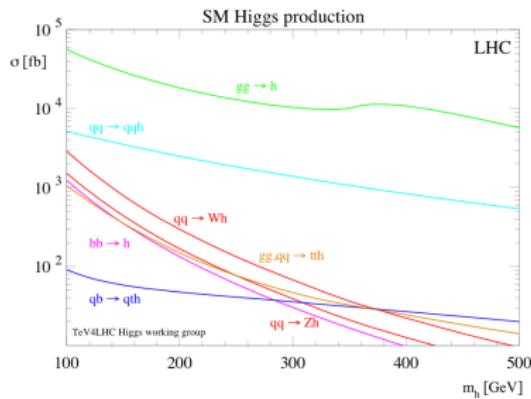
*Vector Boson Fusion*



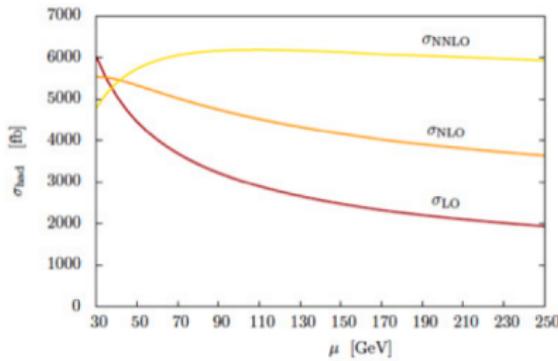
*Gluon Fusion via top loop*

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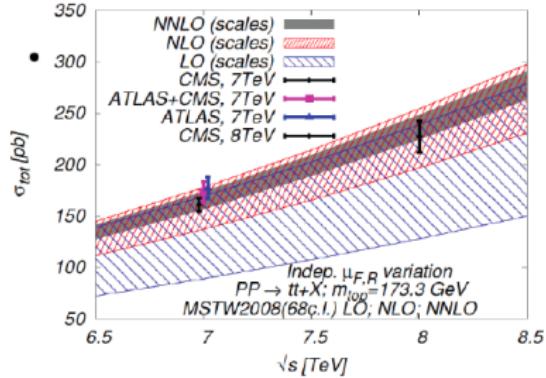


# Motivation for NLO



Higgs plus jet at NNLO

[Boughezal et al. (2013)]

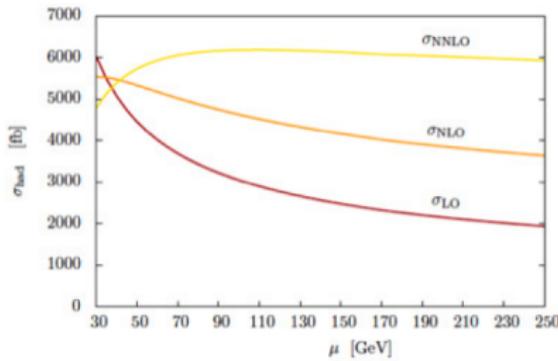


Top pair production at NNLO

[Czakon, Fiedler, Mitov (2013)]

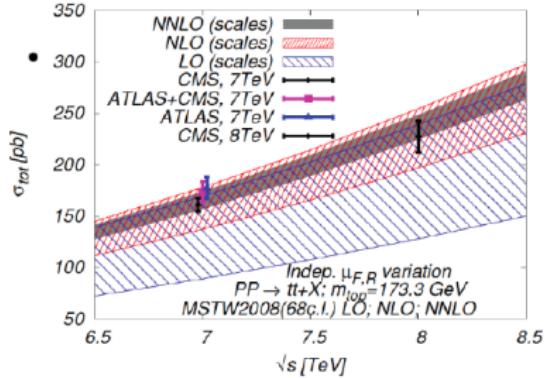
- ▶ Reduce theoretical error
- ▶ Strong dependence on renormalization and factorization scale

# Motivation for NLO



Higgs plus jet at NNLO

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Top pair production at NNLO

[Czakon, Fiedler, Mitov (2013)]

- ▶ Reduce theoretical error
- ▶ Strong dependence on renormalization and factorization scale
- ▶ Development of more general framework for NLO automation

# NLO cross section

$$\sigma^{NLO} = \int_{m+1} \left[ d^{(4)}\sigma^R - d^{(4)}\sigma^A \right] + \int_m \left[ d^{(4)}\sigma^B + \int_{loop} d^{(d)}\sigma^V + \int_1 d^{(d)}\sigma^A \right]$$

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  - ▶ **Real corrections: Radiation**

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  - ▶ **Subtraction terms to regulate infinities**

# NLO cross section

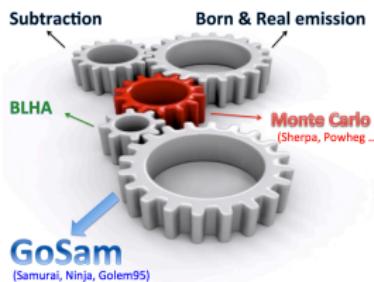
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- ▶ NLO cross section consists of:
  - ▶ Leading Order: Born diagram
  - ▶ Virtual corrections: loop diagrams ← **GoSam provides this part**
  - ▶ Real corrections: Radiation
  - ▶ Subtraction terms to regulate infinities

# GoSam

The GoSam collaboration:

**G. Cullen, HvD, N. Greiner, G. Heinrich, G. Luisoni, P. Mastrolia, E. Mirabella,  
G. Ossola, T. Peraro, J. Reichel, J. Schlenk, J.F. von Soden-Fraunhofen,  
F. Tramontano**



## SAMURAI

$d$ -dimensional Integrand-Level Reduction → **Current default**  
Automated **Model-independent** Computation of the full **Rational Term**  
Mastrolia, Ossola, Reiter, Tramontano

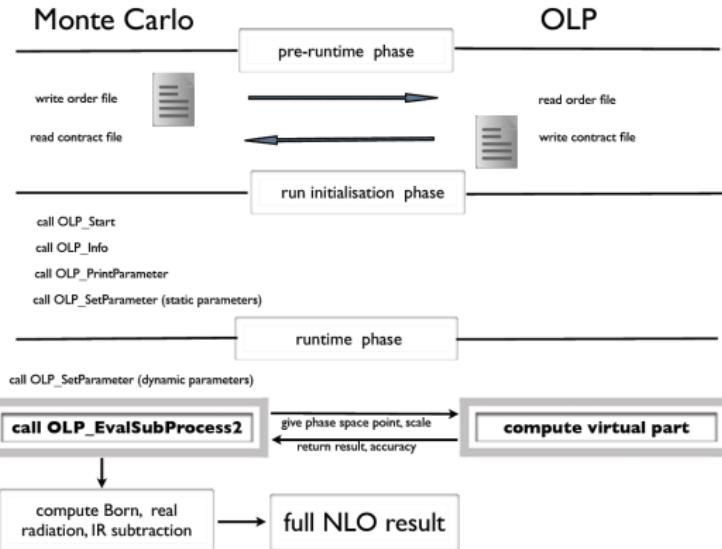
## GOLEM95

Tensorial Reduction → **Rescue System**  
Binoth, Guillet, Heinrich, Pilon, Reiter

## NINJA → Talk of **T. Peraro**

Integrand-Level Reduction + Laurent Expansion → **Stable and Fast!!**  
Mastrolia, Mirabella, Peraro

# Binoth Les Houches Accord



"Update of the Binoth Les Houches Accord for a standard interface between Monte Carlo tools and one-loop programs", arXiv: 1308.3462

[Alioli, Badger, ..., HvD, ... et al. (2013)]

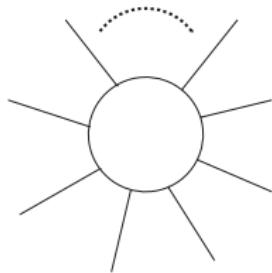
# Interfaces with external MC

- ▶ **GoSam + MadGraph+MadDipole+MadEvent**  
→ ad-hoc interface [Greiner]
- ▶ **GoSam + Sherpa**  
→ via BLHA
- ▶ **GoSam + POWHEG**  
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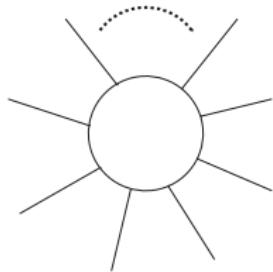
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→ via BLHA [Luisoni, Nason, Oleari, Tramontano]
- ▶ GoSam + HERWIG  
→ *work in progress* [Greiner, Heinrich, von Soden-Fraunhofen]
- ▶ GoSam + AMC@NLO  
→ *work in progress*  
[HvD, Frederix, Frixione, Hirschi, Luisoni, Mastrolia, Ossola, Peraro]

# Scattering amplitudes at one-loop

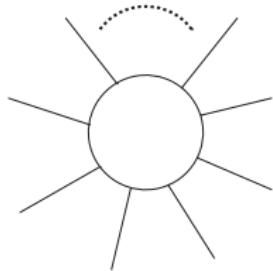


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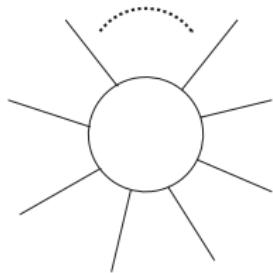
$$\mathcal{M}_n \equiv \int \mathcal{A}_n(\bar{q}) d\bar{q}$$

# Scattering amplitudes at one-loop



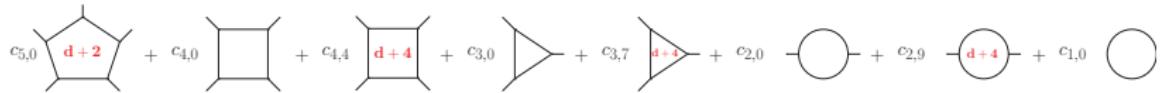
$$\mathcal{M}_n \equiv \int \mathcal{A}_n(\bar{q}) d\bar{q} \equiv \int d^{-2\epsilon}\mu \int d^4 q \frac{N(q, \mu^2)}{\bar{D}_0 \dots \bar{D}_{n-1}}$$

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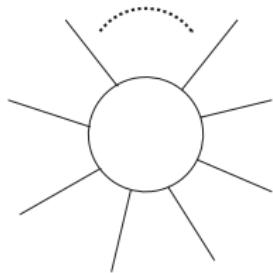


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► Decompose:

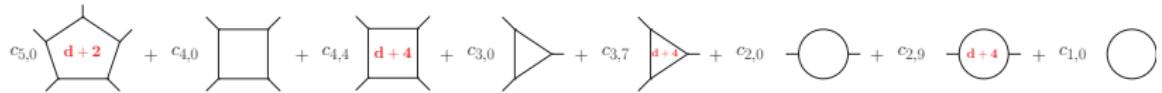


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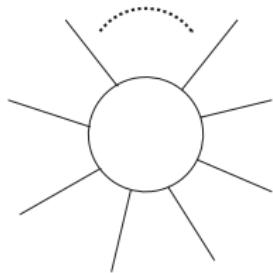
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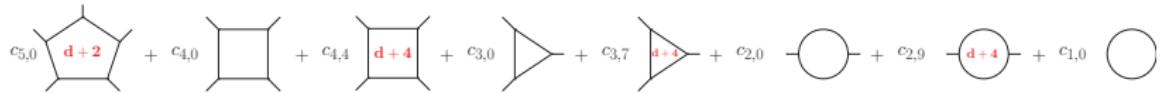
$$\begin{aligned} \int d^{-2\epsilon} \mu^2 d^4 q \mathcal{A}_n(q) &= \int d\bar{q} \frac{c_{5,0} \mu^2}{D_0 D_1 D_2 D_3 D_4} + \int d\bar{q} \frac{c_{4,0} + c_{4,4} \mu^4}{D_0 D_1 D_2 D_3} \\ &\quad + \int d\bar{q} \frac{c_{3,0} + c_{3,7} \mu^2}{D_0 D_1 D_2} + \int d\bar{q} \frac{c_{2,0} + c_{2,9} \mu^2}{D_0 D_1} + \int d\bar{q} \frac{c_{1,0}}{D_0} \end{aligned}$$

# Scattering amplitudes at one-loop



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► computation of  $\mathcal{M}_n \rightarrow$  computation of coefficients

# Integral to Integrand

$$c_{5,0} \text{ (Diagram A)} + c_{4,0} \text{ (Diagram B)} + c_{4,4} \text{ (Diagram C)} + c_{3,0} \text{ (Diagram D)} + c_{3,7} \text{ (Diagram E)} + c_{2,0} \text{ (Diagram F)} - c_{2,9} \text{ (Diagram G)} - c_{1,0} \text{ (Diagram H)}$$

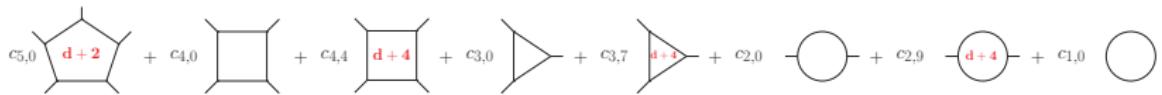
$$\begin{aligned} \int d^{-2\epsilon} \mu^2 d^4 q \mathcal{A}_n(q) = & \int d\bar{q} \frac{c_{5,0} \mu^2}{D_0 D_1 D_2 D_3 D_4} + \int d\bar{q} \frac{c_{4,0} + c_{4,4} \mu^4}{D_0 D_1 D_2 D_3} \\ & + \int d\bar{q} \frac{c_{3,0} + c_{3,7} \mu^2}{D_0 D_1 D_2} + \int d\bar{q} \frac{c_{2,0} + c_{2,9} \mu^2}{D_0 D_1} + \int d\bar{q} \frac{c_{1,0}}{D_0} \end{aligned}$$

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- integral → integrand:

# Integral to Integrand

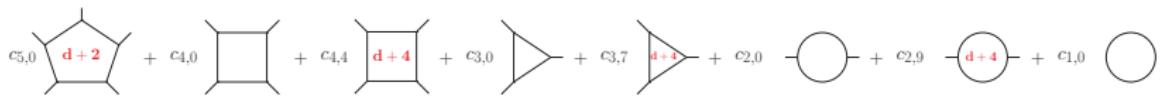


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► integral → integrand:

$$A_n(q) = \frac{c_{5,0}\mu^2 + f_{01234}(q, \mu^2)}{D_0 D_1 D_2 D_3 D_4} + \frac{c_{4,0} + c_{4,4}\mu^4 + f_{0123}(q, \mu^2)}{D_0 D_1 D_2 D_3} \\ + \frac{c_{3,0} + c_{3,7}\mu^2 + f_{012}(q, \mu^2)}{D_0 D_1 D_2} + \frac{c_{2,0} + c_{2,9}\mu^2 + f_{01}(q, \mu^2)}{D_0 D_1} + \frac{c_{1,0} + f_0(q, \mu^2)}{D_0}$$

# Integral to Integrand



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$$\int d^{-2\epsilon} \mu^2 \int d^4 q \frac{f_{ij\dots}(q, \mu^2)}{D_i D_j \dots} = 0$$

## Determining the parametric form of the numerator

# Determining the parametric form of the numerator

$$\mathcal{A}_n = \sum_{ijklm} \frac{\Delta_{ijklm}(q, \mu^2)}{D_i D_j D_k D_l D_m} + \sum_{ijkl} \frac{\Delta_{ijkl}(q, \mu^2)}{D_i D_j D_k D_l} + \sum_{ijk} \frac{\Delta_{ijk}(q, \mu^2)}{D_i D_j D_k} + \sum_{ij} \frac{\Delta_{ij}(q, \mu^2)}{D_i D_j} + \sum_i \frac{\Delta_i(q, \mu^2)}{D_i}$$

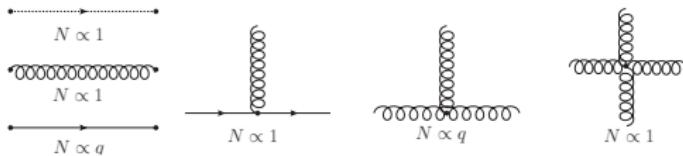
- ▶ Form residues process independent
- ▶ Values of coefficients process dependent

# Determining the parametric form of the numerator

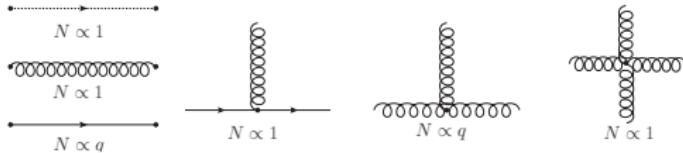
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- ▶ Form residues process independent
- ▶ Values of coefficients process dependent
- ▶ Implemented in **Samurai**  
[Ossola, Reiter, Tramontano, Mastrolia, 2010]

# Rankcounting, normal rank

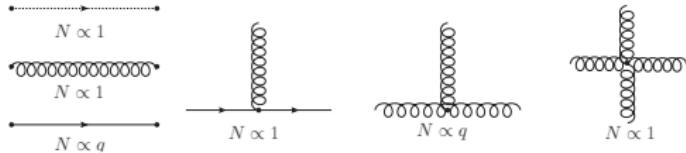


# Rankcounting, normal rank

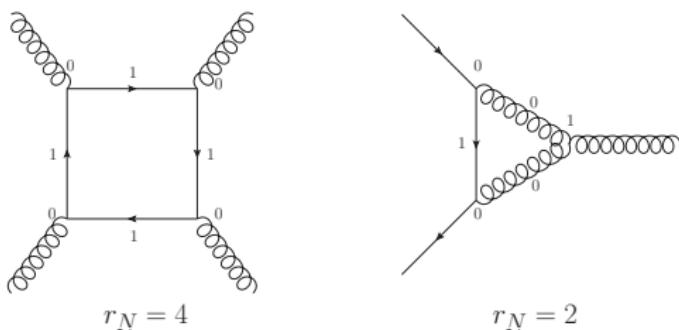


- ▶ Only  $q$  propagators and 3-gluon-vertices contribute one power of  $q$  to numerator

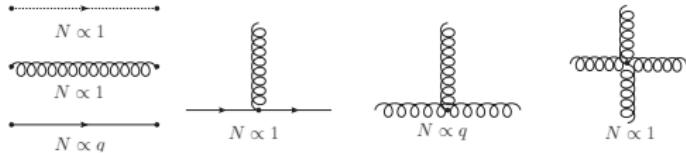
# Rankcounting, normal rank



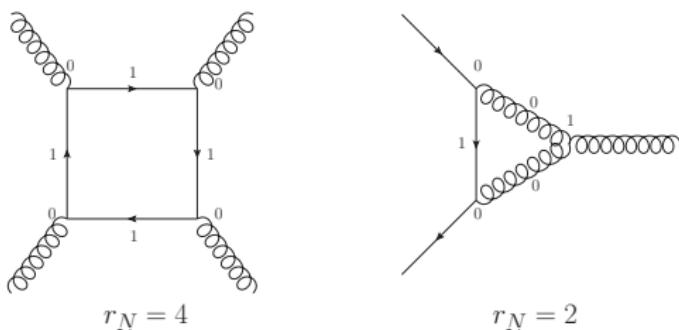
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# Rankcounting, normal rank



- ▶ Only  $q$  propagators and 3-gluon-vertices contribute one power of  $q$  to numerator



- ▶  $r_N \leq \#D$

# Integrand decomposition algorithm



$$\Delta_{ijklm}(\bar{q}) = \text{Res}_{ijklm} \left\{ \frac{N(\bar{q})}{\bar{D}_0 \cdots \bar{D}_{n-1}} \right\}$$

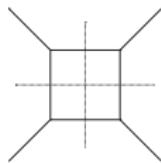
1 coefficient

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1 coefficient



$$\Delta_{ijk\ell}(\bar{q}) = \text{Res}_{ijk\ell} \left\{ \frac{N(\bar{q})}{\bar{D}_0 \cdots \bar{D}_{n-1}} - \sum_{i << m}^{n-1} \frac{\Delta_{ijklm}(\bar{q})}{\bar{D}_i \bar{D}_j \bar{D}_k \bar{D}_\ell \bar{D}_m} \right\}$$

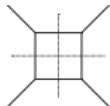
5 coefficients

# Integrand decomposition algorithm



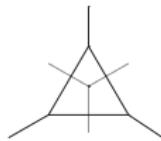
$$\Delta_{ijklm}(\bar{q}) = \text{Res}_{ijklm} \left\{ \frac{N(\bar{q})}{D_0 \cdots D_{n-1}} \right\}$$

1 coefficient



$$\Delta_{ijk\ell}(\bar{q}) = \text{Res}_{ijk\ell} \left\{ \frac{N(\bar{q})}{D_0 \cdots D_{n-1}} - \sum_{i << m}^{n-1} \frac{\Delta_{ijk\ell m}(\bar{q})}{D_i D_j D_k D_\ell D_m} \right\}$$

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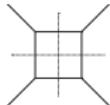
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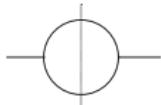
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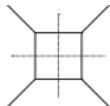
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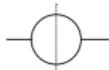
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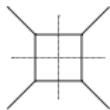
$$\begin{aligned} \Delta_i(\bar{q}) = \text{Res}_i \left\{ \frac{N(\bar{q})}{D_0 \cdots D_{n-1}} - \sum_{i << m}^{n-1} \frac{\Delta_{ijk\ell m}(\bar{q})}{D_i D_j D_k D_\ell D_m} - \sum_{i << \ell}^{n-1} \frac{\Delta_{ijk\ell}(\bar{q})}{D_i D_j D_k D_\ell} + \right. \\ \left. - \sum_{i << k}^{n-1} \frac{\Delta_{ijk}(\bar{q})}{D_i D_j D_k} - \sum_{i < j}^{n-1} \frac{\Delta_{ij}(\bar{q})}{D_i D_j} \right\} \quad 5 \text{ coefficients} \end{aligned}$$

# Integrand decomposition algorithm



$$\Delta_{ijk\ell m}(\bar{q}) = \text{Res}_{ijk\ell m} \left\{ \frac{N(\bar{q})}{D_0 \cdots D_{n-1}} \right\}$$

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10 coefficients

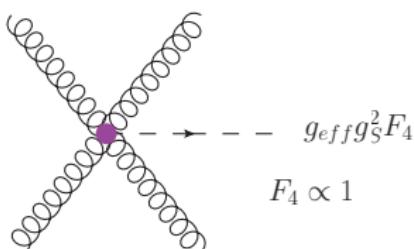
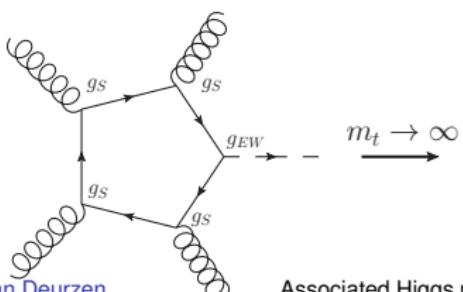
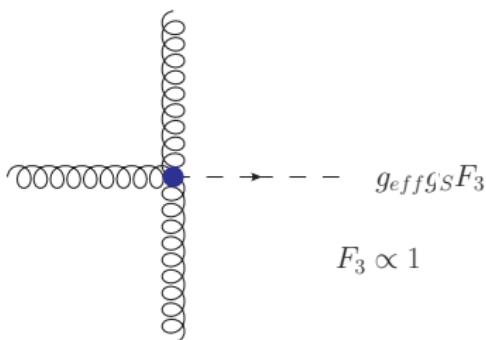
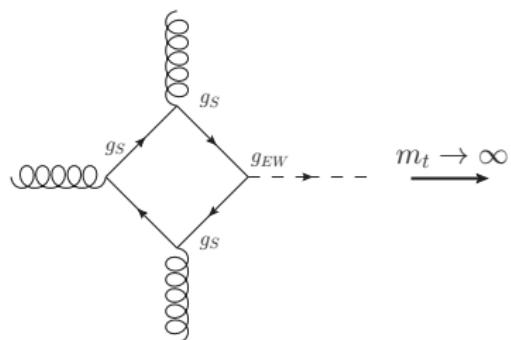
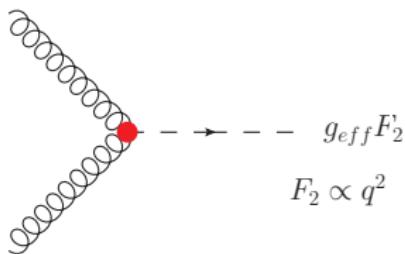
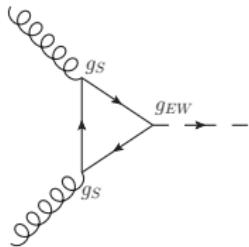


$$\begin{aligned} \Delta_i(\bar{q}) = \text{Res}_i \left\{ \frac{N(\bar{q})}{D_0 \cdots D_{n-1}} - \sum_{i << m}^{n-1} \frac{\Delta_{ijk\ell m}(\bar{q})}{D_i D_j D_k D_\ell D_m} - \sum_{i << \ell}^{n-1} \frac{\Delta_{ijk\ell}(\bar{q})}{D_i D_j D_k D_\ell} + \right. \\ \left. - \sum_{i << k}^{n-1} \frac{\Delta_{ijk}(\bar{q})}{D_i D_j D_k} - \sum_{i < j}^{n-1} \frac{\Delta_{ij}(\bar{q})}{D_i D_j} \right\} \end{aligned}$$

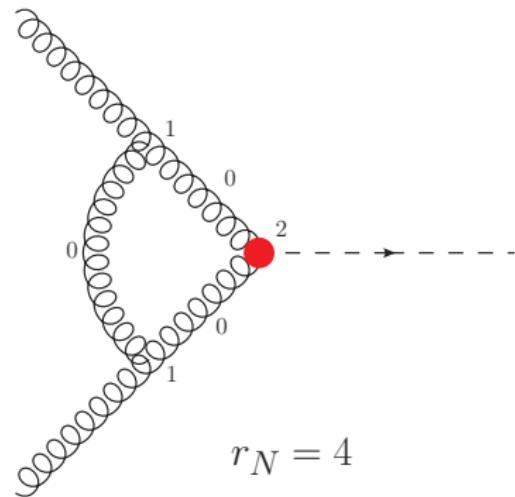
5 coefficients

Hexagon:  $\binom{6}{5} \cdot 1 + \binom{6}{4} \cdot 5 + \binom{6}{3} \cdot 10 + \binom{6}{2} \cdot 10 + \binom{6}{1} \cdot 5 = 386$  coefficients

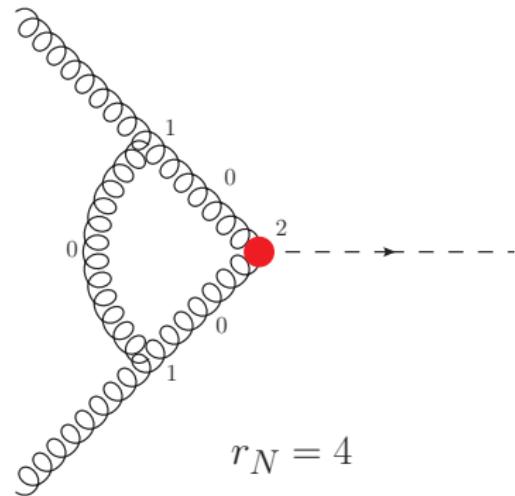
# Effective Vertices



# Rankcounting, higher rank



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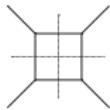
- One effective vertex:  $r_N \leq \#D + 1$

# Extended rank Integrand decomposition algorithm



$$\Delta_{ijk\ell m}(\bar{q}) = \text{Res}_{ijk\ell m} \left\{ \frac{N(\bar{q})}{D_0 \cdots D_{n-1}} \right\}$$

1 → 1 coefficient



$$\Delta_{ijkl}(\bar{q}) = \text{Res}_{ijkl} \left\{ \frac{N(\bar{q})}{D_0 \cdots D_{n-1}} - \sum_{i << m}^{n-1} \frac{\Delta_{ijk\ell m}(\bar{q})}{D_i D_j D_k D_\ell D_m} \right\}$$

5 → 6 coefficients



$$\Delta_{ijk}(\bar{q}) = \text{Res}_{ijk} \left\{ \frac{N(\bar{q})}{D_0 \cdots D_{n-1}} - \sum_{i << m}^{n-1} \frac{\Delta_{ijk\ell m}(\bar{q})}{D_i D_j D_k D_\ell D_m} - \sum_{i << \ell}^{n-1} \frac{\Delta_{ij\ell k}(\bar{q})}{D_i D_j D_k D_\ell} \right\}$$

10 → 15 coefficients



$$\Delta_{ij}(\bar{q}) = \text{Res}_{ij} \left\{ \frac{N(\bar{q})}{D_0 \cdots D_{n-1}} - \sum_{i << m}^{n-1} \frac{\Delta_{ijk\ell m}(\bar{q})}{D_i D_j D_k D_\ell D_m} - \sum_{i << \ell}^{n-1} \frac{\Delta_{ij\ell k}(\bar{q})}{D_i D_j D_k D_\ell} - \sum_{i << k}^{n-1} \frac{\Delta_{ijk}(\bar{q})}{D_i D_j D_k} \right\}$$

10 → 20 coefficients



$$\begin{aligned} \Delta_i(\bar{q}) = \text{Res}_i \left\{ \frac{N(\bar{q})}{D_0 \cdots D_{n-1}} - \sum_{i << m}^{n-1} \frac{\Delta_{ijk\ell m}(\bar{q})}{D_i D_j D_k D_\ell D_m} - \sum_{i << \ell}^{n-1} \frac{\Delta_{ij\ell k}(\bar{q})}{D_i D_j D_k D_\ell} + \right. \\ \left. - \sum_{i << k}^{n-1} \frac{\Delta_{ijk}(\bar{q})}{D_i D_j D_k} - \sum_{i < j}^{n-1} \frac{\Delta_{ij}(\bar{q})}{D_i D_j} \right\} \end{aligned}$$

5 → 15 coefficients

[Mastrolia, Mirabella, Peraro, 2012]

$$\text{Hexagon: } \binom{6}{5} \cdot 1 + \binom{6}{4} \cdot 6 + \binom{6}{3} \cdot 15 + \binom{6}{2} \cdot 20 + \binom{6}{1} \cdot 15 = (386 \rightarrow) 786 \text{ coefficients}$$

► Samurai → XSamurai [HvD et al.]

# Discrete Fourier Transformation (DFT)

- ▶  $\Delta(q, \mu^2)$  multivariate polynomial in  $q$  and  $\mu^2$

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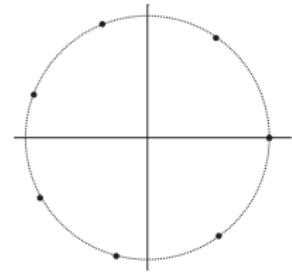
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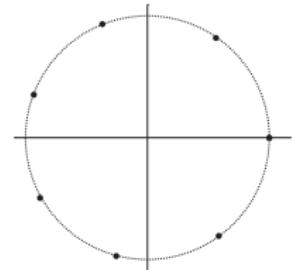
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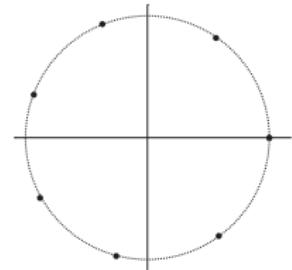
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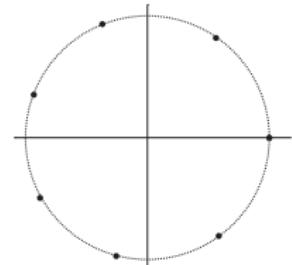
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$$c_l = \frac{\rho^{-l}}{n+1} \sum_{k=0}^n P_k \exp \left[ 2\pi i \frac{k}{n+1} l \right]$$



[Mastrolia, Ossola, Papadopoulos, Pittau (2008)]

# Sampling strategy

- $q = \sum_{i=1}^4 x_i e_i \Rightarrow \mu^2, x_1, x_2, x_3, x_4$  variables

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  - ▶ Use DFT twice,  $\Delta(\mu^2, x_3, C/x_3)$  and  $\Delta(\mu^2 C/x_4, x_4)$  solutions  $\propto \frac{1}{1-C}$ , problem if  $C = 1$

# Sampling strategy

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  - ▶ Use DFT twice,  $\Delta(\mu^2, x_3, C/x_3)$  and  $\Delta(\mu^2 C/x_4, x_4)$  solutions  $\propto \frac{1}{1-C}$ , problem if  $C = 1$
  - ▶ Branching:
    - if( $C=0$ ): Use  $\Delta(\mu^2, x_3, C/x_3)$  and  $\Delta(\mu^2 C/x_4, x_4)$
    - else: Use  $\Delta(\mu^2, x_3, C/x_3)$

# Sampling strategy

- ▶ At double cut:  $\Delta(\mu^2, x_1, x_3, x_4)$  with  $x_3x_4 = F(x_1) = Ax_1^2 + Bx_1 + C$  lot of branchings:

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  - ▶  $F=0$  has no solutions
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# Sampling strategy

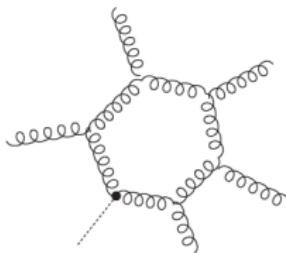
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- ▶ At single cut:  $\Delta(\mu^2, x_1, x_2, x_3, x_4)$  with  $x_3x_4 - x_1x_2 = G$   
similar to the triple cut

# Towards Higgs jets in GF @ NLO

<b>H+0j</b>	<b>1 NLO</b>
$gg \rightarrow H$	1 NLO
<b>H+1j</b>	<b>62 NLO</b>
$qq \rightarrow Hqq$	14 NLO
$qg \rightarrow Hqg$	48 NLO
<b>H+2j</b>	<b>926 NLO</b>
$qq' \rightarrow Hqq'$	32 NLO
$qq \rightarrow Hqq$	64 NLO
$qg \rightarrow Hqg$	179 NLO
$gg \rightarrow Hgg$	651 NLO
<b>H+3j</b>	<b>13179 NLO</b>
$qq' \rightarrow Hqq'g$	467 NLO
$qq \rightarrow Hqgq$	868 NLO
$qg \rightarrow Hqgg$	2519 NLO
$gg \rightarrow Hggg$	9325 NLO

## Computational Challenges:

- ▶ Over 10,000 diagrams
- ▶ Higher-Rank terms
- ▶ 60 Rank-7 hexagons

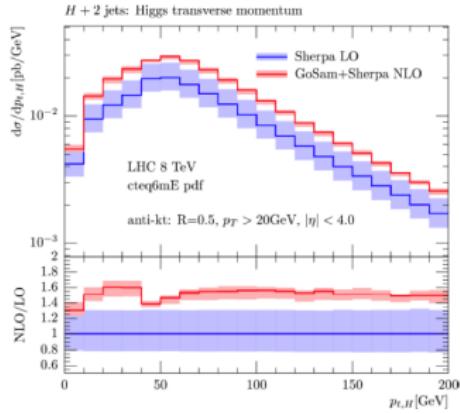
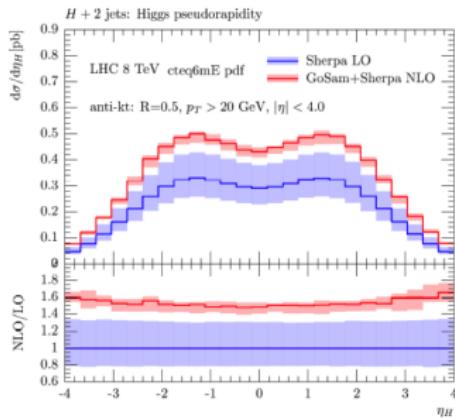


Complex calculations → GoSam enhanced

grouping, optimization through Form4.0, numerical polarization vectors, parallelization

# Higgs + 2 jets in GF @ NLO

- ▶ Results obtained with GoSam+Sherpa
- ▶ Agreement with MCFM (v6.4) [Campbell, Ellis, Williams]



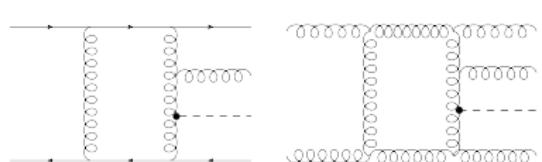
HvD, Greiner, Luisoni, Mastrolia, Mirabella, Ossola, Peraro, von Soden-Fraunhofen, Tramontano (2013)

(also appeared in *Handbook of LHC Higgs Cross Sections: 3. Higgs Properties*)

# Higgs + 3 jets in GF: virtual part

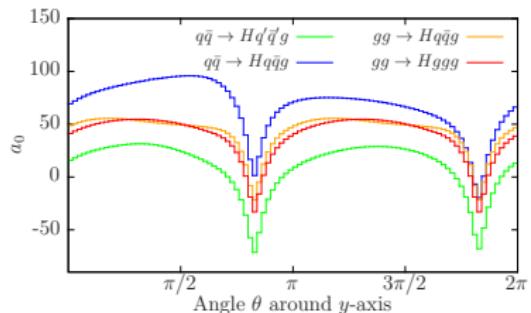
Cullen, HvD, Greiner, Luisoni, Mastrolia, Mirabella, Ossola, Peraro,  
Tramontano, [arXiv:1307.4737](https://arxiv.org/abs/1307.4737)

Virtual parts computed with  
GoSam



SUBPROCESS	DIAGRAMS	TIME/PS-POINT [sec]
$q\bar{q} \rightarrow Hq'\bar{q}'g$	467	0.29
$q\bar{q} \rightarrow Hq\bar{q}g$	868	0.60
$gg \rightarrow Hq\bar{q}g$	2519	3.9
$gg \rightarrow Hggg$	9325	20

$$\frac{2 \operatorname{Re} \left\{ \mathcal{M}^{\text{tree-level}*} \mathcal{M}^{\text{one-loop}} \right\}}{(\alpha_s/2\pi) |\mathcal{M}^{\text{tree-level}}|^2} \equiv \frac{a_{-2}}{\epsilon^2} + \frac{a_{-1}}{\epsilon} + a_0$$



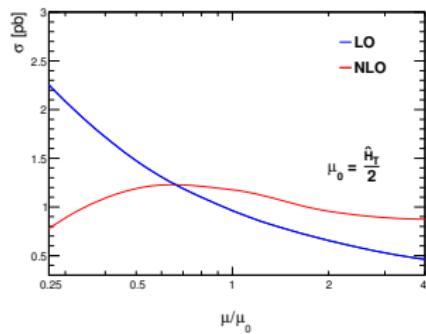
Number of Feynman diagrams and time per  
PS-point point for each subprocess

Tests: gauge invariance and IR poles

# Higgs + 3 jets GF @ NLO: cross-section

Cross sections are obtained with a hybrid setup:

- ▶ GoSam + Sherpa for Born and of the virtual contributions
- ▶ MadGraph+MadDipole+MadEvent for reals/subtraction/integrated dipoles



$$\mu_F = \mu_R = \frac{\hat{H}_T}{2} = \mu_0$$

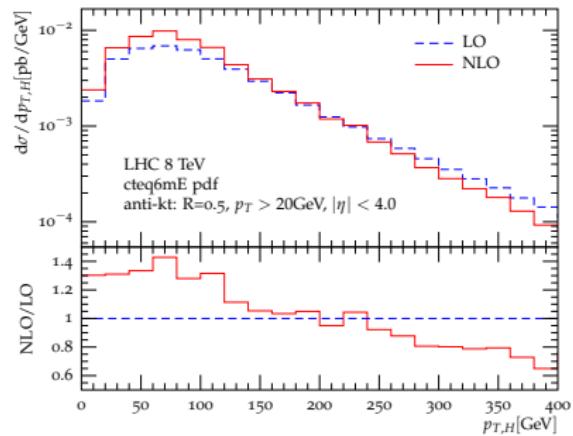
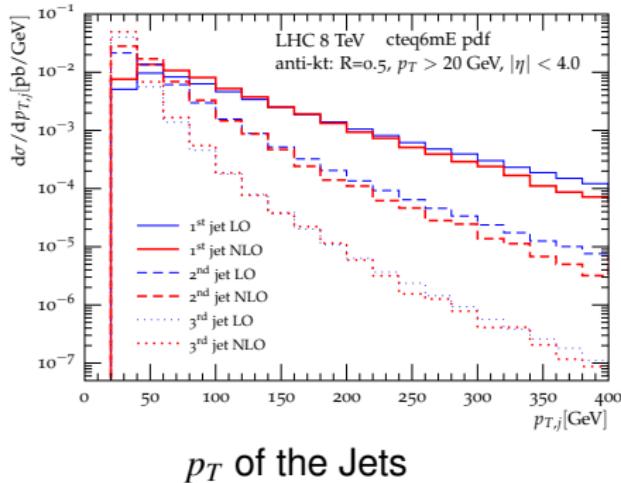
$$\hat{H}_T = \sqrt{m_H^2 + p_{T,H}^2} + \sum_i |p_{T,i}|$$

Tests performed on the cross section:

- ▶ NLO H+2 jets: Agreement between hybrid scheme and GoSam+Sherpa
- ▶ LO H+3 jets: Agreement between MADGRAPH and Sherpa
- ▶ NLO H+3 jets: Independence from  $\alpha$ -parameter (subtraction+int. dipoles)

Cullen, HvD, Greiner, Luisoni, Mastrolia, Mirabella, Ossola, Peraro, Tramontano [arXiv:1307.4737](https://arxiv.org/abs/1307.4737)

# Higgs + 3 jets GF @ NLO: distributions



$pp \rightarrow Hjjj$  generated by GoSam can be paired with available MC programs for further phenomenological analyses.

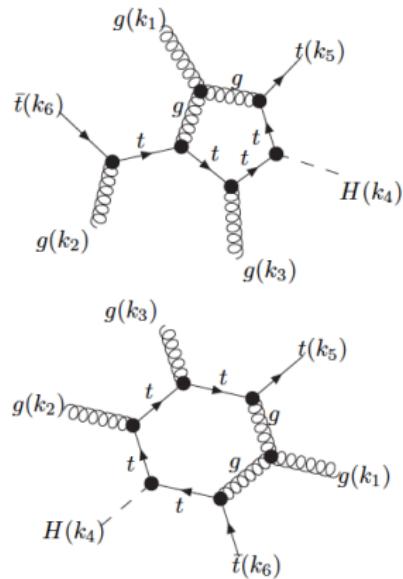
Cullen, HvD, Greiner, Luisoni, Mastrolia, Mirabella, Ossola, Peraro, Tramontano [arXiv:1307.4737](https://arxiv.org/abs/1307.4737)

# $pp \rightarrow Ht\bar{t} + 1 \text{ jet}$ @ NLO

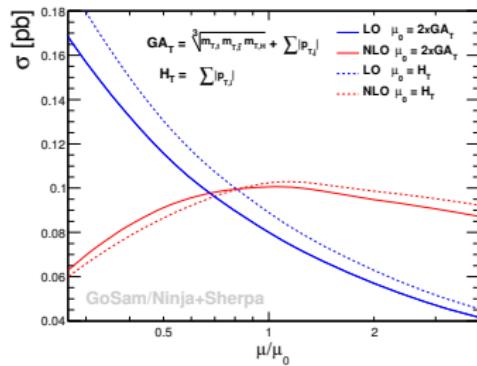
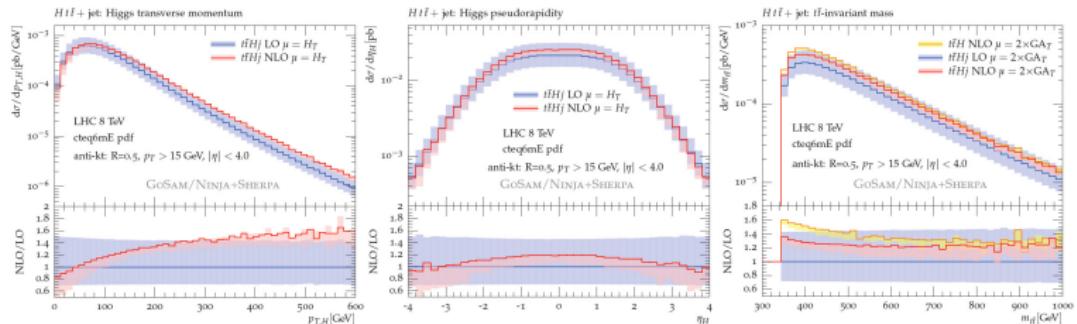
First Application of GoSAM/NINJA + SHERPA → **Talk of T. Peraro**

$t\bar{t}H + 1j$	<b>1895 NLO</b>
$qq \rightarrow Ht\bar{t}g$	320 NLO
$gg \rightarrow Ht\bar{t}g$	1575 NLO

- ▶ Two different mass scales: Higgs and Top
- ▶ 51 hexagons in the gluon-gluon channel



# $H\bar{t}j$ results



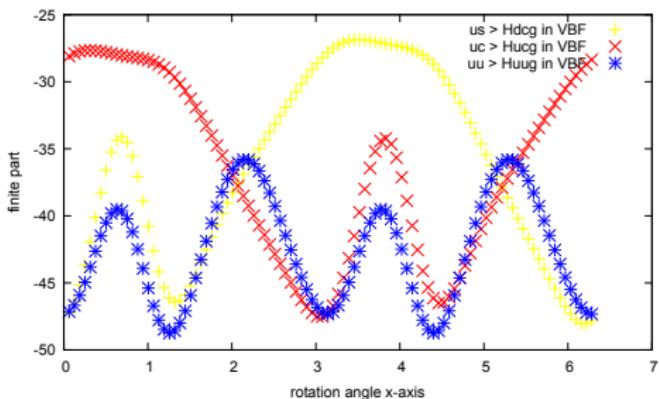
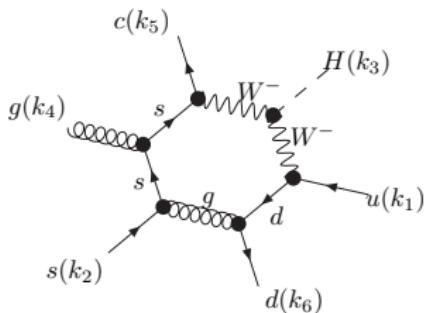
# Vector Boson Fusion

## H+2j      240 NLO

$us \rightarrow Hdc$	24 NLO
$uc \rightarrow Huc$	24 NLO
$us \rightarrow Hus$	24 NLO
$ds \rightarrow Hds$	24 NLO
$ud \rightarrow Hud$	48 NLO
$uu \rightarrow Huu$	48 NLO
$dd \rightarrow Hdd$	48 NLO

## H+3j      2160 NLO

$us \rightarrow Hdgc$	216 NLO
$uc \rightarrow Hucg$	216 NLO
$us \rightarrow Husg$	216 NLO
$ds \rightarrow Hdsg$	216 NLO
$ud \rightarrow Hudg$	432 NLO
$uu \rightarrow Huug$	432 NLO
$dd \rightarrow Hddg$	432 NLO



# Conclusions

- ▶ GoSam is a powerful framework for the automatic computation of one loop virtual amplitudes
- ▶ Samurai has been extended to deal with higher rank numerators: Xsamurai
- ▶ GoSam is interfaced to a lot of Monte Carlo Programs
- ▶ GoSam has been used for a lot of different phenomenology studies, among which
  - ▶ Higgs plus two jet in GF
  - ▶ Higgs plus three jet in GF
  - ▶ Higgs  $t\bar{t}$  jet
  - ▶ Higgs plus two and three jet in VBF in progress

## Outlook

- ▶ Even more processes on the way
- ▶ Interaction with MC and experimental collaborations
- ▶ Additional code improvements towards GoSam 2.0
- ▶ Multi-loop integrand reduction in the making (talk T.Peraro on Thursday)

# BACKUP SLIDES