The 126 GeV Higgs Boson and its Natural Relatives

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What has the LHC Discovery taught us?

Last year (2012) was the year of the Higgs Boson. We now know that there is a particle at 126 GeV which looks very much like the one Higgs boson of the standard model (SM). What has it taught us?

There are two possibilities:
(1) The SM is it, and we can just clean up the details and go home.
(2) There is new physics lurking, but it should naturally give us the 126 GeV particle as observed!

Example of (2):

In supersymmetry (SUSY), there are two Higgs doublets. In general, the lightest neutral physical scalar is not a linear combination equaling that of the SM, unless the SUSY breaking scale M_{SUSY} is rather high, say 10 TeV.

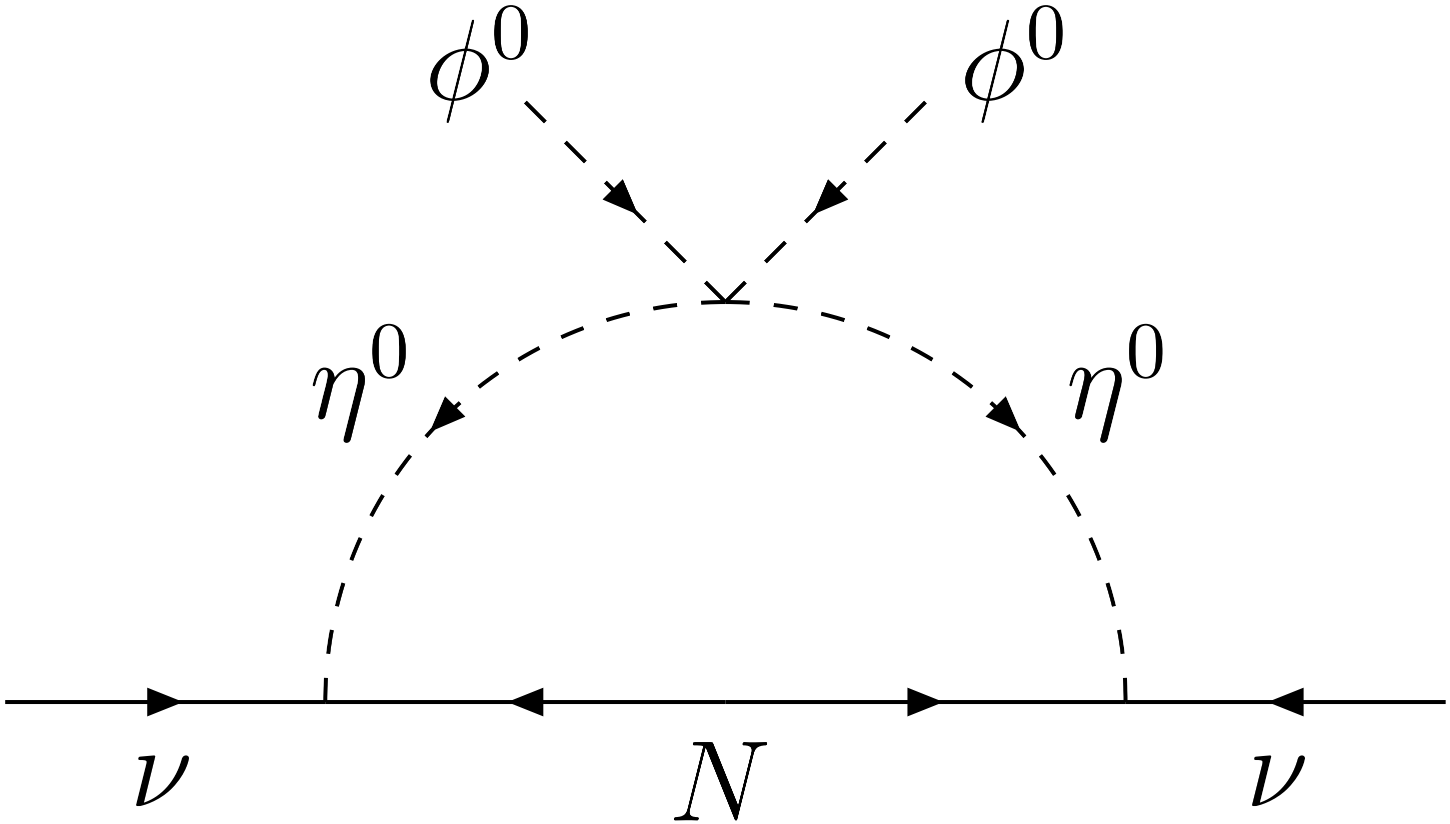
However, $m_H = 126$ GeV $> m_Z$ requires not only large M_{SUSY} but also very fine tuning in MSSM between $m^2_{H_u}$ and μ^2.

The new hope of Natural SUSY is NMSSM
or a gauge extension, i.e. with Z' or more.

Dark Scalar Doublet(s) and Neutrino Mass

A second scalar doublet (η^+, η^0) with an exactly conserved odd Z_2 [Deshpande/Ma(1978)] is good, because it does not mix with the SM Higgs and is a possible dark-matter candidate.

In 2006 I proposed the scotogenic (from the Greek scotos meaning darkness) model of radiative neutrino mass, using the addition of N_i which are also odd under Z_2 together with (η^+, η^0), thereby linking neutrino mass to the existence of dark matter.

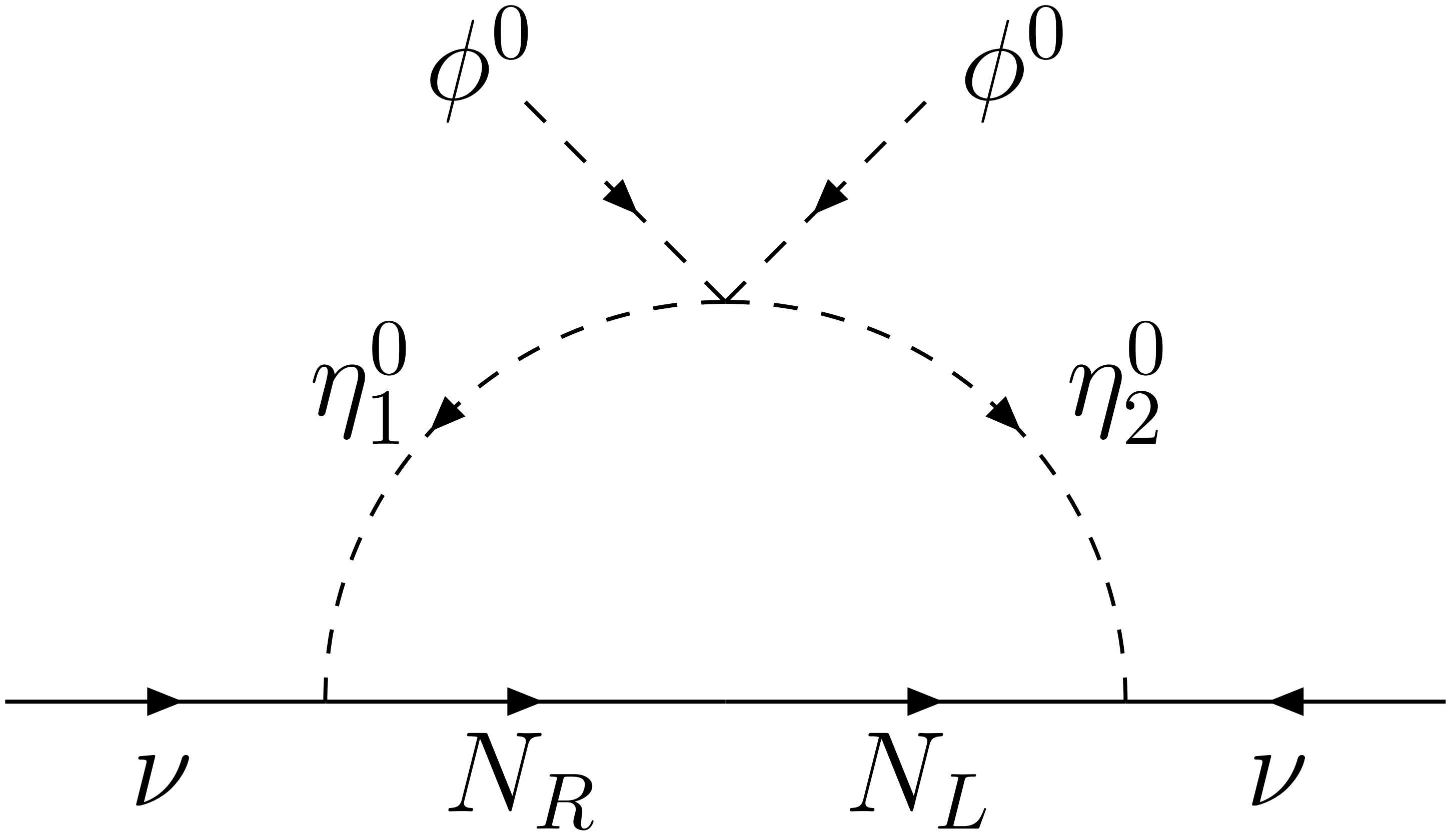


ϕ^0
ϕ^0
η^0
η^0
ν
N
ν

Let $\eta^0 = (\eta_R + i\eta_I)/\sqrt{2}$, then η_R could be (cold) dark matter [Ma(2006)], or N_i of order 10 keV could be (warm) dark matter [Ma(2012)]. Two months later, Barbieri/Hall/Rychkov(2006) proposed (η^+, η^0) by itself and called it the inert Higgs doublet.

Note that η_R and η_I are split in mass by the Z_2 allowed term $(\lambda_5/2)(\Phi^\dagger \eta)^2 + H.c.$ as ϕ^0 acquires a vev. This is important: (1) the one-loop diagram for Majorana neutrino mass is nonzero, and (2) the elastic scattering of η_R off nuclei through Z exchange is kinematically forbidden if η_I is a few hundred keV heavier than η_R.

Ma/Picek/Radovcic(2013): The Z_2 symmetry may be promoted to a local gauge $U(1)_D$ symmetry with two dark scalar doublets: $(\eta^+_{1,2}, \eta^0_{1,2})$ transforming as ± 1 under $U(1)_D$, together with three Dirac N's. The Majorana neutrino mass is then generated by m_N without breaking $U(1)_D$. Such a massless dark photon is consistent with astrophysical observations for Dirac fermion dark matter at the TeV scale. If $U(1)_D$ is broken by a scalar with two units of dark charge, the Z_2 symmetry is recovered. The dark photon is then massive, together with a scalar particle which is also a force carrier for dark matter.



ϕ^0
ϕ^0
η_1^0
η_2^0
ν
N_R
N_L
ν

Lepton (and Quark) Flavor Triality

Ma(2010), Cao/Damanik/Ma/Wegman(2011)

Under non-Abelian flavor symmetries such as A_4, T_7, and $\Delta(27)$, one may assign $L_i = (\nu, l)_i \sim \underline{3}$, $l_i^c \sim \underline{1}_i$ $(i = 1, 2, 3)$, and $\Phi_i = (\phi^+, \phi^0)_i \sim \underline{3}$, then $L_i l_j^c \tilde{\Phi}_k$, where $\tilde{\Phi} = (\bar{\phi}^0, -\phi^-)$, yields

$$\mathcal{M}_l = \begin{pmatrix} v_1^* & 0 & 0 \\ 0 & v_2^* & 0 \\ 0 & 0 & v_3^* \end{pmatrix} \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix} \begin{pmatrix} f_e & 0 & 0 \\ 0 & f_\mu & 0 \\ 0 & 0 & f_\tau \end{pmatrix}.$$

If $v_1 = v_2 = v_3$, then a residual Z_3 symmetry persists in the lepton Yukawa sector:

$$\mathcal{L}_{int} = v^{-1}[m_\tau \bar{L}_\tau \tau_R + m_\mu \bar{L}_\mu \mu_R + m_e \bar{L}_e e_R]\phi_0$$
$$+ v^{-1}[m_\tau \bar{L}_\mu \tau_R + m_\mu \bar{L}_e \mu_R + m_e \bar{L}_\tau e_R]\phi_1$$
$$+ v^{-1}[m_\tau \bar{L}_e \tau_R + m_\mu \bar{L}_\tau \mu_R + m_e \bar{L}_\mu e_R]\phi_2 + H.c.,$$

where $v = \langle \phi_0^0 \rangle$ and

$$\begin{pmatrix} \Phi_1 \\ \Phi_2 \\ \Phi_3 \end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix} \begin{pmatrix} \phi_0 \\ \phi_1 \\ \phi_2 \end{pmatrix}.$$

Thus $e, \mu, \tau \sim 1, \omega^2, \omega$ and $\phi_{0,1,2} \sim 1, \omega, \omega^2$ under Z_3, with $\phi_0 = \Phi_{SM}$.

This flavor triality symmetry allows $\tau^+ \to \mu^+\mu^+e^-$ and $\tau^+ \to e^+e^+\mu^-$ but no others (including $l_1 \to l_2\gamma$ and $\mu \to eee$.)

From $B(\tau^+ \to \mu^+\mu^+e^-) < 2.3 \times 10^{-8}$, we obtain

$$\frac{m_1 m_2}{\sqrt{m_1^2 + m_2^2}} > 22 \text{ GeV} \left(\frac{174 \text{ GeV}}{v}\right),$$

where $m_{1,2}$ are the masses of the mass eigenstates $\psi^0_{1,2} = (\phi^0_1 \pm \bar{\phi}^0_2)/\sqrt{2}$.

S_3 Model (2004)

In 2004, Chen/Frigerio/Ma proposed a flavor model of quarks and leptons based on the non-Abelian discrete symmetry S_3. This is the permutation group of 3 objects, which is also the symmetry group of the equilateral triangle. It has 6 elements divided into 3 equivalence classes, i.e. $C_1 : E = (ABC)$,
$C_2 : R_1 = (BCA), R_2 = (CAB)$, and
$C_3 : F_1 = (ACB), F_2 = (CBA), F_3 = (BAC)$.
It has 3 irreducible representations $\underline{1}$, $\underline{1}'$, and $\underline{2}$, and the multiplication rule $\underline{2} \times \underline{2} = \underline{1} + \underline{1}' + \underline{2}$.

Quark and lepton assignments under S_3:

$L_e = (\nu_e, e),\ Q_1 = (u, d),\ \Phi_3 = (\phi_3^0, \phi_3^-) \sim \underline{1}$,

$e^c, \mu^c, u^c, c^c, d^c, s^c \sim \underline{1}, \quad \tau^c, t^c, b^c \sim \underline{1}'$,

$(L_\mu, L_\tau), (Q_2, Q_3), (\Phi_1, \Phi_2) \sim \underline{2}$.

Yukawa invariants: $\underline{2} \times \underline{1} \times \underline{2}, \quad \underline{2} \times \underline{1}' \times \underline{2}, \quad \underline{1} \times \underline{1} \times \underline{1}$,

thus

$$\mathcal{M}_{u,d} = \begin{pmatrix} g_3^u v_3^* & g_4^u v_3^* & 0 \\ 0 & g_1^u v_1^* & -g_2^u v_1^* \\ 0 & g_1^u v_2^* & g_2^u v_2^* \end{pmatrix}, \begin{pmatrix} g_3^d v_3 & g_4^d v_3 & 0 \\ 0 & g_1^d v_2 & -g_2^d v_2 \\ 0 & g_1^d v_1 & g_2^d v_1 \end{pmatrix}.$$

Note that (v_1, v_2) in $\mathcal{M}_d$ is replaced by (v_2^*, v_1^*) in $\mathcal{M}_u$. This is important in getting a realistic V_{CKM}.

Let $v_3 = 0$ and $v_1 = v_2$ (i.e. $S_3 \to Z_2$), then $\mathcal{M}_{u,d}$ are both rotated by $\pi/4$, so their mismatch is zero, i.e. perfect alignment with $\theta_{23} = 0$. Hence this residual symmetry is a good explanation of why V_{CKM} is almost diagonal. Its breaking occurs when $v_3 \neq 0$ and $v_1 \neq v_2$, which may be assumed to be small naturally.

In the lepton sector, $\mathcal{M}_l$ is just like $\mathcal{M}_d$, but $\mathcal{M}_\nu$ may be chosen to be diagonal if it is Majorana. Hence $\nu_\mu - \nu_\tau$ mixing is predicted to be maximal, i.e. $\theta_{23} = \pi/4$, in agreement with experiment.

Update (2013)

The original 2004 model mainly dealt with the lepton sector and predicted very small θ_{13}, in disagreement with present data. However, it neglected $e - \mu$ mixing which is generally present, so the model is still viable.

Here the quark sector is studied instead.

[Ma/Melic(2013)] Consider first only the 2 heavy quark families with (Φ_1, Φ_2). Let

$$V_{12} = \mu_1^2(\Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2) - \mu_2^2(\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1) +$$

$$\frac{1}{2}\lambda_1(\Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2)^2 + \frac{1}{2}\lambda_2(\Phi_1^\dagger \Phi_1 - \Phi_2^\dagger \Phi_2)^2 + \lambda_3(\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1).$$

This is invariant under S_3 except for the soft μ_2^2 term which breaks S_3 to Z_2 $(\Phi_1 \leftrightarrow \Phi_2)$. The Z_2 symmetry enforces $\langle \phi_1^0 \rangle = \langle \phi_2^0 \rangle = v = 123$ GeV, resulting in the mass eigenstates:

$h^0 = \phi_{1R} + \phi_{2R}, \quad m^2 = 2(2\lambda_1 + \lambda_3)v^2,$
$H^0 = \phi_{1R} - \phi_{2R}, \quad m^2 = 2\mu_2^2 + 2(2\lambda_2 - \lambda_3)v^2,$
$A = \phi_{1I} - \phi_{2I}, \quad m^2 = 2\mu_2^2,$
$H^\pm = (\phi_1^\pm - \phi_2^\pm)/\sqrt{2}, \quad m^2 = 2\mu_2^2 - 2\lambda_3 v^2.$

At this level, h^0 is even under Z_2 and is naturally identified with the SM Higgs. The other scalars are odd under Z_2. Note that if $\mu_2^2 = 0$, then A would be massless.

The $c-t$ and $s-b$ mass matrices are both of the form

$$\mathcal{M} = \begin{pmatrix} f_1 v & -f_2 v \\ f_1 v & f_2 v \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} f_1\sqrt{2}v & 0 \\ 0 & f_2\sqrt{2}v \end{pmatrix}.$$

Consequently, the physical s, b quarks couple to h^0 according to $(m_s/2v)\bar{s}s + (m_b/2v)\bar{b}b$ as in the SM. The other scalar couplings are given by

$$\mathcal{L}_Y = \frac{m_s}{\sqrt{2}v}\left[H^+\bar{t}_L + \left(\frac{H^0 + iA}{\sqrt{2}}\right)\bar{b}_L\right] s_R$$

$$+ \frac{m_b}{\sqrt{2}v}\left[H^+\bar{c}_L + \left(\frac{H^0 + iA}{\sqrt{2}}\right)\bar{s}_L\right] b_R + H.c.,$$

which maintains the Z_2 symmetry with t, b odd and c, s even. This forbids $b \to s\gamma$ but allows $B_s - \bar{B}_s$ mixing. The coefficient of the $(\bar{s}_L b_R)^2$ operator is

$$\frac{m_b^2}{4v^2}\left(\frac{1}{m_H^2} - \frac{1}{m_A^2}\right).$$

The coefficient of the $(\bar{s}_L b_R)(\bar{s}_R b_L)$ operator is

$$\frac{m_s m_b}{4v^2}\left(\frac{1}{m_H^2} + \frac{1}{m_A^2}\right).$$

The hadronic matrix element of the former (latter) gives $-23.87 \times 10^{-6}\ \mathrm{GeV}^3$ and $1.20 \times 10^{-6}\ \mathrm{GeV}^3$.

The experimental value $\Delta m_{B_s} = 1.164 \pm 0.005 \times 10^{-11}$ GeV agrees with the Standard-Model prediction to within 10%, so we obtain

$$\left| -23.87 \left(\frac{1}{m_H^2} - \frac{1}{m_A^2} \right) + 1.20 \left(\frac{1}{m_H^2} + \frac{1}{m_A^2} \right) \right| < 1.16,$$

where $m_{H,A}$ are in units of TeV.

If $m_H = m_A$, then $m_{H,A} > 1.44$ TeV.

If $m_H = 1$ TeV, then $1.03 < m_A < 1.08$ TeV.

If $m_H = 0.7$ TeV, then $0.73 < m_A < 0.75$ TeV.

Add Φ_3 with $\langle \phi_3^0 \rangle = v_3 << v$, then $\mathcal{M}_{d,u}$ are diagonalized on the left by

$$V_d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c' & -s' \\ 0 & s' & c' \end{pmatrix} \begin{pmatrix} c_d & -s_d & 0 \\ s_d & c_d & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

where $s'/c' = v_2/v_1$, and

$$V_u = \begin{pmatrix} 1 & 0 & 0 \\ 0 & s' & -c' \\ 0 & c' & s' \end{pmatrix} \begin{pmatrix} c_u & -s_u e^{i\delta} & 0 \\ s_u e^{-i\delta} & c_u & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Hence $V_{CKM} = V_u^\dagger V_d =$

$$\begin{pmatrix} c_u c_d + c'' s_u s_d e^{i\delta} & -c_u s_d + c'' s_u c_d e^{i\delta} & s'' s_u e^{i\delta} \\ -s_u c_d e^{-i\delta} + c'' c_u s_d & s_u s_d e^{-i\delta} + c'' c_u c_d & s'' c_u \\ -s'' s_d & -s'' c_d & c'' \end{pmatrix},$$

where $s''/c'' = (c'^2 - s'^2)/2s'c'$. Using the 2012 PDG values, we obtain

$$s'' = 0.04135, \;\; s_u = 0.08489, \;\; s_d = 0.20983,$$

with $\cos\delta = -5.47 \times 10^{-3}$, and
$J_{CP} = s_u c_u s_d c_d (s'')^2 c'' \sin\delta = 2.96 \times 10^{-5}$.

This scheme does not predict any precise value of the measured parameters, but it does provide an understanding of why $(s'')^2, (s_u)^2, (s_d)^2$ are small.

To obtain $v_1 \neq v_2$, the Z_2 symmetry must be broken: add $\mu_3^2(\Phi_1^\dagger\Phi_1 - \Phi_2^\dagger\Phi_2)$. This changes h^0. However, in the limit of large $\mu_2^2 > 0$,

$$h^0 - h^0_{SM} \simeq \frac{(\lambda_1 - \lambda_2 + \lambda_3)(v_1^2 - v_2^2)}{2\mu_2^2}\, H^0,$$

Note that $v_1 = v_2$ implies $h^0 = h^0_{SM}$. Without the $(v_1^2 - v_2^2)/4v^2 = 0.0207$ suppression, $\mu_2 > 10$ TeV.

Adding Φ_3 means the addition of 5 quartic terms invariant under S_3, i.e.

$$(\lambda_4/2)(\Phi_3^\dagger\Phi_3)^2 + \lambda_5(\Phi_3^\dagger\Phi_3)(\Phi_1^\dagger\Phi_1 + \Phi_2^\dagger\Phi_2) + \lambda_6\Phi_3^\dagger(\Phi_1\Phi_1^\dagger + \Phi_2\Phi_2^\dagger)\Phi_3$$

$$+ [\lambda_7\Phi_3^\dagger\Phi_1\Phi_3^\dagger\Phi_2 + \lambda_8\Phi_3^\dagger(\Phi_1\Phi_2^\dagger\Phi_1 + \Phi_2\Phi_1^\dagger\Phi_2) + H.c.]$$

The λ_8 term may be eliminated by imposing an extra Z_2 symmetry under which Φ_3 and $(u,d)_L$ are odd, and all others even. This Z_2 symmetry is then allowed to be broken softly by the term $\mu_4^2\Phi_3^\dagger(\Phi_1 + \Phi_2)$.

As a result, for large $m_3^2 > 0$, $v_3 \simeq -\mu_4^2(v_1 + v_2)/m_3^2$.

Hence ϕ_{3R} mixes with $(v_1\phi_{1R} + v_2\phi_{2R})/\sqrt{v_1^2 + v_2^2}$ by $v_3/\sqrt{v_1^2 + v_2^2}$. This means that

$$h^0 - h^0_{SM} \simeq \frac{v_3 m_h^2}{2 v m_3^2}\ \phi_{3R}.$$

If the λ_8 term is present, then $h^0 - h^0_{SM} \simeq (2v_3/v)\ \phi_{3R}$ which means that h^0 exchange itself would contribute too much to $K^0 - \bar{K}^0$ mixing.

With the extra Z_2 symmetry, this problem is alleviated.

The exchange of h^0 could induce $K^0 - \bar{K}^0$ mixing, but its contribution is negligible compared to the direct exchange of ϕ_3^0 which has the effective interaction

$$\frac{s_d^2 c_d^2 m_d m_s}{v_3^2 m_3^2}(\bar{d}_L s_R)(\bar{d}_R s_L).$$

Allowing this to be 20% of the experimental measurement $\Delta m_K = 3.483 \pm 0.006 \times 10^{-15}$ GeV, $v_3 m_3 > 6 \times 10^4$ GeV^2 is obtained.

For example, if $v_3 = 10$ GeV, then $m_3 > 6$ TeV.

The scalar spectrum of this model has only one light Higgs boson h^0 which coincides with the SM Higgs to a very good approximation. As for the other two scalar doublets, they are much heavier. The linear combination $\Phi_1 - \Phi_2$ is constrained by $B_s - \bar{B}_s$ mixing to be heavier than about 0.7 TeV, whereas Φ_3 is constrained by $K^0 - \bar{K}^0$ mixing to be heavier than about 6 TeV if $v_3 = 10$ GeV. With these masses, all rare processes involving only quarks but not leptons such as $b \to s\gamma$ are negligible. However, the $s - b$ sector is connected to the

$\mu - \tau$ sector:

$$\mathcal{L}_Y = \frac{m_\mu}{\sqrt{2}v}\left[H^+\bar{\nu}_{\tau L} + \left(\frac{H^0 + iA}{\sqrt{2}}\right)\bar{\tau}_L\right]\mu_R$$

$$+ \frac{m_\tau}{\sqrt{2}v}\left[H^+\bar{\nu}_{\mu L} + \left(\frac{H^0 + iA}{\sqrt{2}}\right)\bar{\mu}_L\right]\tau_R + H.c.,$$

This means that the decay $b \to s\tau^-\mu^+$ $(B_s \to \tau^+\mu^-)$ proceeds through the exchange of $H^0 + iA$ with a possible branching fraction of 10^{-7}, but $b \to s\tau^+\mu^-$ $(B_s \to \tau^-\mu^+)$ will be suppressed by $(m_\mu/m_\tau)^2$. Given that $B(B_s \to \mu^+\mu^-) \simeq 3.2 \times 10^{-9}$ has been seen at LHCb, our unique prediction is verifiable in the future.

Conclusion

In conclusion, the 126 GeV particle discovered at the LHC could very well be the SM Higgs boson or very close to it. Yet it may have some natural relatives, such as dark scalar doublets, or flavor triality partners, or the extra doublets in an S_3 model, where $h^0 \simeq h^0_{SM}$ because of the residual symmetry Z_2 from S_3, and the extra Z_2 symmetry for Φ_3 and $(u,d)_L$. The unique prediction of this model is $b \to s\tau^-\mu^+$ ($B_s \to \tau^+\mu^-$) which may perhaps be testable at LHCb or Super KEKB.