

125GeV Higgs Boson and Radiative Natural SUSY

Muneyuki Ishida (Meisei Univ.)

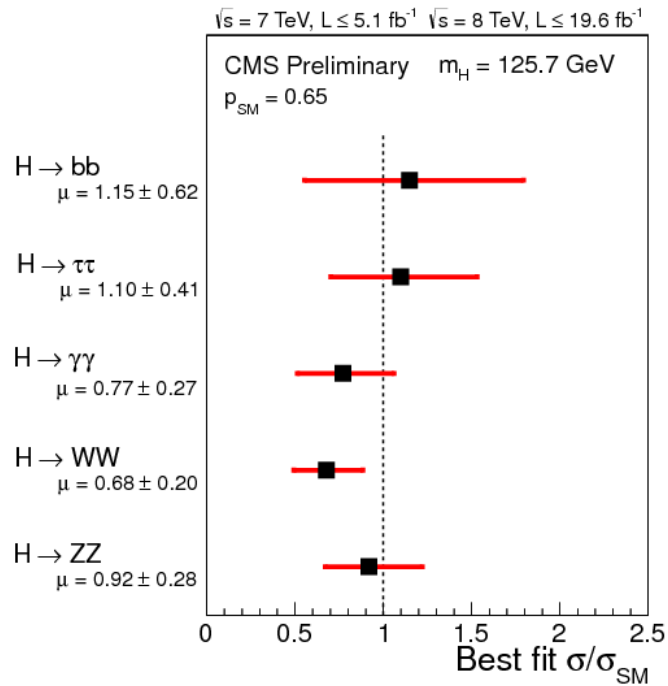
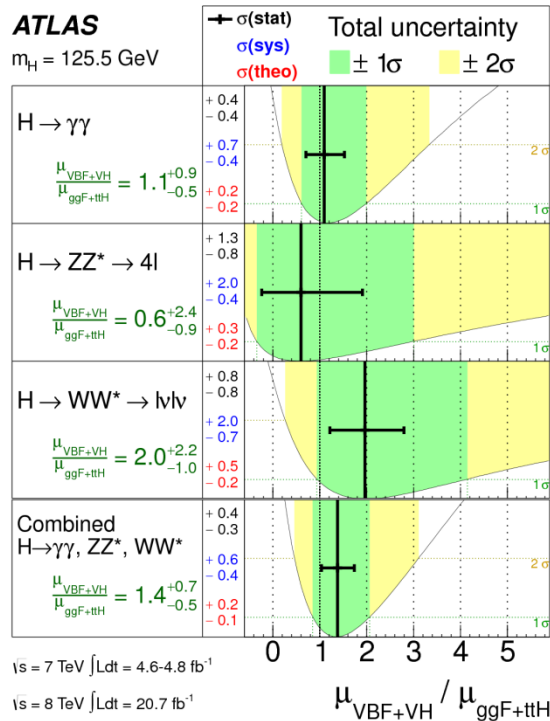
In collaboration with Vernon Barger,
Peisi Huang, Wai-Yee Keung

Higgs Data at LHC

Mass

- $M_h = 125.5 \pm 0.2 \pm 0.6$ GeV (ATLAS)
- $M_h = 125.7 \pm 0.3 \pm 0.3$ GeV (CMS)

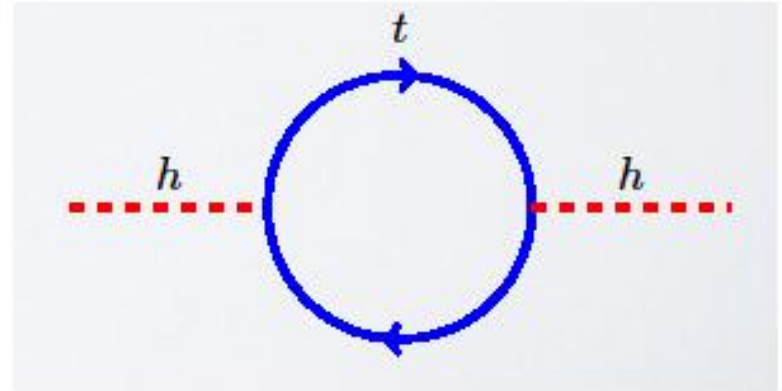
Couplings



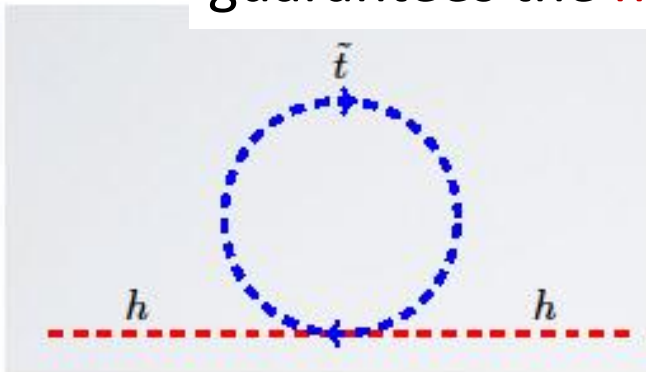
Ratios to the cross sections of SM Higgs are being measured for various channels.

The hierarchy problem

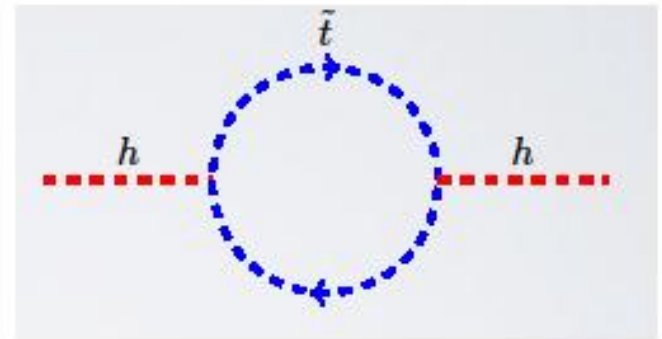
$$\Delta m^2 \sim \frac{y_t^2}{16\pi^2} \Lambda^2$$



- We need extreme fine tuning to give the cancellation
- Introduce Supersymmetry : Automatic cancellation guarantees the **naturalness** of values of parameters.



+



- SUSY is broken in nature.
 - Soft Supersymmetry breaking gives different masses to the SUSY particles and their SM partners.
- It gives different contribution to the Higgs potential.

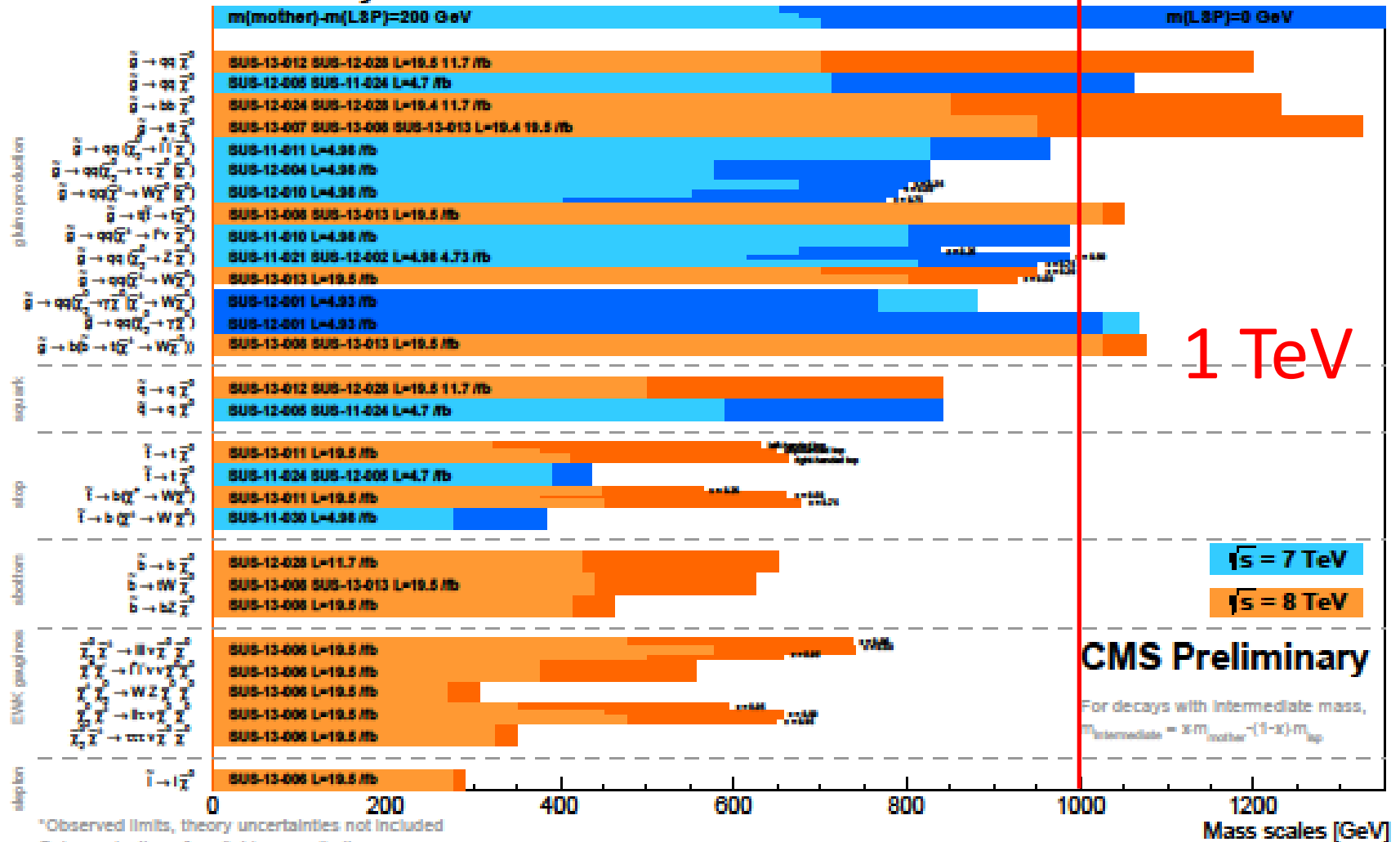
$$V = (\mu^2 + m_{Hd}^2) |h_d^0|^2 + (\mu^2 + m_{Hu}^2) |h_u^0|^2 - (B\mu h_d^{0\dagger} h_u^0 + \text{h.c.}) + \frac{1}{8} (g^2 + g'^2) (|h_u^0|^2 - |h_d^0|^2)^2$$

Radiative correction :

$$\Delta V = \sum_i \frac{(-1)^{2s_i}}{64\pi^2} (2s_i + 1) c_i m_i^4(\Phi) \left[\log \frac{m_i^2(\Phi)}{Q^2} - \frac{3}{2} \right], \quad i: \text{ all fields couple to Higgs}$$

LHC SUSY searches

Summary of CMS SUSY Results* in SMS framework EPSHEP 2013



*Observed limits, theory uncertainties not included
 Only a selection of available mass limits
 Probe "up to" the quoted mass limit

How can the heavy SUSY particles accommodate the Higgs boson of mass 125 GeV ?

Natural SUSY

$$-\frac{m_h^2}{2} = \mu^2 + m_{Hu}^2(\Lambda) + \delta m_{Hu}^2(M_{SUSY})$$

- Fine-tuning measure $\Delta_i = C_i / \left(\frac{m_h^2}{2}\right)$,

$$C_i = |\mu^2|, m_{Hu}^2(\Lambda), -\delta m_{Hu}^2(M_{SUSY})$$

Requiring all $\Delta_i < 5$ ($\Delta^{-1} = 20\%$ tuning : $-1 = 4 - 5$)



Low Higgsino mass : $|\mu| < \sim 300 \text{ GeV}$ is required.

$$-\delta m_{Hu}^2(M_{SUSY}) \cong \frac{3h_t^2}{8\pi^2} (m_{Q3}^2 + m_{U3}^2 + A_t^2) \log \frac{\Lambda}{M_{SUSY}} < \left(\frac{m_h^2}{2}\right) 5$$

$$\rightarrow m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2 (= m_{Q3}^2 + m_{U3}^2) < \frac{8\pi^2}{3h_t^2} \frac{\frac{m_h^2}{2} 5}{(1+R_t^2) \log \frac{\Lambda}{M_{SUSY}}} \frac{\Lambda}{5} = (700 \text{ GeV})^2 \frac{3}{\log \frac{\Lambda}{M_{SUSY}}} \frac{\Lambda}{5}$$

Light 3rd generations : Stop mass $< \sim 700 \text{ GeV}$.

Natural SUSY

- $\mu \approx 100 - 250 \text{ GeV}$
- $m(\tilde{t}_1, \tilde{t}_2, \tilde{b}_1) \approx \sim 500 \text{ GeV}$
- $m(\tilde{g}) \lesssim 1.5 \text{ TeV}$
- $m(\tilde{q}, \tilde{l}) \approx 10 - 20 \text{ TeV}$

Arkani-Hamed, Pappucci et al., Brust et al., Essig et al.,
Baer, Barger, Huang, Tata, Wymant...

Low High-Scale fine-tuning $\Delta_i < 20\%$

$\sim 10 \text{ TeV}$ 1st 2nd squarks

$\sim 1.5 \text{ TeV}$ gluino

$\sim 500 \text{ GeV}$ stop

$\sim 200 \text{ GeV}$ Higgsino

What about the Higgs mass
in this scenario?

MSSM Higgs mass

- Two Higgs doublet model : H_u and H_d
- D-term $\frac{1}{8}(g^2 + g'^2)(|H_u^0|^2 - |H_d^0|^2)^2$
 - Tree-level bound $M_h \leq M_Z |\cos 2\beta|$

Large quantum correction is needed to reproduce $M_h \cong 125$ GeV .

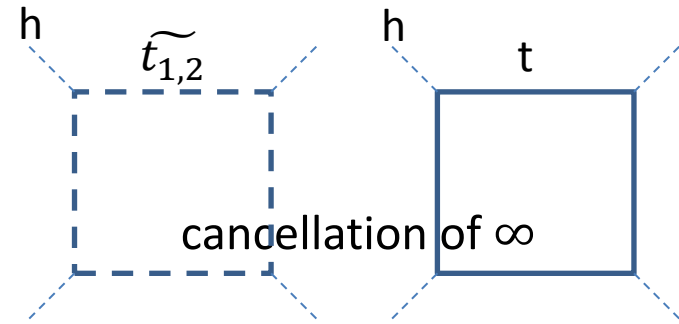
the largest contribution comes from

stop and top loop

Loop correction to MSSM Higgs mass

- *Stop squared mass matrix*

$$M_{\tilde{t}}^2 = \begin{pmatrix} m_t^2 + m_{Q_3}^2 + D_L & m_t X_t \\ m_t X_t & m_t^2 + m_{U_3}^2 + D_R \end{pmatrix}$$



$$M_{SUSY} = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}} \quad \text{stop mass gives SUSY breaking scale}$$

- *Higgs mass formula*

$$M_h^2 = M_Z^2 \cos^2 2\beta + \frac{3m_t^4}{4\pi^2 v^2} \left[\log \frac{M_{SUSY}^2}{m_t^2} + x_t^2 \left(1 - \frac{x_t^2}{12} \right) \right];$$

Logarithmic dependence of stop mass M_{SUSY}

polynomial dependence of stop mixing parameter $x_t = \frac{X_t}{M_{SUSY}}$

Large CP-odd higgs mass m_A , moderate and large values of $\tan\beta$

Tremendous efforts to calculate M_h in MSSM.

Carena, Wagner, Heinemeyer, Hempfling, Espinoza, Zhang,
 Degrassi, Hollik, Zhang, Weiglein, Zwirner, Okada, Yamaguchi, Ellis, Brignole, Martin,

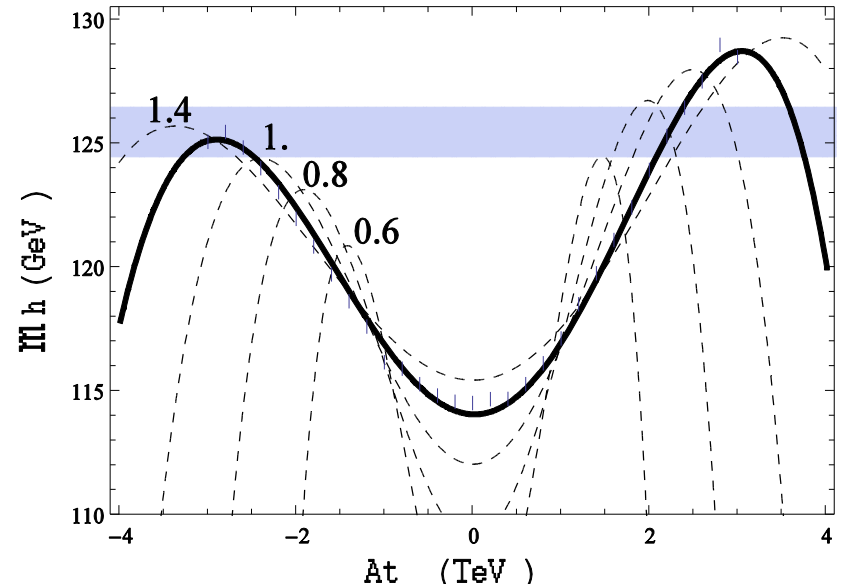
MSSM Higgs mass

- 3-loop analysis of M_h by H3m package
- Analysis by Isajet

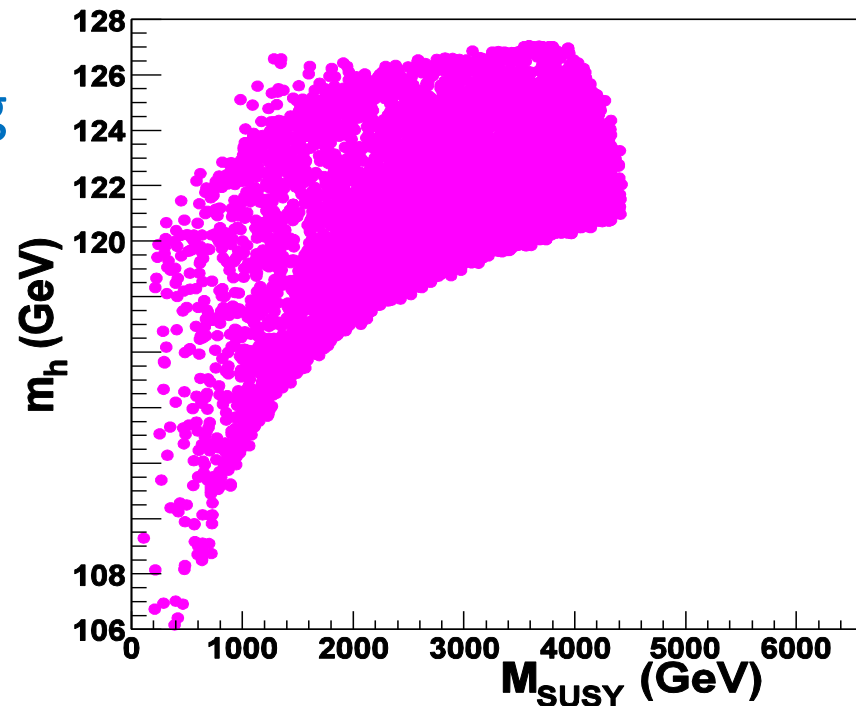
➔ Natural SUSY scenario has great difficulty in accomodating a Higgs at 125 GeV.

Stop masses $M_{SUSY} < 600$ GeV is

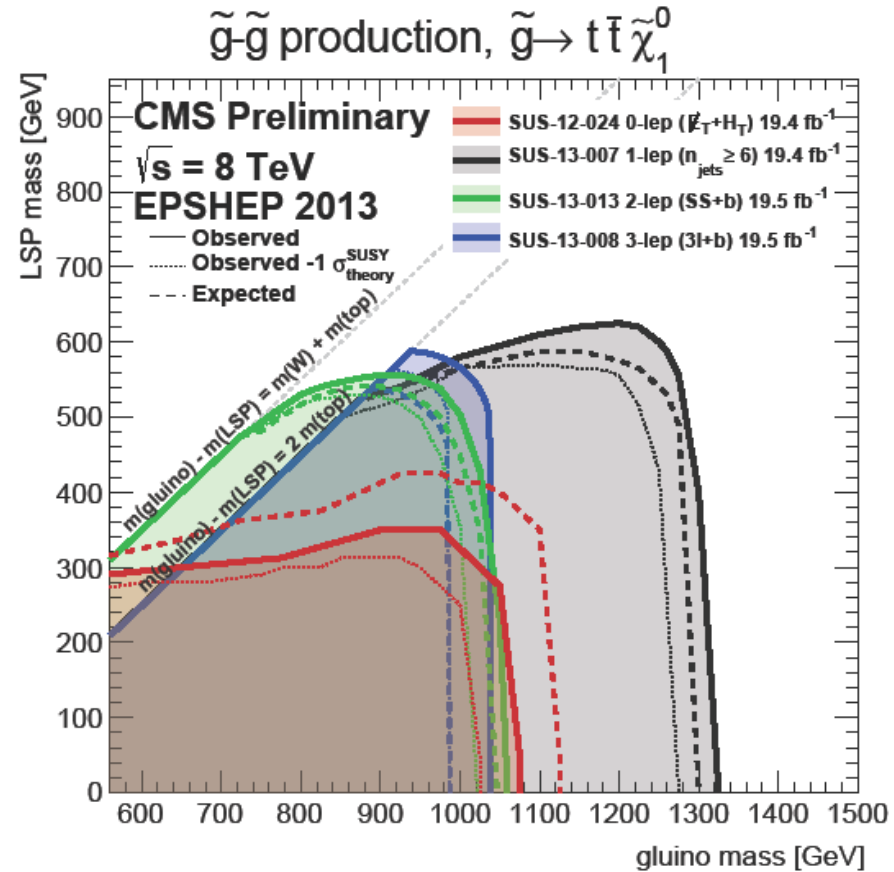
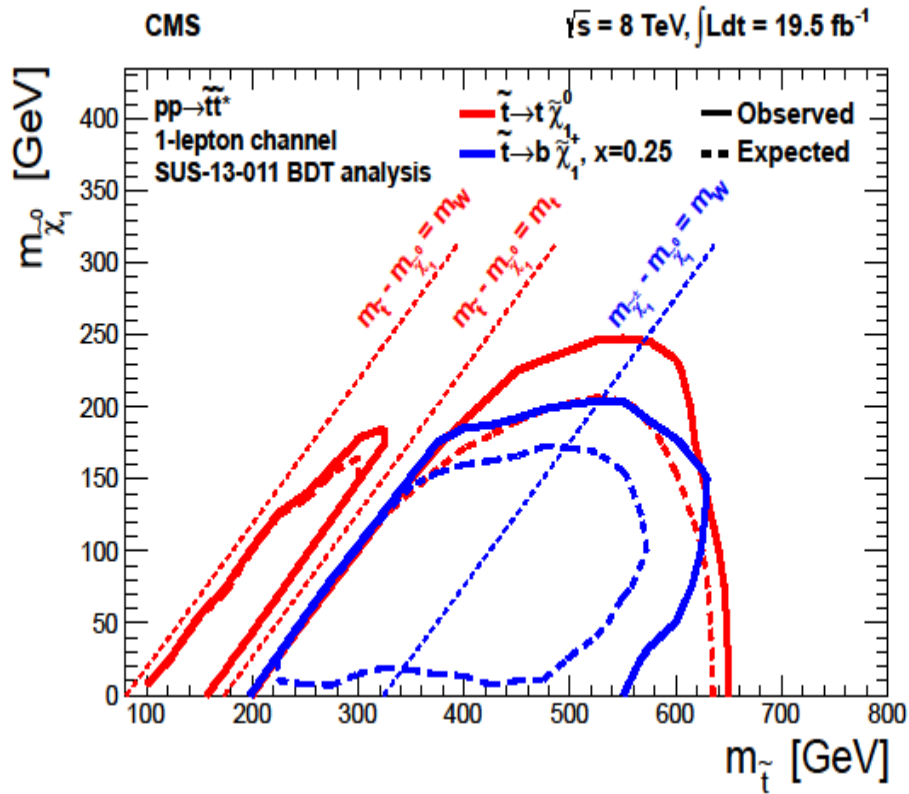
- inconsistent with $M_h = 125$ GeV
- too small $b \rightarrow s \gamma$ ratio .



V. Barger, MI, P. Huang, W-Y. Keung, PRD (2012)



SUSY searches at LHC



stop $m_{\tilde{t}_1} > \sim 0.65 \text{ TeV}$

gluino $m_{\tilde{g}} > 1.4(0.9) \text{ TeV}$ when $m_{\tilde{q}} \sim (\gg) m_{\tilde{g}}$,

→ Natural SUSY seems to be incorrect .

A weaker condition

- We do not know exactly what is the High-Scale SUSY model at present.
- Some cancellation mechanism between

$m_{Hu}^2(\Lambda)$ and $\delta m_{Hu}^2(M_{SUSY})$ is needed in the HS model.

We stand on a phenomenological viewpoint:
don't specify HS model, and
require **Naturalness only at EW scale** .

Radiative Natural SUSY (RNS)

- A kind of SUSY GUT model : $\Lambda = M_{GUT}$.
- $m_{Hu}^2(M_{GUT})$ is adjusted to reproduce a small negative

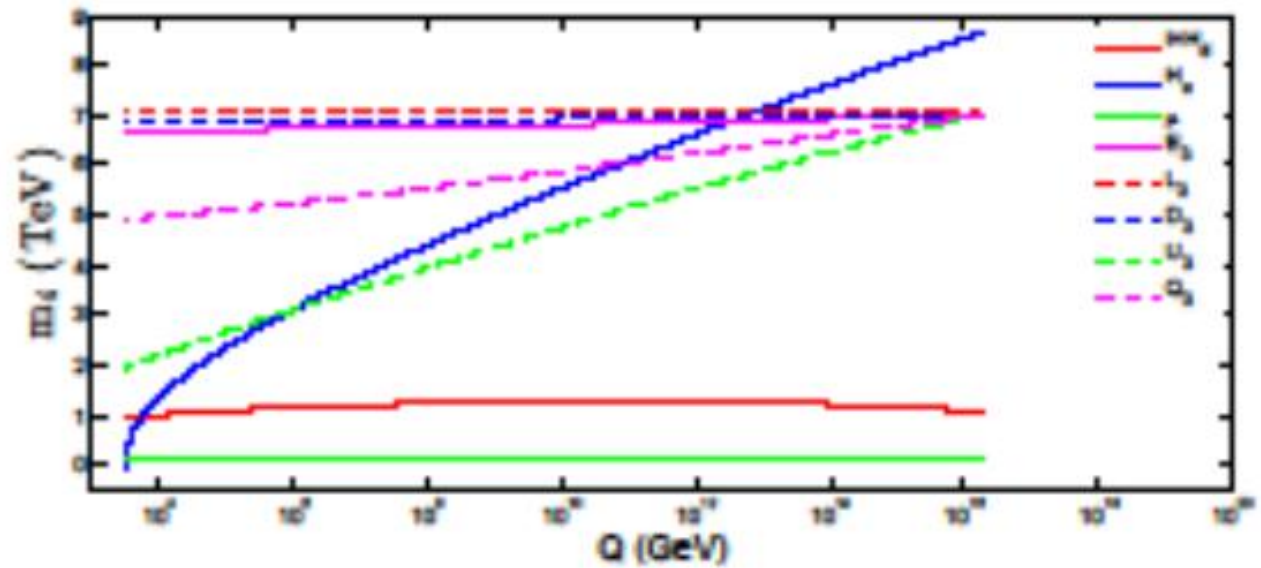
$$m_{Hu}^2(M_{SUSY}) (= m_{Hu}^2(M_{GUT}) + \delta m_{Hu}^2) \sim -\frac{M_Z^2}{2} < 0$$

EW symmetry breaking occurs by large radiative correction $\delta m_{Hu}^2 < 0$.

Non universal Higgs mass model

2 parameters : $m_{Hu}^2(M_{GUT}) \gg m_{Hd}^2(M_{GUT})$, traded as μ and m_A .

The model specified by the parameters: $m_0, m_{1/2}, \tan\beta, \mu, m_A, A_0$



How to get Electroweak Naturalness

Minimization of Higgs potential gives another relation.

$$\frac{M_Z^2}{2} = \frac{m_{Hd}^2 + \Sigma_d^d - t_\beta^2 (m_{Hu}^2 + \Sigma_u^u)}{t_\beta^2 - 1} - \mu^2 \approx -m_{Hu}^2 - \Sigma_u^u - \mu^2$$

ElectroWeak Fine-Tuning measure:

$$\Delta_{EW} = \max C_i / \left(\frac{M_Z^2}{2} \right).$$
$$C_i = m_{Hd}^2 / (t_\beta^2 - 1), \Sigma_d^d / (t_\beta^2 - 1), m_{Hu}^2, \Sigma_u^u, \mu^2$$

Requiring Low $\Delta_{EW} < 10-30$, 3-10% FT

→ Small Higgsino mass $|\mu| \sim 100-300 \text{ GeV}$

Radiative correction

- $$\sum_u^u = \frac{\partial \Delta V}{\partial (|h_u|^2)}$$

includes **stop** and other SUSY particle **loop effects**.

gives the largest Δ_{EW} in the relevant parameter region.

- The largest contribution comes from stops.

$$\Sigma_u^u(\tilde{t}_{1,2}) = \frac{3}{16\pi^2} H(m_{\tilde{t}_{1,2}}^2) \times \left[h_t^2 - g_Z^2 \mp \frac{h_t^2 A_t^2 - 8g_Z^2 \left(\frac{1}{4} - \frac{2}{3}x_W\right) \Delta_t}{m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2} \right]$$

$$H(m^2) = m^2 \left(\log \frac{m^2}{Q^2} - 1 \right), \quad Q^2 = m_{\tilde{t}_2} m_{\tilde{t}_1}$$

Low Δ_{EW} requires fairly light stops.

larger $m_{\tilde{t}_1}$ than that of NS is due to cancellation by A_t .

larger \tilde{t}_2 : its contribution can be suppressed in large mass splitting.

Predicted Mass spectra in RNS

parameter	RNS2
$m_0(1, 2)$	7025.0
$m_0(3)$	7025.0
$m_{1/2}$	568.3
A_0	-11426.6
$\tan\beta$	8.55
μ	150

m_A	1000
$m_{\tilde{g}}$	1562.8
$m_{\tilde{u}L}$	7020.9
$m_{\tilde{u}R}$	7256.2
$m_{\tilde{e}R}$	6755.4
$m_{\tilde{t}1}$	1843.4
$m_{\tilde{t}2}$	4921.4
$m_{\tilde{b}1}$	4962.6
$m_{\tilde{b}2}$	6914.9
$m_{\tilde{\tau}1}$	6679.4
$m_{\tilde{\tau}2}$	7116.9
$m_{\tilde{\nu}\tau}$	7128.3
$m_{\tilde{W}2}$	513.9
$m_{\tilde{W}1}$	152.7
$m_{\tilde{Z}4}$	525.2
$m_{\tilde{Z}3}$	268.8
$m_{\tilde{Z}2}$	159.2
$m_{\tilde{Z}1}$	135.4
m_h	125.0

- $m_{\tilde{t}_1} = 1\sim 2\text{TeV}$ heavier light stop than that in generic NS
- $m_{\tilde{t}_2} = 2\sim 5\text{TeV}$
- $m_{\tilde{g}} = 1\sim 4\text{TeV}$
- Light higgsino-like $\tilde{W}_1, \tilde{Z}_{1,2}$ have masses $\approx |\mu| = 100 - 300\text{GeV}$ from Naturalness.
- 1st, 2nd generation squarks & sleptons masses 1-8 or 20-30TeV.

Successfully reproduces the Higgs mass 125 GeV.
Consistent with the present LHC data.

Consistent with $\text{BF}(b \rightarrow s\gamma)$
(because of the heavier stops) .

Bs → μ⁺μ⁻

- Recent measurement by LHCb: $(3.2_{-1.2}^{+1.5}) \times 10^{-9}$ time-integrated cross section

Consistent with the SM prediction

$$BF_{SM}(Bs \rightarrow \mu^+ \mu^-) = (3.54 \pm 0.30) \times 10^{-9}$$

CP-averaged branching fraction [R.Fleischer arXiv:1212.4967](#)

$$y_s = \frac{\Gamma_{L^-} - \Gamma_H}{2\Gamma} = 0.088 \pm 0.014 \text{ light/heavy mass eigenstates}$$

$$BF_{SM}(Bs \rightarrow \mu^+ \mu^-) = (3.53 \pm 0.38) \times 10^{-9}.$$

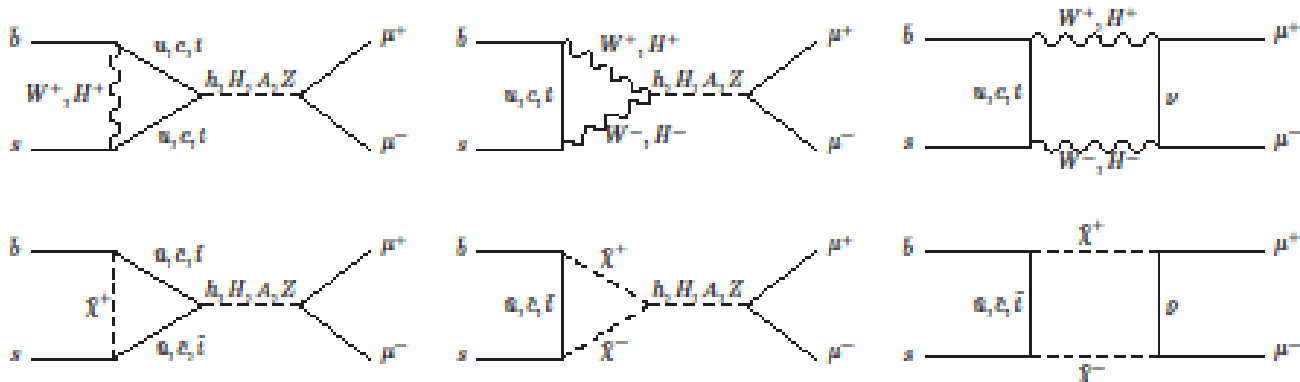
using $C_{10} = -4.16 \pm 0.04$ [A.Arbey, M. Battaglia et al., arXiv:1212.4887](#)

→ New physics contributions must be small.

$$BF(B_s \rightarrow \mu^+ \mu^-) = \frac{G_F^2 \alpha^2}{64 \pi^3} f_{B_s}^2 m_{B_s}^3 |V_{tb} V_{ts}^*|^2 \tau_{B_s} \sqrt{1 - \frac{4m_\mu^2}{m_{B_s}^2}} \times \left\{ \left(1 - \frac{4m_\mu^2}{m_{B_s}^2}\right) |C_{Q1} - C'_{Q1}|^2 + \left| (C_{Q2} - C'_{Q2}) + 2(C_{10} - C'_{10}) \frac{m_\mu}{m_{B_s}} \right|^2 \right\}$$

$$C_{Q1} \approx -C_{Q2} \approx -\bar{\mu} a_t \frac{\tan^3 \beta}{(1 + \epsilon_b \tan \beta)^2} \frac{m_\xi^2 m_b m_\mu}{4x_W M_W^2 m_A^2} f\left(\frac{1}{\bar{\mu}^2}\right) \propto \frac{\tan^3 \beta}{m_A^2}, \quad \bar{\mu} = \frac{\mu}{M_{SUSY}}, \quad a_t = \frac{A_t}{M_{SUSY}}, \quad f(x) = -\frac{x}{1-x} - \frac{x}{(1-x)^2} \log x > 0$$

Large $\tan \beta (> \sim 50)$ and small $m_A < 0.5$ TeV region is disfavored.



LHC $gg \rightarrow A \rightarrow bb, \tau\tau$
 Enhanced compared with
 Standard model in large $\tan\beta$
 LEP II pair production of $H^+ H^-$

Combining $B_s \rightarrow \mu\mu$,
 Small m_A region has
 been almost excluded.

$\rightarrow m_A \geq \sim 500 \text{ GeV}$
 $3 < \tan\beta \leq \sim 30$

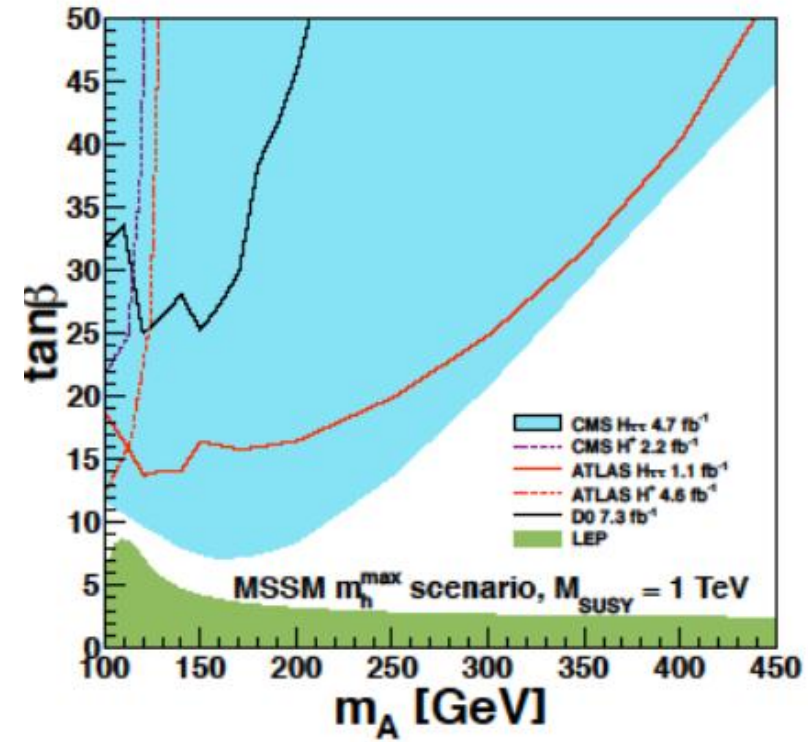
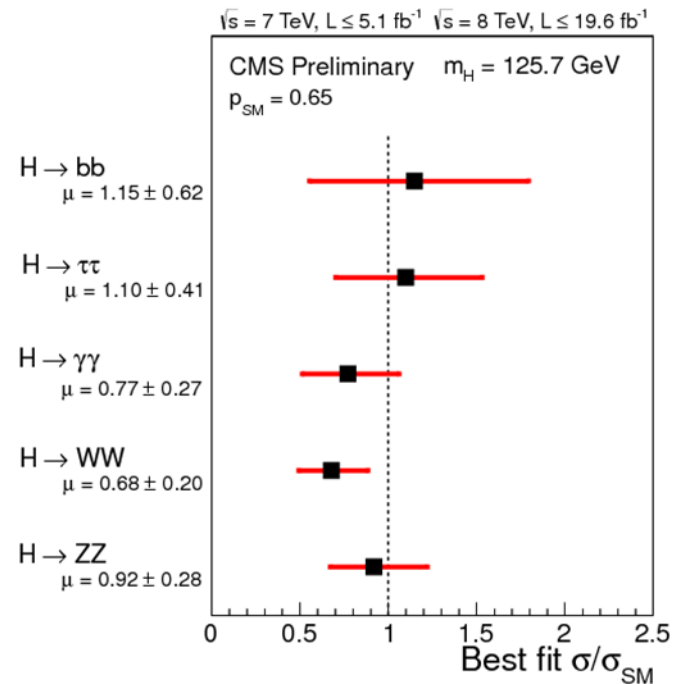
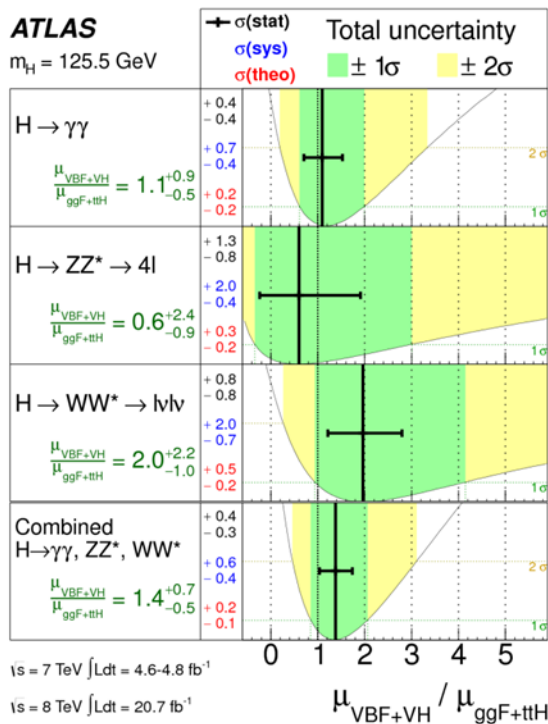


Figure 15: The 95% C.L. MSSM exclusion contours m_h -max benchmark scenario obtained by the ATLAS [234], CMS [138], and DØ [232] collaborations. The LHC collaborations contribute searches for $H \rightarrow \tau^+\tau^-$ and $H^\pm \rightarrow \tau\nu_\tau$ while DØ combines $H \rightarrow \tau^+\tau^-$ with $H \rightarrow b\bar{b}$ searches for these results. Also shown is the region excluded by LEP searches [34], assuming a top quark mass of 174.3 GeV.

Cross section ratios of 125 GeV Higgs in MSSM/RNS



Its cross sections have no deviation from SM predictions at present.

→ We must wait for future measurements with higher statistics to get a definite conclusion.

- RNS can be tested from Higgs cross section ratios.

Cross section ratio: σ_P

- Production and Decay Cross section of $XX \rightarrow h \rightarrow PP$

$$\sigma(XX \rightarrow h \rightarrow PP) = \sigma_{XX \rightarrow h} \times BF(h \rightarrow PP)$$

Prod.cross sect. from XX \times Branching Fraction to PP

$$BF(h \rightarrow PP) = \frac{\Gamma_{h \rightarrow PP}}{\Gamma_h^{tot}}, \quad \sigma_{XX \rightarrow h} \propto \Gamma_{h \rightarrow XX}$$

$$\sigma_P = \frac{\sigma(XX \rightarrow h \rightarrow PP)}{\sigma(XX \rightarrow h_{SM} \rightarrow PP)} = \frac{r_{XX}^2 r_{PP}^2}{R}$$

Ratios of the couplings:

$$r_{XX} = \frac{g_{hXX}}{g_{h_{SM}XX}}$$

Ratio of the total width: Assuming no exotic channels

$$R = \frac{\Gamma_h^{tot}}{\Gamma_{h_{SM}}^{tot}} = 0.57r_{bb}^2 + 0.06r_{\tau\tau}^2 + 0.25r_{VV}^2 + 0.09r_{gg}^2 + 0.03r_{cc}^2$$

Tree-level Higgs mass matrix

MSSM : a kind of 2 Higgs Doublet Model(2HDM).

- $$H_u = \begin{pmatrix} H_u^+ \\ h_u^0 = (H_u^0 + iA_u^0)/\sqrt{2} \end{pmatrix} \quad H_d = \begin{pmatrix} H_d^+ \\ h_d^0 = (H_d^0 + iA_d^0)/\sqrt{2} \end{pmatrix}$$

H_u couples to **up**-type quarks, H_d couples to **down**-type quarks.

$$\begin{pmatrix} M_Z^2 s_\beta^2 + m_A^2 c_\beta^2 & -(M_Z^2 + m_A^2) s_\beta c_\beta \\ -(M_Z^2 + m_A^2) s_\beta c_\beta & M_Z^2 c_\beta^2 + m_A^2 s_\beta^2 \end{pmatrix} : (H_u^0, H_d^0) \text{ base}$$
- $$\begin{pmatrix} h_u^0 \\ h_d^0 \end{pmatrix} = \begin{pmatrix} v_u \\ v_d \end{pmatrix} + \frac{1}{\sqrt{2}} \begin{pmatrix} c_\alpha & s_\alpha \\ -s_\alpha & c_\alpha \end{pmatrix} \begin{pmatrix} h \\ H \end{pmatrix} + \frac{i}{\sqrt{2}} \begin{pmatrix} s_\beta & c_\beta \\ -c_\beta & s_\beta \end{pmatrix} \begin{pmatrix} G \\ A \end{pmatrix}$$

two CP-even neutral Higgs : **h and H** $v_u/v_d = \tan\beta$
with **mixing angle α** defined in (h_u^0, h_d^0) basis.
a CP-odd neutral Higgs : **A** , a charged Higgs : **H[±]**
- When m_A is large, $\alpha \rightarrow \beta - \frac{\pi}{2}$ decoupling limit.

Couplings to weak gauge bosons

Close to the decoupling limit $\alpha \rightarrow \beta - \frac{\pi}{2}$

Moderate value of $\tan\beta > 5$, (where $\beta \approx \frac{\pi}{2}$).

- $r_{VV} = \frac{g_{hVV}}{g_{h_{SM}VV}} = \frac{v_u c_\alpha - v_d s_\alpha}{v} = \sin(\beta - \alpha) = s_{\beta - \alpha} \cong 1$

No deviation from SM prediction.

Couplings to up-type fermions

- $r_{tt} = \frac{c_\alpha}{s_\beta} \cong 1$

No deviation from SM prediction.

Couplings to down-type fermions

- $r_{bb} = \frac{-s_\alpha}{c_\beta} \cong 1$ in tree-level. But $-s_\alpha, c_\beta \ll 1$.

r_{bb} can deviate from unity by quantum correction.

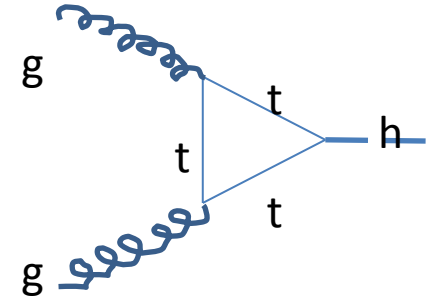
Loop induced gg and $\gamma\gamma$ couplings

gg fusion

$$r_{tt} = \frac{c_\alpha}{s_\beta} = 1, \quad r_{bb}$$

$$r_{gg} = \frac{1.03 \left(\frac{c_\alpha}{s_\beta} \right) + (-0.059 + i0.081)(r_{bb})}{1.03 + (-0.059 + i0.081)}$$

top
bottom



top loop is dominant.

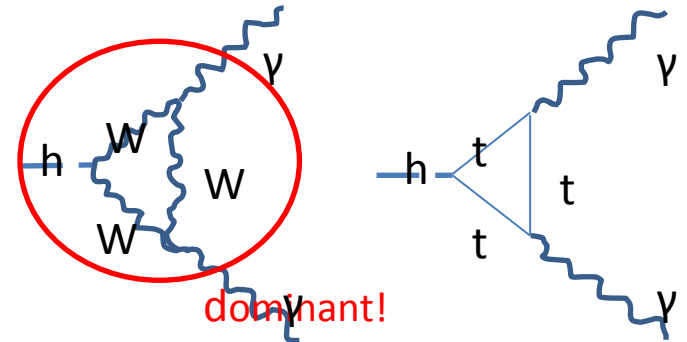
$\gamma\gamma$ production

$$r_{\gamma\gamma} =$$

$$\frac{\left(\frac{7}{4}\right)1.19 \sin(\beta-\alpha) - \left(\frac{4}{9}\right)1.03 \left(\frac{c_\alpha}{s_\beta}\right) - (1/9)(-0.059 + i0.081)(r_{bb})}{\left(\frac{7}{4}\right)1.19 - \left(\frac{4}{9}\right)1.03 - (1/9)(-0.059 + i0.081)}$$

W
top
bottom

W loop is dominant



Sum rule of Cross section ratio σ_P


- In large $m_A (\geq 500\text{GeV})$ region close to the decoupling limit,

$$\alpha = \beta - \frac{\pi}{2} + \epsilon < 0, \quad \epsilon < \frac{\pi}{2} - \beta \ll 1, \quad -\alpha \ll 1$$

$$r_{VV} = s_{\beta-\alpha} \cong 1, \quad r_{tt} = r_{cc} = \frac{c_\alpha}{s_\beta} \cong 1 + \frac{\epsilon}{t_\beta} \cong 1,$$

$$r_{gg} \cong r_{\gamma\gamma} \cong 1, \quad R \left(= \frac{\Gamma_h^{\text{tot}}}{\Gamma_{h_{SM}}^{\text{tot}}} \right) \cong 0.6r_{bb}^2 + 0.4$$

Prediction for cross section ratio:


$$\sigma_\gamma = \sigma_W = \sigma_Z = \frac{1}{0.6r_{bb}^2 + 0.4},$$

$$0.4 \sigma_\gamma + 0.6 \sigma_b = 1$$

$$\sigma_b = \frac{r_{bb}^2}{0.6r_{bb}^2 + 0.4}$$

It can be used to check MSSM.

$\gamma\gamma$ enhancement/suppression

means bb suppression/enhancement

bb and $\tau\tau$ couplings

$$r_{\tau\tau} = \frac{-s_\alpha}{c_\beta} \cong 1 - \epsilon t_\beta \cong 1 - \frac{\epsilon}{\epsilon_\beta}$$

$$r_{bb} = \frac{-s_\alpha}{c_\beta} \left[1 - \frac{\Delta_b}{1+\Delta_b} \left(1 + \frac{1}{t_\alpha t_\beta} \right) \right] \cong 1 - \frac{1}{1+\Delta_b} \epsilon t_\beta \cong 1 - \frac{1}{1+\Delta_b} \frac{\epsilon}{\epsilon_\beta}$$

$$\alpha = -\epsilon_\beta + \epsilon, \quad \frac{\pi}{2} - \beta = \epsilon_\beta \cong \frac{1}{\tan\beta} \ll 1$$

$$\Delta_b = \bar{\mu} t_\beta \left[\frac{2\alpha_s}{3\pi} \widehat{m}_g I(\widehat{m}_g^2, \widehat{m}_{b1}^2, \widehat{m}_{b2}^2) + \frac{h_t^2}{16\pi^2} a_t I(\bar{\mu}^2, \widehat{m}_{t1}^2, \widehat{m}_{t2}^2) \right]$$

sbottom-gluino + stop-chargino loops

- $\Delta_b \propto \mu$: small, but not negligible in Natural SUSY

$\epsilon > 0$: bb suppression, $\epsilon < 0$: bb enhancement

Its correction $-\frac{\epsilon}{\epsilon_\beta}$: small/small delicate problem!

Improved 2LL formula

$$\begin{pmatrix} M_Z^2 s_\beta^2 + m_A^2 c_\beta^2 + \delta_{11} & -(M_Z^2 + m_A^2) s_\beta c_\beta + \delta_{12} \\ -(M_Z^2 + m_A^2) s_\beta c_\beta + \delta_{12} & M_Z^2 c_\beta^2 + m_A^2 s_\beta^2 + \delta_{22} \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} c_\alpha & s_\alpha \\ -s_\alpha & c_\alpha \end{pmatrix} \begin{pmatrix} h \\ H \end{pmatrix}$$

$$\delta_{11} = F_3 \frac{3\overline{m}_t^4}{4\pi^2 v^2 s_\beta^2} \left[t \left(1 - G_{\frac{15}{2}} t \right) + a_t x_t \left(1 - \frac{a_t x_t}{12} \right) \left(1 - 2G_{\frac{9}{2}} t \right) \right] - M_Z^2 s_\beta^2 (1 - F_3)$$

$$\delta_{22} = -F_3 \frac{\overline{m}_t^4}{2 \cdot 16\pi^2 v^2 s_\beta^2} \left[\left(1 - 2G_{\frac{9}{2}} t \right) (x_t \bar{\mu})^2 \right]$$

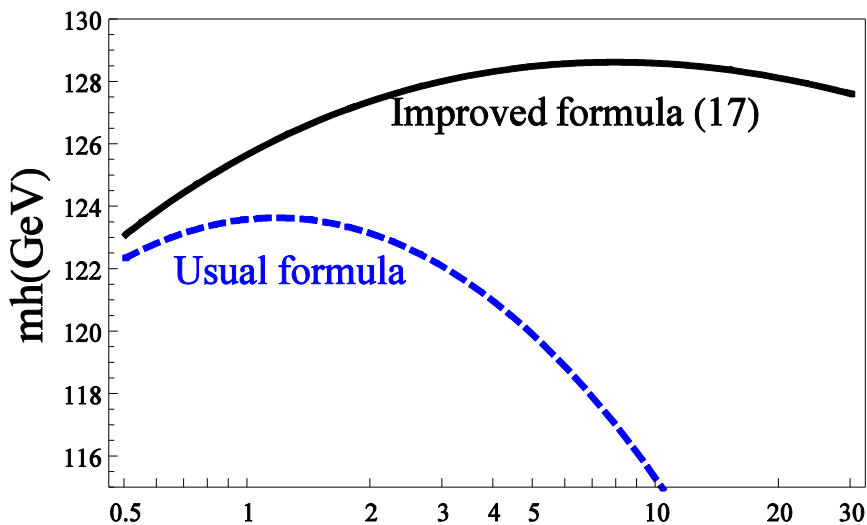
$$\delta_{12} = -F_9 \frac{3\overline{m}_t^4}{4 \cdot 8\pi^2 v^2 s_\beta^2} \left[\left(1 - 2G_{\frac{9}{2}} t \right) x_t \bar{\mu} \left(1 - \frac{a_t x_t}{6} \right) \right] + M_Z^2 s_\beta c_\beta (1 - F_{\frac{3}{2}})$$

$$F_l = \frac{1}{1 + l h_t^2 t / (8\pi^2)} : \text{Higgs WF ren.}$$

$$G_l = -\frac{1}{16\pi^2} (l h_t^2 - 32\pi\alpha_s)$$

$$x_t = \frac{X_t}{M_{SUSY}}, \quad a_t = \frac{A_t}{M_{SUSY}}$$

$$\bar{\mu} = \frac{\mu}{M_{SUSY}}, \quad t = \ln \frac{M_{SUSY}^2}{m_t^2}$$



In decoupling limit

$$m_h^2 = M_Z^2 c_{2\beta}^2 + F_3 \frac{3\overline{m}_t^4}{4\pi^2 v^2} \left[t \left(1 - G_{\frac{15}{2}} t \right) + x_t^2 \left(1 - \frac{x_t^2}{12} \right) \left(1 - 2G_{\frac{9}{2}} t \right) \right] - M_Z^2 s_\beta^4 (1 - F_3) \dots (17)$$

Decoupling base and Flavor-Tuned (FT) Higgs boson.

- $\alpha = -\epsilon_\beta + \epsilon$

$$\rightarrow \begin{pmatrix} c_\alpha & s_\alpha \\ -s_\alpha & c_\alpha \end{pmatrix} = \begin{pmatrix} c_{\epsilon\beta} & -s_{\epsilon\beta} \\ s_{\epsilon\beta} & c_{\epsilon\beta} \end{pmatrix} \begin{pmatrix} c_\epsilon & s_\epsilon \\ -s_\epsilon & c_\epsilon \end{pmatrix} = \begin{pmatrix} s_\beta & -c_\beta \\ c_\beta & s_\beta \end{pmatrix} \begin{pmatrix} c_\epsilon & s_\epsilon \\ -s_\epsilon & c_\epsilon \end{pmatrix}$$

Rotating $-\epsilon_\beta$: decoupling base

- $$\frac{1}{\sqrt{2}} \begin{pmatrix} c_\alpha & -s_\alpha \\ s_\alpha & c_\alpha \end{pmatrix} \begin{pmatrix} M_Z^2 s_\beta^2 + m_A^2 c_\beta^2 + \delta_{11} & -(M_Z^2 + m_A^2) s_\beta c_\beta + \delta_{12} \\ -(M_Z^2 + m_A^2) s_\beta c_\beta + \delta_{12} & M_Z^2 c_\beta^2 + m_A^2 s_\beta^2 + \delta_{22} \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} c_\alpha & s_\alpha \\ -s_\alpha & c_\alpha \end{pmatrix} \begin{pmatrix} h \\ H \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{pmatrix} c_\epsilon & -s_\epsilon \\ s_\epsilon & c_\epsilon \end{pmatrix} \begin{pmatrix} M_Z^2 c_\beta^2 + \delta_{11} s_\beta^2 + \delta_{22} c_\beta^2 & (M_{12}^\beta)^2 \\ +2\delta_{12} s_\beta c_\beta & m_A^2 + M_Z^2 s_\beta^2 + \delta_{22} s_\beta^2 \\ (M_{12}^\beta)^2 & +\delta_{11} c_\beta^2 - 2\delta_{12} s_\beta c_\beta \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} c_\epsilon & s_\epsilon \\ -s_\epsilon & c_\epsilon \end{pmatrix} \begin{pmatrix} h \\ H \end{pmatrix}$$

- $$(M_{12}^\beta)^2 = -\frac{M_Z^2}{2} (-s_{4\beta}) + (\delta_{22} - \delta_{11}) s_\beta c_\beta + \delta_{12} (s_\beta^2 - c_\beta^2) \cong -\frac{2M_Z^2 + \delta_{11} - \delta_{22}}{\tan\beta} + \delta_{12}$$

$$\rightarrow \frac{\epsilon}{\epsilon_\beta} \approx \frac{(M_{12}^\beta)^2}{m_A^2 \epsilon_\beta} \cong -\frac{2M_Z^2 + \delta_{11} - \delta_{22} - \delta_{12} \tan\beta}{m_A^2}$$

In tree-level, $\frac{\epsilon}{\epsilon_\beta} = -\frac{2M_Z^2}{m_A^2}$. ϵ to be small negative \rightarrow bb enhancement.

When large μ , Cancellation by the off-diagonal element $\delta_{12} \tan\beta$ is possible. \rightarrow bb suppression.

Flavor-Tuned (FT) Higgs boson.

Barger, Huang, Ishida, Keung, PRD (2012)

Carena et al., PRD62, 055008(2000)

$$\delta_{12} \cong -\frac{3\overline{m}_t^4}{8\Pi^2 v^2 s_\beta^2} \left[\left(1 - 2G_{\frac{9}{2}} t\right) x_t \bar{\mu} \left(1 - \frac{a_t x_t}{6}\right) \right]$$

For small μ in RNS, its effect is small, but not negligible.

Prediction for the cross section ratios

- $124\text{GeV} < m_h$: constraint.

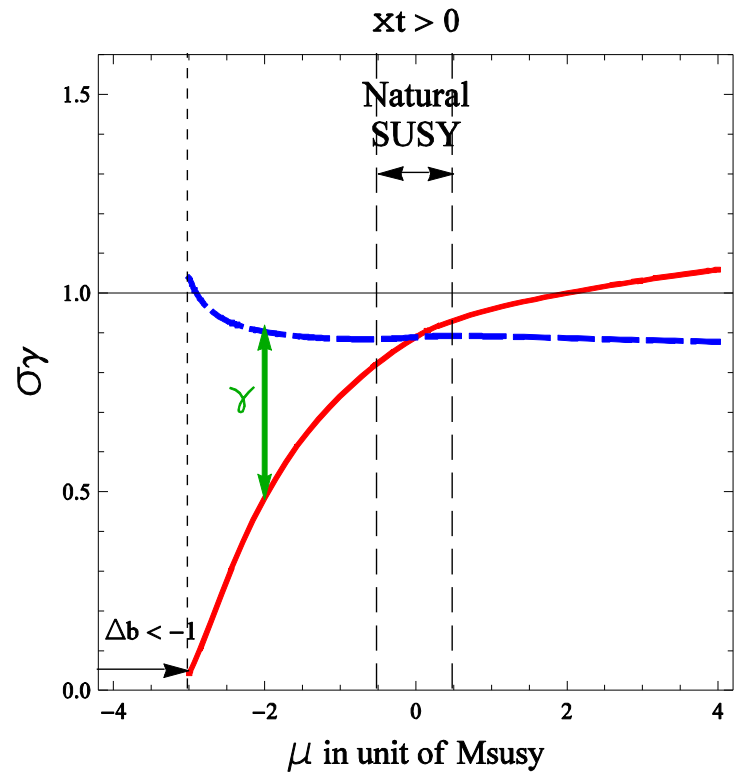
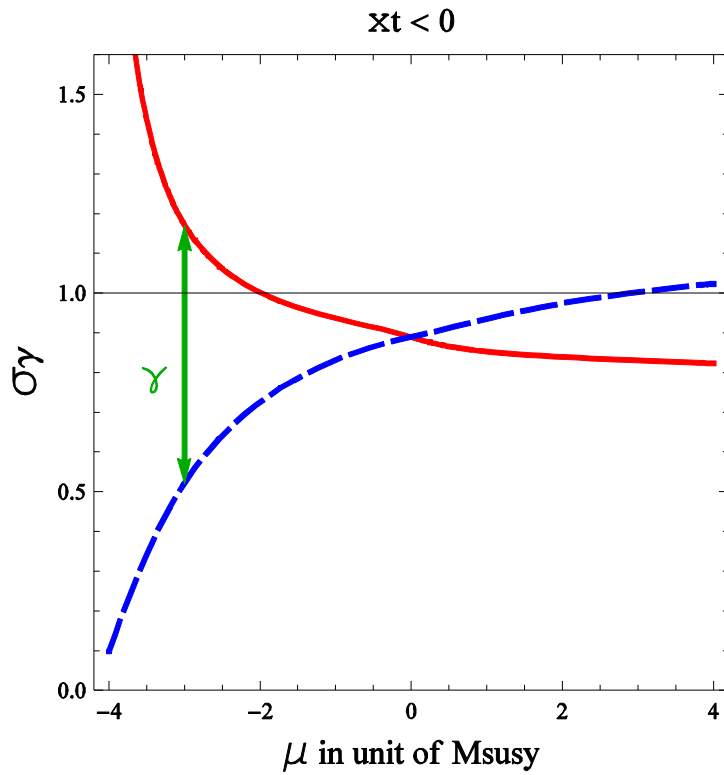
By using Improved 2LL formula

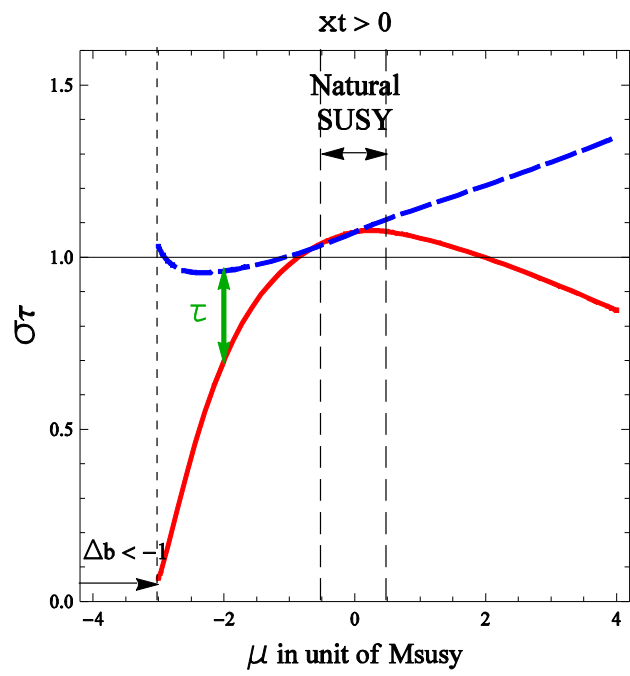
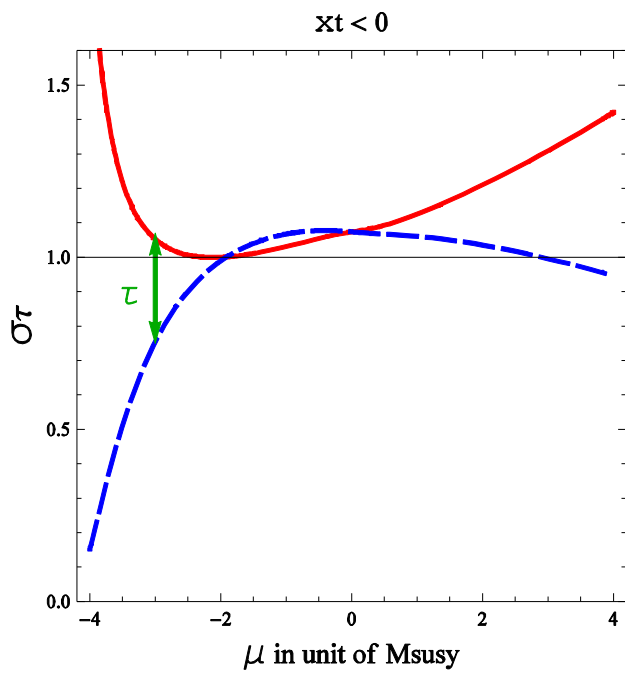
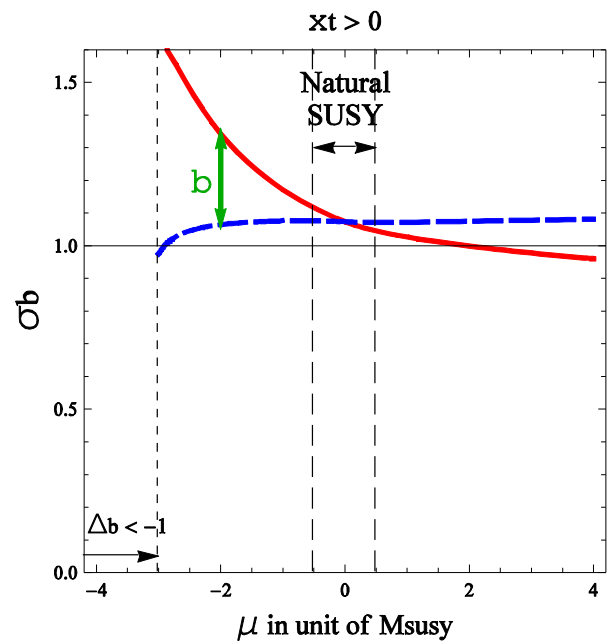
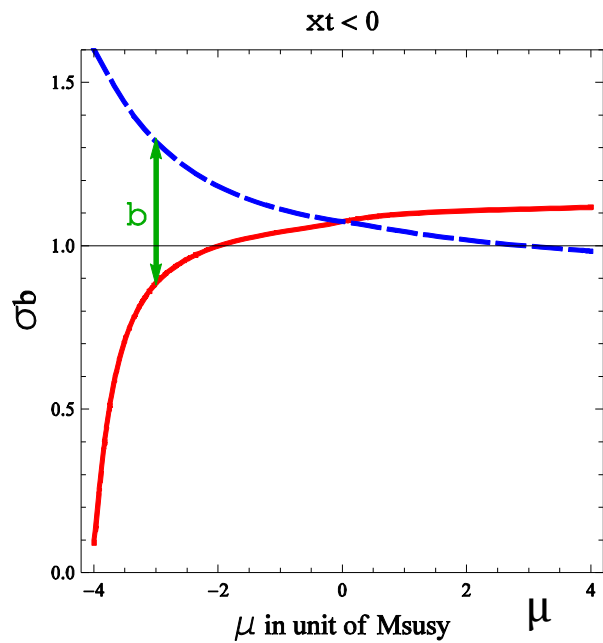
→ $1.95 < |x_t| < 2.86$: nearby the maximal mixing $\sqrt{6}$

$m_A=500\text{GeV}$, $\tan\beta=20$.

- $x_t > 0$ is favored in RNS : the running down from the GUT scale.
- predict $\bar{\mu} = \frac{\mu}{M_{SUSY}}$ dependence
of $\sigma_\gamma, \sigma_b, \sigma_\tau$

Prediction

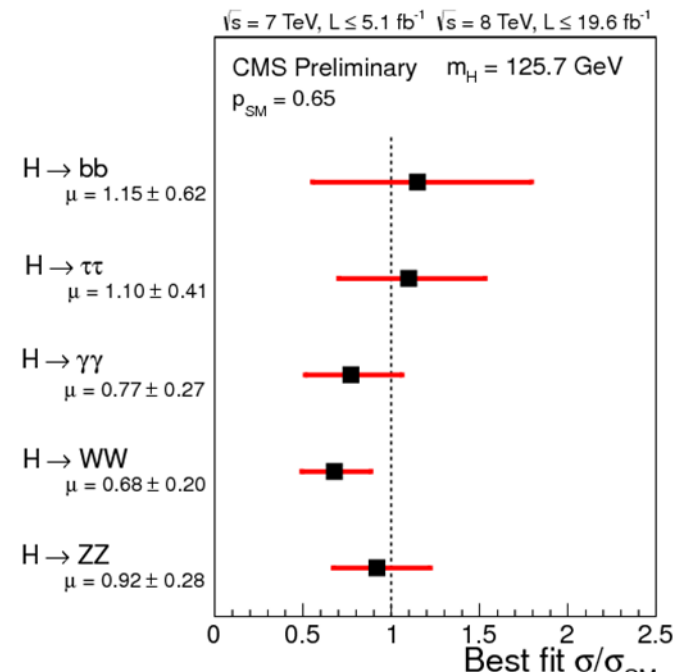




- FT Higgs boson can give $\gamma\gamma$ enhancement $\sigma\gamma \sim 1.5$ when $\bar{\mu} \approx -3 \rightarrow \mu = -3 \text{ TeV}$ (Large!) when $M_{SUSY} = 1 \text{ TeV}$ case.
- Natural SUSY which requires small μ always predicts $\gamma\gamma$ suppression.

	$\sigma\gamma$	σb	$\sigma\tau$
$m_A = 500 \text{ GeV}$	$0.82 \sim 0.91$	$1.06 \sim 1.12$	$1.04 \sim 1.08$
$m_A = 1000 \text{ GeV}$	$0.95 \sim 0.98$	$1.01 \sim 1.03$	$1.01 \sim 1.02$

Its deviation from unity is negligible for $m_A > 1 \text{ TeV}$ case.



Other method of testing RNS

- Light Higgsino-like chargino and neutralino are main feature of RNS. (Small $\mu \sim 150\text{GeV}$.)
→ RNS can be fully tested by future ILC experiment.

$$e^+ e^- \rightarrow \widetilde{W}_1^+ \widetilde{W}_1^- , \widetilde{Z}_1 \widetilde{Z}_2 \quad \text{ILC is Higgsino factory.}$$

- $\widetilde{W}_1 \widetilde{Z}_2, \widetilde{W}_1 \widetilde{Z}_1, \widetilde{W}_1 \widetilde{W}_1, \widetilde{Z}_1 \widetilde{Z}_2$, Large production cross section at LHC.

$$m_{\widetilde{W}_1} - m_{\widetilde{Z}_1} \text{ and } m_{\widetilde{Z}_2} - m_{\widetilde{Z}_1} \text{ are typically}$$

5 ~ 20 GeV : Higgsinos are almost degenerate.

Very low visible energy release from \widetilde{W}_1 and \widetilde{Z}_2 decays.

→ beneath SM background at LHC.

Other method testing RNS

- Assuming $M_1 = M_2 = M_3 = m_{\frac{1}{2}}$ at GUT scale.
→ $M_1 : M_2 : M_3 \approx 1 : 2 : 7$ at weak scale.
- $m_{\tilde{g}} > 1.4\text{TeV}$ at LHC.
→ $M_2 > 400\text{ GeV}$, $M_1 > 200\text{ GeV}$.

- Wino-like $\tilde{W}_2 \tilde{Z}_4$ production.

and Same-sign diboson signals at LHC is promising.

Baer, Barger, Huang, Mickelson, Mustafayev, Sreethawong, Tata [arXiv:1302.5816](https://arxiv.org/abs/1302.5816)

$$\tilde{W}_2^+ \tilde{Z}_4 \rightarrow W^+ \tilde{Z}_{1,2} + W^+ \tilde{W}_1^-$$

For integrated luminosity 100(1000) fb^{-1}

M_2 can be tested up to 550(800) GeV at **LHC14**.

Concluding Remarks

- MSSM and Natural SUSY can be tested by the Measurements of the Cross section ratios of the 125GeV Higgs boson to the SM predictions at LHC.
- Test of MSSM: Sum rule : $\sigma_\gamma = \sigma_W = \sigma_Z$, $0.4 \sigma_\gamma + 0.6 \sigma_b = 1$
- Radiative Natural SUSY requires small $|\mu| < \sim 0.5$ TeV.
- 2 Mechanism of leading $\gamma\gamma$ enhancement : large μ required
 FT model $\sigma_\gamma = 1.5$ possible for large $\mu=2$ TeV for $M_{SUSY}=1TeV$.
 light stau $\mu \tan\beta = 30$ TeV or more is needed.
- Rad. Natural SUSY (RNS) always predicts $\gamma\gamma$ suppression.

	σ_γ	σ_b	σ_τ
mA = 500 GeV	0.82 ~ 0.91	1.06 ~ 1.12	1.04 ~ 1.08
mA = 1000 GeV	0.95 ~ 0.98	1.01 ~ 1.03	1.01 ~ 1.02

- Light higgsino contribution is expected to be negligible in higgs $\gamma\gamma$ decay in RNS.
- Wino-like $\widetilde{W}_2 \widetilde{Z}_4$ production.
 and Same-sign diboson signals at LHC is promising

SUSY RGE

$$L = \lambda_2(M_{SUSY}^+) \Phi(M_{SUSY}^+)^4, \quad \lambda_2(M_{SUSY}^+) = \frac{g^2(t) + g'^2(t)}{4}$$



$\mu = M_{SUSY}$

$$L = \lambda_2(M_{SUSY}^-) \Phi(M_{SUSY}^-)^4$$

$$= \lambda_2(M_{SUSY}^+) \Phi(M_{SUSY}^-)^4 + \Delta\lambda_2^{th}(M_{SUSY}) \Phi(M_{SUSY}^-)^4$$

non-SUSY
2HDM RGE



does not run by RGE
'Frozen' at M_{SUSY}

$$L = \lambda_2(m_t) \Phi(m_t)^4 + \Delta\lambda_2^{th}(M_{SUSY}) \Phi(M_{SUSY}^-)^4$$

$\mu = m_t$

Scale down from $t \rightarrow 0$ ($\mu = M_{SUSY} \rightarrow m_t$)

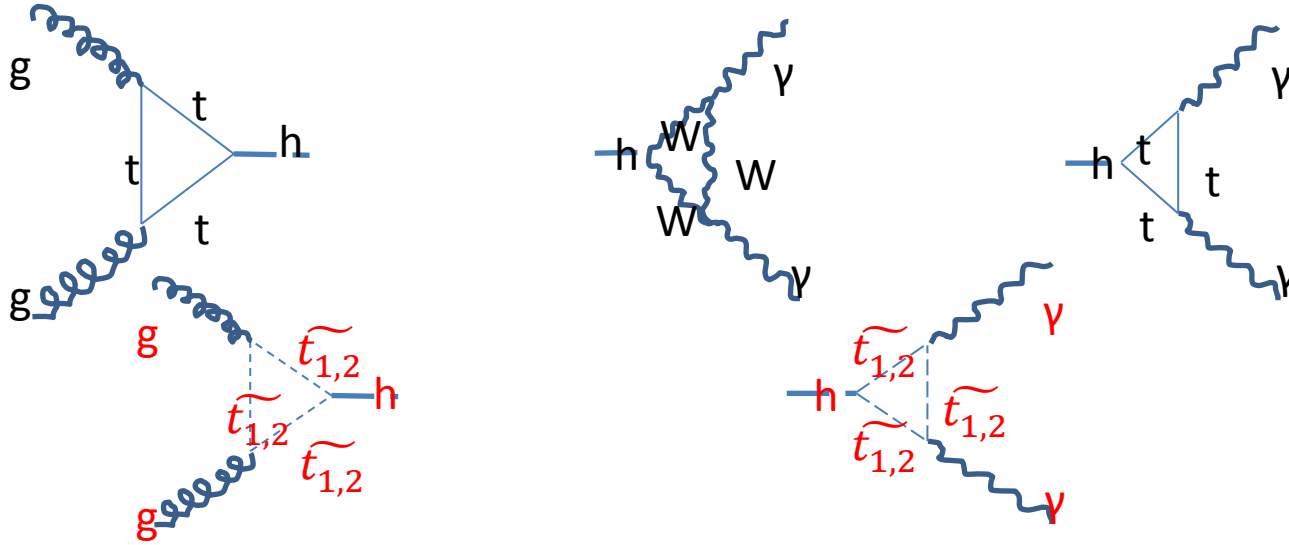
$$\Phi(m_t)^4 = \left(1 + t \frac{3h_t^2}{8\pi^2}\right) \Phi(M_{SUSY}^-)^4 : \text{ Higgs WF renormalization factor}$$

$$\rightarrow \Phi(M_{SUSY}^-)^4 = \frac{1}{1 + t \frac{3h_t^2}{8\pi^2}} \Phi(m_t)^4 = \frac{1}{1 - 2\gamma_2 t} \Phi(m_t)^4 \neq (1 + 2\gamma_2 t) \Phi(m_t)^4$$

improvement

$$\Delta\lambda_2^{th}(M_{SUSY}) \Phi(M_{SUSY}^-)^4 = F_3 \Delta\lambda_2^{th}(M_{SUSY}) \Phi(m_t)^4, \quad F_3 = \frac{1}{1 + t \frac{3h_t^2}{8\pi^2}}$$

Exotic loops : Effect from stop



$$r_{gg} = 1 - \frac{m_t^2(X_t^2 - m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)}{4m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2} \approx 1 - \frac{m_t^2(m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2)}{2m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2} \quad \text{where } X_t^2 = 6(m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2)/2$$

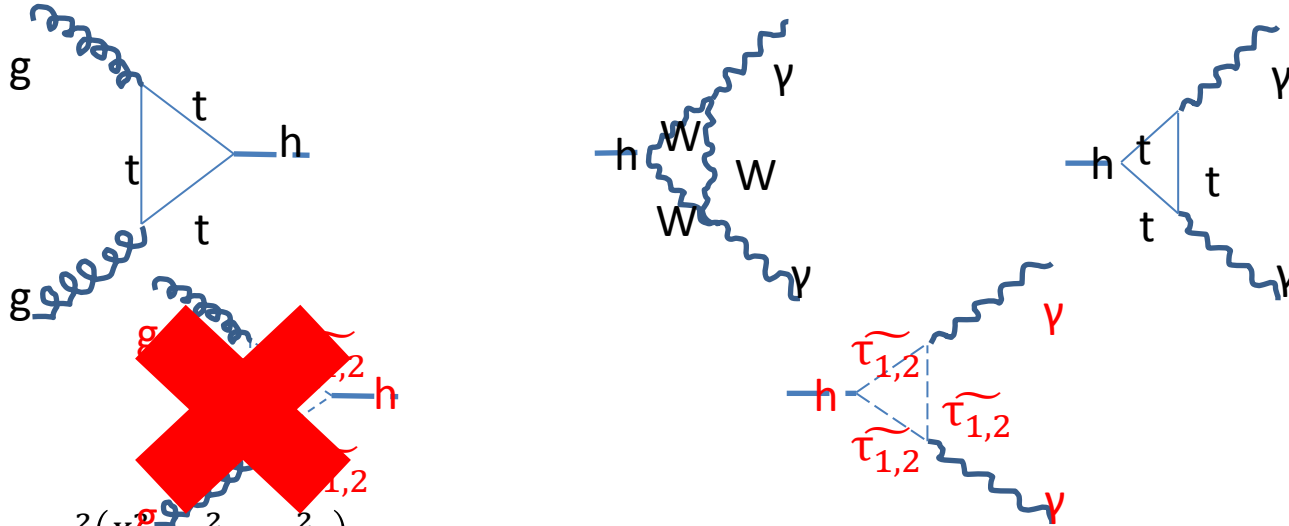
$$1 - \frac{8 Ft}{3 F_W} = 1 - \frac{8 \frac{2.06}{3}}{3 \cdot 8.32} = \frac{7}{9}, \quad \frac{F_S}{F_W} = \frac{-0.5}{8.32}$$

$$r_{\gamma\gamma} = \frac{1 - \frac{8 Ft}{3 F_W} + \frac{2m_t^2(X_t^2 - m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2) F_S}{3m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2 F_W}}{1 - \frac{8 Ft}{3 F_W}} \approx 1 + \frac{m_t^2(m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2)}{7.28 m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2}$$

$$r_{gg} r_{\gamma\gamma} \approx 1 - 0.36 \frac{m_t^2(m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2)}{m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2} \approx 1 - 0.36 \frac{m_t^2}{m_{\tilde{t}_1}^2} = 0.99 \quad \text{for } m_{\tilde{t}_1} = 1 \text{ TeV} \ll m_{\tilde{t}_2}$$

Negligible

Exotic loops : Effect from stau



$$r_{\gamma\gamma} = \frac{1 - \frac{8Ft}{3FW} \frac{m_t^2 (X_t^2 - m_{\tilde{\tau}_1}^2 - m_{\tilde{\tau}_2}^2) F_S}{m_{\tilde{\tau}_1}^2 m_{\tilde{\tau}_2}^2}}{1 - \frac{8Ft}{3FW}} \approx 1 + 0.052 \frac{m_t^2 (X_t^2 - m_{\tilde{\tau}_1}^2 - m_{\tilde{\tau}_2}^2)}{m_{\tilde{\tau}_1}^2 m_{\tilde{\tau}_2}^2}$$

Vacuum stability of Scalar Higgs potential. [Hisano,Sugiyama2011](#)

$$|\mu \tan\beta| < 76.9\sqrt{m_{L3}m_{E3}} + 38.7(m_{L3} + m_{E3}) - 10.4\text{TeV}$$

Rough estimate $m_{\tilde{\tau}_1} \sim m_{E3} = 0.1\text{TeV}$, $m_{\tilde{\tau}_2} \sim m_{L3} \sim 0.6\text{TeV}$, $X_\tau = |\mu \tan\beta| < 36\text{TeV}$

→ $r_{\gamma\gamma} = 1.1$

→ $\sigma_\gamma \leq 1.25$ for $m_{\tilde{\tau}_1} > 100\text{ GeV}$ [Kitahara,arXiv:1208.4792](#)

$\sigma_\gamma \leq 1.5$ for very large $\tan\beta \approx 100$, [Carena,Gori,Low,Shah,Wagner,arXiv:1211.6136](#)