Current status of constraints on the elements of the neutrino mass matrix

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Outline of the talk

- The light-neutrino mass matrix
- The elements of the neutrino mass matrix — Majorana case
- The elements of the neutrino mass matrix — Dirac case
- Summary

The light-neutrino mass matrix

In this talk by neutrino mass matrix we always mean the

'3×3 light-neutrino mass matrix':

- Neutrinos Majorana particles $\rightarrow$ assume that there is a (effective) mass term

$$\mathcal{L} = -\frac{1}{2}\overline{\nu_L^c} M_\nu \nu_L + \text{H.c.} = \frac{1}{2}\nu_L^T C^{-1} M_\nu \nu_L + \text{H.c.}$$

  with a symmetric 3×3-matrix M_ν. Typical origin: seesaw mechanism.

- Neutrinos Dirac particles $\rightarrow$ assume three right-handed neutrinos leading to the mass term

$$\mathcal{L} = -\overline{\nu_R} M_D \nu_L + \text{H.c.},$$

  where M_D is an arbitrary 3×3-matrix.

I. Majorana neutrinos

The Majorana neutrino mass matrix

Choose a basis in which the charged-lepton mass matrix is given by

$$M_\ell = \mathrm{diag}(m_e,\, m_\mu,\, m_\tau).$$

$$\Rightarrow M_\nu = U^*_{\mathrm{PMNS}} \mathrm{diag}(m_1,\, m_2,\, m_3) U^\dagger_{\mathrm{PMNS}}.$$

Lepton mixing matrix: can be parameterized by six phases and three mixing angles:

$$U_{\mathrm{PMNS}} \equiv U = D_1 V D_2$$

$$D_1 = \mathrm{diag}(e^{i\alpha},\, e^{i\beta},\, e^{i\gamma}), \quad D_2 = \mathrm{diag}(e^{i\rho},\, e^{i\sigma},\, 1),$$

$$V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

$s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$; three mixing angles $\theta_{12}, \theta_{23}, \theta_{13} \in [0^\circ, 90^\circ]$.

Absolute values of the elements of M_ν

$$|(M_\nu)_{\alpha\beta}| = \left| \sum_{k=1}^{3} m_k e^{2i\sigma_k} V_{\alpha k} V_{\beta k} \right|$$

depends on the nine parameters:

$$m_0,\ \Delta m^2_{21},\ \Delta m^2_{31}, \theta_{12},\ \theta_{23},\ \theta_{13}, \delta,\ \rho,\ \sigma.$$

- m_0: bounds from cosmology, $(\beta\beta)_{0\nu}$, ^3H-decay: $m_0 \lesssim 0.3\,\text{eV}$
- $\Delta m^2_{21},\ \Delta m^2_{31}, \theta_{12},\ \theta_{23},\ \theta_{13}, \delta$: global fits of oscillation data,
- $\rho,\ \sigma$: totally unconstrained: treated as free parameters in $[0, 2\pi)$.

Global fits of neutrino oscillation data

Current situation:

parameter	best fit	1σ-range	3σ-range
Δm^2_{21} [10^{-5} eV^2]	7.62	7.43 – 7.81	7.12 – 8.20
$\lvert\Delta m^2_{31}\rvert$ [10^{-3} eV^2]	2.55	2.46 – 2.61	2.31 – 2.74
	2.43	2.37 – 2.50	2.21 – 2.64
$\sin^2\theta_{12}$	0.320	0.303 – 0.366	0.27 – 0.37
$\sin^2\theta_{23}$	$0.427 \oplus 0.613$	$(0.400 - 0.461) \oplus (0.573 - 0.635)$	0.36 – 0.68
	0.600	0.569 – 0.626	0.37 – 0.67
$\sin^2\theta_{13}$	0.0246	0.0218 – 0.0275	0.017 – 0.033
	0.0250	0.0223 – 0.0276	0.017 – 0.033
δ	0.80π	$0 - 2\pi$	$0 - 2\pi$
	1.97π	$0 - 2\pi$	$0 - 2\pi$

[Forero et al., Phys. Rev. D 86 (2012) 073012]

Upper line: normal spectrum, lower line: inverted spectrum.

Analytical upper bound on $|(M_\nu)_{\alpha\beta}|$

Upper bound on matrix elements

The absolute value of an element of a matrix is smaller (or equal) than its largest singular value.

$$\Rightarrow |(M_\nu)_{\alpha\beta}| \leq \max_k m_k.$$

(Holds also for Dirac neutrinos.)

Current cosmological bounds imply

$$\sum_\nu m_\nu \lesssim \mathcal{O}(1\ \text{eV}) \Rightarrow |(M_\nu)_{\alpha\beta}| \lesssim 0.3\ \text{eV}.$$

Analytical lower bound on $|(M_\nu)_{\alpha\beta}|$

Can also give an analytical expression for a lower bound $|(M_\nu)_{\alpha\beta}|$:

$$\text{Majorana neutrinos:} \quad |(M_\nu)_{\alpha\beta}| = \left| \sum_{k=1}^{3} m_k e^{2i\sigma_k} V_{\alpha k} V_{\beta k} \right|$$

$(\sigma_1, \sigma_2, \sigma_3) = (\rho, \sigma, 0)$ ... *unconstrained phases.* $\Rightarrow$ treated as free parameters.

$$a_k \equiv m_k |V_{\alpha k}||V_{\beta k}| \Rightarrow |(M_\nu)_{\alpha\beta}| = \left| \sum_{k=1}^{3} e^{i\varphi_k} a_k \right|$$

with *free parameters* φ_k. $\Rightarrow$

Lower bound

$$|(M_\nu)_{\alpha\beta}| \geq 2 \max_k a_k - \sum_k a_k .$$

Independent of Majorana phases!

Evaluation of the lower bound on $|(M_\nu)_{\alpha\beta}|$

Numerical evaluation of the lower bounds.

→ Non-trivial lower bounds only for two matrix elements (all values in units of eV):

		1σ	2σ	3σ
$|(M_\nu)_{ee}|$ (inv. spectrum)	Forero *et al.*	1.52×10^{-2}	1.36×10^{-2}	1.14×10^{-2}
	Fogli *et al.*	1.62×10^{-2}	1.44×10^{-2}	1.24×10^{-2}
$|(M_\nu)_{\tau\tau}|$ (norm. spectrum)	Forero *et al.*	0	0	0
	Fogli *et al.*	1.86×10^{-2}	1.27×10^{-2}	0

[Forero *et al.*, Phys. Rev. D 86 (2012) 073012; arXiv:1205.4018v3,
Fogli *et al.*, Phys. Rev. D 86 (2012) 013012; arXiv:1205.5254v3].

⇒ Bound on $|(M_\nu)_{ee}|$ for an inverted spectrum still very far away from bounds delivered by searches for $(\beta\beta)_{0\nu}$:

$$m_{\beta\beta} \lesssim 0.4\,\text{eV}$$

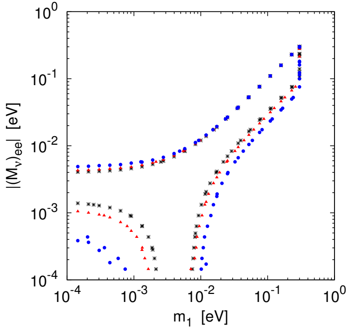
[W. Rodejohann, J.Phys. G39 (2012) 124008; M. Auger *et al.* (EXO Coll.), Phys. Rev. Lett. **109** (2012) 032505]

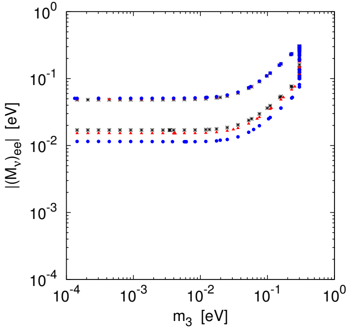
Allowed ranges for the $|(M_\nu)_{\alpha\beta}|$

A. Merle and W. Rodejohann (2006):[1] Plots of the allowed ranges of $|(M_\nu)_{\alpha\beta}|$ versus the smallest neutrino mass m_0.

2012: W. Grimus, POL repeated analysis with new data:[2]

→ At 3σ plots still in agreement with plots of Merle and Rodejohann.





[1] A. Merle and W. Rodejohann, Phys. Rev. D **73** (2006) 073012.

[2] W. Grimus and P.O. Ludl, JHEP **1212** (2012) 117.

Correlations of the elements of the neutrino mass matrix

Idea: Due to improved data $\rightarrow$ correlation plots of $|(M_\nu)_{\alpha\beta}|$

Majorana neutrinos $\Rightarrow$ 6 independent matrix elements

$\Rightarrow$ 15 pairings, 2 spectra $\Rightarrow$ 30 correlation plots.

Among these 30 correlations: Five manifest at 3σ:

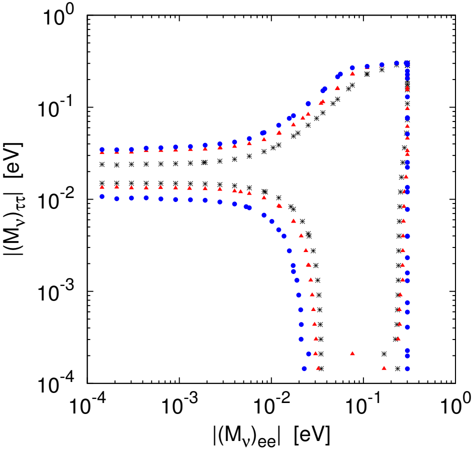
$\|(M_\nu)_{ee}\|$	vs.	$\|(M_\nu)_{\mu\mu}\|$	(normal spectrum)
$\|(M_\nu)_{ee}\|$	vs.	$\|(M_\nu)_{\mu\tau}\|$	(normal spectrum)
$\|(M_\nu)_{ee}\|$	vs.	$\|(M_\nu)_{\tau\tau}\|$	(normal spectrum)
$\|(M_\nu)_{\mu\mu}\|$	vs.	$\|(M_\nu)_{\mu\tau}\|$	(normal spectrum)
$\|(M_\nu)_{\mu\tau}\|$	vs.	$\|(M_\nu)_{\tau\tau}\|$	(normal spectrum)

May be subsumed as:

"If one matrix element is small, the other one must be large."

Correlations of the elements of the neutrino mass matrix

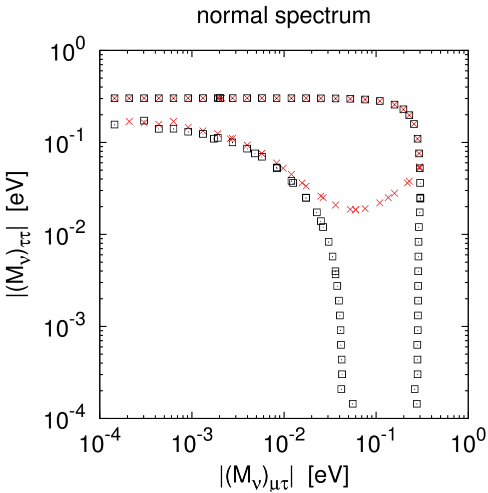
Forero et al.:[3] best fit: $*$, 1σ : ▲, 3σ : ●; normal spectrum



At 3σ the results for the different global fits (Fogli, Forero) agree.

[3] Forero et al., Phys. Rev. D 86 (2012) 073012.

1σ : Fogli et al.[4] ($\times$) compared to Forero et al. ($\boxdot$)



[4] Fogli et al., Phys. Rev. D 86 (2012) 013012.

II. Dirac neutrinos

The Dirac neutrino mass matrix

Remember Majorana case:

$$M_\ell = \mathrm{diag}(m_e,\, m_\mu,\, m_\tau) \Rightarrow M_\nu = U^*_{\mathrm{PMNS}} \mathrm{diag}(m_1,\, m_2,\, m_3) U^\dagger_{\mathrm{PMNS}}.$$

$\Rightarrow |(M_\nu)_{\alpha\beta}|$ has 9 real parameters
(7 restricted through experiments/observations).

Dirac neutrino mass matrix:

$$M_\ell = \mathrm{diag}(m_e,\, m_\mu,\, m_\tau) \Rightarrow M_D = V_R \, \mathrm{diag}(m_1,\, m_2,\, m_3) U^\dagger_{\mathrm{PMNS}}.$$

V_R ... completely unconstrained unitary 3×3-matrix.
$\Rightarrow$ Even if U_{PMNS} known, at least 9 free parameters in M_D.

V_R can be eliminated by studying

$$H_D \equiv M_D^\dagger M_D = U_{\mathrm{PMNS}} \mathrm{diag}(m_1^2,\, m_2^2,\, m_3^2) U^\dagger_{\mathrm{PMNS}}.$$

$\rightarrow$ H_D contains all physical parameters but does not tell too much about M_D itself.

The Dirac neutrino mass matrix

$\Rightarrow$ Hard to put bounds on the Dirac neutrino mass matrix.

Freedom of choosing V_R even allows to set several elements of M_D to zero!

Example: QR decomposition

Let M be a complex $n \times n$-matrix, then there exists a unitary matrix V such that

$$VM = \begin{pmatrix} m_{11} & m_{12} & m_{13} & \dots & m_{1n} \\ 0 & m_{22} & m_{23} & \dots & m_{2n} \\ 0 & 0 & m_{33} & \dots & m_{3n} \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & m_{nn} \end{pmatrix}$$

(upper triangular matrix.)

The Dirac neutrino mass matrix

Furthermore, due to the freedom of choosing V_R, one can arbitrarily permute the rows of M_D without changing physical predictions. E.g.

$$M_D = \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{pmatrix}$$

and

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} M_D = \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{31} & m_{32} & m_{33} \\ m_{21} & m_{22} & m_{23} \end{pmatrix}$$

lead to the same H_D and thus to the same physical predictions.

$\rightarrow$ much more freedom than in the Majorana case.

Nevertheless, studies are possible, for example the quest for texture zeros in M_D.

Texture zeros in the Dirac neutrino mass matrix

Usual assumption: $M_\ell = \mathrm{diag}(m_e,\, m_\mu,\, m_\tau)$

→ Studied by C. Hagedorn and W. Rodejohann in 2005:[5]

→ Up to 5 texture zeros in M_D possible.

→ Repeating this analysis with the current data:

Results coincide with findings of Hagedorn and Rodejohann.

Only difference: Due to $\sin^2\theta_{13} \gg 10^{-4}$ some texture zeros are now excluded.

In particular: Five texture zeros in M_D no longer allowed.

[5] C. Hagedorn and W. Rodejohann, JHEP 07 (2005) 034.

Summary

- Majorana neutrinos: in basis where M_ℓ diagonal: $|(M_\nu)_{\alpha\beta}|$ has 9 parameters.
  $\rightarrow$ Numerical study using the improved oscillation data:
  - Allowed ranges and correlations of the elements of the neutrino mass matrix.
  - Five correlations stringent at 3σ [all normal spectrum]: "If one matrix element is small, the other one must be large."
  - More interesting correlations at 1σ.
  - Results from different global fits agree at 3σ. Differences at 1σ.
  - Correlation plots may be helpful tools for model building.
- Dirac neutrinos: much more freedom in M_D. Physically relevant parameters contained in $H_D = M_D^\dagger M_D$.
  - Hard to put bounds on M_D itself.
  - Nevertheless statements about texture zeros possible. Results of Hagedorn and Rodejohann still valid.

Thank you for your attention!