

Phenomenology of Neutrino Oscillations and Mixing

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Neutrino Unbound: <http://www.nu.to.infn.it>

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Matter To The Deepest: Recent Developments In Physics of
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Experimental Evidences of Neutrino Oscillations

Solar
 $\nu_e \rightarrow \nu_\mu, \nu_\tau$

VLBL Reactor
 $\bar{\nu}_e$ disappearance

$\left(\begin{array}{l} \text{SNO, BOREXino} \\ \text{Super-Kamiokande} \\ \text{GALLEX/GNO, SAGE} \\ \text{Homestake, Kamiokande} \\ \text{(KamLAND)} \end{array} \right)$

$\rightarrow \left\{ \begin{array}{l} \Delta m_S^2 \simeq 7.6 \times 10^{-5} \text{ eV}^2 \\ \sin^2 \vartheta_S \simeq 0.30 \end{array} \right.$

Atmospheric
 $\nu_\mu \rightarrow \nu_\tau$

LBL Accelerator
 ν_μ disappearance

LBL Accelerator
 $\nu_\mu \rightarrow \nu_\tau$

$\left(\begin{array}{l} \text{Super-Kamiokande} \\ \text{Kamiokande, IMB} \\ \text{MACRO, Soudan-2} \\ \text{(K2K, MINOS, T2K)} \\ \text{(Opera)} \end{array} \right)$

$\rightarrow \left\{ \begin{array}{l} \Delta m_A^2 \simeq 2.4 \times 10^{-3} \text{ eV}^2 \\ \sin^2 \vartheta_A \simeq 0.50 \end{array} \right.$

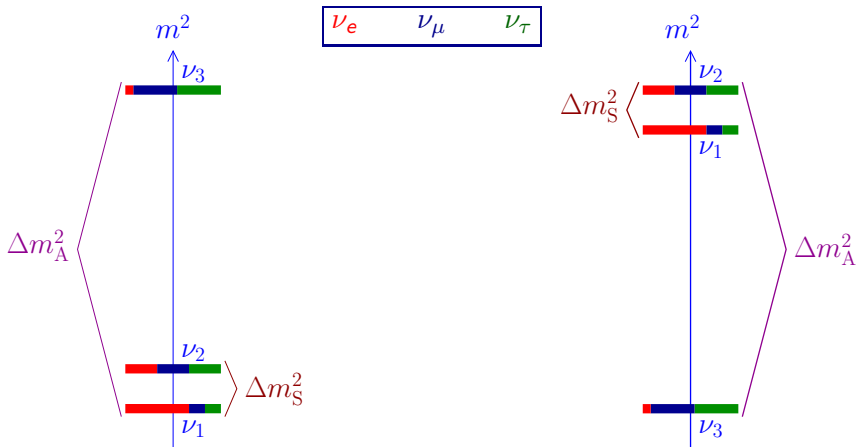
LBL Accelerator
 $\nu_\mu \rightarrow \nu_e$

LBL Reactor
 $\bar{\nu}_e$ disappearance

$\left(\begin{array}{l} \text{(T2K, MINOS)} \\ \text{Daya Bay, RENO} \\ \text{Double Chooz} \end{array} \right)$

$\rightarrow \left\{ \begin{array}{l} \Delta m_A^2 \\ \sin^2 \vartheta_{13} \simeq 0.023 \end{array} \right.$

Three-Neutrino Mixing Paradigm



Normal Spectrum

$$\Delta m_S^2 = \Delta m_{21}^2 = 7.50 \pm 0.20 \times 10^{-5} \text{ eV}^2 \quad \text{uncertainty} \simeq 2.6\%$$

$$\Delta m_A^2 = |\Delta m_{31}^2| \simeq |\Delta m_{32}^2| = 2.32_{-0.08}^{+0.12} \times 10^{-3} \text{ eV}^2 \quad \text{uncertainty} \simeq 5\%$$

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_2} & 0 \\ 0 & 0 & e^{i\lambda_3} \end{pmatrix}$$

$$\vartheta_{23} = \vartheta_A$$

$$\sin^2 \vartheta_{23} \simeq 0.4 - 0.6$$

Chooz, Palo Verde

T2K, MINOS

Daya Bay, RENO

$$\sin^2 \vartheta_{13} = 0.023 \pm 0.002$$

$$\vartheta_{12} = \vartheta_S$$

$$\sin^2 \vartheta_{12} = 0.30 \pm 0.01$$

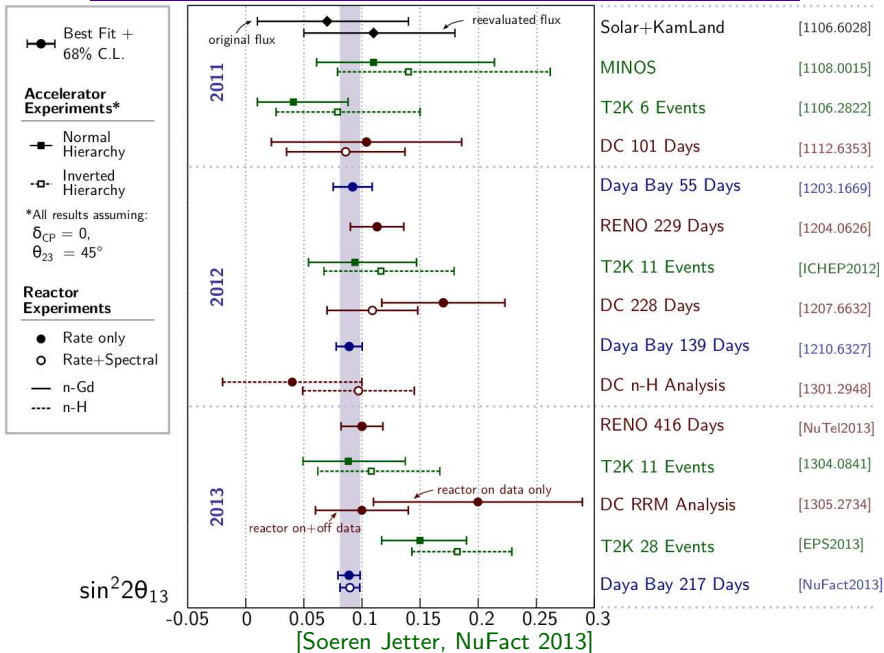
$\beta\beta_{0\nu}$

$$\frac{\delta \sin^2 \vartheta_{23}}{\sin^2 \vartheta_{23}} \simeq 40\%$$

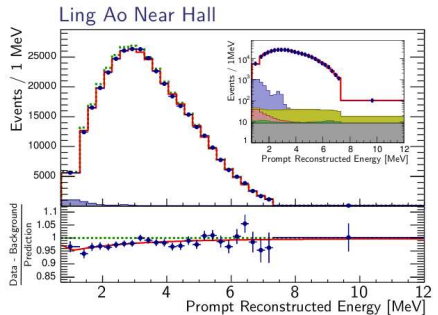
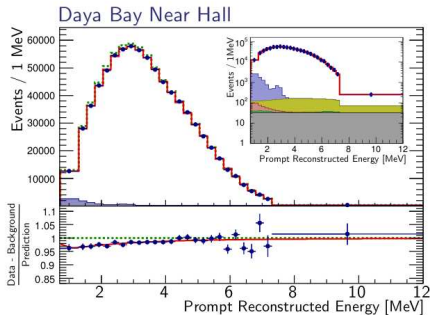
$$\frac{\delta \sin^2 \vartheta_{13}}{\sin^2 \vartheta_{13}} \simeq 10\%$$

$$\frac{\delta \sin^2 \vartheta_{12}}{\sin^2 \vartheta_{12}} \simeq 5\%$$

Global Comparison of ν_{13} Measurements



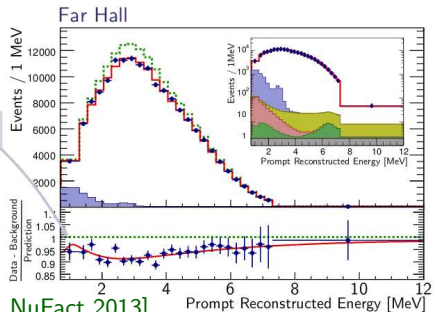
Daya Bay - 22 August 2013



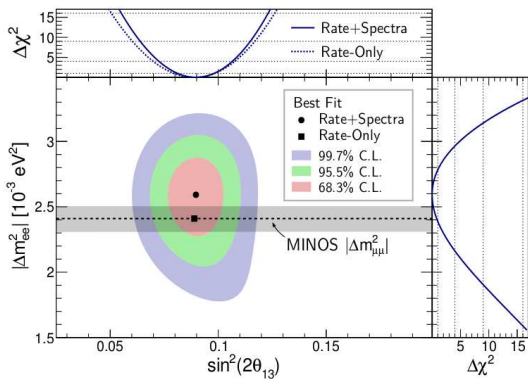
Spectral distortion
consistent with oscillation

- Both background and predicted no-oscillation spectra from best fit
- Statistical errors only

Shape distortion from energy losses in acrylic



[Soeren Jetter, NuFact 2013]

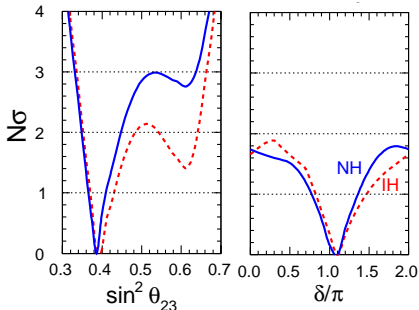


$$\sin^2 2\theta_{13} = 0.090^{+0.008}_{-0.009}$$

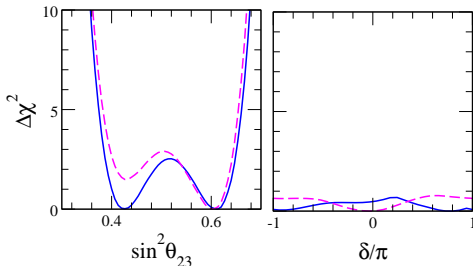
$$|\Delta m_{ee}^2| = 2.59^{+0.19}_{-0.20} \cdot 10^{-3} \text{eV}^2$$

$$\chi^2/N_{\text{DoF}} = 162.7/153$$

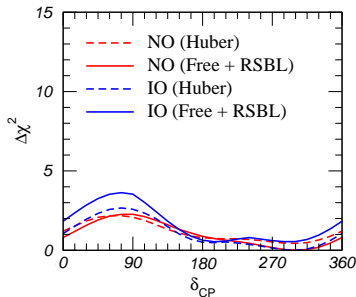
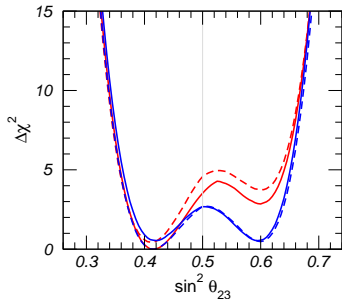
[Soeren Jetter, NuFact 2013]



[Fogli, Lisi, Marrone, Montanino, Palazzo, Rotunno, PRD 86 (2012) 013012]



[Forero, Tortola, Valle, PRD 86 (2012) 073012]



[Gonzalez-Garcia, Maltoni, Salvado, Schwetz, JHEP 12 (2012) 123; <http://www.nu-fit.org>]

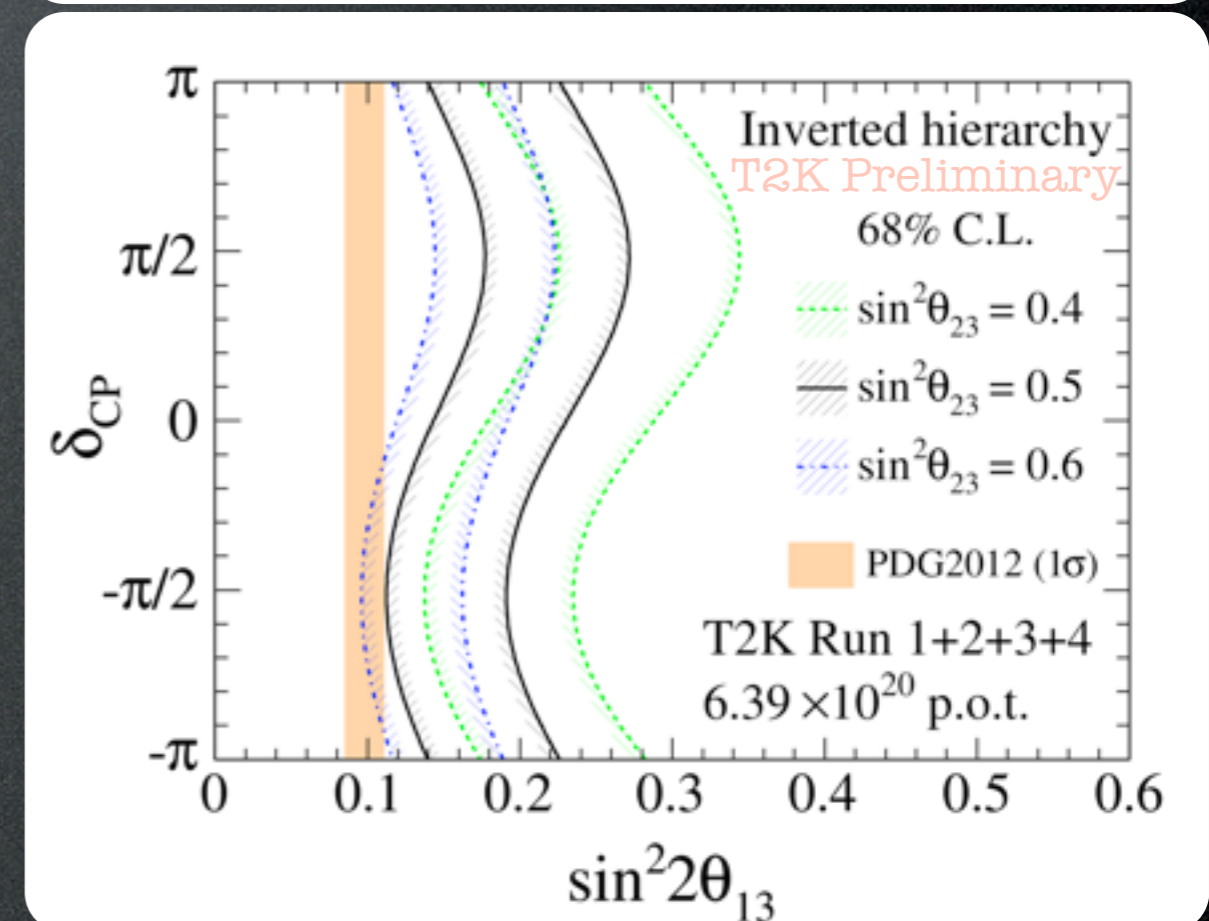
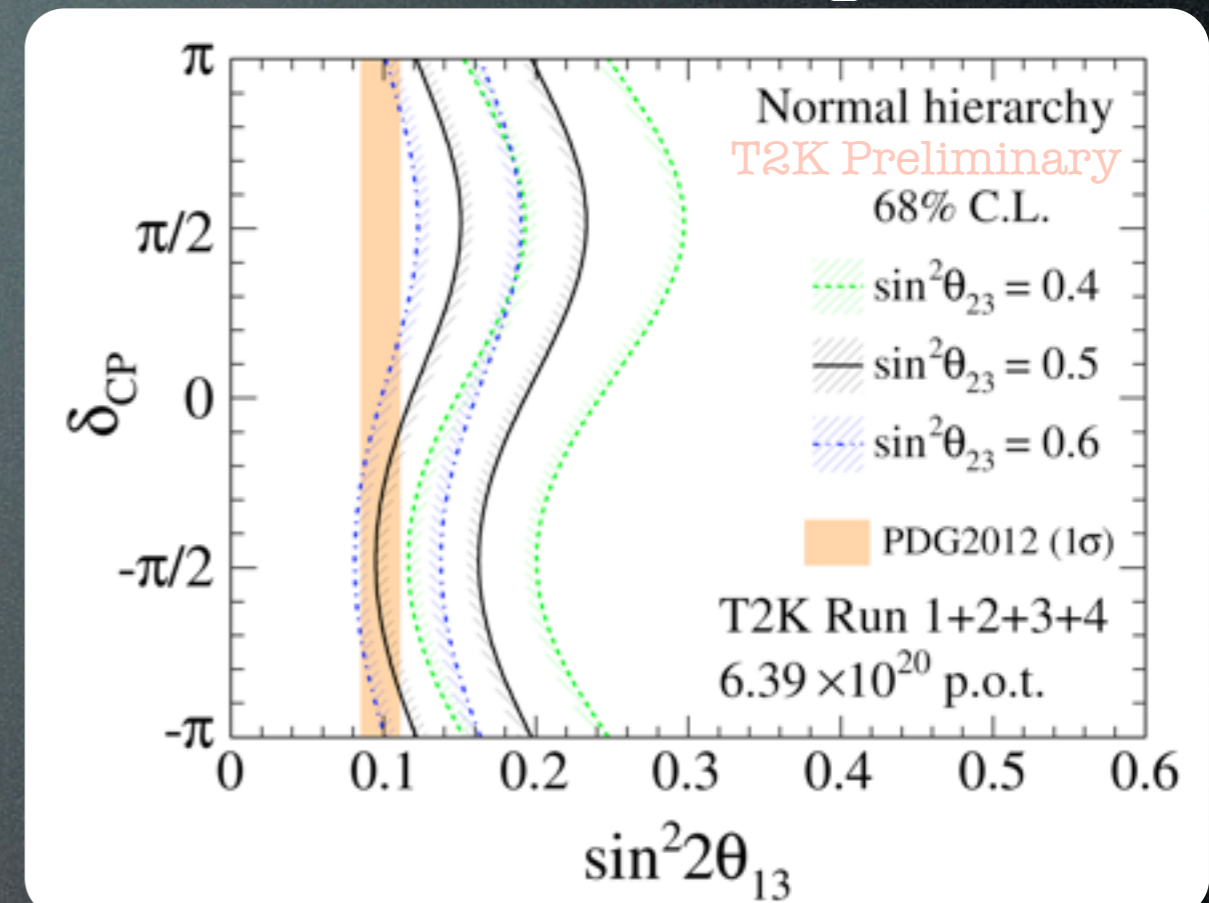
Open Problems

- ▶ $\vartheta_{23} \stackrel{?}{\leq} 45^\circ$
 - ▶ Atmospheric ν , T2K, NO ν A,
- ▶ Mass Hierarchy ?
 - ▶ NO ν A, Atmospheric ν , JUNO (Daya Bay II), RENO-50, Supernova ν , ...
- ▶ CP violation ?
 - ▶ NO ν A, LAGUNA-LBNO, LBNE (USA), HyperK, ...
- ▶ Absolute Mass Scale ?
 - ▶ β Decay, Neutrinoless Double- β Decay, Cosmology, ...
- ▶ Dirac or Majorana ?
 - ▶ Neutrinoless Double- β Decay, ...
- ▶ Beyond Three-Neutrino Mixing ? Sterile Neutrinos ?

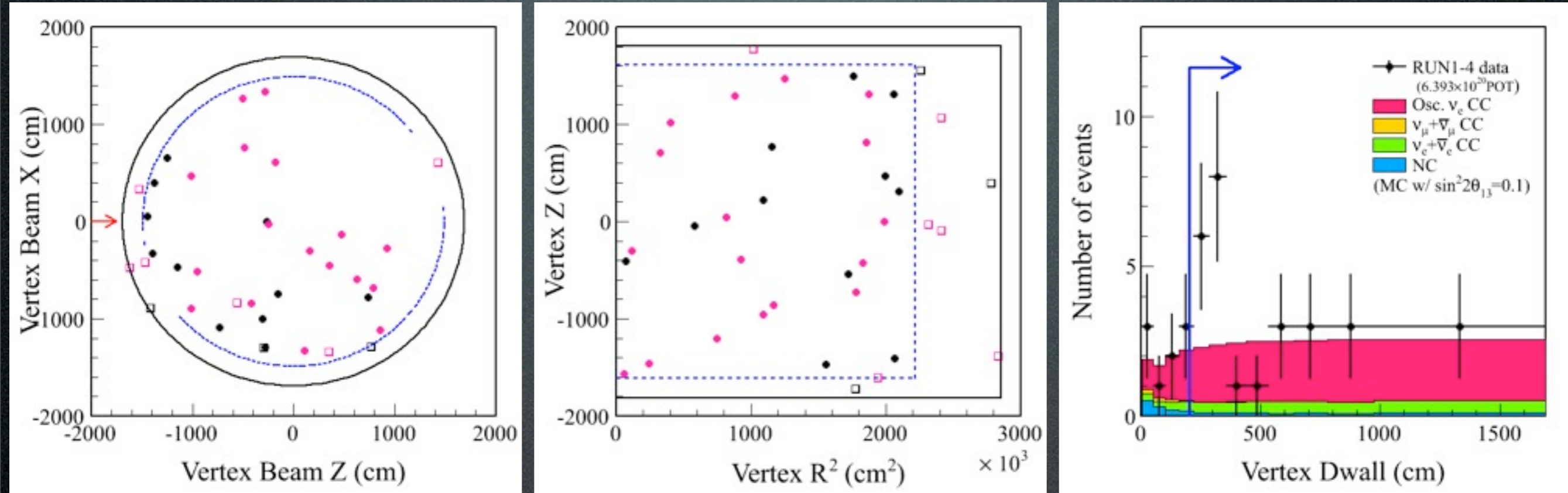
Effect of θ_{23} Uncertainty

- ν_e appearance probability also depends on the value of θ_{23}
- If θ_{23} is fixed at values near the edge of the current allowed region, the fit contours shift
- Future improved measurements of θ_{23} will be important to extract information about other oscillation parameters (including δ_{CP}) in long-baseline experiments
 - A T2K combined $\nu_e + \nu_\mu$ analysis is underway

Note: these are 1D contours for various values of δ_{CP} , not 2D contours



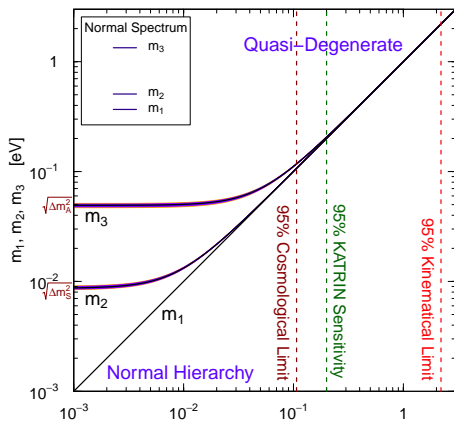
Far Detector ν_e Vertex Distribution



	RUN1+2+3	RUN4	RUN1+2+3+4
D_{wall}	34.4%	54.7%	20.9%
$F_{romwall} beam_{ }$	6.04%	85.6%	8.93%
$R^2 + Z$	32.4%	98.1%	64.5%

- With increased statistics, the p-values for the test distributions have increased

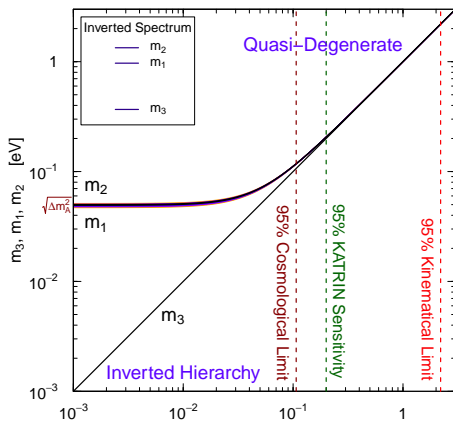
Absolute Scale of Neutrino Masses



Lightest mass: m_1 [eV]

$$m_2^2 = m_1^2 + \Delta m_{21}^2 = m_1^2 + \Delta m_S^2$$

$$m_3^2 = m_1^2 + \Delta m_{31}^2 = m_1^2 + \Delta m_A^2$$



Lightest mass: m_3 [eV]

$$m_1^2 = m_3^2 - \Delta m_{31}^2 = m_3^2 + \Delta m_A^2$$

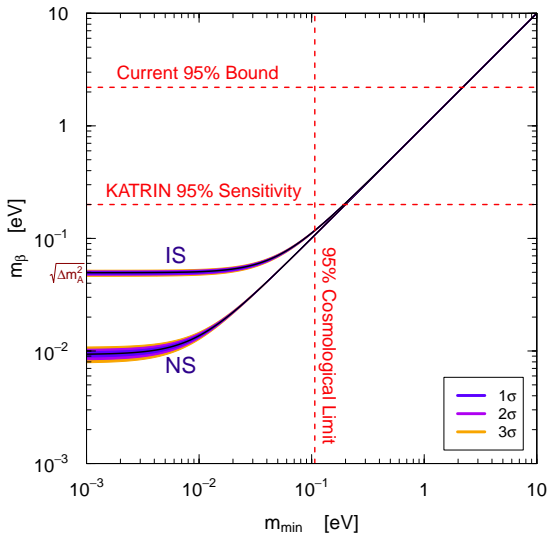
$$m_2^2 = m_1^2 + \Delta m_{21}^2 \simeq m_3^2 + \Delta m_A^2$$

Quasi-Degenerate for $m_1 \simeq m_2 \simeq m_3 \simeq m_\nu \gtrsim \sqrt{\Delta m_A^2} \simeq 5 \times 10^{-2}$ eV

95% Cosmological Limit: Planck + WMAP9 + highL + BAO [arXiv:1303.5076]

Effective Neutrino Mass in Beta-Decay

$$m_\beta^2 = |U_{e1}|^2 m_1^2 + |U_{e2}|^2 m_2^2 + |U_{e3}|^2 m_3^2$$



- ▶ Quasi-Degenerate:

$$m_\beta^2 \simeq m_\nu^2 \sum_k |U_{ek}|^2 = m_\nu^2$$

- ▶ Inverted Hierarchy:

$$m_\beta^2 \simeq (1 - s_{13}^2) \Delta m_A^2 \simeq \Delta m_A^2$$

- ▶ Normal Hierarchy:

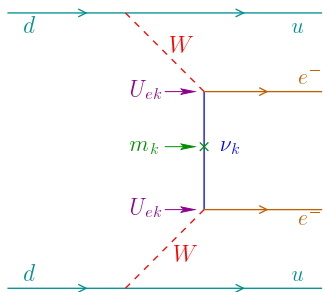
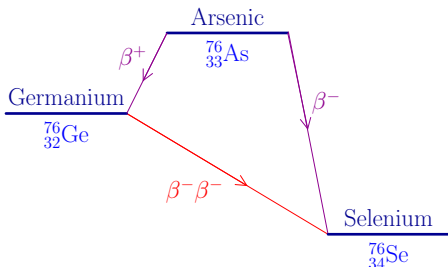
$$\begin{aligned} m_\beta^2 &\simeq s_{12}^2 c_{13}^2 \Delta m_S^2 + s_{13}^2 \Delta m_A^2 \\ &\simeq 2 \times 10^{-5} + 6 \times 10^{-5} \text{ eV}^2 \end{aligned}$$

- ▶ $m_\beta \lesssim 4 \times 10^{-2} \text{ eV}$



Normal Spectrum

Majorana ν : Neutrinoless Double-Beta Decay



$$(T_{1/2}^{0\nu})^{-1} = G_{0\nu} |\mathcal{M}_{0\nu}|^2 m_{\beta\beta}^2$$

Effective Majorana Mass

$$m_{\beta\beta} = \left| \sum_{k=1}^3 U_{ek}^2 m_k \right|$$

EXO + KamLAND-Zen



[PRL 109 (2012) 032505; PRL 110 (2013) 062502]

$$|m_{\beta\beta}| \lesssim 0.12 - 0.25 \text{ eV} \quad (90\% \text{C.L.})$$

GERDA

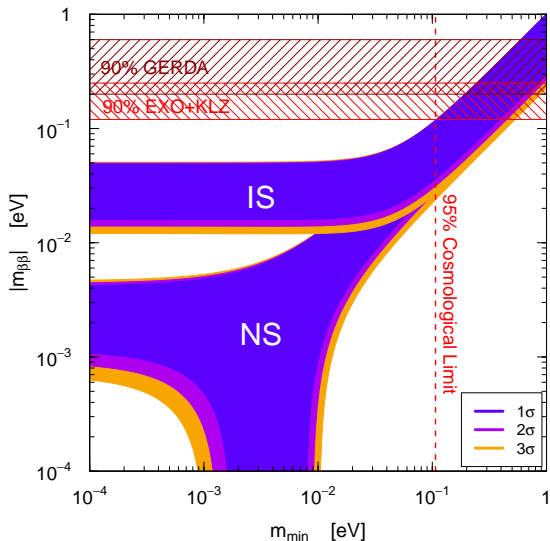


[arXiv:1307.4720]

$$|m_{\beta\beta}| \lesssim 0.2 - 0.6 \text{ eV} \quad (90\% \text{C.L.})$$

Effective Majorana Neutrino Mass

$$m_{\beta\beta} = |U_{e1}|^2 m_1 + |U_{e2}|^2 e^{i\alpha_2} m_2 + |U_{e3}|^2 e^{i\alpha_3} m_3$$



► Quasi-Degenerate:

$$|m_{\beta\beta}| \simeq m_\nu \sqrt{1 - s_{2\vartheta_{12}}^2 s_{\alpha_2}^2}$$

► Inverted Hierarchy:

$$|m_{\beta\beta}| \simeq \sqrt{\Delta m_A^2 (1 - s_{2\vartheta_{12}}^2 s_{\alpha_2}^2)}$$

► Normal Hierarchy:

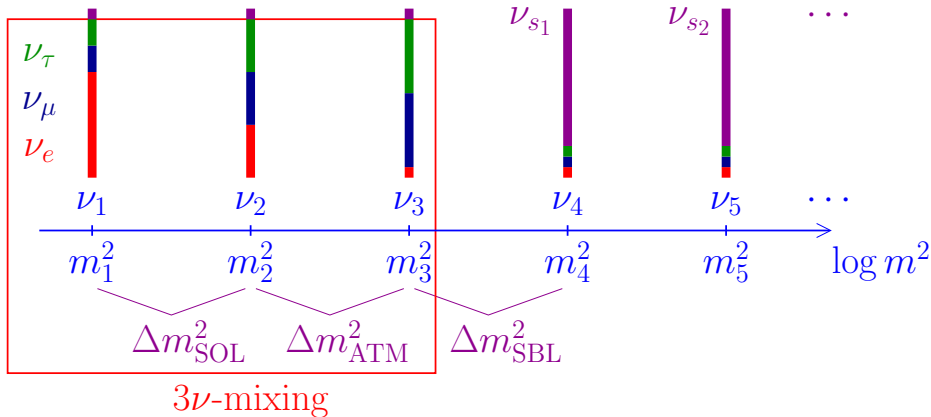
$$|m_{\beta\beta}| \simeq |s_{12}^2 \sqrt{\Delta m_S^2} + e^{i\alpha} s_{13}^2 \sqrt{\Delta m_A^2}|$$

$$\simeq |2.7 + 1.2e^{i\alpha}| \times 10^{-3} \text{ eV}$$

$$m_1 \gtrsim 10^{-3} \text{ eV} \Rightarrow \text{cancellation?}$$

$$|m_{\beta\beta}| \lesssim 10^{-2} \text{ eV} \Rightarrow \text{Normal Spectrum}$$

Beyond Three-Neutrino Mixing: Sterile Neutrinos



Light Sterile Neutrinos

- ▶ Sterile means **no standard model interactions**
[Pontecorvo, Sov. Phys. JETP 26 (1968) 984]
- ▶ Active neutrinos (ν_e, ν_μ, ν_τ) can oscillate into light sterile neutrinos (ν_s)
- ▶ Observables:
 - ▶ **Disappearance** of active neutrinos (neutral current deficit)
 - ▶ Indirect evidence through **combined fit of data** (current indication)
- ▶ Short-baseline anomalies + 3ν -mixing:

$$\Delta m_{21}^2 \ll |\Delta m_{31}^2| \ll |\Delta m_{41}^2| \leq \dots$$

ν_1	ν_2	ν_3	ν_4	...
ν_e	ν_μ	ν_τ	ν_{s1}	...

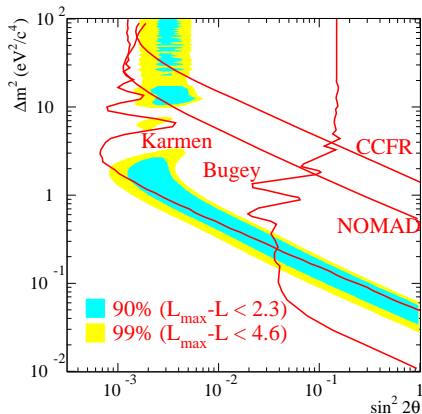
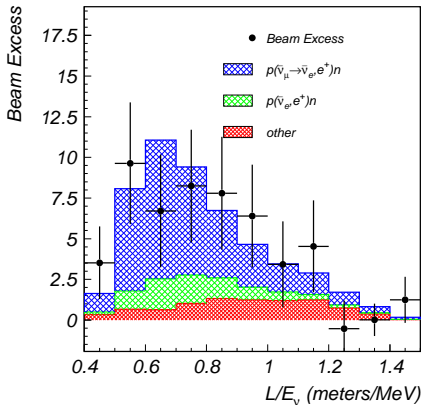
LSND

[PRL 75 (1995) 2650; PRC 54 (1996) 2685; PRL 77 (1996) 3082; PRD 64 (2001) 112007]

$$\bar{\nu}_\mu \rightarrow \bar{\nu}_e$$

$$L \simeq 30 \text{ m}$$

$$20 \text{ MeV} \leq E \leq 200 \text{ MeV}$$



3.8 σ excess

$$\Delta m_{\text{LSND}}^2 \gtrsim 0.2 \text{ eV}^2 \quad (\gg \Delta m_{\text{A}}^2 \gg \Delta m_{\text{S}}^2)$$

MiniBooNE

$L \simeq 541 \text{ m}$

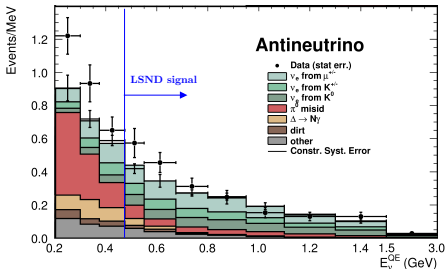
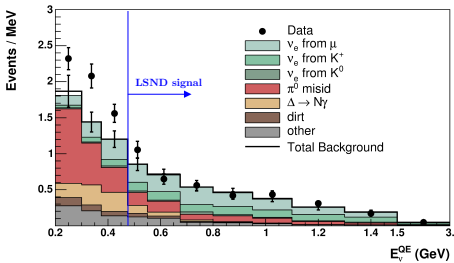
$200 \text{ MeV} \leq E \lesssim 3 \text{ GeV}$

$\nu_\mu \rightarrow \nu_e$

[PRL 102 (2009) 101802]

$\bar{\nu}_\mu \rightarrow \bar{\nu}_e$

[PRL 110 (2013) 161801]



► Agreement with LSND signal?

► CP violation?

► Low-energy anomaly!

Reactor Electron Antineutrino Anomaly

[Mention et al, PRD 83 (2011) 073006]

[update in White Paper, arXiv:1204.5379]

new reactor $\bar{\nu}_e$ fluxes

[Mueller et al, PRC 83 (2011) 054615]

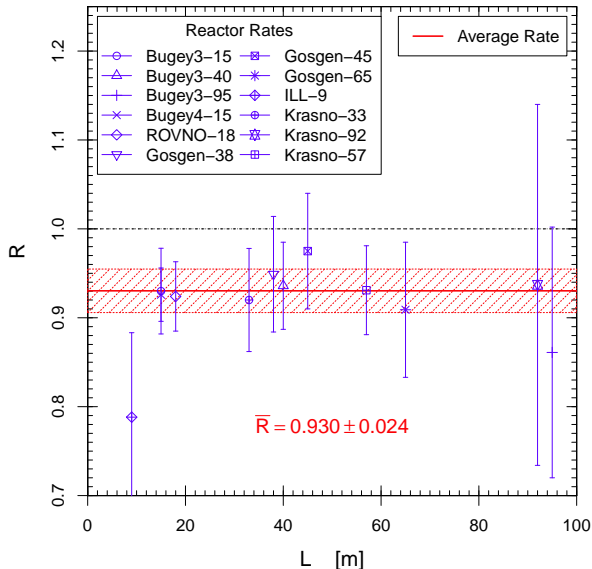
[Huber, PRC 84 (2011) 024617]

$\sim 2.8\sigma$ anomaly

[see also: Ciuffoli, Evslin, Li, JHEP 12 (2012)

110; Zhang, Qian, Vogel, PRD 87 (2013)

073018; Ivanov et al, arXiv:1306.1995]



Gallium Anomaly

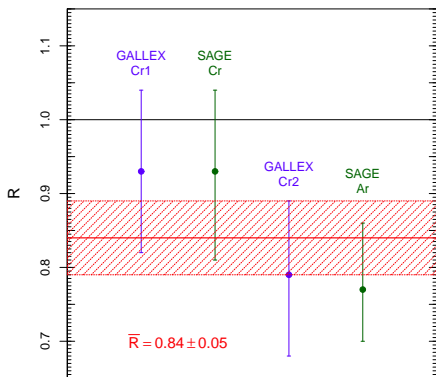
Gallium Radioactive Source Experiments: GALLEX and SAGE

Detection Process: $\nu_e + {}^{71}\text{Ga} \rightarrow {}^{71}\text{Ge} + e^-$

ν_e Sources: $e^- + {}^{51}\text{Cr} \rightarrow {}^{51}\text{V} + \nu_e$ $e^- + {}^{37}\text{Ar} \rightarrow {}^{37}\text{Cl} + \nu_e$

Anomaly supported by new ${}^{71}\text{Ga}({}^3\text{He}, {}^3\text{H}){}^{71}\text{Ge}$ cross section measurement

[Frekers et al., PLB 706 (2011) 134]



$E \sim 0.7 \text{ MeV}$

$\langle L \rangle_{\text{GALLEX}} = 1.9 \text{ m}$

$\langle L \rangle_{\text{SAGE}} = 0.6 \text{ m}$

$\sim 2.9\sigma$ anomaly

Effective SBL Oscillation Probabilities in 3+1 Schemes

$$P_{\nu_\alpha \rightarrow \nu_\beta}^{(-) \quad (-)} = \sin^2 2\vartheta_{\alpha\beta} \sin^2 \left(\frac{\Delta m_{41}^2 L}{4E} \right)$$

$$\sin^2 2\vartheta_{\alpha\beta} = 4|U_{\alpha 4}|^2 |U_{\beta 4}|^2$$

No CP Violation!

$$P_{\nu_\alpha \rightarrow \nu_\alpha}^{(-) \quad (-)} = 1 - \sin^2 2\vartheta_{\alpha\alpha} \sin^2 \left(\frac{\Delta m_{41}^2 L}{4E} \right)$$

$$\sin^2 2\vartheta_{\alpha\alpha} = 4|U_{\alpha 4}|^2 (1 - |U_{\alpha 4}|^2)$$

Perturbation of 3 ν Mixing

$$|U_{e4}|^2 \ll 1, \quad |U_{\mu 4}|^2 \ll 1, \quad |U_{\tau 4}|^2 \ll 1, \quad |U_{s4}|^2 \simeq 1$$

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \end{pmatrix}$$

↑
SBL

$$\sin^2 2\vartheta_{\alpha\alpha} \ll 1$$

↓

$$|U_{\alpha 4}|^2 \simeq \frac{\sin^2 2\vartheta_{\alpha\alpha}}{4}$$

3+1: Appearance vs Disappearance

- ▶ ν_e disappearance experiments:

$$\sin^2 2\vartheta_{ee} = 4|U_{e4}|^2 (1 - |U_{e4}|^2) \simeq 4|U_{e4}|^2$$

- ▶ ν_μ disappearance experiments:

$$\sin^2 2\vartheta_{\mu\mu} = 4|U_{\mu4}|^2 (1 - |U_{\mu4}|^2) \simeq 4|U_{\mu4}|^2$$

- ▶ $\nu_\mu \rightarrow \nu_e$ experiments:

$$\sin^2 2\vartheta_{e\mu} = 4|U_{e4}|^2 |U_{\mu4}|^2 \simeq \frac{1}{4} \sin^2 2\vartheta_{ee} \sin^2 2\vartheta_{\mu\mu}$$

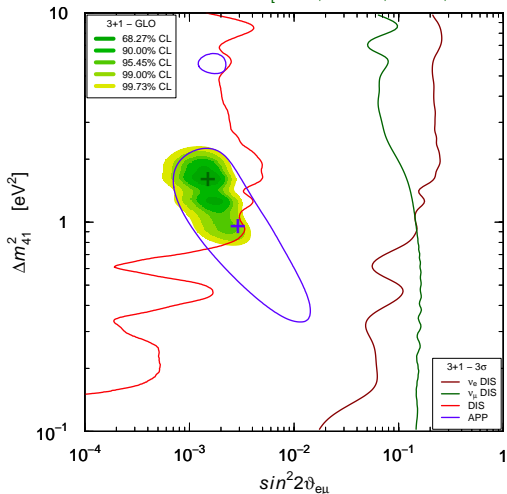
- ▶ Upper bounds on $\sin^2 2\vartheta_{ee}$ and $\sin^2 2\vartheta_{\mu\mu} \implies$ strong limit on $\sin^2 2\vartheta_{e\mu}$

[Okada, Yasuda, Int. J. Mod. Phys. A12 (1997) 3669-3694]

[Bilenky, Giunti, Grimus, Eur. Phys. J. C1 (1998) 247]

3+1 Global Fit

[Giunti, Laveder, Y.F. Li, H.W. Long, arXiv:1308.5288]



MiniBooNE $E > 475$ MeV
GoF = 29% PGoF = 9%

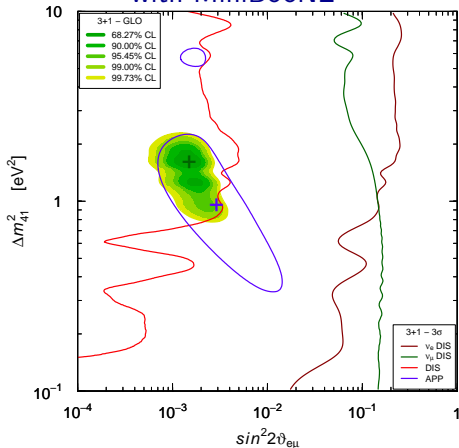
- ▶ APP $\nu_\mu \rightarrow \nu_e$ & $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$:
LSND (Y), MiniBooNE (?),
OPERA (N), ICARUS (N),
KARMEN (N), NOMAD (N),
BNL-E776 (N)
- ▶ DIS ν_e & $\bar{\nu}_e$: Reactors (Y),
Gallium (Y), $\nu_e C$ (N),
Solar (N)
- ▶ DIS ν_μ & $\bar{\nu}_\mu$: CDHSW (N),
MINOS (N),
Atmospheric (N),
MiniBooNE/SciBooNE (N)

No Osc. excluded at 6.2σ
 $\Delta\chi^2/\text{NDF} = 46.2/3$

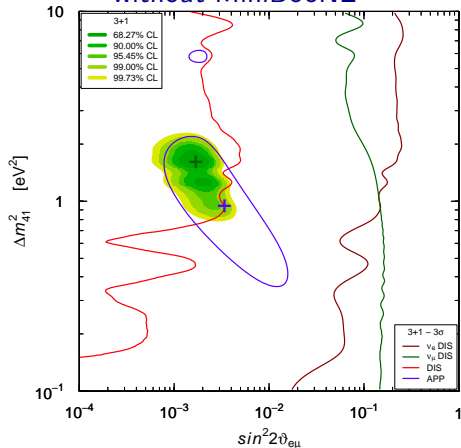
[different approach and conclusions: Kopp, Machado, Maltoni, Schwetz, JHEP 1305 (2013) 050]

MiniBooNE Impact on SBL Oscillations?

with MiniBooNE



without MiniBooNE



GoF = 29% PGoF = 9%

No Osc. excluded at 6.2 σ

$\Delta\chi^2/\text{NDF} = 46.2/3$

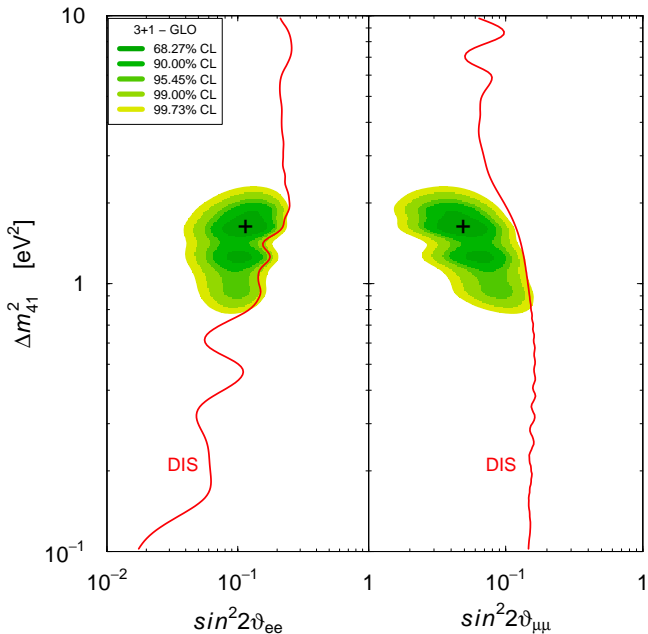
GoF = 19% PGoF = 8%

No Osc. excluded at 6.3 σ

$\Delta\chi^2/\text{NDF} = 47.1/3$

Without LSND: No Osc. excluded only at 2.1 σ ($\Delta\chi^2/\text{NDF} = 8.3/3$)

ν_e and ν_μ Disappearance



Many Exciting New Experiments and Projects

- ▶ Reactor $\bar{\nu}_e$ Disappearance:
 - ▶ Nucifer (OSIRIS, Saclay), Stereo (ILL, Grenoble) [arXiv:1204.5379]
 - ▶ DANSS (Kalinin Nuclear Power Plant, Russia) [arXiv:1304.3696], POSEIDON (PIK, Gatchina, Russia) [arXiv:1204.2449]
 - ▶ SCRAAM (San Onofre, California) [arXiv:1204.5379]
 - ▶ CARR (China Advanced Research Reactor) [arXiv:1303.0607]
 - ▶ Neutrino-4 (SM-3, Dimitrovgrad, Russia), SOLID (BR2, Belgium), Hanaro (Korea) [D. Lhuillier, EPSHEP 2013]
- ▶ Radioactive Source ν_e and $\bar{\nu}_e$ Disappearance:
 - ▶ SOX (Borexino, Gran Sasso, Italy) [arXiv:1304.7721]
 - ▶ CeLAND (^{144}Ce @KamLAND, Japan) [arXiv:1107.2335]
 - ▶ SAGE (Baksan, Russia) [arXiv:1006.2103]
 - ▶ IsoDAR (DAE δ ALUS, USA) [arXiv:1210.4454, arXiv:1307.2949]
 - ▶ SNO+, JUNO, RENO [T. Lasserre, Neutrino 2012]
- ▶ Accelerator $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ Appearance:
 - ▶ ICARUS/NESSIE (CERN) [arXiv:1304.2047, arXiv:1306.3455]
 - ▶ nuSTORM [arXiv:1308.0494]
 - ▶ OscSNS (Oak Ridge, USA) [arXiv:1305.4189, arXiv:1307.7097]

Conclusions

- ▶ Robust Three-Neutrino Mixing Paradigm. Open problems: $\vartheta_{23} \lesseqgtr 45^\circ?$, CP Violation, Mass Hierarchy, Absolute Mass Scale, Dirac or Majorana?
- ▶ Short-Baseline ν_e and $\bar{\nu}_e$ 3+1 Disappearance:
 - ▶ Reactor $\bar{\nu}_e$ anomaly is alive and exciting
 - ▶ Gallium ν_e anomaly strengthened by new cross-section measurements
 - ▶ Many promising projects to test short-baseline ν_e and $\bar{\nu}_e$ disappearance in a few years with reactors and radioactive sources
 - ▶ Independent tests through effect of m_4 in β -decay and $(\beta\beta)_{0\nu}$ -decay
- ▶ Short-Baseline $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ LSND Signal:
 - ▶ MiniBooNE experiment has been inconclusive
 - ▶ If $|U_{e4}| > 0$ why not $|U_{\mu4}| > 0$? \implies Maybe LSND luckily observed a fluctuation of a small $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ transition probability with amplitude $\sin^2 2\vartheta_{e\mu} = 4|U_{e4}|^2|U_{\mu4}|^2$, which has not been seen by other appearance experiments
 - ▶ Better experiments are needed to check LSND signal
- ▶ Cosmology:
 - ▶ Important effects of sterile neutrinos
 - ▶ Implications depend on theoretical framework and considered data set
 - ▶ Cosmological indications must be checked by laboratory experiments

Backup Slides

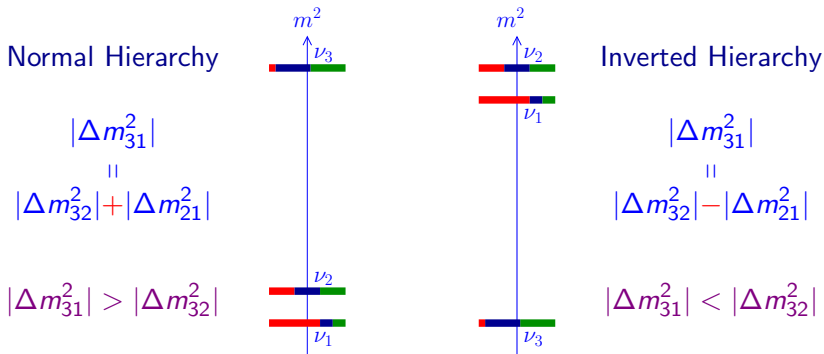
Mass Hierarchy

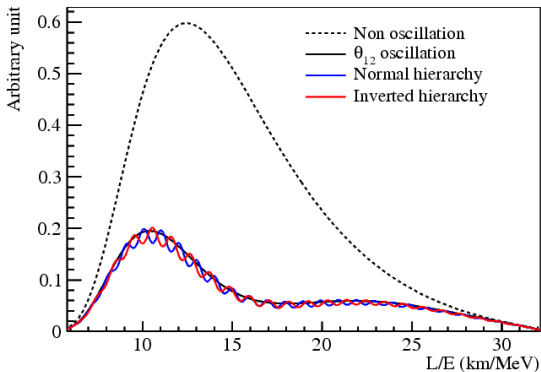
1. Matter Effect: Atmospheric, Long-Baseline, Supernova Experiments

▶ $\nu_e \leftrightarrow \nu_\mu$ MSW resonance: $V = \frac{\Delta m_{13}^2 \cos 2\vartheta_{13}}{2E} \Leftrightarrow \Delta m_{13}^2 > 0$ NH

▶ $\bar{\nu}_e \leftrightarrow \bar{\nu}_\mu$ MSW resonance: $V = -\frac{\Delta m_{13}^2 \cos 2\vartheta_{13}}{2E} \Leftrightarrow \Delta m_{13}^2 < 0$ IH

2. Phase Difference: Reactor $\bar{\nu}_e \rightarrow \bar{\nu}_e$: JUNO - Daya Bay II (China), RENO-50 (Korea)





$$F(L/E) = \phi(E)\sigma(E)P_{ee}(L/E)$$

$$P_{ee}(L/E) = 1 - P_{21} - P_{31} - P_{32}$$

$$P_{21} = \cos^4(\theta_{13}) \sin^2(2\theta_{12}) \sin^2(\Delta_{21})$$

$$P_{31} = \cos^2(\theta_{12}) \sin^2(2\theta_{13}) \sin^2(\Delta_{31})$$

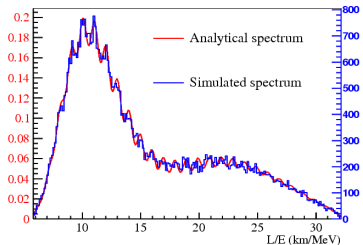
$$P_{32} = \sin^2(\theta_{12}) \sin^2(2\theta_{13}) \sin^2(\Delta_{32})$$

$$\Delta_{21} \ll \Delta_{31} \approx \Delta_{32}$$

S.T. Petcov et al., PLB533(2002)94
 S.Choubey et al., PRD68(2003)113006
 J. Learned et al., hep-ex/0612022

L. Zhan, Y. Wang, J. Cao, L. Wen,
 PRD78:111103, 2008
 PRD79:073007, 2009

**Precision energy spectrum
 measurement: Looking for
 interference between P_{31} and P_{32}
 → relative measurement**



[Miao He, NuFact 2013]

CP Violation

$$P_{\nu_\alpha \rightarrow \nu_\beta} - P_{\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta} = -16 J_{\alpha\beta} \sin\left(\frac{\Delta m_{21}^2 L}{4E}\right) \sin\left(\frac{\Delta m_{31}^2 L}{4E}\right) \sin\left(\frac{\Delta m_{32}^2 L}{4E}\right)$$

$$J_{\alpha\beta} = \text{Im}(U_{\alpha 1} U_{\alpha 2}^* U_{\beta 1}^* U_{\beta 2}) = \pm J$$

$$J = s_{12} c_{12} s_{23} c_{23} s_{13} c_{13}^2 \sin \delta_{13}$$

Necessary conditions for observation of CP violation:

- ▶ Sensitivity to all mixing angles: ϑ_{12} , ϑ_{23} and smaller ϑ_{13}
- ▶ Sensitivity to oscillations due to small Δm_{21}^2 and large Δm_{31}^2
($\Delta m_{32}^2 = \Delta m_{31}^2 - \Delta m_{21}^2$)

Sterile Neutrinos from Physics Beyond the SM

- ▶ Neutrinos are special in the Standard Model: the only **neutral fermions**
- ▶ In extensions of SM neutrinos can mix with non-SM fermions

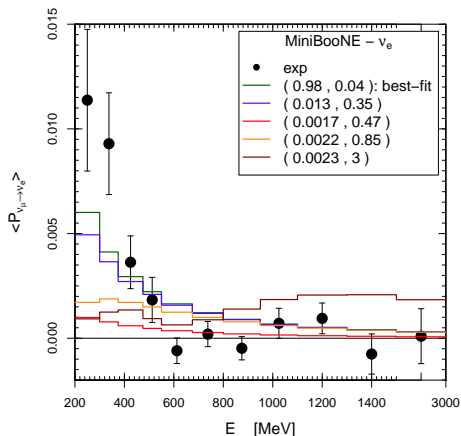
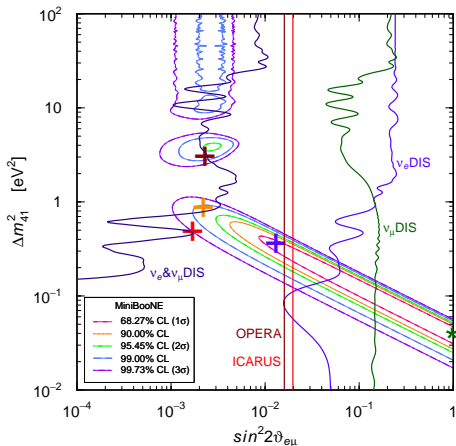
▶ SM: $L_L = \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} \quad \tilde{\Phi} = i\sigma_2 \Phi^* = \begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix} \xrightarrow[\text{Breaking}]{\text{Symmetry}} \begin{pmatrix} \nu/\sqrt{2} \\ 0 \end{pmatrix}$

- ▶ SM singlet $\overline{L}_L \tilde{\Phi}$ can couple to new singlet chiral fermion field ν_R (**right-handed neutrino**) related to physics beyond the SM
- ▶ Known examples: SUSY, new symmetries, extra dimensions, mirror world, ... [see http://www.nu.to.infn.it/Sterile_Neutrinos/]
- ▶ **Dirac mass term** $\sim \overline{L}_L \tilde{\Phi} \nu_R$ + **Majorana mass term** $\sim \overline{\nu_R^c} \nu_R$
- ▶ Diagonalization of mass matrix \implies massive Majorana neutrinos

		LOW	HIG	noMB	noLSND
No	χ^2	339.2	308.0	283.2	286.7
Osc.	NDF	259	253	221	255
	GoF	0.06 %	1 %	0.3 %	8 %
3+1	χ_{\min}^2	291.7	261.8	236.1	278.4
Osc.	NDF	256	250	218	252
	GoF	6 %	29 %	19 %	12 %
	$\Delta m_{41}^2 [\text{eV}^2]$	1.6	1.6	1.6	1.7
	$ U_{e4} ^2$	0.033	0.03	0.03	0.024
	$ U_{\mu 4} ^2$	0.012	0.013	0.014	0.0073
	$\sin^2 2\vartheta_{e\mu}$	0.0016	0.0015	0.0017	0.0007
	$\sin^2 2\vartheta_{ee}$	0.13	0.11	0.12	0.093
	$\sin^2 2\vartheta_{\mu\mu}$	0.048	0.049	0.054	0.03
	$(\chi_{\min}^2)_{\text{APP}}$	99.3	77.0	50.9	91.8
	$(\chi_{\min}^2)_{\text{DIS}}$	180.1	180.1	180.1	180.1
	$\Delta\chi_{\text{PG}}^2$	12.7	4.8	5.1	6.4
	NDF_{PG}	2	2	2	2
	GoF_{PG}	0.2 %	9 %	8 %	4 %
	$p\text{-val}_{\text{No Osc.}}$	3×10^{-10}	5×10^{-10}	3×10^{-10}	4×10^{-2}
	$n\sigma_{\text{No Osc.}}$	6.3σ	6.2σ	6.3σ	2.1σ

[Giunti, Laveder, Y.F. Li, H.W. Long, arXiv:1308.5288]

MiniBooNE Low-Energy Excess?



- ▶ No fit of low-energy excess for realistic $\Delta m_{41}^2 \gtrsim 0.8 \text{ eV}^2$ and $\sin^2 2\theta_{e\mu} \lesssim 5 \times 10^{-3}$
- ▶ APP-DIS PGoF = 0.1%
- ▶ Neutrino energy reconstruction problem?

[Martini, Ericson, Chanfray, PRD 85 (2012) 093012; PRD 87 (2013) 013009]

Effective SBL Oscillation Probabilities in 3+2 Schemes

$$\phi_{kj} = \Delta m_{kj}^2 L / 4E$$

$$\eta = \arg[U_{e4}^* U_{\mu 4} U_{e5} U_{\mu 5}^*]$$

$$P_{\nu_{\mu} \rightarrow \nu_e}^{(-) \quad (-)} = 4|U_{e4}|^2 |U_{\mu 4}|^2 \sin^2 \phi_{41} + 4|U_{e5}|^2 |U_{\mu 5}|^2 \sin^2 \phi_{51} + 8|U_{\mu 4} U_{e4} U_{\mu 5} U_{e5}| \sin \phi_{41} \sin \phi_{51} \cos(\phi_{54} \overset{(+)}{-} \eta)$$

$$P_{\nu_{\alpha} \rightarrow \nu_{\alpha}}^{(-) \quad (-)} = 1 - 4(1 - |U_{\alpha 4}|^2 - |U_{\alpha 5}|^2)(|U_{\alpha 4}|^2 \sin^2 \phi_{41} + |U_{\alpha 5}|^2 \sin^2 \phi_{51}) - 4|U_{\alpha 4}|^2 |U_{\alpha 5}|^2 \sin^2 \phi_{54}$$

[Sorel, Conrad, Shaevitz, PRD 70 (2004) 073004; Maltoni, Schwetz, PRD 76 (2007) 093005; Karagiorgi et al, PRD 80 (2009) 073001; Kopp, Maltoni, Schwetz, PRL 107 (2011) 091801; Giunti, Laveder, PRD 84 (2011) 073008; Donini et al, JHEP 07 (2012) 161; Conrad, Ignarra, Karagiorgi, Shaevitz, Spitz, AHEP 2013 (2013) 163897; Kopp, Machado, Maltoni, Schwetz, JHEP 1305 (2013) 050]

▶ Good: CP violation

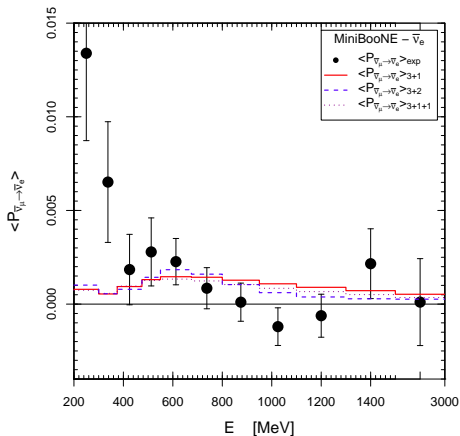
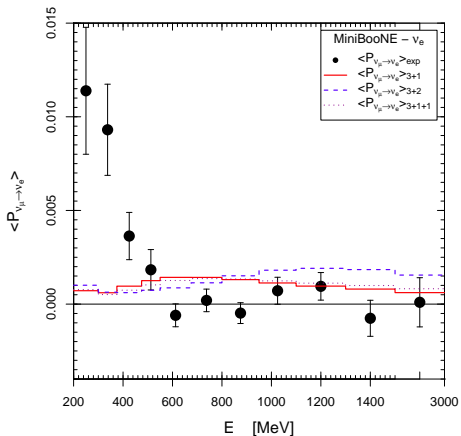
▶ Bad: Two massive sterile neutrinos at the eV scale!

4 more parameters: $\underbrace{\Delta m_{41}^2, |U_{e4}|^2, |U_{\mu 4}|^2, \Delta m_{51}^2, |U_{e5}|^2, |U_{\mu 5}|^2, \eta}_{3+1}$

3+2

- ▶ 3+2 should be preferred to 3+1 only if
 - ▶ there is evidence of two peaks of the probability corresponding to two Δm^2 's
 - or
 - ▶ there is CP-violating difference of $\nu_\mu \rightarrow \nu_e$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ transitions
- ▶ 2008 ν + 2010 $\bar{\nu}$ MiniBooNE data indicated ν - $\bar{\nu}$ difference
 - ⇓
 - reasonable and useful to consider 3+2
- ▶ ν - $\bar{\nu}$ difference almost disappeared with 2012 $\bar{\nu}$ data
- ▶ Okkam razor: 3+1 is enough!
- ▶ Different approach and conclusions:
 - ▶ Kopp, Machado, Maltoni, Schwetz, JHEP 1305 (2013) 050:
Use all MiniBooNE data. No 3+1 global fit. 3+2 slightly preferred? Small allowed region.
 - ▶ Conrad, Ignarra, Karagiorgi, Shaevitz, Spitz, AHEP 2013 (2013) 163897:
Use all MiniBooNE data. 3+2 strongly preferred. Very small allowed regions.

MiniBooNE Low-Energy Excess?



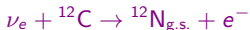
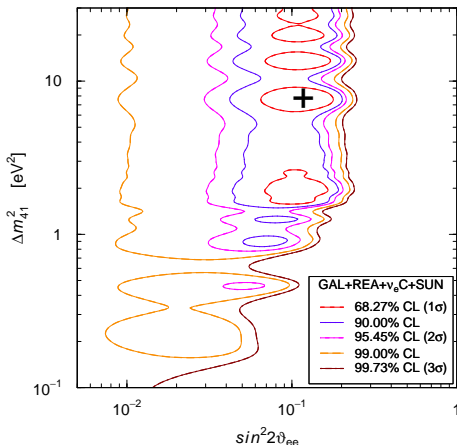
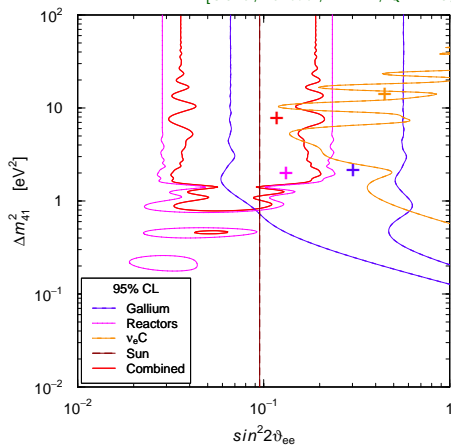
- ▶ $3+1$: GoF = 6% PGoF = 0.2%
- ▶ $3+2$: GoF = 8% PGoF = 0.1%
- ▶ $3+1+1$: GoF = 6% PGoF = 0.2%

	3+2 LOW	3+2 HIG	3+1+1 LOW	3+1+1 HIG
χ_{\min}^2	284.4	256.4	289.8	259.0
NDF	252	246	253	247
GoF	8 %	31 %	6 %	29 %
$\Delta m_{41}^2 [\text{eV}^2]$	1.9	0.93	1.6	1.6
$ U_{e4} ^2$	0.03	0.015	0.026	0.023
$ U_{\mu 4} ^2$	0.012	0.0097	0.011	0.012
$\Delta m_{51}^2 [\text{eV}^2]$	4.1	1.6		
$ U_{e5} ^2$	0.013	0.018	0.0088	0.0092
$ U_{\mu 5} ^2$	0.0065	0.0091	0.0049	0.0052
η/π	0.51	1.6	0.4	0.45
$(\chi_{\min}^2)_{\text{APP}}$	87.7	69.8	94.8	75.5
$(\chi_{\min}^2)_{\text{DIS}}$	179.1	179.1	180.1	180.1
$\Delta\chi_{\text{PG}}^2$	17.7	7.5	14.9	3.4
NDF _{PG}	4	4	3	3
GoF _{PG}	0.1 %	11 %	0.2 %	34 %
p-val ₃₊₁	0.12	0.25	0.59	0.42
$n\sigma_{3+1}$	1.6σ	1.2σ	0.54σ	0.81σ

[Giunti, Laveder, Y.F. Li, H.W. Long, arXiv:1308.5288]

Global ν_e and $\bar{\nu}_e$ Disappearance

[Giunti, Laveder, Y.F. Li, Q.Y. Liu, H.W. Long, PRD 86 (2012) 113014]



KARMEN + LSND

[Conrad, Shaevitz, PRD 85 (2012) 013017]

[Giunti, Laveder, PLB 706 (2011) 200]

solar ν_e + KamLAND $\bar{\nu}_e$ + ϑ_{13}

[Giunti, Li, PRD 80 (2009) 113007]

[Palazzo, PRD 83 (2011) 113013; PRD 85 (2012) 077301]

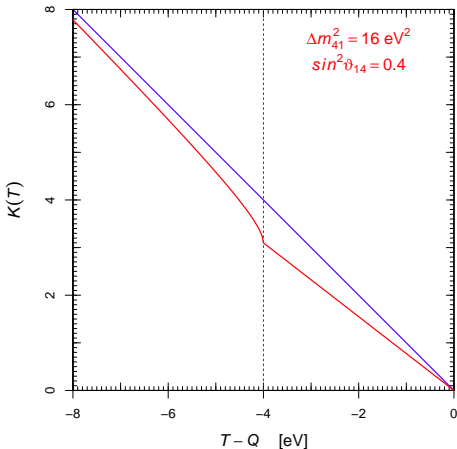
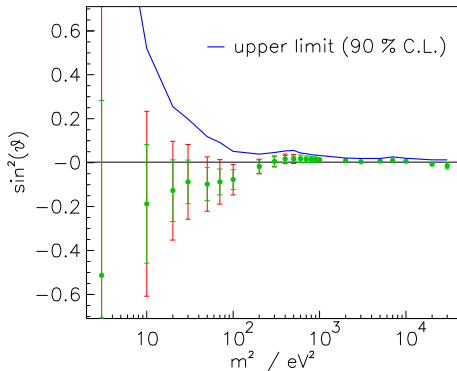
GoF = 62% PGoF = 4%

No Osc. excluded at 2.7σ

$\Delta\chi^2/\text{NDF} = 10.1/2$

Mainz Limit on m_4^2

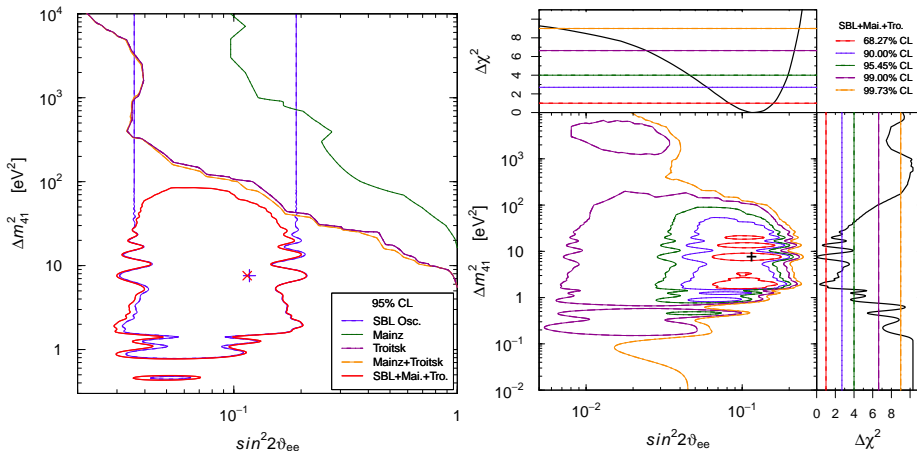
[Kraus, Singer, Valerius, Weinheimer, EPJC 73 (2013) 2323]



$$m_4 \gg m_1, m_2, m_3 \implies \Delta m_{41}^2 \equiv m_4^2 - m_1^2 \simeq m_4^2$$

Troitsk: Surprising Much Better Limit on m_{41}^2

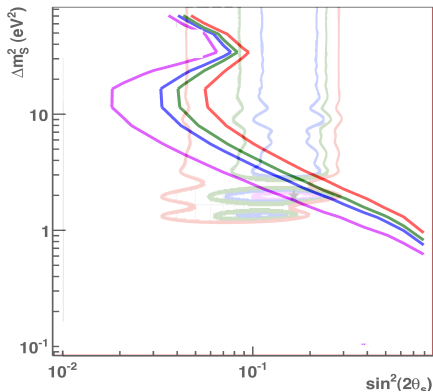
[Belesev et al, JETP Lett. 97 (2013) 67; arXiv:1307.5687]



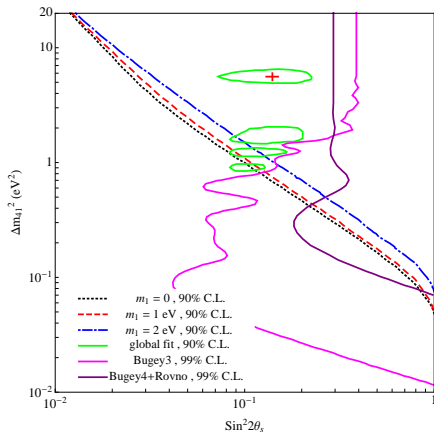
$$2\sigma : 0.85 \lesssim \Delta m_{41}^2 \lesssim 43 \text{ eV}^2 \implies 6 \text{ cm} \lesssim \frac{L_{41}^{\text{osc}}}{E [\text{MeV}]} \lesssim 3 \text{ m}$$

[Giunti, Laveder, Y.F. Li, H.W. Long, PRD 87 (2013) 013004]

KATRIN Sensitivity



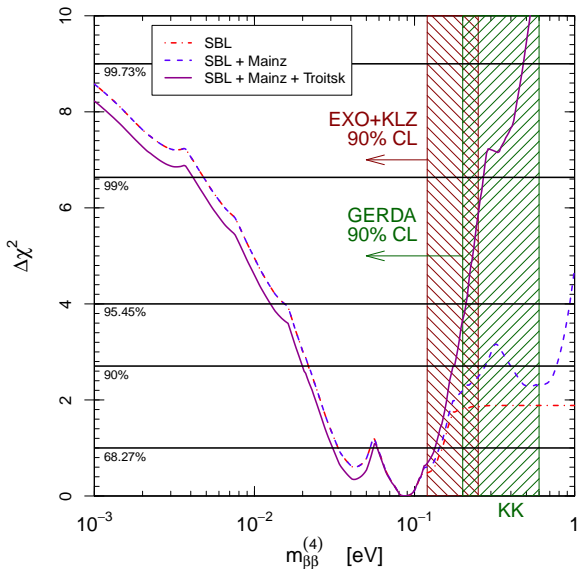
[Formaggio, Barrett, PLB 706 (2011) 68]



[Esmaili, Peres, PRD 85 (2012) 117301]

[see also Sejersen Riis, Hannestad, JCAP (2011) 1475; Sejersen Riis, Hannestad, Weinheimer, PRC 84 (2011) 045503]

Neutrinoless Double- β Decay



$$|m_{\beta\beta}| = \left| \sum_{k=1}^4 U_{ek}^2 m_k \right|$$

$$m_{\beta\beta}^{(4)} = |U_{e4}|^2 \sqrt{\Delta m_{41}^2}$$

caveat:

possible cancellation
with $m_{\beta\beta}^{(3\nu-IH)}$

[Barry et al, JHEP 07 (2011) 091]

[Li, Liu, PLB 706 (2012) 406]

[Rodejohann, JPG 39 (2012) 124008]