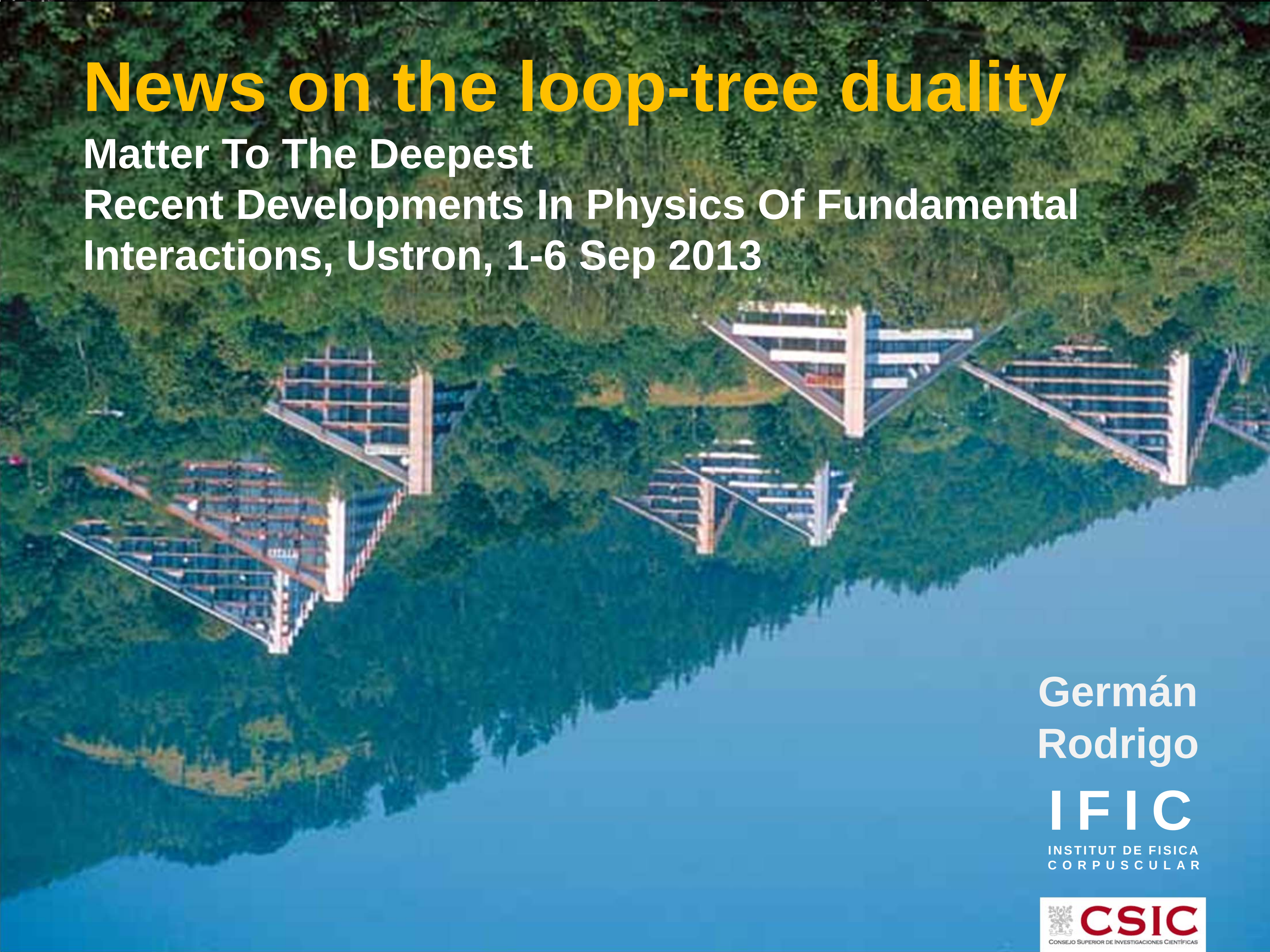

News on the loop-tree duality

Matter To The Deepest
Recent Developments In Physics Of Fundamental Interactions, Ustron, 1-6 Sep 2013

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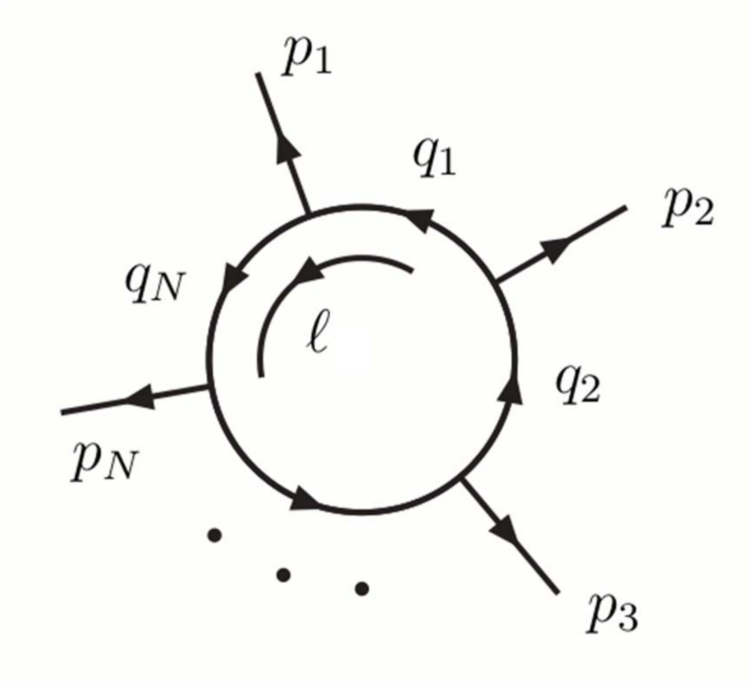
Loop radiative corrections

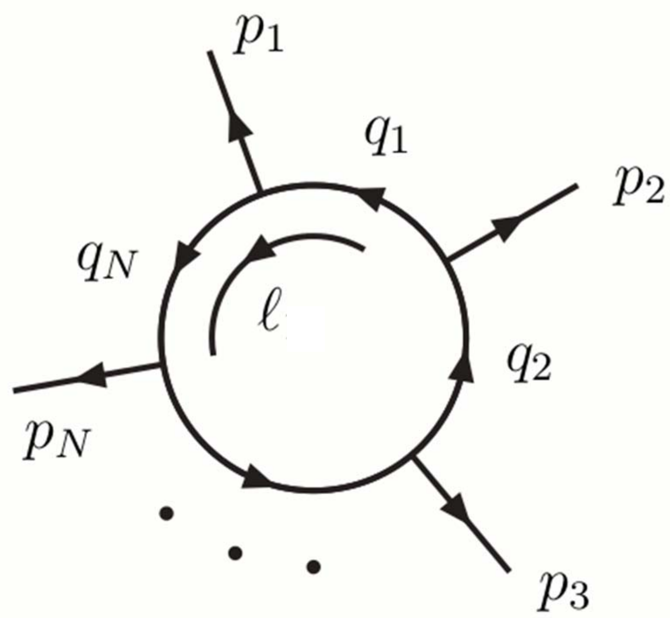


- Main bottleneck in higher-order calculations for many years, in particular for large number of external particles

- Huge progress and gain in efficiency from experienced **Feynmanists** (stable tensor reduction [next talks]) and **Generalized Unitarity** (OPP, …): **multiple-cuts** over a loop basis (at one-loop and even higher-orders)

- Still a great effort to cancel IR (and UV) divergences with real corrections

Loop radiative corrections



To fix the notation:

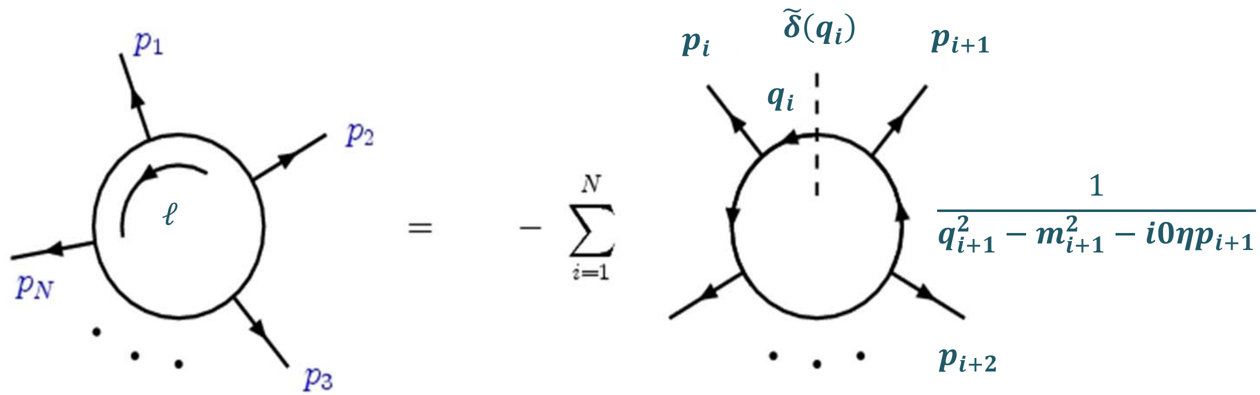
$q_i = \ell + k_i$ with $k_i = p_1 + \cdots + p_i$

$G_F(q_i) = \frac{1}{q_i^2 - m_i^2 + i0}$ and $\int_\ell = -i \int \frac{d^d\ell}{(2\pi)^d}$

Single cuts only (by-passing the Feynman Tree Theorem)

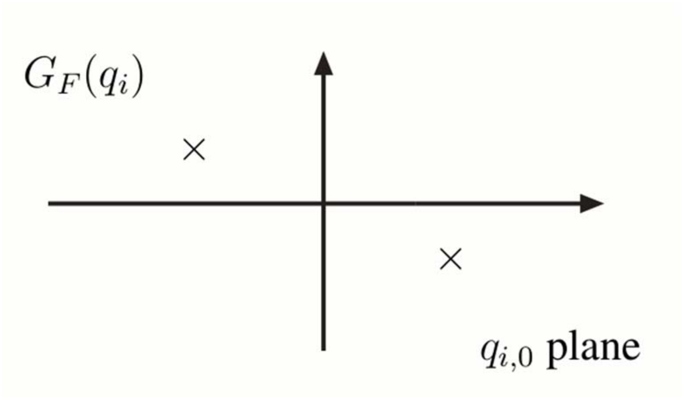
[Catani, Gleisberg, Krauss, GR, Winter, JHEP 09(2008)064]

duality relation between one-loop integrals (one-loop scattering amplitudes) with an arbitrary number of external legs (momenta) and corresponding **single-cut** bremsstrahlung integrals.

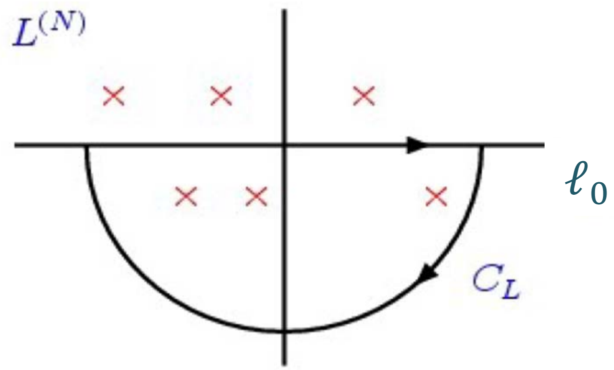


- the duality relation is realised by modifying the customary **+i0 prescription** of the Feynman propagators;
- the new +i0 prescription thus compensates for the absence of multiple-cut contributions that appear in the Feynman Tree Theorem;
- in any relativistic, local and unitary quantum field theory;
- **recast virtual corrections** in a form that closely parallels the contribution of real corrections.

The loop-tree duality theorem



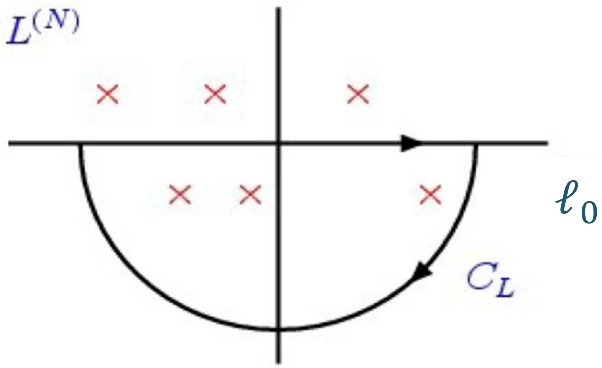
Cauchy residue theorem
in the loop energy complex plane



Feynman Propagator +i0: positive frequencies are propagated forward in time, and negative backward

selects residues with definite **positive** energy and negative imaginary part (indeed in any coordinate system)

The loop-tree duality theorem



- the one-loop integral represented as a linear combination of *N* single-cut **phase-space** integrals

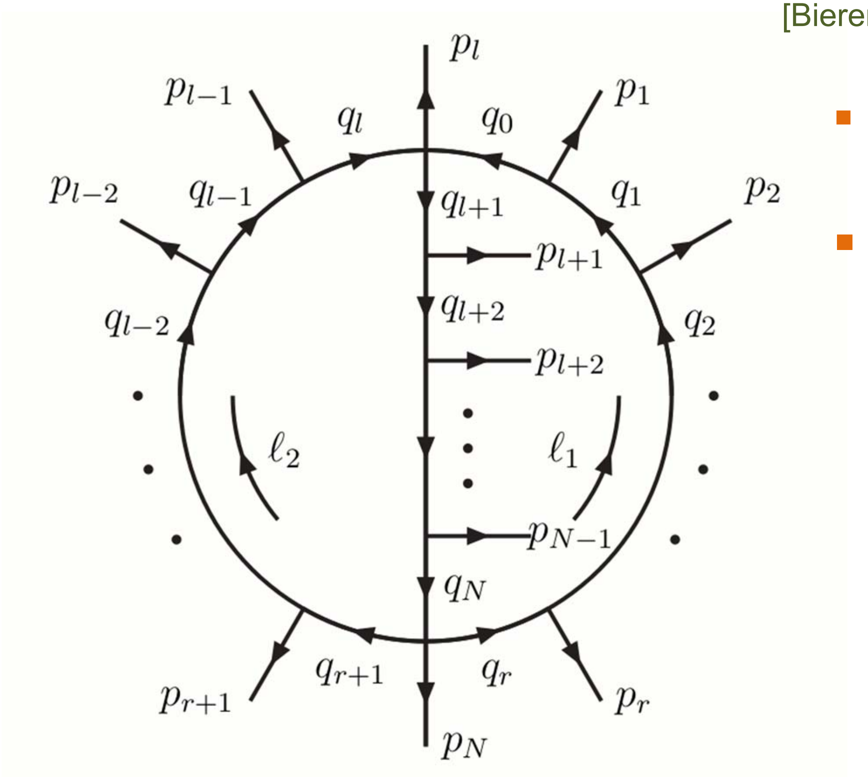
$$\boxed{\int_\ell \prod G_F(q_i) = -\int_\ell \sum \tilde{\delta}(q_i) \prod_{j \neq i} G_D(q_i; q_j)}$$

- where $\tilde{\delta}(q_i) = 2\pi i\, \delta_+(\boldsymbol{q_i^2 - m_i^2})$ sets internal line on-shell;
- $G_D(q_i; q_j) = \frac{1}{\boldsymbol{q_j^2 - m_j^2 - i0\eta(q_j - q_i)}}$ dual propagator;
- Lorentz-covariant dual prescription with *η* is a **future-like** vector: $\eta^2 \geq 0$ with $\eta_0 > 0$;
- different choices of *η* are equivalent to different choices of the coordinate system;
- the dependence on *η* cancels in the sum of dual integrals.

Master topology at two loops

[Bierenbaum, Catani, Draggiotis, GR, JHEP 10(2010)073]

[Bierenbaum, Buchta, Draggiotis, Malamos, GR, JHEP 03(2013)025]



- Iterative application of the duality theorem

- **Loop lines:**

$$\alpha_1 = \{0,1,\ldots,r\}$$
$$\alpha_2 = \{r+1,\ldots,l\}$$
$$\alpha_3 = \{l+1,\ldots,N\}$$

$$G_D(\alpha_k) = \sum_{i\in\alpha_k} \tilde{\delta}(q_i) \prod_{j\neq i} G_D(q_i;q_j)$$

$$G_F(\alpha_k) = \prod_{i\in\alpha_k} G_F(q_i)$$

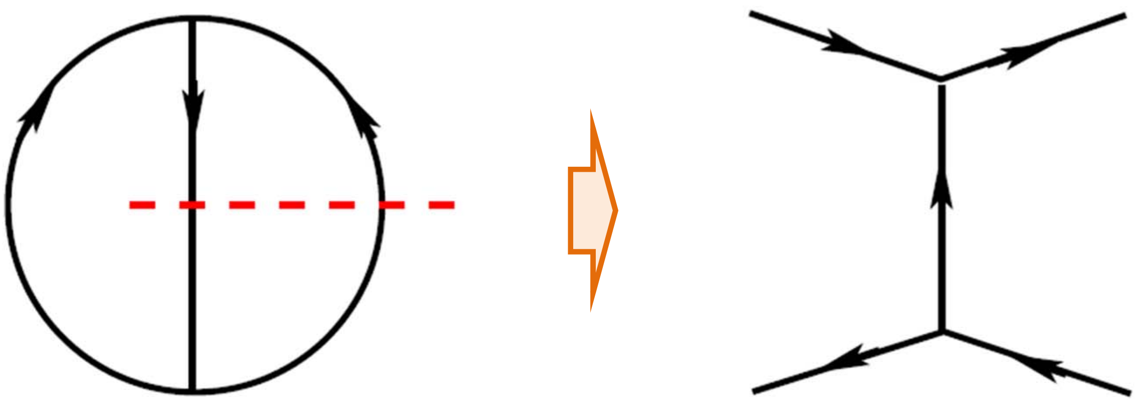
- duality applies to Feynman propagators depending on the same loop momentum

$$G_D(\alpha_1 \cup \alpha_2) = G_D(\alpha_1)G_D(\alpha_2) + G_D(\alpha_1)G_F(\alpha_2) + G_F(\alpha_1)G_D(\alpha_2)$$

Duality at two loops

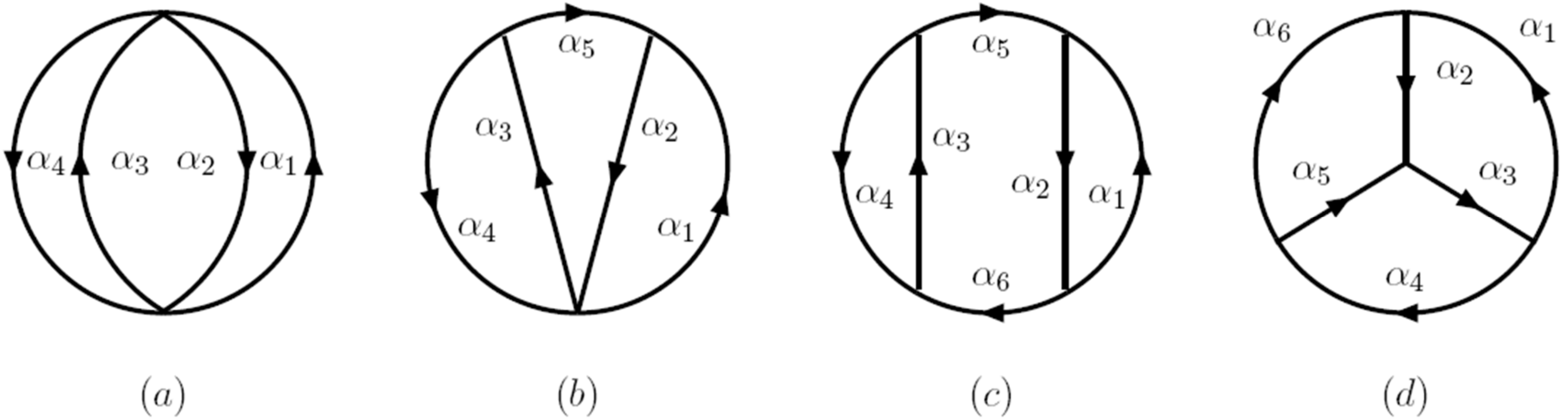
- **Two cuts only:** open any two-loop diagram to a **tree-level diagram** (sign in $-\alpha_1$ indicates a change of momentum flow)

$$L^{(2)}(p_1, \dots, p_N) = \int_{\ell_1} \int_{\ell_2} [-G_D(\alpha_1) G_F(\alpha_2) G_D(\alpha_3) + G_D(\alpha_1)\, G_D(\alpha_2 \cup \alpha_3) + G_D(-\alpha_1 \cup \alpha_2) G_D(\alpha_3)]$$



- at one-loop the complex dual prescription depends on external momenta only, however, at two loops it might depend on the integration momenta: complex dual prescription on external momenta only requires to introduce disconnected diagrams

Master topologies at three loops



- **Three cuts only:** such that the three-loop diagrams are open to a **tree-level diagram**

- Or multiple cuts (starting from three) leading to disconnected tree diagrams, with complex dual prescription depending on external momenta only.

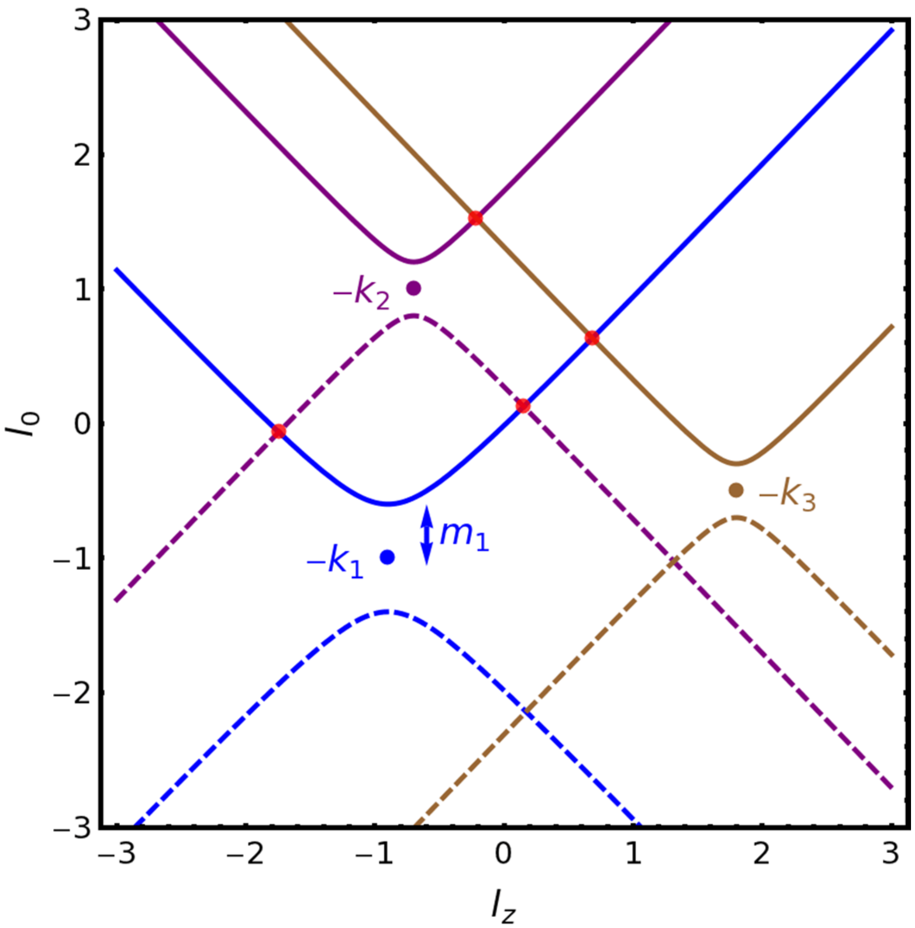
- One single cut per "loop line"

Singularities of the loop integrand

[Buchta, Chachamis, Malamos, in preparation]

- The loop-momentum space approach is attractive because it allows a direct physical interpretation of loop singularities.
- Loop singularities arise when subsets of internal lines go on-shell. (assuming UV singularities have been subtracted)
- Although the existence of singular points in the loop momentum-space is not enough to ensure the presence of singularities:
    - threshold singularities are integrable: contour deformation for numerically stable integration [Soper, Nagy, Weinzierl, …]
    - IR singularities remain and are cancelled by coherent sum with real emission partonic configurations [subtraction methods at NLO and higher orders]

Singularities of the loop integrand



The loop integrand becomes singular

$$G_F^{-1}(q_i) = q_i^2 - m_i^2 + i0 = 0$$

at **hyperboloids** with origin in $-k_{i,\mu}$

Forward light-cone (solid)

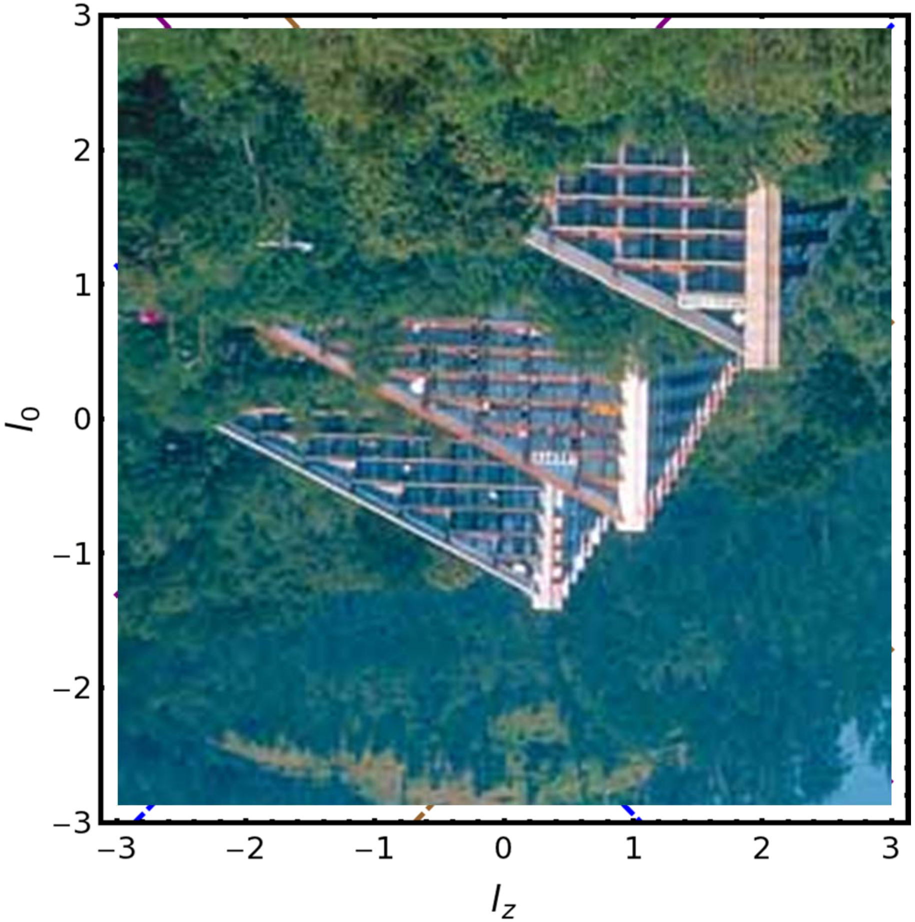
$$q_{i,0}^{(+)} = \sqrt{\vec{q}_i^{\,2} + m_i^2 - i0}$$

Backward light-cone (dashed)

$$q_{i,0}^{(-)} = -q_{i,0}^{(+)}$$

Duality: equivalent to integrate along the **forward** light-cones

Singularities of the loop integrand



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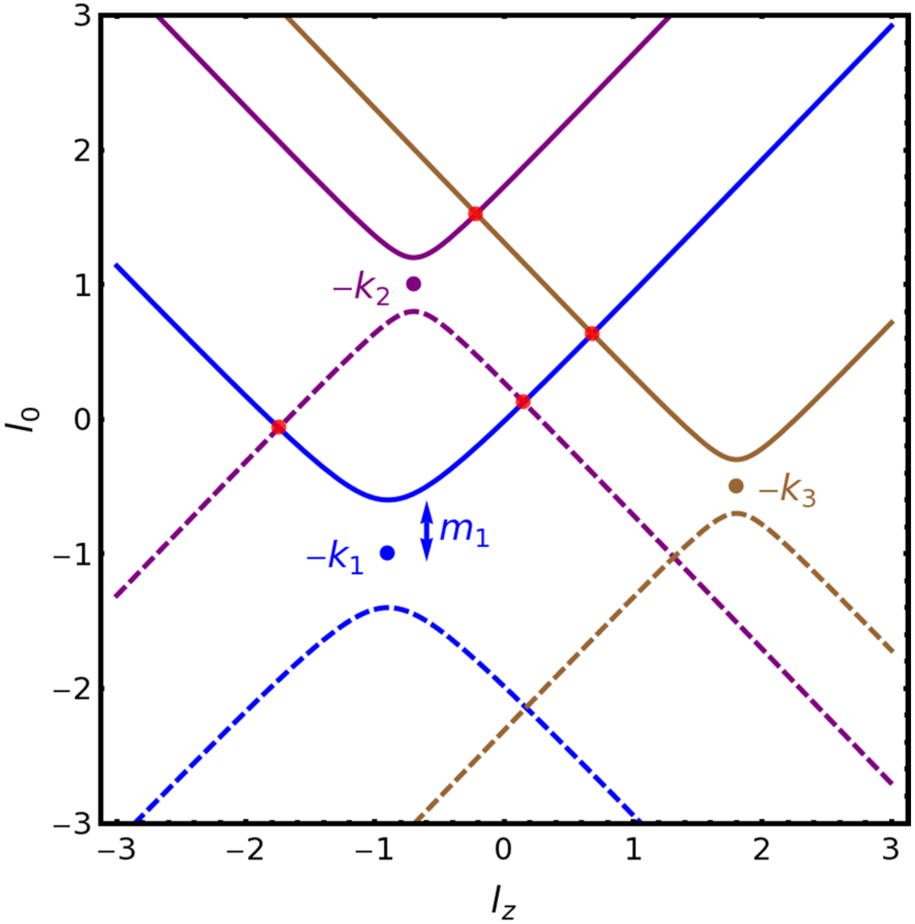
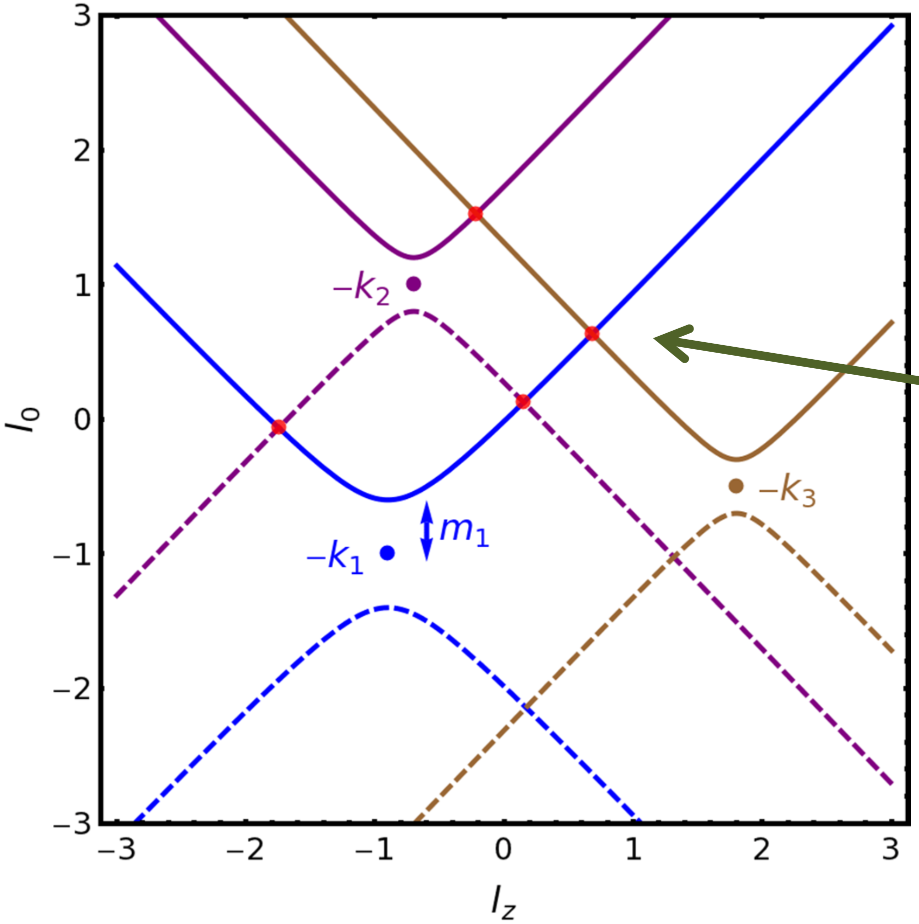
Threshold singularities



Feynman and Dual propagators become positive inside the respective light-cone hyperboloid, and negative outside

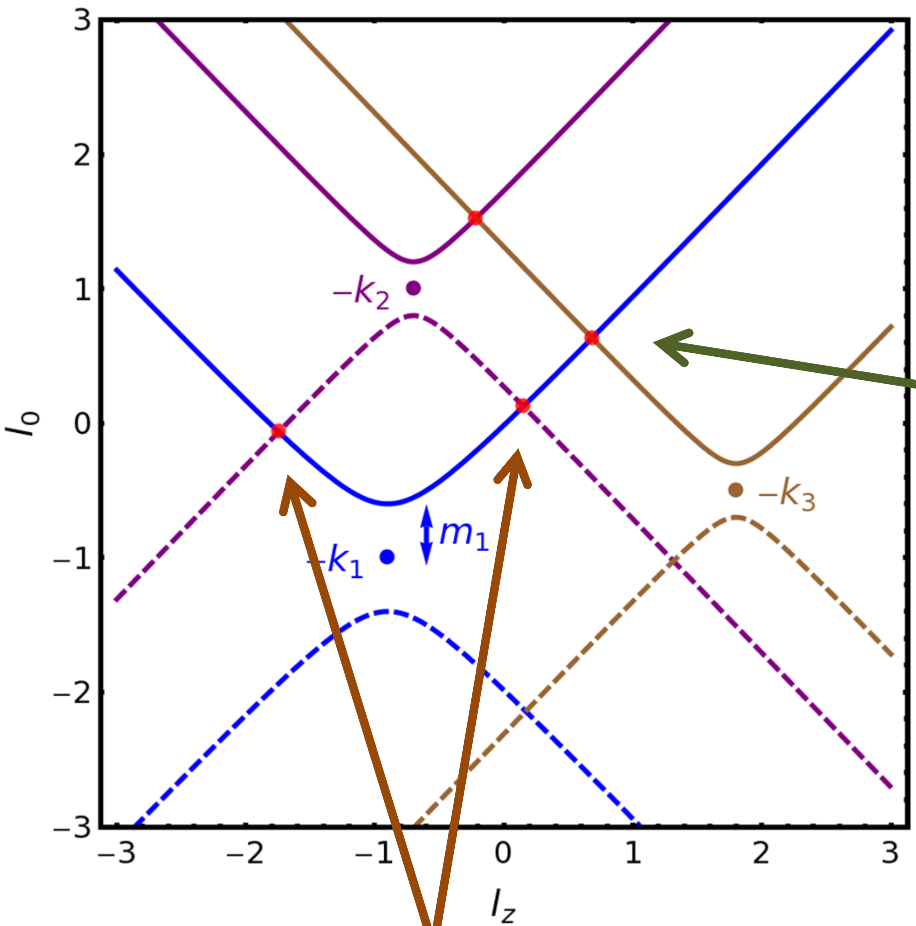
Threshold singularities



Feynman and Dual propagators become positive inside the respective light-cone hyperboloid, and negative outside

- Integrating along the forward light-cone $\tilde{\delta}(q_1)$ one pass **from outside to inside** the light-cone hyperboloid of $-k_3$
- Integrating along the forward light-cone $\tilde{\delta}(q_3)$ one pass **from inside to outside** the light-cone hyperboloid of $-k_1$

Threshold singularities



Feynman and Dual propagators become positive inside the respective light-cone hyperboloid, and negative outside

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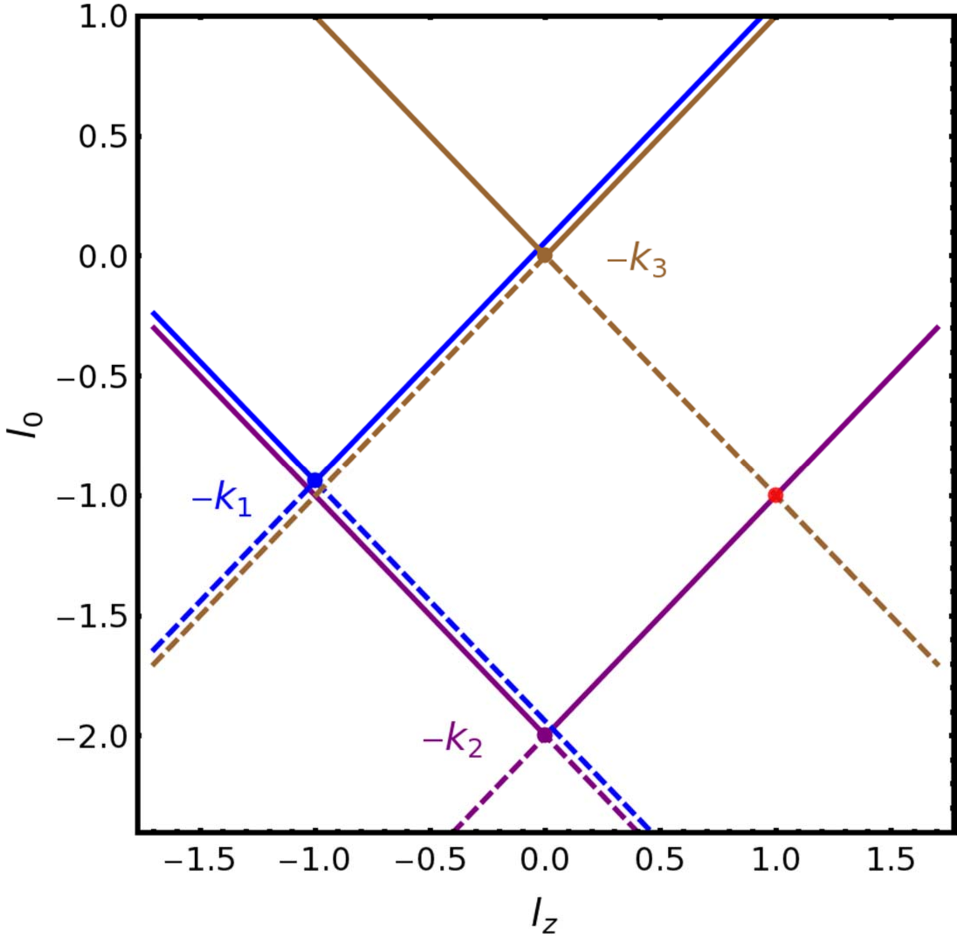
- Integrating along the forward light-cone $\tilde{\delta}(q_3)$ one pass **from inside to outside** the light-cone hyperboloid of $-k_1$

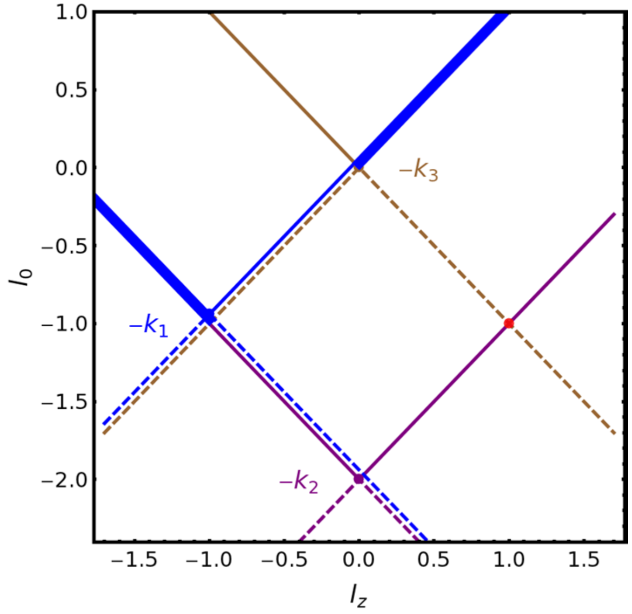
- Only singular along the forward light-cone $\tilde{\delta}(q_1)$ but not along the forward light-cone $\tilde{\delta}(q_2)$

IR singularities



- Massless internal line and on-shell adjacent external momenta
- The light-cone hyperboloids intersect **tangentially** over an **infinite** interval
- e.g. $e^+e^- \to q(p_1)\bar{q}(p_2)$ with $k_1 = p_1$, $k_2 = p_1 + p_2$

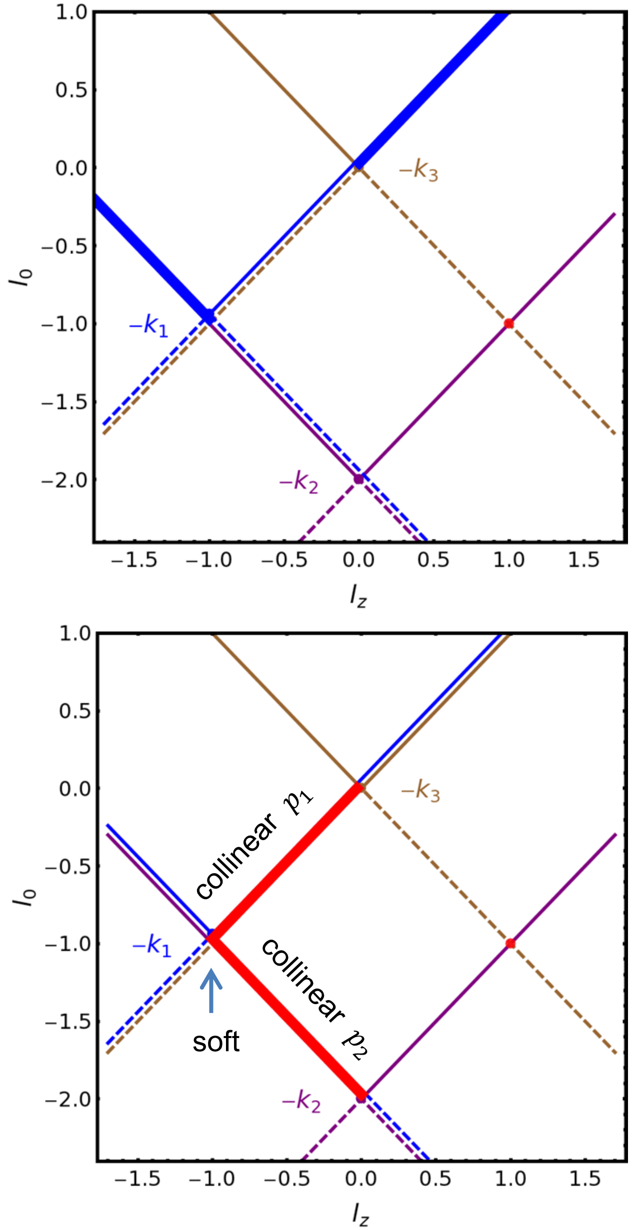
IR singularities



Forward with forward light-cone:

- collinear singular behaviour cancels among dual integrals

IR singularities



Forward with forward light-cone:

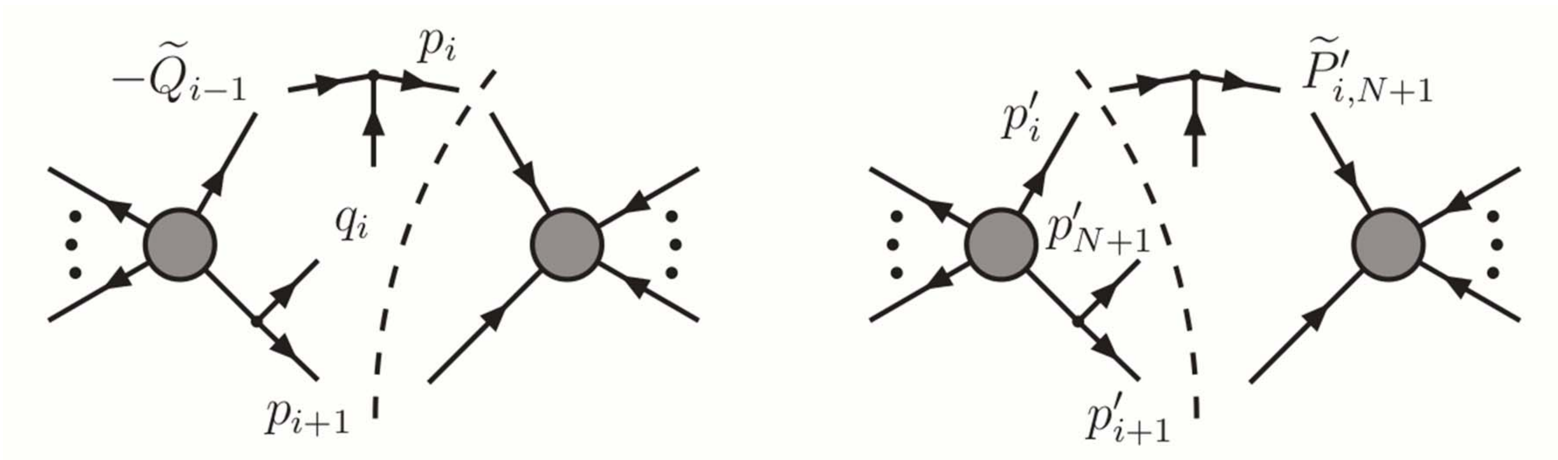
- collinear singular behaviour cancels among dual integrals

Forward with backward light-cone:

- Collinear and soft singular behaviour remains, and
- Is restricted to a **finite region** of the loop-momentum space, which is of the order of the magnitude of external momenta
- Mapping with finite real emission phase-space

Factorization in the collinear limit

[Catani, de Florian, GR, PLB586(2004), JHEP 07(2012)026]



- dual scattering amplitude

$$\left|\boldsymbol{M}_N^{(1)}(p_1,\ldots,p_N)\right\rangle \to \left|\boldsymbol{M}_{N+2}^{(0)}(\ldots,p_i,-q_i,q_i,\ldots)\right\rangle$$
$$\simeq \boldsymbol{Sp}^{(0)}(p_i,-q_i;-\tilde{Q}_{i-1})\left|\overline{\boldsymbol{M}}_{N+1}^{(0)}(\ldots,p_{i-1},-\tilde{Q}_{i-1},q_i,\ldots)\right\rangle$$

- tree-level scattering amplitude

$$\left|\boldsymbol{M}_{N+1}^{(0)}(p'_1,\ldots,p'_{N+1})\right\rangle$$
$$\simeq \boldsymbol{Sp}^{(0)}(p'_i,p'_{N+1};\tilde{P}_{i,N+1})\left|\overline{\boldsymbol{M}}_N^{(0)}(\ldots,p'_{i-1},\tilde{P}_{i,N+1},p'_{i+1},\ldots)\right\rangle$$

with on-shell momenta $\widetilde{\boldsymbol{Q}}_{i-1}^2=\boldsymbol{0}$, $\widetilde{\boldsymbol{P}}_{i,N+1}^2=\boldsymbol{0}$,

and restricted loop momenta $\boldsymbol{q}_{i,0}^{(+)}<\boldsymbol{p}_{i,0}$

Conclusions and outlook

- the loop-tree duality method exhibits attractive theoretical issues and nice properties
- threshold and infrared singularities occurring in the intersection of forward light-cones cancel among dual integrals
- remaining loop singularities are restricted to a **finite region** of the loop momentum space, which is of the size of external momenta

Outlook

- numerical implementation at one-loop
- singularities at higher orders