Models for Sterile Neutrinos

Matter To The Deepest, Ustron, 2 Sep 2013

Contents:

- Neutrino masses and light sterile neutrinos
- Mechanisms for sterile neutrino mass generation
- Example models: flavor symmetries, extra dimensions, seesaw extensions and Froggatt-Nielsen mechanism

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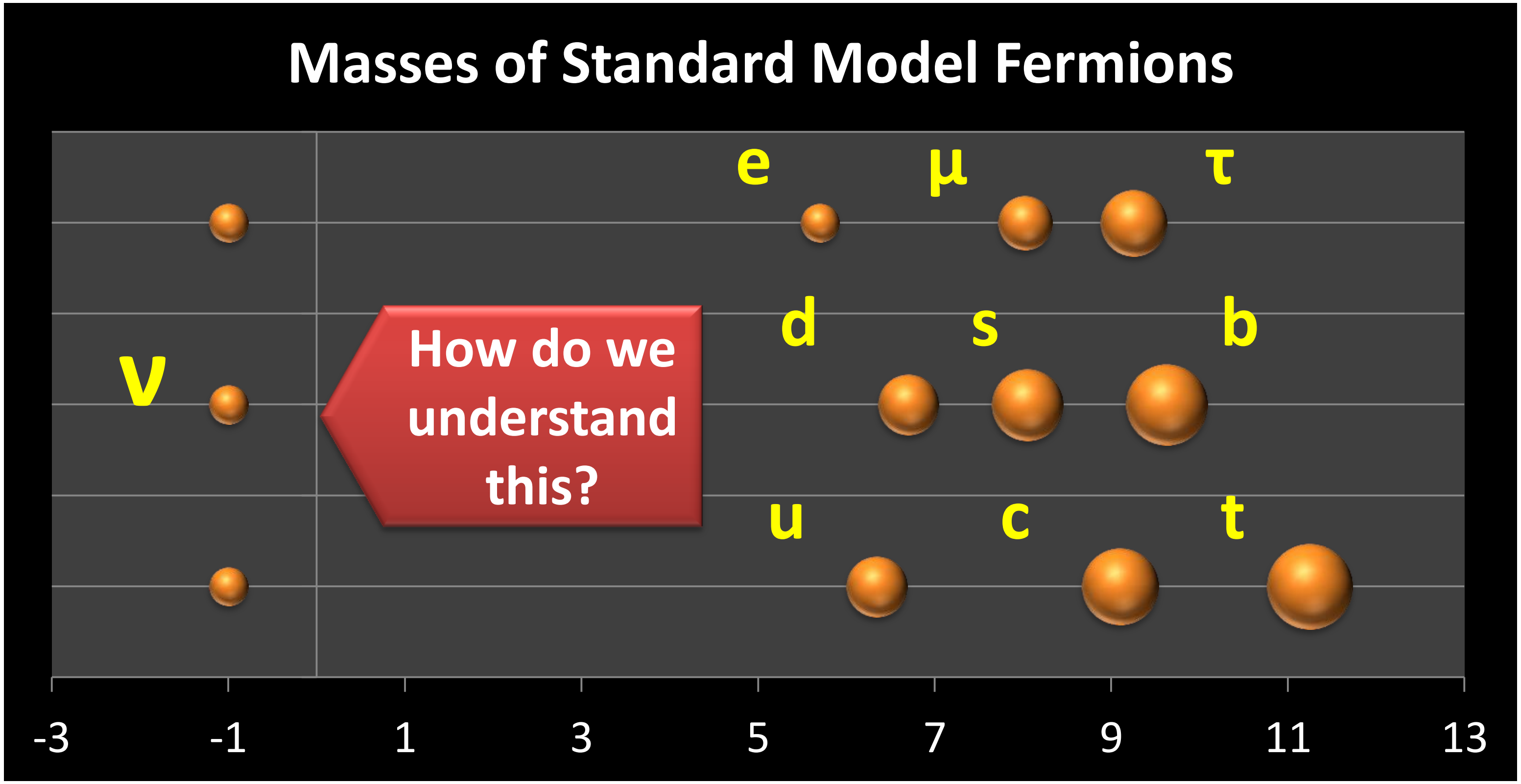
I. Neutrino masses and light sterile neutrinos

Neutrinos are **massless** in the **SM** as a result of the model's simple structure:

- $SU(2)_L \times U(1)_Y$ gauge symmetry and Lorentz invariance; Fundamentals of the model, mandatory for its consistency as a **QFT**.
- Economical particle content: No right-handed neutrinos → a Dirac mass term is not allowed. Only one Higgs doublet → a Majorana mass term is not allowed.
- Renormalizability: No dimension ≥ 5 operators --- a Majorana mass term is forbidden.



I. Neutrino masses and light sterile neutrinos

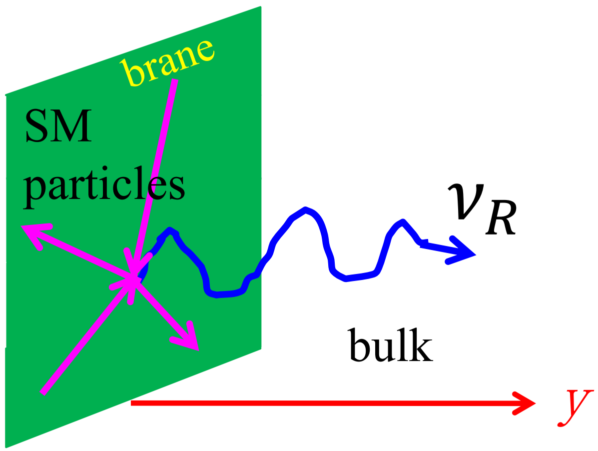
Dirac neutrinos

Neutrinos are Dirac particles
ν_R + a pure Dirac mass term
Extremely tiny Yukawa coupling $\sim 10^{-11}$, (hierarchy puzzle)

$$\mathscr{L} = \mathscr{L}_{\rm SM} + \left\{ Y \bar{l}_{\rm L} \nu_{\rm R} \tilde{\phi} + {\rm h.c.} \right\}$$

The smallness of Dirac masses is ascribed to the assumption that ν_R have access to an extra spatial dimension

(Dienes, Dudas, Gherghetta 98; Arkani-Hamed, Dimopoulos, Dvali, March-Russell 98)



The wavefunction of ν_R spreads out over the extra dimension y , giving rise to a suppressed Yukawa interaction at $y = 0$.

$$\left[\overline{l_{\rm L}} Y_\nu \tilde{H} N_{\rm R} \right]_{y=0} \sim \frac{1}{\sqrt{L}} \left[\overline{l_{\rm L}} Y_\nu \tilde{H} N_{\rm R} \right]_{y=L}$$

I. Neutrino masses and light sterile neutrinos

Majorana neutrinos

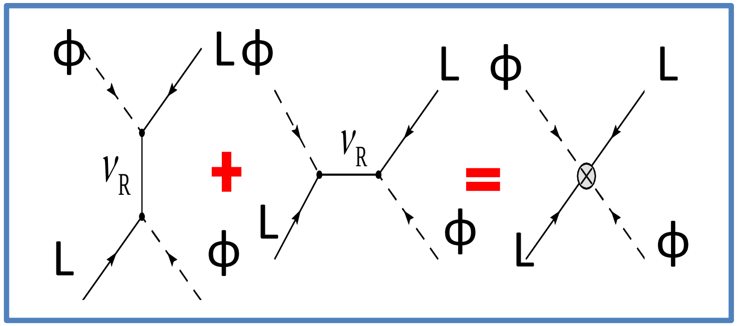
Neutrinos are Majorana particles

ν_R + Majorana & Dirac masses + **seesaw**

Natural description of the smallness of ν-masses

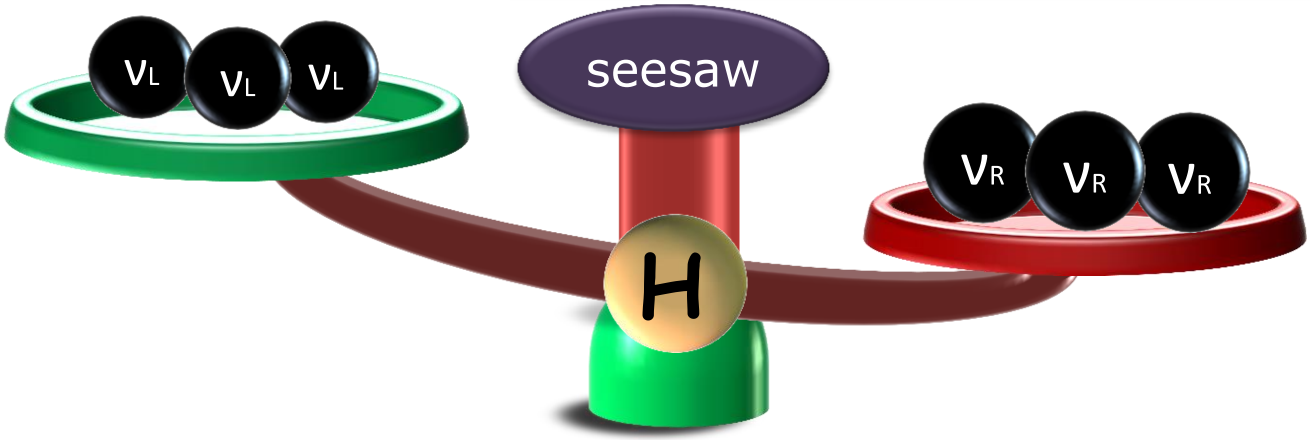
Integrate out right-handed neutrinos

$$\mathscr{L} = \mathscr{L}_{SM} + \left\{ Y \bar{l}_L \nu_R \tilde{\phi} + \boxed{\frac{1}{2} M_R \bar{\nu}_R \nu_R^C} + \text{h.c.} \right\}$$



$$-iY^T \frac{\not{p} + M_R}{p^2 - M_R^2} Y \left(\varepsilon_{cd}\varepsilon_{ba} + \varepsilon_{ca}\varepsilon_{bd} \right) P_L = i\kappa \left(\varepsilon_{cd}\varepsilon_{ba} + \varepsilon_{ca}\varepsilon_{bd} \right) P_L$$

$$p^2 << M_R^2 \Rightarrow \boxed{Y^T M_R^{-1} Y = \kappa} \Rightarrow \boxed{m_\nu = -m_D^T M_R^{-1} m_D}$$

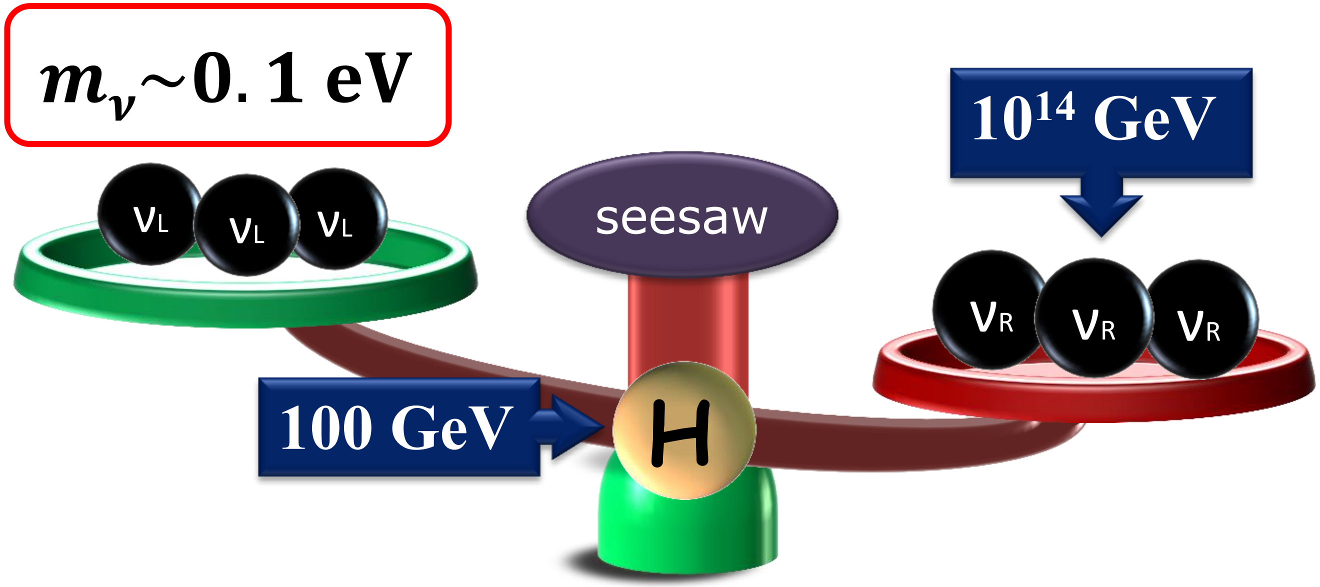


I. Neutrino masses and light sterile neutrinos

The scale of seesaw

Typical choice of the seesaw scale:

$M_R \sim \Lambda_{GUT} \gg \Lambda_{EW}$ & $M_D \sim \Lambda_{EW}$



I. Neutrino masses and light sterile neutrinos

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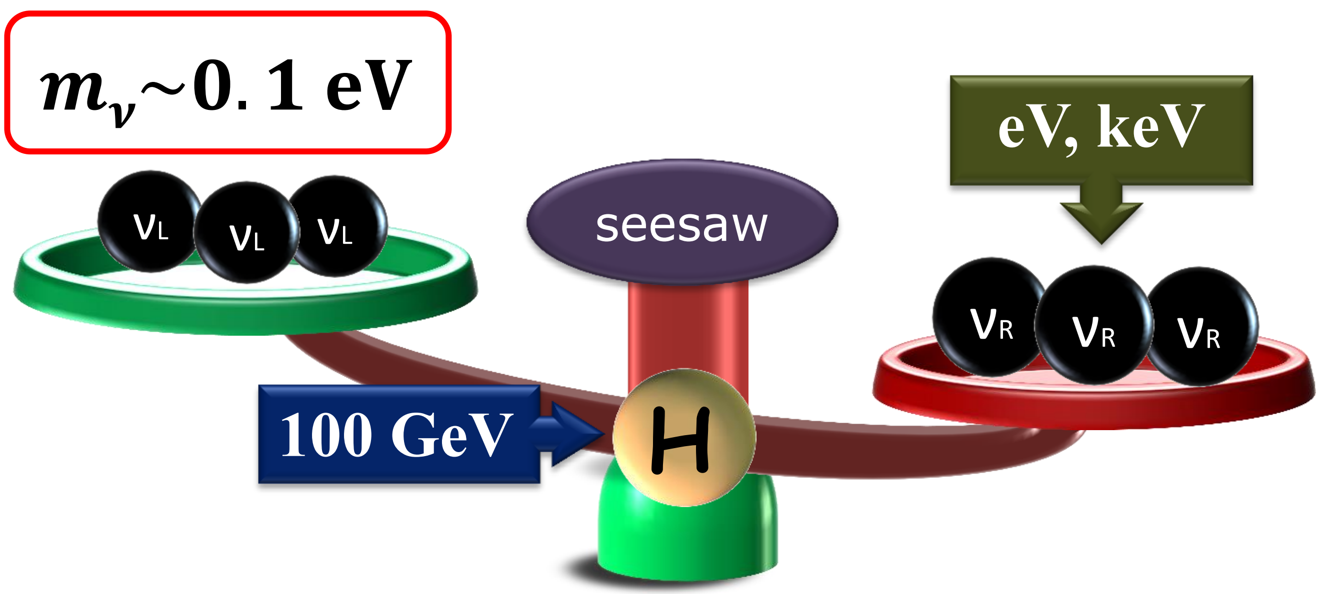
Alternatively, **eV** scale or **keV** scale right-handed neutrinos could also be nature



I. Neutrino masses and light sterile neutrinos

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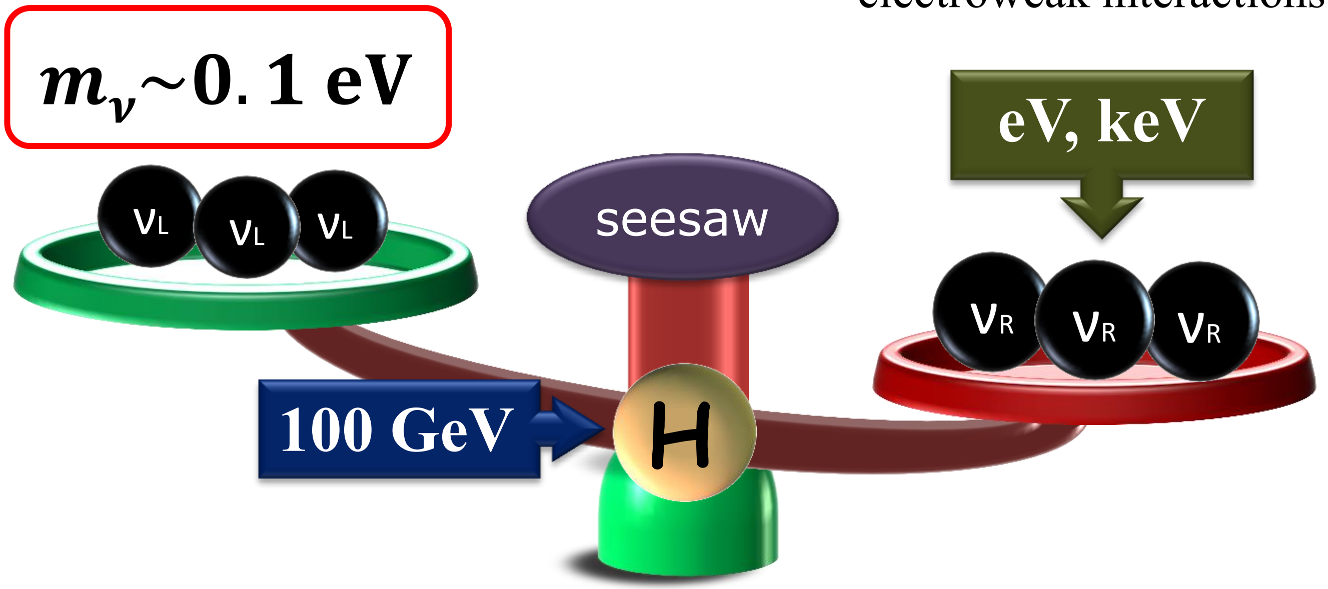
sterile neutrino: ν_s

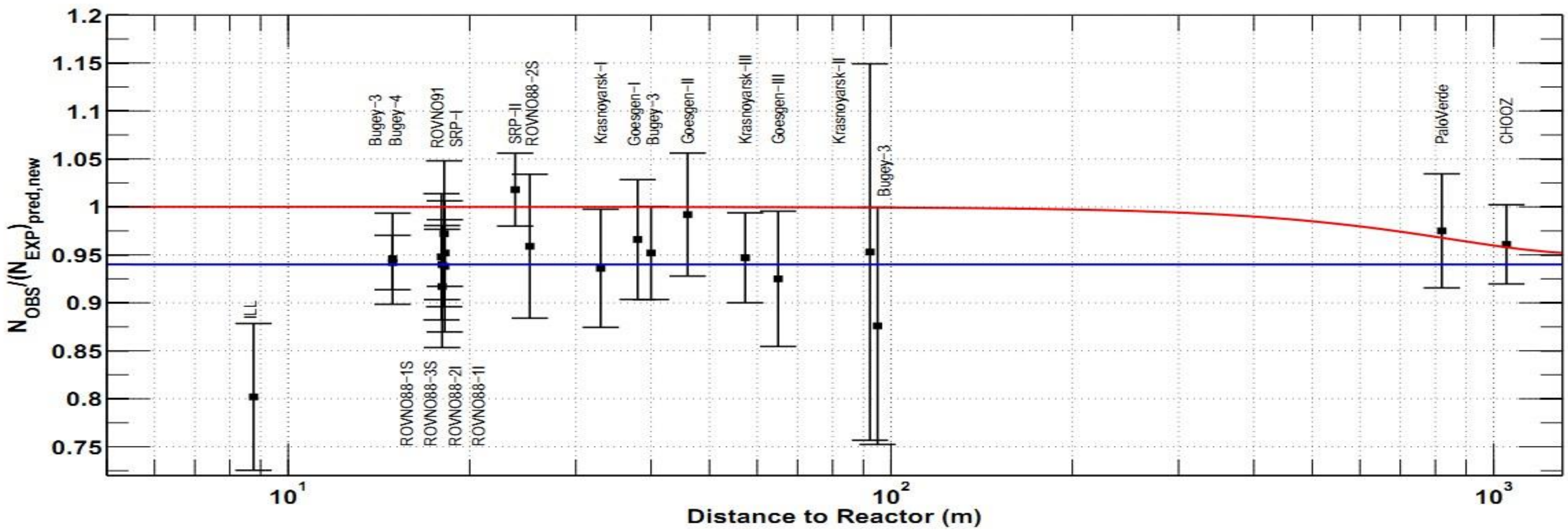
- SU(2) singlet: does not participant electroweak interactions



I. Neutrino masses and light sterile neutrinos

Reactor neutrino anomaly

Mention, Fechner, Lasserre, Mueller, Lhuillier, Cribier, Letourneau, 2011



Recent re-evaluation of the anti-neutrino spectra of nuclear reactors indicates increased fluxes, which can be explained by additional sterile neutrinos with masses at the **eV** scale

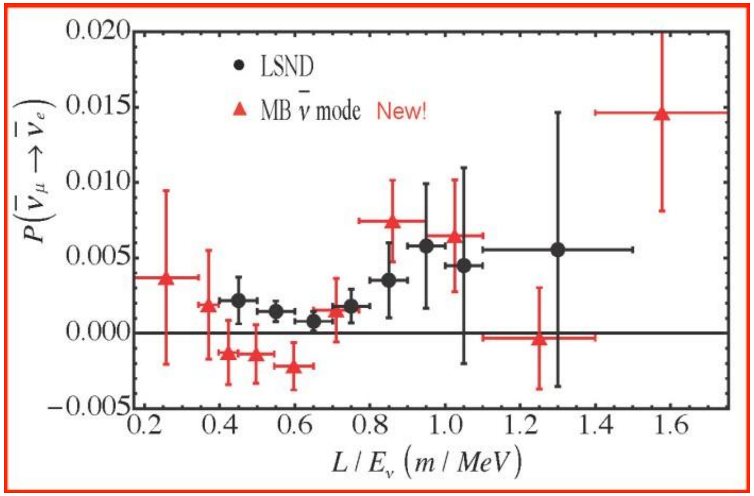
$$P_{\overline{\nu}_e \to \overline{\nu}_e} \simeq 1 - 4\,|U_{e4}|^2 \sin^2\left(\frac{\Delta_{41} L}{4E}\right) = 1 - \sin^2(2\theta_{14}) \sin^2\left(\frac{\Delta_{41} L}{4E}\right)$$

I. Neutrino masses and light sterile neutrinos

Reactor neutrino anomaly

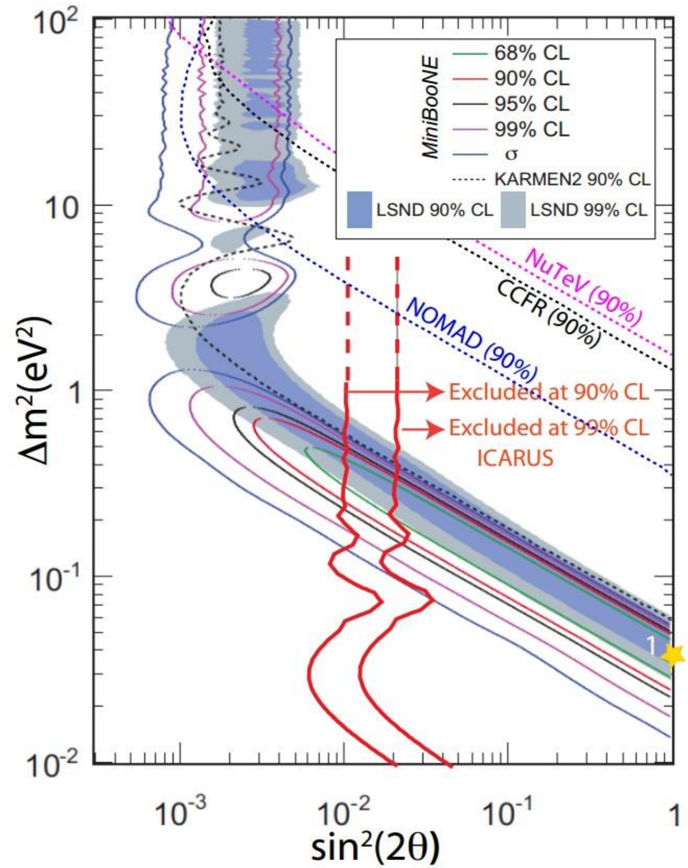
Short-baseline experiments

LSND



3.8σ excess at low energies:
$\Delta m^2 = 1\ \mathrm{eV}^2$

MiniBooNE & ICARUS

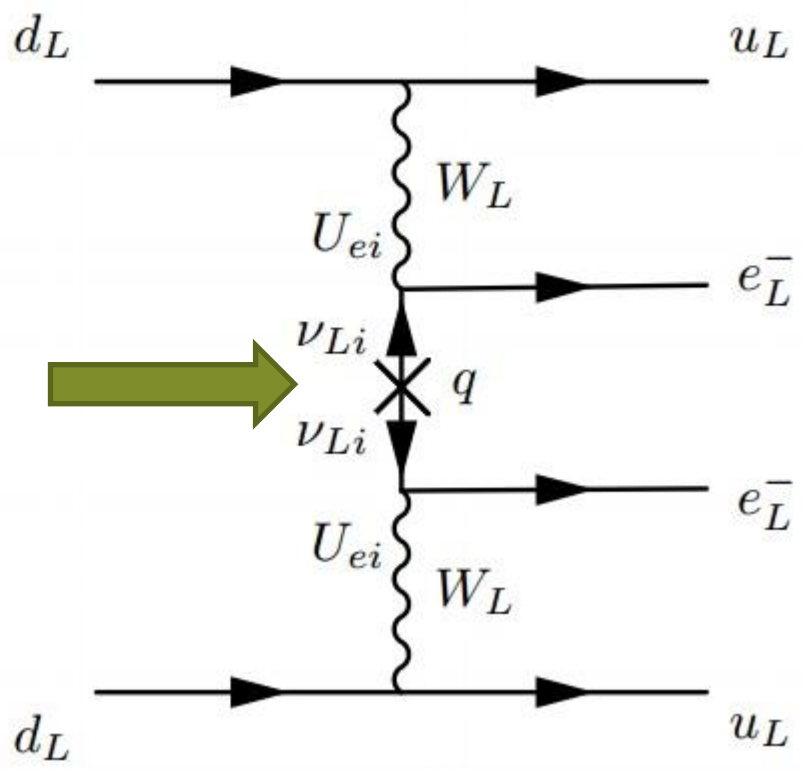


$$P_{\bar{\nu}_\mu \to \bar{\nu}_e} \simeq \underline{4|U_{e4}|^2|U_{\mu 4}|^2 \sin^2\left(\frac{\Delta_{41}L}{4E}\right)} + 4|U_{e5}|^2|U_{\mu 5}|^2 \sin^2\left(\frac{\Delta_{51}L}{4E}\right)$$
$$+\ 8|U_{e4}U_{\mu 4}U_{e5}U_{\mu 5}| \sin\left(\frac{\Delta_{41}L}{4E}\right) \sin\left(\frac{\Delta_{51}L}{4E}\right) \cos\left(\frac{\Delta_{54}L}{4E} + \delta\right)$$

I. Neutrino masses and light sterile neutrinos

neutrino-less double beta decay

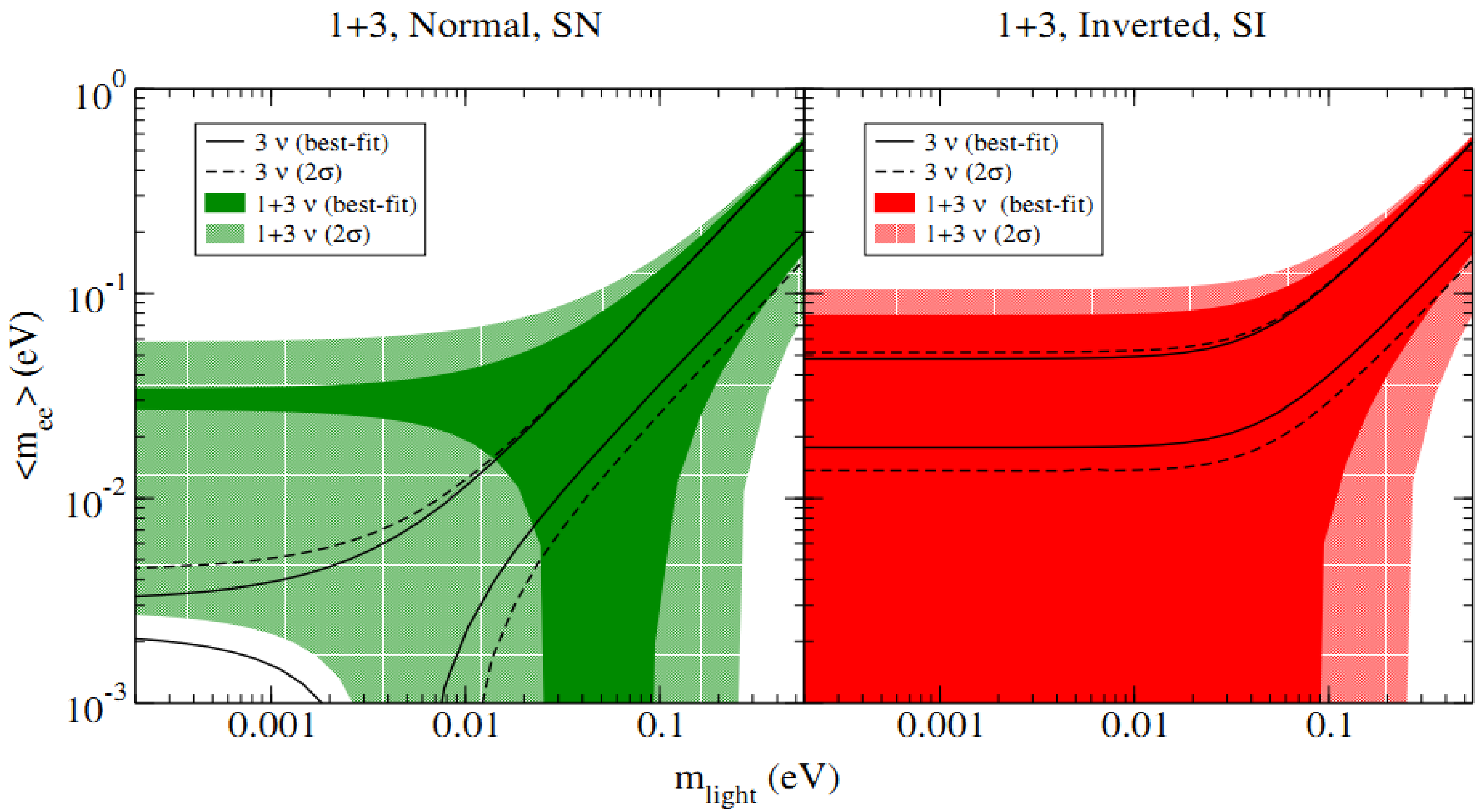
sterile
neutrinos



I. Neutrino masses and light sterile neutrinos

neutrino-less double beta decay

The allowed ranges in the $\langle m_{ee}\rangle - m_{\rm light}$ parameter space



$$\langle m_{ee}\rangle_{(1+3)\nu} \simeq \left| c_{14}^2 \langle m_{ee}\rangle_{3\nu} + s_{14}^2 \sqrt{\Delta m_{41}^2}\, e^{i\gamma} \right|$$

I. Neutrino masses and light sterile neutrinos

Best-fit and estimated 2σ values from **neutrino oscillations**.

Kopp, Maltoni, Schwetz, 1103.4570

	parameter	Δm^2_{41} [eV]	$\lvert U_{e4}\rvert^2$	Δm^2_{51} [eV]	$\lvert U_{e5}\rvert^2$
3+1/1+3	best-fit	1.78	0.023		
	2σ	1.61–2.01	0.006–0.040		
3+2/2+3	best-fit	0.47	0.016	0.87	0.019
	2σ	0.42–0.52	0.004–0.029	0.77–0.97	0.005–0.033
1+3+1	best-fit	0.47	0.017	0.87	0.020
	2σ	0.42–0.52	0.004–0.029	0.77–0.97	0.005–0.035

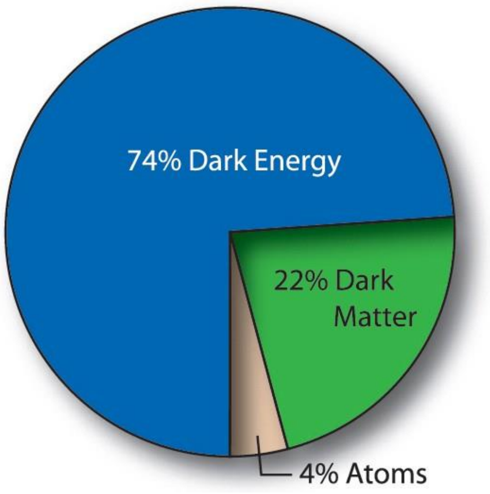
I. Neutrino masses and light sterile neutrinos

sterile neutrino Warm Dark Matter

WDM – relativistic at decoupling, non-relativistic at radiation to matter dominance transition

reduces small scale structure:

- smoother profiles
- less Dwarf Satellites



keV sterile neutrino **WDM**: works very well

- Right-handed neutrinos probably exist (seesaw)
- $M_R \approx$ **keV**
- Only one light ν_s is enough, the other two could still be heavy (**thermal leptogenesis**)

The **νMSM** Asaka, Blanchet, Shaposhnikov, 05; Asaka, Shaposhnikov, 05

✓ A **keV** ν_{R1} can be **WDM** (production: active-sterile oscillation, etc)

✓ GeV-scale ν_{R2} & ν_{R3} generate light neutrino masses via seesaw

✓ ν_{R2} and ν_{R3} are quasi-degenerate

✓ ν_{R2} & ν_{R3} account for the Baryon Asymmtry of the Universe

II. Mechanisms for light sterile neutrino mass

$$m_\nu = M_D \, M_R^{-1} M_D^T$$

0.1 eV — 1 eV (or keV)

- Why are they so **light**?
- Why do they not form the Dirac particles as heavy as the charged fermions?

Solution:

mechanisms (eg., discrete symmetries, extra dimensions, seesaw) suppressing M_R and M_D simultaneously

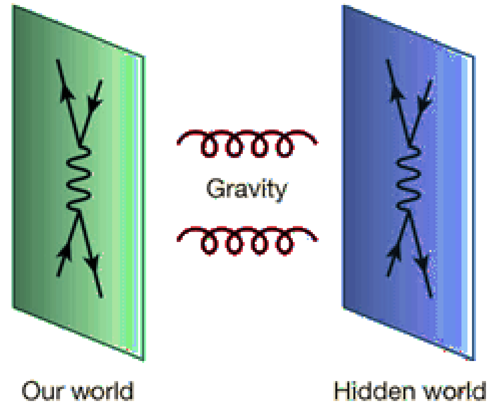
III. Example: split seesaw

Extra Dimension Theories

(Kusenko, Takahashi, Yanagida, **10**)

- Splitting between the SM brane and a **hidden** brane

active neutrinos



sterile neutrinos

$$S = \int d^4x\, dy\, M \left(i\bar{\Psi}\Gamma^A \partial_A \Psi + m\bar{\Psi}\Psi \right)$$

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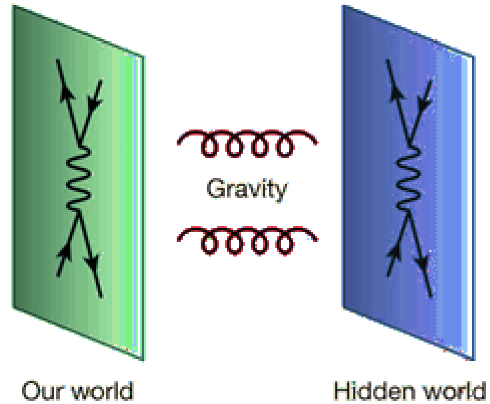

sterile neutrinos

- Sterile neutrino zero mode develops an **exponential profile** in the bulk

$$\Psi_R^{(0)}(y,x) = \sqrt{\frac{2m}{e^{2m\ell}-1}}\frac{1}{\sqrt{M}}e^{my}\psi_R^{(4D)}(x)$$

$$S = \int d^4x\,dy\left\{M\left(i\bar{\Psi}_{iR}^{(0)}\Gamma^A\partial_A\Psi_{iR}^{(0)} + m_i\bar{\Psi}_{iR}^{(0)}\Psi_{iR}^{(0)}\right)\right.$$
$$\left. +\delta(y)\left(\frac{\kappa_i}{2}v_{\rm B-L}\bar{\Psi}_{iR}^{(0)c}\Psi_{iR}^{(0)} + \tilde{\lambda}_{i\alpha}\bar{\Psi}_{iR}^{(0)}L_\alpha\,\phi + {\rm h.c.}\right)\right\}$$

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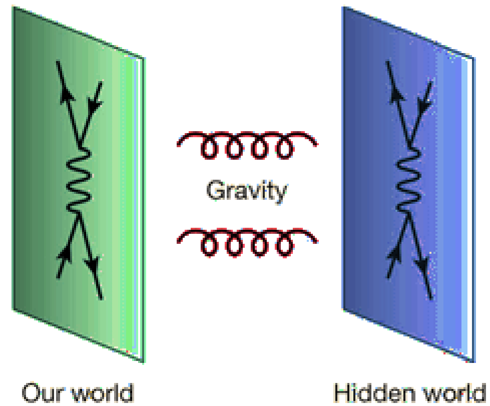

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$$\Psi_R^{(0)}(y,x) = \sqrt{\frac{2m}{e^{2m\ell}-1}}\frac{1}{\sqrt{M}}e^{my}\psi_R^{(4D)}(x)$$

sterile mass

$$M_{Ri} = \kappa_i v_{\mathrm{B-L}}\frac{2m_i}{M(e^{2m_i\ell}-1)}$$

Yukawa coupling

$$\lambda_{i\alpha} = \frac{\tilde{\lambda}_{i\alpha}}{\sqrt{M}}\sqrt{\frac{2m_i}{e^{2m_i\ell}-1}} = \tilde{\lambda}_{i\alpha}\sqrt{\frac{M_{Ri}}{\kappa_i v_{\mathrm{B-L}}}}$$

$$(m_\nu)_{\alpha\beta} = \left(\sum_i \frac{1}{\kappa_i}\tilde{\lambda}_{i\alpha}\tilde{\lambda}_{i\beta}\right)\frac{\langle\phi^0\rangle^2}{v_{\mathrm{B-L}}}$$

III. Example: flavor symmetries

Flavor symmetries

$L_e - L_\mu - L_\tau$ symmetry:

	L_{eL}	$L_{\mu L}$	$L_{\tau L}$	e_R	μ_R	τ_R	N_{1R}	N_{2R}	N_{3R}	ϕ	Δ
$\mathcal{F}$	1	−1	−1	1	−1	−1	1	−1	−1	0	0

(Lindner, Merle, Niro, **10**)

$$\mathcal{M}_\nu = \left(\begin{array}{ccc|ccc} 0 & m_L^{e\mu} & m_L^{e\tau} & m_D^{e1} & 0 & 0 \\ m_L^{e\mu} & 0 & 0 & 0 & m_D^{\mu 2} & m_D^{\mu 3} \\ m_L^{e\tau} & 0 & 0 & 0 & m_D^{\tau 2} & m_D^{\tau 3} \\ \hline m_D^{e1} & 0 & 0 & 0 & M_R^{12} & M_R^{13} \\ 0 & m_D^{\mu 2} & m_D^{\tau 2} & M_R^{12} & 0 & 0 \\ 0 & m_D^{\mu 3} & m_D^{\tau 3} & M_R^{13} & 0 & 0 \end{array}\right)$$

$$\begin{pmatrix} \lambda_+ & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda_- & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Lambda_+ & 0 & 0 \\ 0 & 0 & 0 & 0 & \Lambda_- & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

two heavy + one massless right-handed neutrinos

$$\Lambda_\pm = \pm\sqrt{2} M_R$$
$$\lambda_\pm = \pm\sqrt{2}[m_L - m_D^2/M_R]$$

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two heavy + one massless right-handed neutrinos

$$\Lambda_\pm = \pm\sqrt{2}M_R$$
$$\lambda_\pm = \pm\sqrt{2}[m_L - m_D^2/M_R]$$

light sterile neutrino from explicit symmetry breaking

$$\left(\begin{array}{ccc|ccc} s_L^{ee} & m_L^{e\mu} & m_L^{e\tau} & m_D^{e1} & 0 & 0 \\ m_L^{e\mu} & s_L^{\mu\mu} & 0 & 0 & m_D^{\mu 2} & m_D^{\mu 3} \\ m_L^{e\tau} & 0 & s_L^{\tau\tau} & 0 & m_D^{\tau 2} & m_D^{\tau 3} \\ \hline m_D^{e1} & 0 & 0 & S_R^{11} & M_R^{12} & M_R^{13} \\ 0 & m_D^{\mu 2} & m_D^{\tau 2} & M_R^{12} & S_R^{22} & 0 \\ 0 & m_D^{\mu 3} & m_D^{\tau 3} & M_R^{13} & 0 & S_R^{33} \end{array}\right)$$

III. Example: flavor symmetries

Flavor symmetries

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$\mathcal{F}$	1	−1	−1	1	−1	−1	1	−1	−1	0	0

(Lindner, Merle, Niro, **10**)

$$\mathcal{M}_\nu = \left(\begin{array}{ccc|ccc} 0 & m_L^{e\mu} & m_L^{e\tau} & m_D^{e1} & 0 & 0 \\ m_L^{e\mu} & 0 & 0 & 0 & m_D^{\mu 2} & m_D^{\mu 3} \\ m_L^{e\tau} & 0 & 0 & 0 & m_D^{\tau 2} & m_D^{\tau 3} \\ \hline m_D^{e1} & 0 & 0 & 0 & M_R^{12} & M_R^{13} \\ 0 & m_D^{\mu 2} & m_D^{\tau 2} & M_R^{12} & 0 & 0 \\ 0 & m_D^{\mu 3} & m_D^{\tau 3} & M_R^{13} & 0 & 0 \end{array}\right)$$

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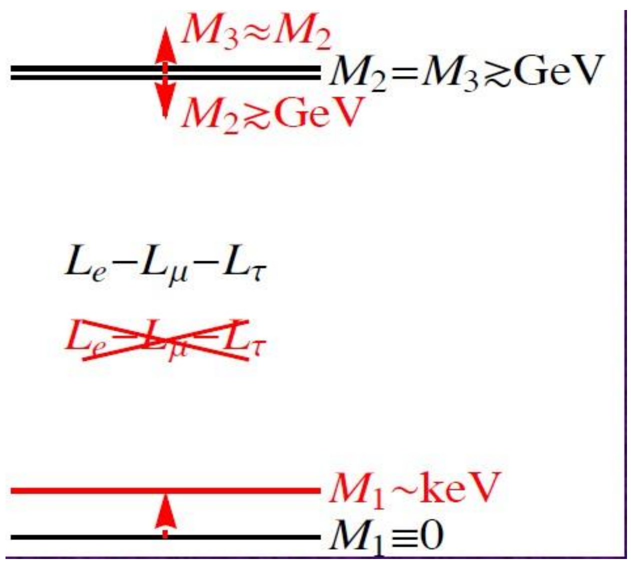

$m_s \sim S \sim$ keV

$\Delta M = M_3 - M_2 \sim m_s$

drawback: maximum solar mixing!

III. Example: flavor symmetries

Flavor symmetries

Friedberg-Lee symmetry: R.Friedberg & T.D.Lee, 2006

Neutrino mass operator is invariant under the transformation

$$\nu_e \rightarrow \nu_e + z; \qquad \nu_\mu \rightarrow \nu_\mu + z; \qquad \nu_\tau \rightarrow \nu_\tau + z;$$

$z \rightarrow$ Grassmann number

III. Example: flavor symmetries

Flavor symmetries

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$z \to$ Grassmann number

$$a(\overline{\nu}_\tau - \overline{\nu}_\mu)(\nu_\tau - \nu_\mu) + b(\overline{\nu}_\mu - \overline{\nu}_e)(\nu_\mu - \nu_e) + c(\overline{\nu}_e - \overline{\nu}_\tau)(\nu_e - \nu_\tau)$$

$$\overline{M} = \begin{pmatrix} b+c & -b & c \\ -b & a+b & a \\ c & a & c+a \end{pmatrix}$$

❖ Rank 2 → one massless eigenstate

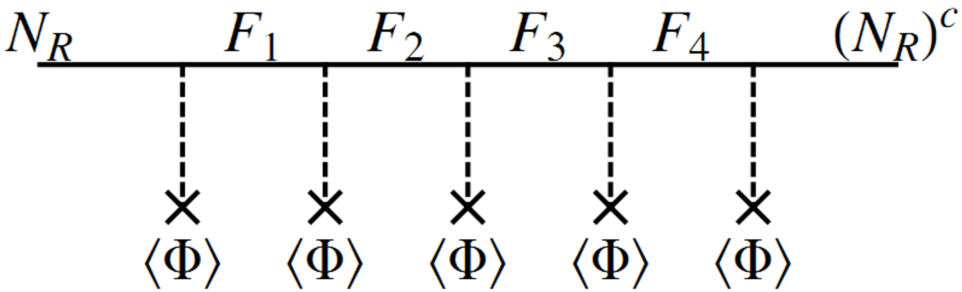
- ✓ Applied to the right-handed neutrino sector
- ✓ One massless sterile neutrino before symmetry breaking

He, Li, Liao, 2009

III. Example: FN mechanism

Froggatt-Nielsen mechanism

- Fermion flavors are differently charged under a $U(1)_{\mathrm{FN}}$ symmetry
- Right-handed neutrino masses receive a suppression factor
  $M \to M\lambda^{2F}$ $\left(\lambda = \frac{\langle \phi \rangle}{\Lambda} < 1\right)$



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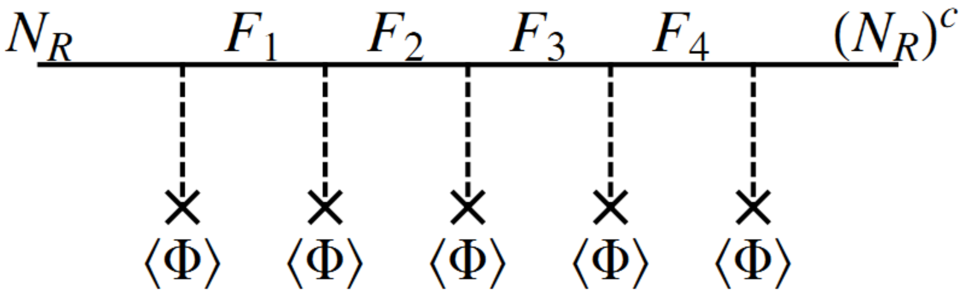
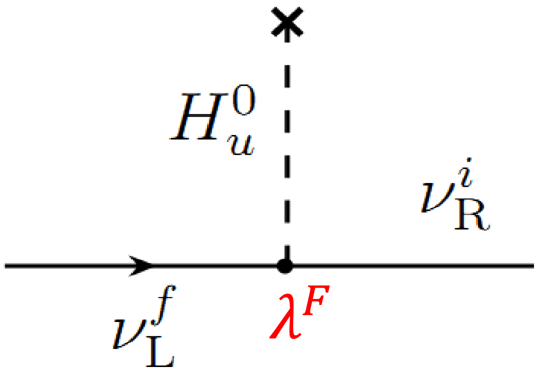


- Vertex suppressed by a factor λ^F



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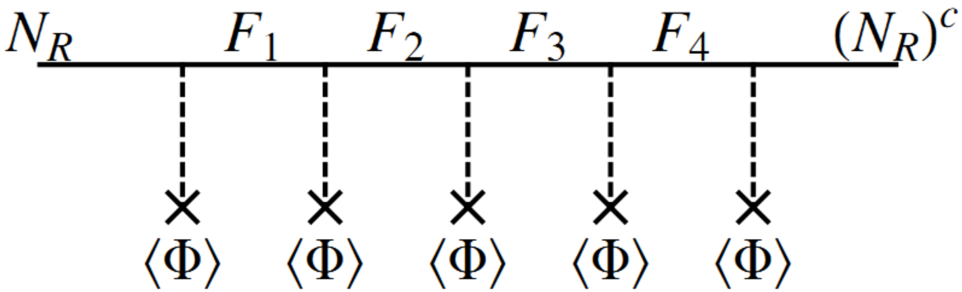
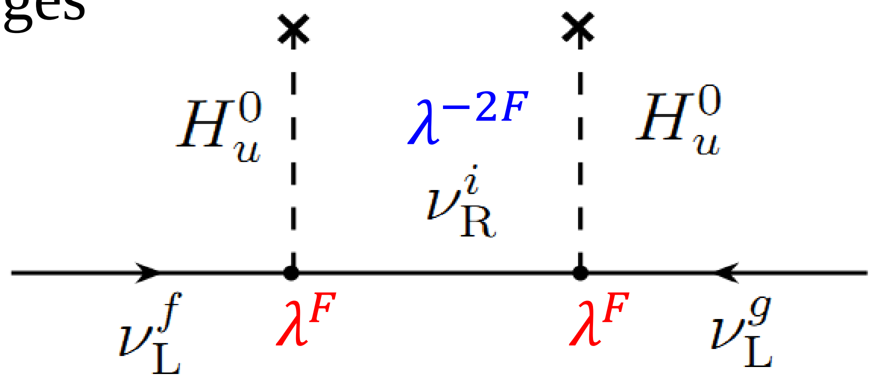


- Seesaw formula and the active neutrino masses are **not** affected by the FN charges



III. Example: FN mechanism

Froggatt-Nielsen mechanism

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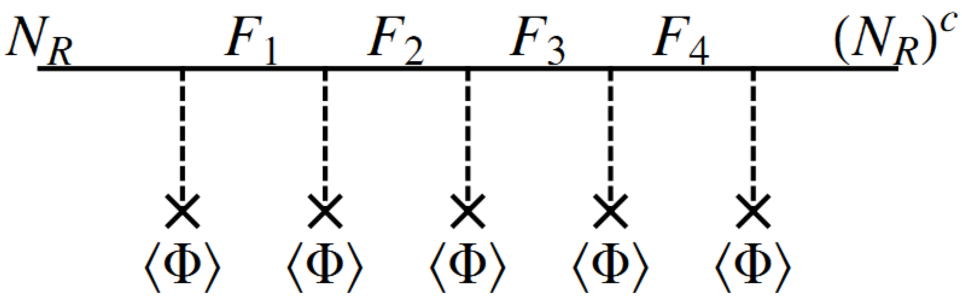
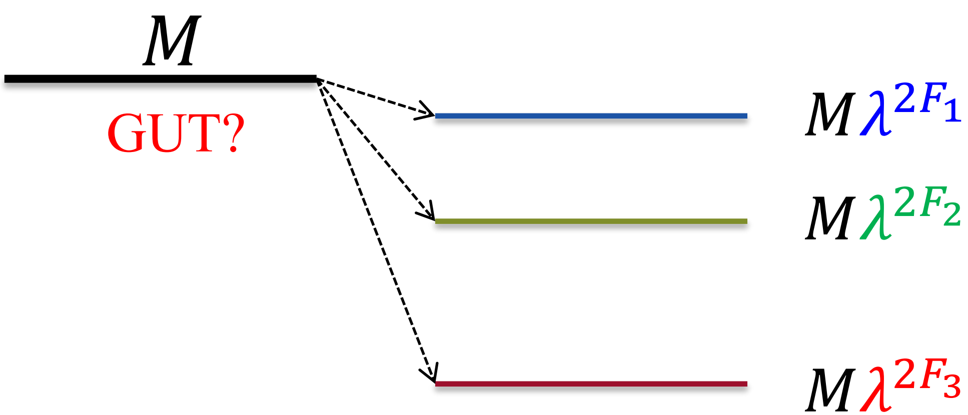


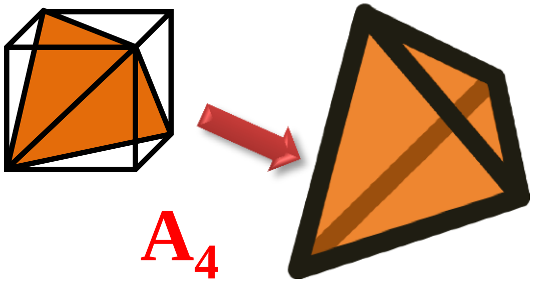
- Seesaw formula and the active neutrino masses are **not** affected by the FN charges

- sterile neutrino mass spectrum



III. Example: FN mechanism

A flavor model based on A_4 + FN mechanism



- Symmetry group of tetrahedron
- Even permutations of four objects
- 12 elements; 4 irreducible represents: $1, 1', 1''$, and 3

(Wyler 79', Ma, Rajasekaran 01', Babu, Ma, Valle 03', Altarelli, Feruglio, 05')

Field	L	e^c	μ^c	τ^c	$h_{u,d}$	φ	φ'	ξ	ν_s
$\mathrm{SU}(2)_L$	2	1	1	1	2	1	1	1	1
A_4	$\underline{3}$	$\underline{1}$	$\underline{1}''$	$\underline{1}'$	$\underline{1}$	$\underline{3}$	$\underline{3}$	$\underline{1}$	$\underline{1}$
Z_3	ω	ω^2	ω^2	ω^2	1	1	ω	ω	1
$\mathrm{U}(1)_{FN}$	–	3	1	0	–	–	–	–	6

⇦ particle assignments

Barry, Rodejohann, HZ, JHEP 2011; JCAP 2012

$$\mathcal{L}_{\mathrm{Y}} = \frac{y_e}{\Lambda} e^c (\varphi L) h_d + \frac{y_\mu}{\Lambda} \mu^c (\varphi L)' h_d + \frac{y_\tau}{\Lambda} \tau^c (\varphi L)'' h_d + \frac{x_a}{\Lambda^2} \xi (L h_u L h_u) + \frac{x_d}{\Lambda^2} (\varphi' L h_u L h_u) + \text{h.c.} + \dots$$

$$\mathcal{L}_{\mathrm{Y}_s} = \frac{x_e}{\Lambda^2} \xi (\varphi' L h_u) \nu_s + \frac{x_f}{\Lambda^2} (\varphi' \varphi' L h_u) \nu_s + m_s \nu_s^c \nu_s^c + \text{h.c.}$$

III. Example: FN mechanism

Field	L	e^c	μ^c	τ^c	$h_{u,d}$	φ	φ'	ξ	ν_s
$SU(2)_L$	2	1	1	1	2	1	1	1	1
A_4	$\underline{3}$	$\underline{1}$	$\underline{1}''$	$\underline{1}'$	$\underline{1}$	$\underline{3}$	$\underline{3}$	$\underline{1}$	$\underline{1}$
Z_3	ω	ω^2	ω^2	ω^2	1	1	ω	ω	1
$U(1)_{FN}$	$-$	3	1	0	$-$	$-$	$-$	$-$	6

particle assignments

Barry, Rodejohann, HZ, JHEP 2011; JCAP 2012

Adopting the VEV alignments

$\langle \xi \rangle = u; \quad \langle \varphi \rangle = (v, 0, 0)$
$\langle \varphi' \rangle = (v', v', v')$

$$M_\nu^{4\times 4} = \begin{pmatrix} a + \frac{2d}{3} & -\frac{d}{3} & -\frac{d}{3} & e \\ \cdot & \frac{2d}{3} & a - \frac{d}{3} & e \\ \cdot & \cdot & \frac{2d}{3} & e \\ \cdot & \cdot & \cdot & m_s \end{pmatrix}$$

$$a = 2x_a \frac{uv_u^2}{\Lambda^2}, \quad d = 2x_d \frac{v' v_u^2}{\Lambda^2}, \quad e = \sqrt{2} x_e \frac{uv' v_u}{\Lambda^2}$$

Numerical example: assuming Yukawa couplings are of order 1 and $\lambda = 10^{-1.5} \cong 0.03$

$$a \sim d \simeq 0.1 \left(\frac{u}{10^{11}\,\mathrm{GeV}} \right) \left(\frac{v_u}{10^2\,\mathrm{GeV}} \right)^2 \left(\frac{10^{12.5}\,\mathrm{GeV}}{\Lambda} \right)^2 \mathrm{eV},$$

$$e \simeq 0.1 \left(\frac{\lambda}{10^{-1.5}} \right)^6 \left(\frac{u}{10^{11}\,\mathrm{GeV}} \right) \left(\frac{v'}{10^{11}\,\mathrm{GeV}} \right) \left(\frac{v_u}{10^2\,\mathrm{GeV}} \right) \left(\frac{10^{12.5}\,\mathrm{GeV}}{\Lambda} \right)^2 \mathrm{eV}$$

$$m_s \simeq 10^{0.5} \left(\frac{\lambda}{10^{-1.5}} \right)^{12} \left(\frac{v}{10^{11}\,\mathrm{GeV}} \right)^2 \left(\frac{10^{12.5}\,\mathrm{GeV}}{\Lambda} \right) \mathrm{eV}$$

an eV-scale ν_s is accommodated

III. Example: FN mechanism

Extension to the seesaw framework

Field	L	e^c	μ^c	τ^c	$h_{u,d}$	φ	φ'	φ''	ξ	ξ'	ξ''	Θ	ν_1^c	ν_2^c	ν_3^c
$SU(2)_L$	2	1	1	1	2	1	1	1	1	1	1	1	1	1	1
A_4	$\underline{3}$	$\underline{1}$	$\underline{1}''$	$\underline{1}'$	$\underline{1}$	$\underline{3}$	$\underline{3}$	$\underline{3}$	$\underline{1}$	$\underline{1}'$	$\underline{1}$	$\underline{1}$	$\underline{1}$	$\underline{1}'$	$\underline{1}$
Z_3	ω	ω^2	ω^2	ω^2	1	1	ω	ω^2	ω^2	ω	1	1	ω^2	ω	1
$U(1)_{\rm FN}$	-	3	1	0	-	-	-	-	-	-	-	-1	F_1	F_2	F_3

- ✓ Right-handed neutrinos are A_4 singlets so as to assign different FN charges (mass splitting)
- ✓ One of the sterile neutrinos is located at **eV** or **keV** scale
- ✓ The other two right-handed neutrinos generate active neutrino masses via seesaw (reproducing the ***ν*MSM** structure)
- ✓ Tri-bimaximal mixing (**TBM**) is obtained at leading order from vacuum alignments of flavons
- ✓ Charged-lepton corrections → θ_{13}

III. Example: extended seesaw

Minimal Extended Seesaw (MES)

Light sterile neutrinos: suppressed by **seesaw** as well?

HZ, Phys.Lett.B 714 (2012) 262

III. Example: extended seesaw

Minimal Extended Seesaw (MES)

The model: **SM** + **three** right-handed neutrinos + **one** singlet S

$$-\mathcal{L}_m = \overline{\nu_L} M_D \nu_R + \overline{S^c} M_S \nu_R + \frac{1}{2} \overline{\nu_R^c} M_R \nu_R + \text{h.c.}$$

$$M_S = (\times \quad \times \quad \times)$$

$$M_\nu^{7\times 7} = \begin{pmatrix} 0 & M_D & 0 \\ M_D^T & M_R & M_S^T \\ 0 & M_S & 0 \end{pmatrix}$$

✓ The full **7×7** neutrino mass matrix is of rank **6**, and therefore, one active neutrino is massless.

III. Example: extended seesaw

Minimal Extended Seesaw (MES)

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✓ The full **7 × 7** neutrino mass matrix is of rank **6**, and therefore, one active neutrino is massless.

If $M_R \gg M_S, M_D$, we can integrate out ν_R

$$m_\nu \simeq M_D M_R^{-1} M_S^T \left(M_S M_R^{-1} M_S^T\right)^{-1} M_S \left(M_R^{-1}\right)^T M_D^T - M_D M_R^{-1} M_D^T$$

$$m_s \simeq -M_S M_R^{-1} M_S^T$$

III. Example: extended seesaw

Minimal Extended Seesaw (MES)

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$$m_s \simeq -M_S M_R^{-1} M_S^T$$

Do not cancel with each other

⇒ **two** massive active neutrinos

III. Example: extended seesaw

Minimal Extended Seesaw (MES)

The model: **SM** + **three** right-handed neutrinos + **one** singlet S

$$-\mathcal{L}_m = \overline{\nu_L} M_D \nu_R + \overline{S^c} M_S \nu_R + \frac{1}{2}\overline{\nu_R^c} M_R \nu_R + \text{h.c.}$$

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$$m_s \simeq -M_S M_R^{-1} M_S^T$$

$M_D \sim 100$ GeV;
$M_S \sim 500$ GeV; $M_R \sim 2 \times 10^{14}$ GeV

⇒

$m_\nu \sim 0.05$ eV;
$m_s \sim 1.3$ eV; $U_{e4} \sim 0.2$

III. Example: extended seesaw

Minimal Extended Seesaw (MES)

The model: **SM** + **three** right-handed neutrinos + **one** singlet *S*

$$-\mathcal{L}_m = \overline{\nu_L} M_D \nu_R + \overline{S^c} M_S \nu_R + \frac{1}{2}\overline{\nu_R^c} M_R \nu_R + \text{h.c.}$$

$$M_S = (\times \quad \times \quad \times)$$

$$M_\nu^{7\times 7} = \begin{pmatrix} 0 & M_D & 0 \\ M_D^T & M_R & M_S^T \\ 0 & M_S & 0 \end{pmatrix}$$

✓ The full **7 × 7** neutrino mass matrix is of rank **6**, and therefore, one active neutrino is massless.

If $M_R \gg M_S, M_D$, we can integrate out ν_R

$$m_\nu \simeq M_D M_R^{-1} M_S^T \left(M_S M_R^{-1} M_S^T\right)^{-1} M_S \left(M_R^{-1}\right)^T M_D^T - M_D M_R^{-1} M_D^T$$

$$m_s \simeq -M_S M_R^{-1} M_S^T$$

✓ No need to artificially insert tiny mass scales and Yukawa couplings
✓ Thermal leptogenesis works.
✓ Only **one** singlet *S* is allowed (minimal extension).

III. Example: extended seesaw

Minimal Extended Seesaw (MES)

How to realize the **MES** structure?

1. **Flavor symmetry (e.g. A_4)**

TABLE I: Particle assignments in the flavor A_4 model.

Field	ℓ	e_R	μ_R	τ_R	H	φ	φ'	φ''	ξ	ξ'	χ	ν_{R1}	ν_{R2}	ν_{R3}	S
SU(2)	2	1	1	1	2	1	1	1	1	1	1	1	1	1	1
A_4	$\underline{3}$	$\underline{1}$	$\underline{1}''$	$\underline{1}'$	$\underline{1}$	$\underline{3}$	$\underline{3}$	$\underline{3}$	$\underline{1}$	$\underline{1}'$	$\underline{1}$	$\underline{1}$	$\underline{1}'$	$\underline{1}$	$\underline{1}$
Z_4	1	1	1	1	1	1	i	-1	1	-1	$-$i	1	$-$i	-1	i

$$\begin{aligned}
\mathcal{L} \;=\;& \frac{y_e}{\Lambda}\left(\bar{\ell}H\varphi\right)_{\underline{1}} e_R + \frac{y_\mu}{\Lambda}\left(\bar{\ell}H\varphi\right)_{\underline{1}'} \mu_R + \frac{y_\tau}{\Lambda}\left(\bar{\ell}H\varphi\right)_{\underline{1}''} \tau_R \\
&+ \frac{y_1}{\Lambda}\left(\bar{\ell}\tilde{H}\varphi\right)_{\underline{1}} \nu_{R1} + \frac{y_2}{\Lambda}\left(\bar{\ell}\tilde{H}\varphi'\right)_{\underline{1}''} \nu_{R2} + \frac{y_3}{\Lambda}\left(\bar{\ell}\tilde{H}\varphi''\right)_{\underline{1}} \nu_{R3} \\
&+ \frac{1}{2}\lambda_1\xi\overline{\nu^c_{R1}}\nu_{R1} + \frac{1}{2}\lambda_2\xi'\overline{\nu^c_{R2}}\nu_{R2} + \frac{1}{2}\lambda_3\xi\overline{\nu^c_{R3}}\nu_{R3} \\
&+ \frac{1}{2}\rho\chi\overline{S^c}\nu_{R1} + \text{h.c.}\,,
\end{aligned}$$

III. Example: extended seesaw

Minimal Extended Seesaw (MES)

How to realize the **MES** structure?

1. **Flavor symmetry (e.g. A_4)**

- charged-lepton mass matrix is diagonal

$$m_\ell = \frac{\langle H \rangle v}{\Lambda} \mathrm{diag}\,(y_e, y_\mu, y_\tau)$$

- Dirac mass terms

$$M_D = \frac{\langle H \rangle v}{\Lambda} \begin{pmatrix} y_1 & y_2 & 0 \\ 0 & y_2 & y_3 \\ 0 & y_2 & -y_3 \end{pmatrix}$$

$$M_S = \begin{pmatrix} \rho u & 0 & 0 \end{pmatrix}$$

- right-handed neutrino mass matrix

$$M_R = \mathrm{diag}\,(\lambda_1 v, \lambda_2 v, \lambda_3 v)$$

$$m_\nu = -\frac{\langle H \rangle^2 v}{\Lambda^2} \begin{pmatrix} \frac{y_2^2}{\lambda_2} & \frac{y_2^2}{\lambda_2} & \frac{y_2^2}{\lambda_2} \\ \frac{y_2^2}{\lambda_2} & \frac{y_2^2\lambda_3 + y_3^2\lambda_2}{\lambda_2\lambda_3} & \frac{y_2^2\lambda_3 - y_3^2\lambda_2}{\lambda_2\lambda_3} \\ \frac{y_2^2}{\lambda_2} & \frac{y_2^2\lambda_3 - y_3^2\lambda_2}{\lambda_2\lambda_3} & \frac{y_2^2\lambda_3 + y_3^2\lambda_2}{\lambda_2\lambda_3} \end{pmatrix}$$

diagonalized by TBM

sterile parameters:

$$m_s \simeq \frac{\rho^2 u^2}{\lambda_1 v} \qquad R \simeq \begin{pmatrix} \frac{y_1 \langle H \rangle v}{\rho u \Lambda} & 0 & 0 \end{pmatrix}^T$$

$v = 10^{13}$ GeV, $u = 10^2$ GeV
$\Lambda = 10^{14}$ GeV

$m_s \sim 1.2$ eV $\qquad R \sim 0.16$

III. Example: extended seesaw

How to realize the **MES** structure in the $U(1)'$ extension?

2. $\mathbf{U(1)'}$ model

Julian Heeck, HZ, 1211.0538

Assuming only singlet fields S_i are charged under $U(1)'$, the anomaly-free conditions reduce to

$$\sum_f Y'(f) = 0 \quad \text{and} \quad \sum_f (Y'(f))^3 = 0$$

$[U(1)'] \times [\text{graviton}]^2$ $\qquad\qquad$ $[U(1)']^3$

III. Example: extended seesaw

How to realize the **MES** structure in the $U(1)'$ extension?

2. $U(1)'$ model — Julian Heeck, HZ, 1211.0538

Assuming only singlet fields S_i are charged under $U(1)'$, the anomaly-free conditions reduce to

$$\sum_f Y'(f) = 0 \quad \text{and} \quad \sum_f (Y'(f))^3 = 0$$

solutions for five singlets and $Z \leq 25$

Z_1	Z_2	Z_3	Z_4	Z_5
−9	−5	−1	7	8
−9	−7	2	4	10
−18	−17	1	14	20
−21	−12	5	6	22
−25	−8	−7	18	22

Nakayama, Takahashi, Yanagida, 11’

III. Example: extended seesaw

How to realize the **MES** structure in the $U(1)'$ extension?

2. $\mathbf{U(1)'}$ model Julian Heeck, HZ, 1211.0538

Our principle:

- anomaly-free U(1)'
- new fermions are all massive at tree-level
- only one extra scalar is introduced
- couplings like $\overline{L}HS$ and $\overline{S^c}S$ are forbidden.

The model contents: in total ten "right-handed neutrinos"

	$\nu_{R,1}$	$\nu_{R,2}$	$\nu_{R,3}$	S_1	S_2	S_3	S_4	S_5	S_6	S_7	ϕ
Y'	0	0	0	11	-5	-6	1	-12	2	9	11

Lagrangian:

$$\mathcal{L}_m = (m_D)_{ij}\overline{\nu_{L,i}}\nu_{R,j} + \frac{1}{2}(M_R)_{ij}\overline{\nu^c_{R,i}}\nu_{R,j} + w_i\phi^\dagger\,\overline{S^c_1}\nu_{R,i}$$
$$+ y_1\phi\,\overline{S^c_3}S_2 + y_2\phi\,\overline{S^c_4}S_5 + y_3\phi^\dagger\,\overline{S^c_6}S_7 + \text{h.c.},$$

III. Example: extended seesaw

How to realize the **MES** structure in the $U(1)'$ extension?

2. $\mathbf{U(1)'}$ model Julian Heeck, HZ, 1211.0538

Our principle:

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	$\nu_{R,1}$	$\nu_{R,2}$	$\nu_{R,3}$	S_1	S_2	S_3	S_4	S_5	S_6	S_7	ϕ
Y'	0	0	0	11	-5	-6	1	-12	2	9	11

mass matrix: block structure (13×13)

$$\mathcal{M} = \begin{pmatrix} (\mathcal{M}_{\rm MES})_{7\times7} & 0 \\ 0 & (\mathcal{M}_S)_{6\times6} \end{pmatrix}$$

$$\mathcal{M}_S = \begin{pmatrix} 0 & y_1\langle\phi\rangle & 0 & 0 & 0 & 0 \\ y_1\langle\phi\rangle & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & y_2\langle\phi\rangle & 0 & 0 \\ 0 & 0 & y_2\langle\phi\rangle & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & y_3\langle\phi\rangle \\ 0 & 0 & 0 & 0 & y_3\langle\phi\rangle & 0 \end{pmatrix}$$

III. Example: radiative inverse seesaw

Dev, Pilaftsis, **PRD** 13

SM + 3 heavy right-handed neutrinos + 3 SM gauge singlet neutrinos

$$-\mathcal{L}_{\rm m} = \overline{\nu_{\rm L}} M_{\rm D} \nu_{\rm R} + \overline{S} M_{\rm R} \nu_{\rm R} + \frac{1}{2}\overline{S}\mu S^c + {\rm H.c.}$$

9x9 ν-mass

$\{\nu_L, \nu_R^c, S^c\}$
$$M_\nu = \begin{pmatrix} 0 & M_{\rm D} & 0 \\ M_{\rm D}^T & 0 & M_{\rm R}^T \\ 0 & M_{\rm R} & \mu \end{pmatrix}$$

Light neutrino mass matrix:

$$m_\nu \simeq M_{\rm D} M_{\rm R}^{-1} \mu (M_{\rm R}^T)^{-1} M_{\rm D}^T = F \mu F^T$$

III. Example: radiative inverse seesaw

Dev, Pilaftsis, **PRD** 13

SM + 3 heavy right-handed neutrinos + 3 SM gauge singlet neutrinos

$$-\mathcal{L}_{\rm m} = \overline{\nu_{\rm L}} M_{\rm D} \nu_{\rm R} + \overline{S} M_{\rm R} \nu_{\rm R} + \frac{1}{2}\overline{S}\mu S^c + {\rm H.c.}$$

9x9 ν-mass $\{\nu_L, \nu_R^c, S^c\}$

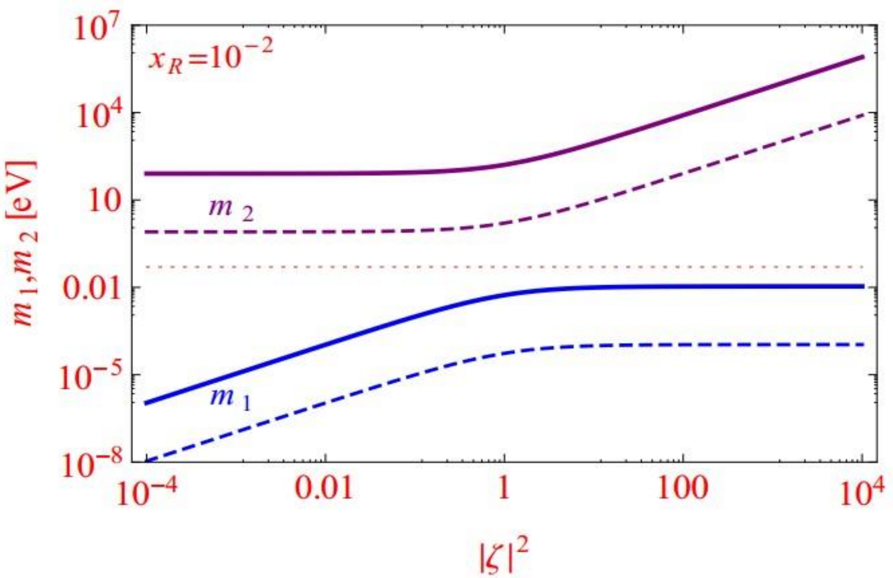
$$M_\nu = \begin{pmatrix} 0 & M_{\rm D} & 0 \\ M_{\rm D}^T & 0 & M_{\rm R}^T \\ 0 & M_{\rm R} & \mu \end{pmatrix} \Rightarrow \begin{pmatrix} \mathbf{0} & M_D & \mathbf{0} \\ M_D^{\sf T} & \mu_R & M_N^{\sf T} \\ \mathbf{0} & M_N & \mu_S \end{pmatrix}$$

active neutrino mass (**1-loop**):

$$M_D \mu_R^{-1} x_R f(x_R) M_D^{\sf T}$$

sterile neutrino mass (**seesaw**):

$$M_N \mu_R^{-1} M_N^{\sf T}$$



III. Example: non-standard models

Non-standard Approaches

Mirror model

Berezhiania, Mohapatra 95; Foot, Volkas, 95;
Berezinsky, Narayan, Vissani 02

$$\mathbf{SU(3)} \times \mathbf{SU(2)} \times \mathbf{U(1)} \quad \otimes \quad \mathbf{SU(3)'} \times \mathbf{SU(2)'} \times \mathbf{U(1)'}$$

Quarks (B=1/3) & Leptons (L=1) — Quarks (B'=1/3) & Leptons (L'=1)

Yukawa interactions

$-L = Y\overline{f}_L H f_R$

$\langle H \rangle = v$

$m_\nu \sim v^2/M$

Yukawa interactions

$-L = Y'\overline{f}_L{}' H' f_R{}'$

$\langle H' \rangle = v'$

$m_s \sim v'^2/M$

$$m_{\nu s} \sim \frac{LL'HH'}{\Lambda_{PL}}$$

Different inflation, reheating temp

V. Summary

Light sterile neutrinos present in: short-baseline neutrino oscillation experiments; effective mass measured in neutrino-less double beta decays; keV Warm Dark Matter; …

Mechanisms are needed to understand the smallness light sterile neutrinos

a) suppress M_D and M_R simultaneously via flavor symmetries, warped extra dimensions; FN mechanism;

b) non-standard approaches: mirror models; SUSY; …

c) extended seesaw models; $U(1)'$; radiative seesaw…

Thanks