Perturbative corrections to rare B decays

Mikołaj Misiak
(University of Warsaw)

1. **Introduction**

2. **NNLO QCD corrections to $B_s \to \mu^+\mu^-$**
   [T. Hermann, MM, M. Steinhauser, to be published]

3. **NLO EW corrections to $B_s \to \mu^+\mu^-$**
   [C. Bobeth, M. Gorbahn, E. Stamou, to be published]

   } Common update of the branching ratio in the SM (work in progress)

4. **NNLO QCD corrections to $\bar{B} \to X_s\gamma$**
   [M. Czakon, P. Fiedler, T. Huber, MM, T. Schutzmeier, M. Steinhauser, to be published]
   [H. M. Asatrian *et al*, to be published]

5. **Summary**

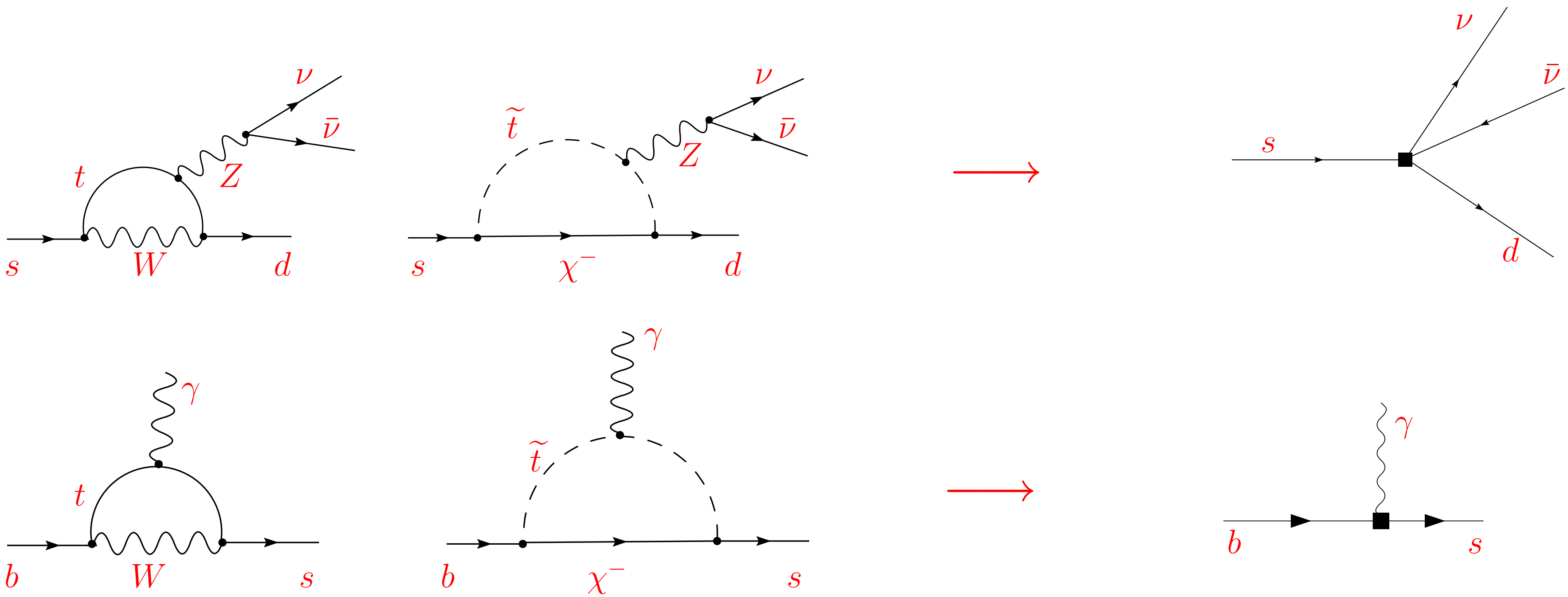
B-meson or Kaon decays occur at low energies, at scales $\mu \ll M_W$.

We pass from the full theory of electroweak interactions to an effective theory by removing the high-energy degrees of freedom, i.e. integrating out the W-boson and all the other particles with $m \sim M_W$.

$$\mathcal{L}_{\text{(full EW}\times\text{QCD)}} \longrightarrow \mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QED}\times\text{QCD}} \begin{pmatrix} \text{quarks} \neq t \\ \text{\& leptons} \end{pmatrix} + N \sum_n C_n(\mu)\, Q_n$$

Q_n – local interaction terms (operators), C_n – coupling constants (Wilson coefficients)

Information on the electroweak-scale physics is encoded in the values of $C_i(\mu)$, e.g.,



This is a modern version of the Fermi theory for weak interactions. It is "nonrenormalizable" in the traditional sense but actually renormalizable. It is also predictive because all the C_i are calculable, and only a finite number of them is necessary at each given order in the (external momenta)/M_W expansion.

Advantages: Resummation of $\left(\alpha_s \ln \frac{M_W^2}{\mu^2}\right)^n$ using renormalization group, easier account for symmetries.

Operators (dim 6) that matter for $B_s \to \mu^+\mu^-$ read

$$Q_A = \left(\bar{b}\gamma^\alpha\gamma_5 s\right)\left(\bar{\mu}\gamma_\alpha\gamma_5\mu\right)$$

$$Q_{S(P)} = \left(\bar{b}\gamma_5 s\right)\left(\bar{\mu}(\gamma_5)\mu\right) = \frac{i(\bar{b}\gamma^\alpha\gamma_5 s)\partial_\alpha(\bar{\mu}(\gamma_5)\mu)}{m_b+m_s} + \boxed{E} + \boxed{T}$$

E: vanishing by EOM

T: total derivative

Necessary non-perturbative input: $\langle 0|\bar{b}\gamma^\alpha\gamma_5 s|B_s(p)\rangle = i p^\alpha f_{B_s}$

Recent lattice determinations of the B_s-meson decay constant:

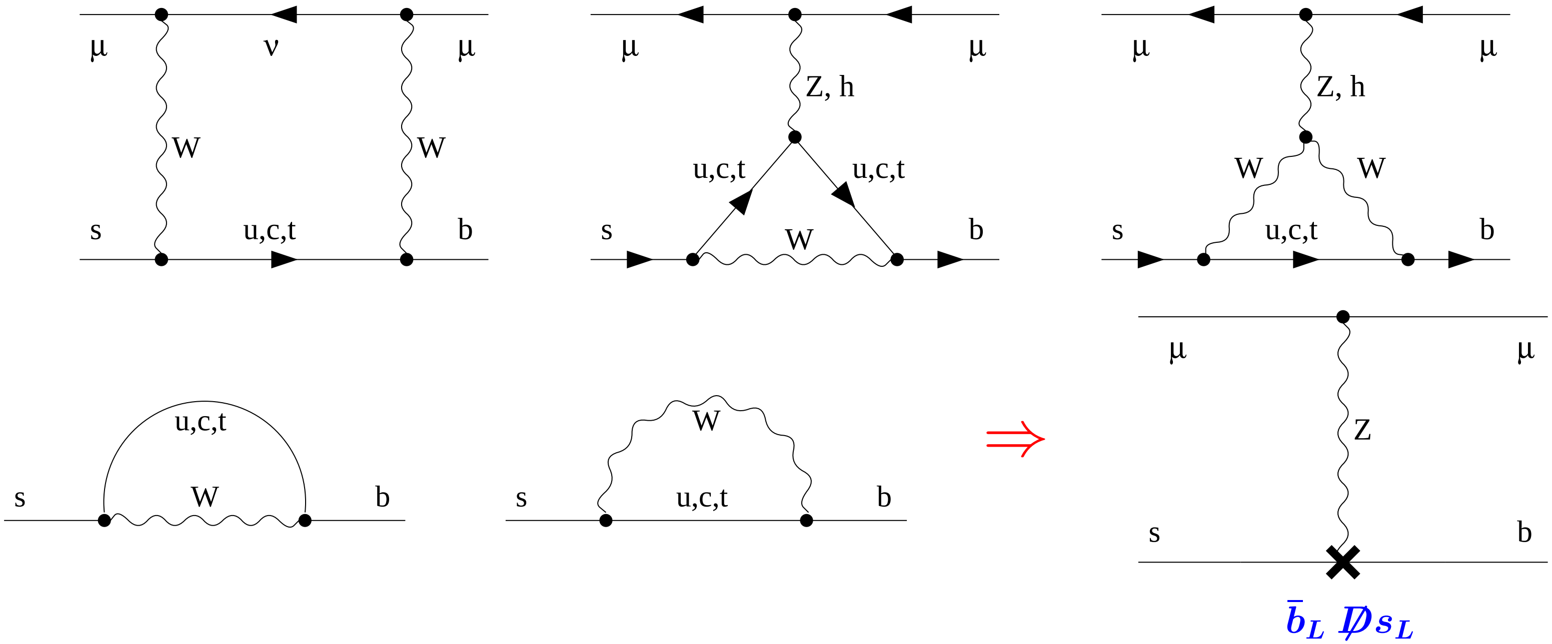
$$f_{B_s} = \begin{cases} 225.0(4.0)\text{ MeV}, & \text{HPQCD (r)}, & \text{arXiv:1110.4150} \\ 224.0(5.0)\text{ MeV}, & \text{HPQCD (nr)}, & \text{arXiv:1302.2644} \\ 234.0(6.0)\text{ MeV}, & \text{ROME}, & \text{arXiv:1212.0301} \\ 242.0(9.5)\text{ MeV}, & \text{FNAL/MILC}, & \text{arXiv:1112.3051} \\ 232(10)\text{ MeV}, & \text{ETM}, & \text{arXiv:1107.1141} \\ 219(12)\text{ MeV}, & \text{ALPHA}, & \text{arXiv:1210.6524} \end{cases}$$

Instantaneous branching ratio (with $B_s\bar{B}_s$ mixing ignored):

$$\mathcal{B}^{[t=0]}(B_s \to \mu^+\mu^-) = \frac{|N|^2}{8\pi} M_{B_s}^3 f_{B_s}^2 \tau_{B_s} s \left(|rC_A - uC_P|^2 + |usC_S|^2 \right) + \mathcal{O}(\alpha_{em}),$$

where $N = \frac{V_{tb}^* V_{ts} G_F^2 M_W^2}{\pi^2}$, $r = \frac{2m_\mu}{M_{B_s}}$, $s = \sqrt{1-r^2}$ and $u = \frac{M_{B_s}}{m_b+m_s}$.

Evaluation of the LO Wilson coefficients in the SM:



$$C_A^{(0)} = \frac{1}{2} Y_0\left(m_t^2/M_W^2\right), \qquad Y_0(x) = \frac{3x^2}{8(x-1)^2}\ln x + \frac{x^2-4x}{8(x-1)},$$

$$C_{S,P} = \mathcal{O}\left(\frac{m_\mu}{M_W}\right).$$

Effects of $C_{S,P}$ are on the branching ratio are suppressed by $M_{B_s}^2/M_W^2$ $\Rightarrow$ negligible.

Thus, only C_A matters in the SM.

Experimental world average for the CP-averaged and time-integrated branching ratio (LHCb arXiv:1307.5024 & CMS arXiv:1307.5025):

$$\overline{\mathcal{B}}(B_s \to \mu^+\mu^-)_{\rm exp} = (2.9 \pm \underbrace{0.7}_{24\%}) \times 10^{-9}$$

Prospects: $\pm 10\%$ **in 2018,** $\pm 7\%$ **in 2021, ...**

S. Hansmann-Menzemer, talk at the EPS 2013 conference, Stockholm, July 2013;
J. Serrano, talk given at the LPCC seminar at CERN, August 6th, 2013.

TH uncertainties before including the new QCD and EW corrections:

$$\overline{\mathcal{B}}(B_s \to \mu^+\mu^-)_{\rm SM} = \frac{G_F^4 M_W^4 m_\mu^2 M_{B_s}}{8\pi^5} \times$$

$\left(\kappa^{-1} = 1 - \tau_{B_s}\Delta\Gamma_s/2\right)$
K. de Bruyn *et al.*,
Phys. Rev. Lett. 109 (2012) 041801.

$$\times \underbrace{|V_{tb}^* V_{ts}|^2}_{\pm 3.5\%} \underbrace{\kappa \tau_{B_s}}_{\pm 1.1\%} \left\{ \underbrace{f_{B_s}^2}_{\pm(2.7 \div 7.9)\%} \left[\underbrace{Y_0\left(\frac{m_t^2}{M_W^2}\right) + \mathcal{O}(\alpha_s)}_{\pm 1.7\%[\text{ for } M_t = (173.2 \pm 1.0)\text{GeV}]} + \underbrace{\mathcal{O}(\alpha_{\rm em}) + \mathcal{O}(\alpha_s^2)}_{\pm 4\%\text{(to be removed)}} \right]^2 + \underbrace{\mathcal{O}(\alpha_{\rm em})}_{< 1\%} \right\}$$

$$= \begin{cases} (3.59 \pm \underbrace{0.24}_{6.6\%}) \times 10^{-9} & \text{for } f_{B_s} = 225.0(3.0)\text{ MeV [HPQCD+, arXiv:1302.2644]} \\ (4.16 \pm \underbrace{0.41}_{10\%}) \times 10^{-9} & \text{for } f_{B_s} = 242.0(9.5)\text{ MeV [FNAL/MILC, arXiv:1112.3051]} \end{cases}$$

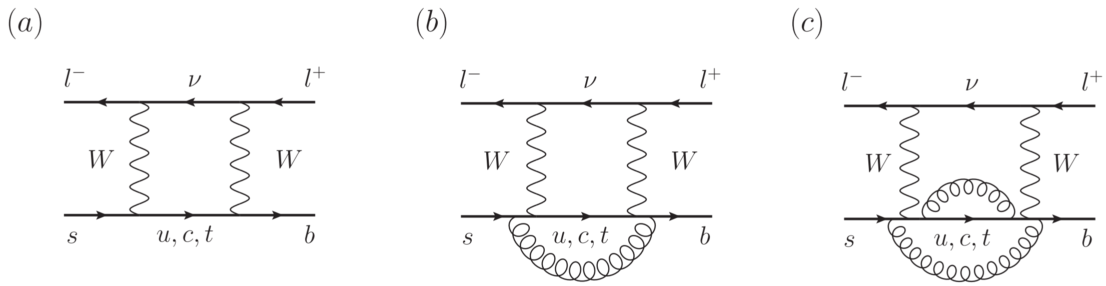
The $\mathcal{O}(\alpha_s)$ corrections enhance $\overline{\mathcal{B}}$ by around $+2.2\%$ when $\overline{m}_t(\overline{m}_t)$ is used at the leading order.
G. Buchalla, A.J. Buras, NPB 400 (1993) 225, NPB 548 (1999) 309; MM, J. Urban, PLBB 451 (1999) 161.

Logarithmically $\left(\ln\left(m_t^2/m_b^2\right)\right)$ enhanced electromagnetic corrections suppress $\overline{\mathcal{B}}$ by around -1.5%.
C. Bobeth, P. Gambino, M. Gorbahn, U. Haisch, JHEP 0404 (2004) 071; T. Huber, E. Lunghi, MM, D. Wyler, NPB 740 (2006) 105;
MM, arXiv:1112.5978.

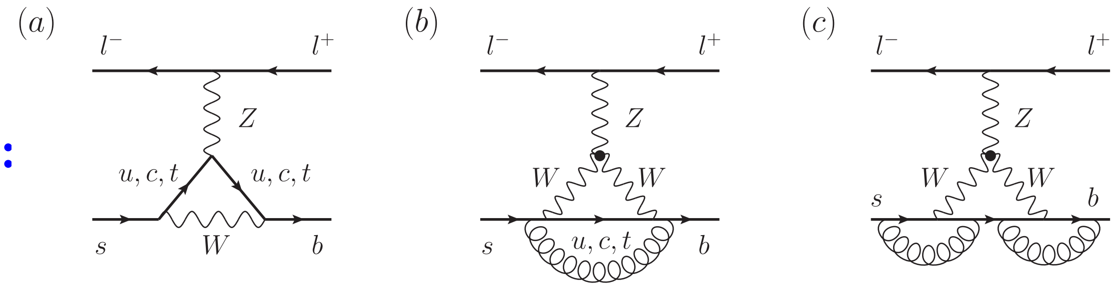
Evaluation of the NNLO QCD matching corrections

[T. Hermann, MM, M. Steinhauser, to be published]

W-boxes:
(1LPI)



Z-penguins:
(1LPI)

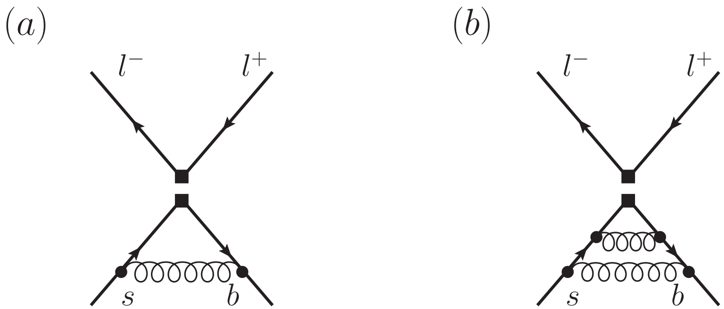


All the external momenta and light masses have been set to zero ⇒ No loop diagrams on the effective theory side.

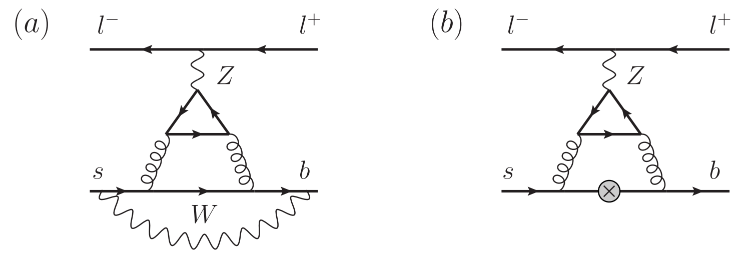
Subtleties: (i) counterterms with finite parts $\sim \bar{b}_L \not{D} s_L$

(ii) evanescent operators: $E_B = (\bar{b}\gamma_\nu\gamma_\rho\gamma_\sigma\gamma_5 s)(\bar{\mu}\gamma^\sigma\gamma^\rho\gamma^\nu\gamma_5\mu) - 4(\bar{b}\gamma_\alpha\gamma_5 s)(\bar{\mu}\gamma^\alpha\gamma_5\mu)$

$E_T = \mathrm{Tr}\,(\gamma^\nu\gamma^\rho\gamma^\sigma\gamma^\alpha\gamma_5)(\bar{b}\gamma_\nu\gamma_\rho\gamma_\sigma s)(\bar{\mu}\gamma_\alpha\gamma_5\mu) + 24(\bar{b}\gamma_\alpha\gamma_5 s)(\bar{\mu}\gamma^\alpha\gamma_5\mu)$



Renormalization of E_B

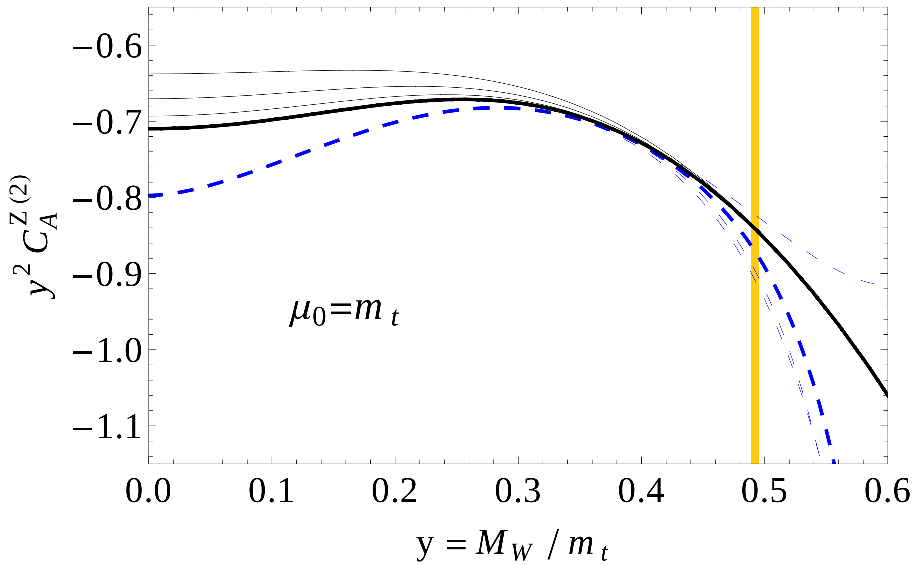


Diagrams generating E_T

Perturbative series for the Wilson coefficient at $\mu = \mu_0 \sim m_t, M_W$:

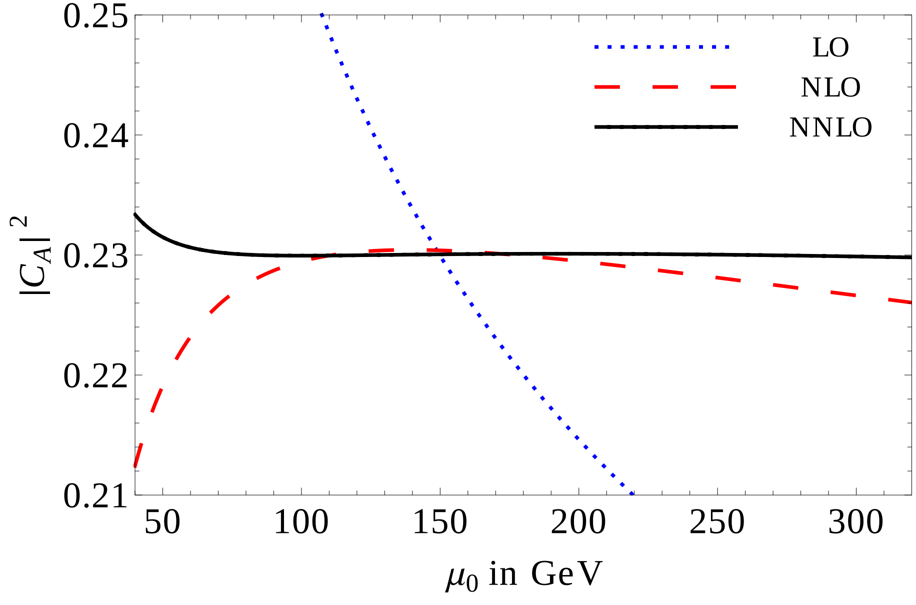
$$C_A(\mu_0) = C_A^{(0)}(\mu_0) + \frac{\alpha_s}{4\pi} C_A^{(1)}(\mu_0) + \left(\frac{\alpha_s}{4\pi}\right)^2 C_A^{(2)}(\mu_0) + \frac{\alpha_{em}}{4\pi} \Delta_{\rm EW} C_A(\mu_0) + \ldots$$

The top quark mass is $\overline{\rm MS}$-renormalized at μ_0 with respect to QCD, and on shell with respect to the EW interactions. Both α_s and α_{em} are $\overline{\rm MS}$-renormalized at μ_0 in the effective theory.



$$C_A^{(n)} = C_A^{W,(n)} + C_A^{Z,(n)}$$

To deal with single-scale tadpole integrals, we expand around $y = 1$ (solid lines) and around $y = 0$ (dashed lines), where $y = M_W/m_t$. The expansions reach $(1-y^2)^{16}$ and y^{12}, respectively. The blue band indicates the physical region.



Matching scale dependence of $|C_A|^2$ gets significantly reduced. The plot corresponds to $\Delta_{\rm EW} C_A(\mu_0) = 0$. However, with our conventions for m_t and the global normalization, μ_0-dependence is due to QCD only.

NNLO fit:

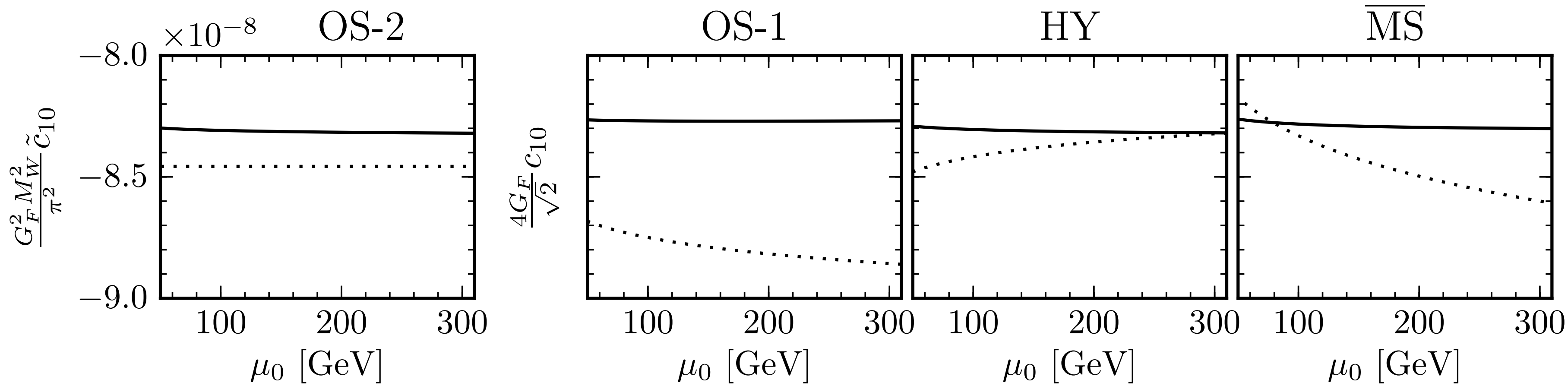
$$|C_A(m_t)|^2 = 0.2301 \left(\frac{M_t}{173.18}\right)^{3.09} \left(\frac{\alpha_s(M_Z)}{0.1184}\right)^{-0.17} + \mathcal{O}(\alpha_{em})$$

Evaluation of the NLO EW matching corrections

[C. Bobeth, M. Gorbahn, E. Stamou, to be published]

Method: similar to the NNLO QCD case. Two-loop integrals with three mass scales are present.

Dependence of the final result on μ_0 in various renormalization schemes (dotted – LO, solid – NLO):



In all the four plots: no QCD corrections to C_A included, $m_t(m_t)$ w.r.t. QCD used.

OS-2 scheme: **Global normalization factor in $\mathcal{L}_{\text{eff}}$ set to $N = V_{tb}^* V_{ts}\, G_F^2 M_W^2 / \pi^2$**

Masses at the LO renormalized on-shell w.r.t. EW interactions (including M_W in N)

Plotted quantity: $-2 C_A\, G_F^2 M_W^2 / \pi^2$ in GeV^{-2}

NLO EW matching correction to the BR: -3.7%

other schemes: **Global normalization factor in $\mathcal{L}_{\text{eff}}$ set to $4 V_{tb}^* V_{ts}\, G_F / \sqrt{2}$**

At the LO, $\alpha_{em}(\mu_0)$ used

$\overline{\text{MS}}$: Masses and $\sin^2\theta_W$ renormalized at μ_0

OS-1: Masses as in OS-2, $\sin^2\theta_W$ on-shell

HY (hybrid): Masses as in OS-2, $\sin^2\theta_W$ as in $\overline{\text{MS}}$.

NNLO QCD corrections to $\bar{B} \to X_s \gamma$

$b \in \bar{B} \equiv (\bar{B}^0 \text{ or } B^-)$

The inclusive decay rate is approximated by the corresponding partonic decay rate of the b quark.

$$\Gamma(\bar{B} \to X_s \gamma)_{E_\gamma > E_0} = \Gamma(b \to X_s^p \gamma)_{E_\gamma > E_0} + \left(\begin{array}{c} \text{non-perturbative effects} \\ \sim (2 \pm 5)\% \text{ for } E_0 = 1.6\,\text{GeV} \\ \text{Benzke et al., arXiv:1003.5012} \end{array} \right)$$

Previous SM estimate [hep-ph/0609232]:

$$\mathcal{B}(\bar{B} \to X_s \gamma)^{\text{SM}}_{E_\gamma > 1.6\text{ GeV}} = (3.15 \pm 0.23) \times 10^{-4}$$

Contributions to the total TH uncertainty (summed in quadrature):

5% non-perturbative, **3%** from the interpolation in m_c

3% higher order $\mathcal{O}(\alpha_s^3)$, **3%** parametric

Experimental world average (HFAG, 2.08.2012):

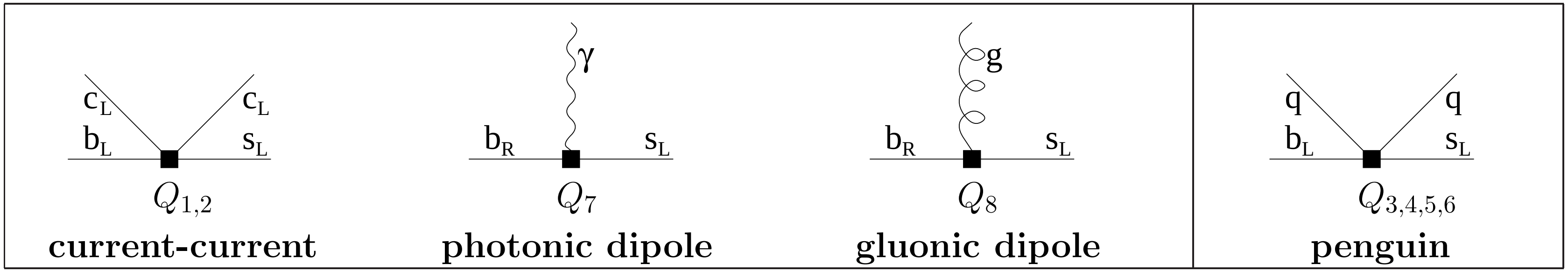
$$\mathcal{B}(\bar{B} \to X_s \gamma)^{\text{EXP}}_{E_\gamma > 1.6\text{ GeV}} = (3.43 \pm 0.21 \pm 0.07) \times 10^{-4}$$

Experiment agrees with the SM at better than $\sim 1\sigma$ level. Uncertainties: TH $\sim 7\%$, EXP $\sim 6.5\%$.

Decoupling of W, Z, t, H^0 $\Rightarrow$ effective weak interaction Lagrangian:

$$L_{\text{weak}} \sim \sum C_i(\mu)\, Q_i$$

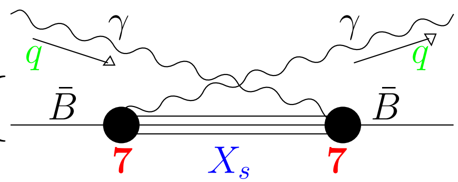
8 operators matter in the SM when the higher-order EW and/or CKM-suppressed effects are neglected:



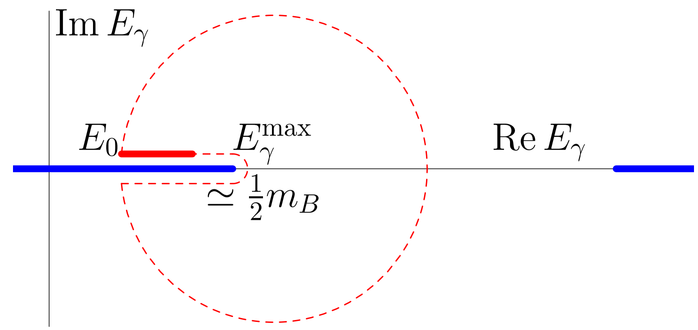
$$\Gamma(\bar{B} \to X_s \gamma)_{E_\gamma > E_0} = |C_7|^2\, \Gamma_{77}(E_0) + (\text{other})$$

Optical theorem:

$$\frac{d\Gamma_{77}}{dE_\gamma} \sim \text{Im}\{\ \ \}\equiv \text{Im}\, A$$



Integrating the amplitude A over E_γ:



OPE on the ring $\Rightarrow$ Non-perturbative corrections to $\Gamma_{77}(E_0)$ form a series in $\frac{\Lambda_{\text{QCD}}}{m_b}$ and α_s that begins with

$$\frac{\mu_\pi^2}{m_b^2},\ \frac{\mu_G^2}{m_b^2},\ \frac{\rho_D^3}{m_b^3},\ \frac{\rho_{LS}^3}{m_b^3}, \ldots;\ \frac{\alpha_s \mu_\pi^2}{(m_b - 2E_0)^2},\ \frac{\alpha_s \mu_G^2}{m_b(m_b - 2E_0)}; \ldots,$$

where $\mu_\pi, \mu_G, \rho_D, \rho_{LS} = \mathcal{O}(\Lambda_{\text{QCD}})$ are extracted from the semileptonic $\bar{B} \to X_c e \bar{\nu}$ spectra and the B–$B^\star$ mass difference.

The relevant perturbative quantity:

$$\frac{\Gamma[b \to X_s \gamma]_{E_\gamma > E_0}}{|V_{cb}/V_{ub}|^2 \, \Gamma[b \to X_u e \bar{\nu}]} = \left| \frac{V_{ts}^* V_{tb}}{V_{cb}} \right|^2 \frac{6 \alpha_{\rm em}}{\pi} \underbrace{\sum_{i,j} C_i \, C_j \, K_{ij}}_{P(E_0)}$$

Expansions of the Wilson coefficients and K_{ij}:

$$C_i(\mu_b) = C_i^{(0)}(\mu_b) + \frac{\alpha_s(\mu_b)}{4\pi} C_i^{(1)}(\mu_b) + \left(\frac{\alpha_s(\mu_b)}{4\pi}\right)^2 C_i^{(2)}(\mu_b) + \ldots$$

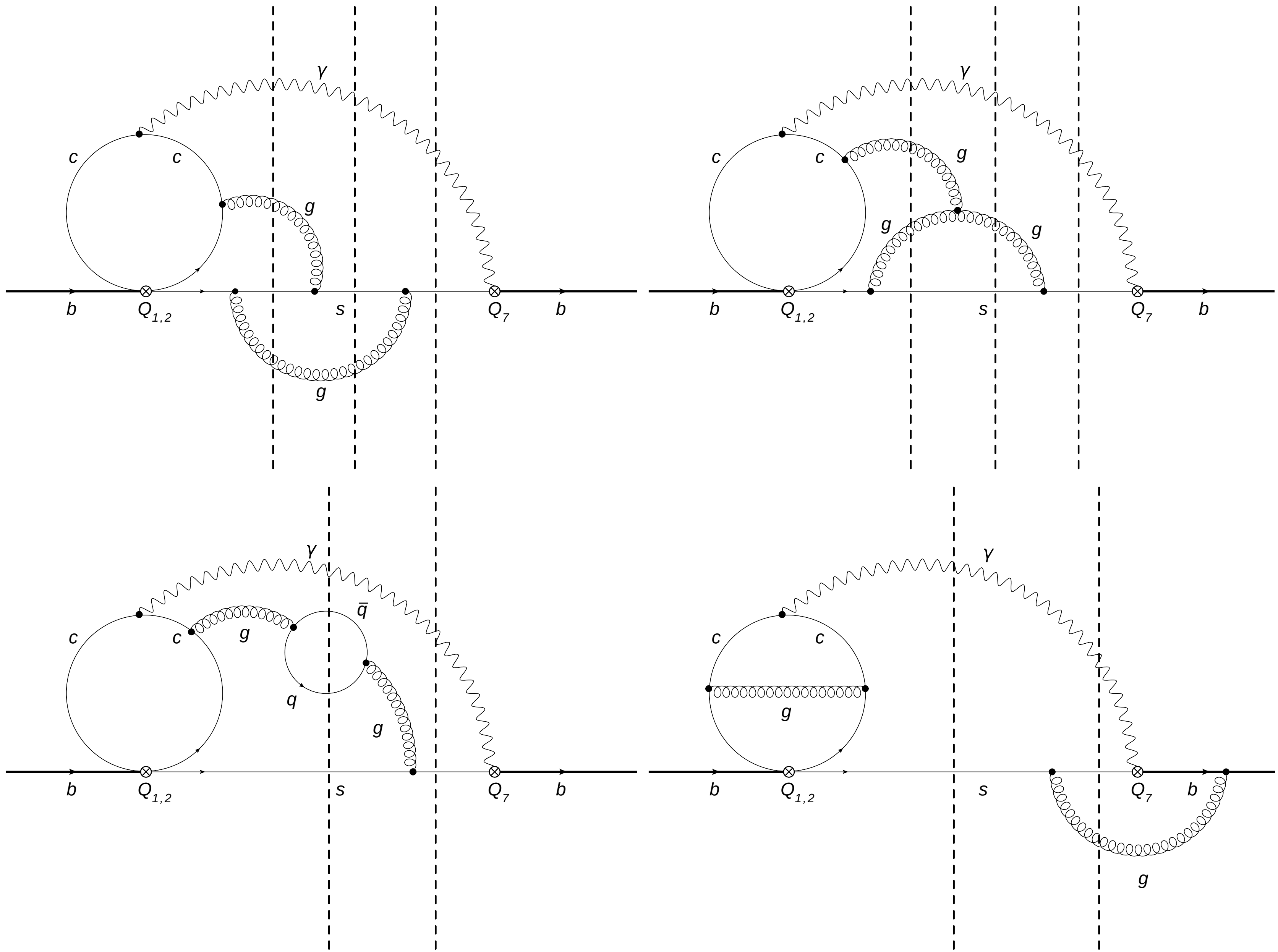
$$K_{ij} = K_{ij}^{(0)} + \frac{\alpha_s(\mu_b)}{4\pi} K_{ij}^{(1)} + \left(\frac{\alpha_s(\mu_b)}{4\pi}\right)^2 K_{ij}^{(2)} + \ldots \qquad\qquad \mu_b \sim \frac{m_b}{2}$$

Most important at the NNLO: $K_{77}^{(2)}$, $K_{27}^{(2)}$ **and** $K_{17}^{(2)}$.

They depend on $\frac{\mu_b}{m_b}$, $\frac{E_0}{m_b}$ **and** $r = \frac{m_c}{m_b}$.

Evaluation of $K_{27}^{(2)}$ and $K_{17}^{(2)}$ for $m_c = 0$:

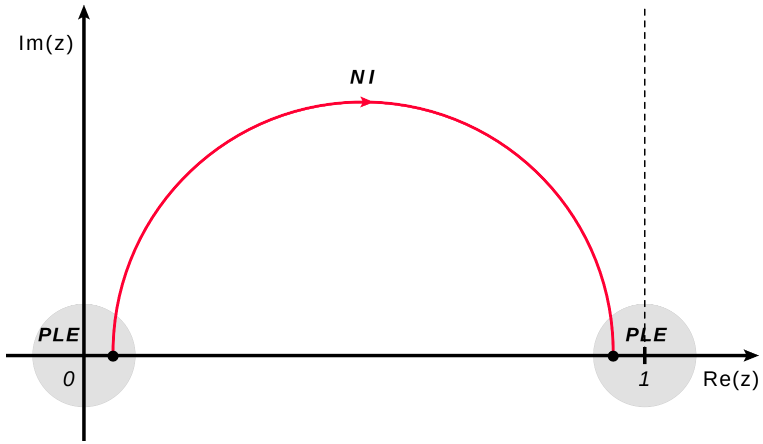
[M. Czakon, P. Fiedler, T. Huber, MM, T. Schutzmeier, M. Steinhauser, to be published]



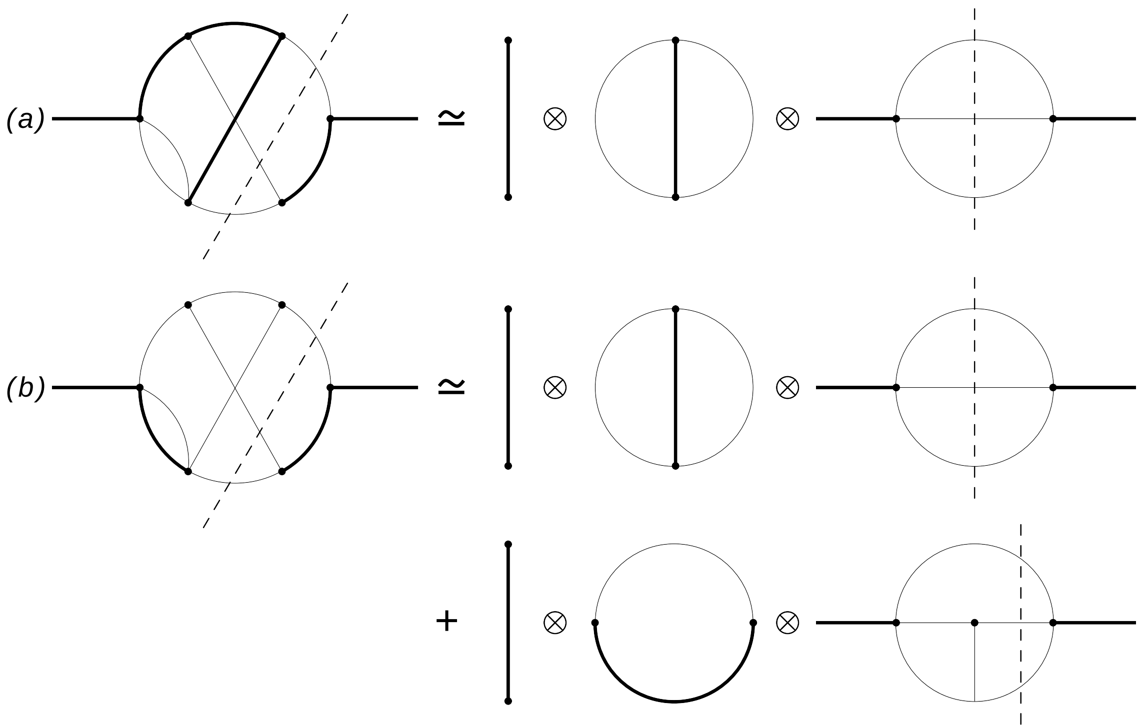
Master integrals and differential equations:

	n_D	n_{OS}	n_{eff}	$n_{massless}$
2-particle cuts	**292**	**92**	**143**	**9**
3-particle cuts	**267**	**54**	**110**	**11**
4-particle cuts	**292**	**17**	**37**	**7**

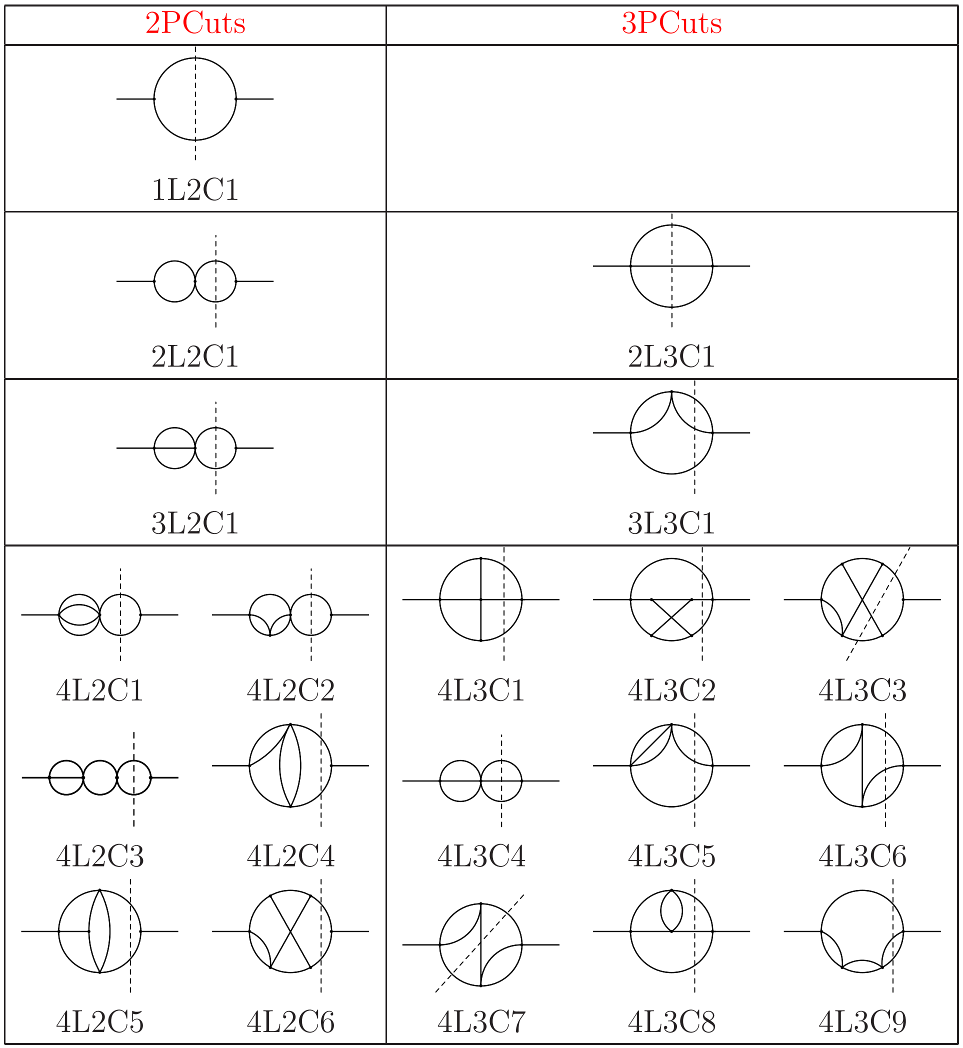
$$\frac{d}{dz}I_i(z) = \sum_j R_{ij}(z)I_j(z), \qquad z = \frac{p^2}{m_b^2}.$$

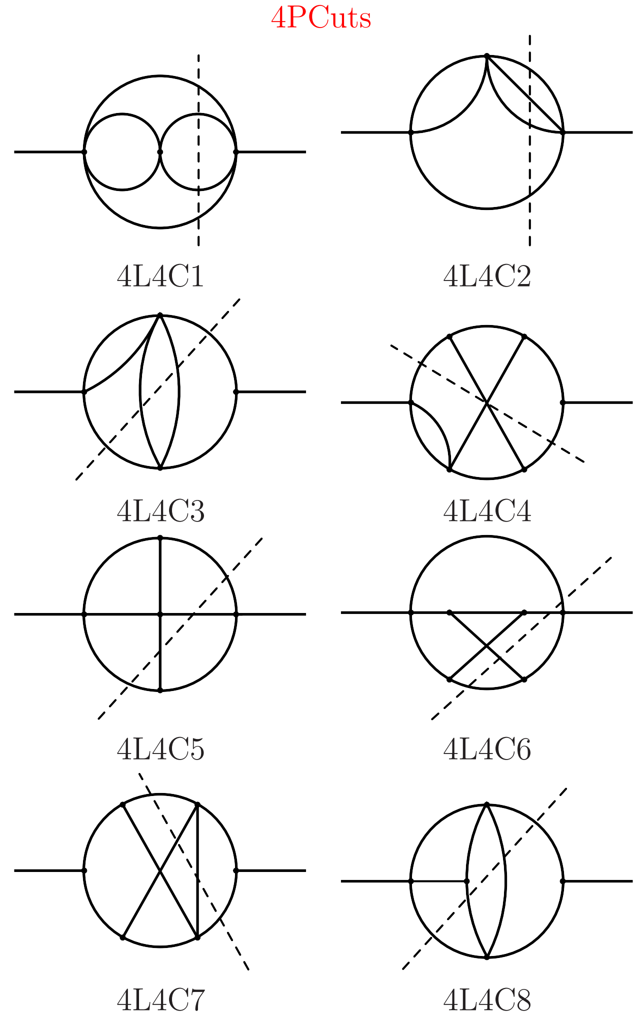


Boundary conditions in the vicinity of $z = 0$:



Massless integrals for the boundary conditions:





Results for the NNLO corrections:

$$K_{27}^{(2)}(r,E_0) = A_2 + F_2(r,E_0) + 3f_q(r,E_0) + f_b(r) + f_c(r) + \frac{8}{3}\phi_{27}^{(1)}(r,E_0)\ln r$$
$$+ \left[(4L_c - x_m)\, r\frac{d}{dr} + x_m E_0 \frac{d}{dE_0}\right] f_{NLO}(r,E_0) + \frac{416}{81}x_m$$
$$+ \left(\frac{10}{3}K_{27}^{(1)} - \frac{2}{3}K_{47}^{(1)} - \frac{208}{81}K_{77}^{(1)} - \frac{35}{27}K_{78}^{(1)} - \frac{254}{81}\right) L_b - \frac{5948}{729}L_b^2,$$

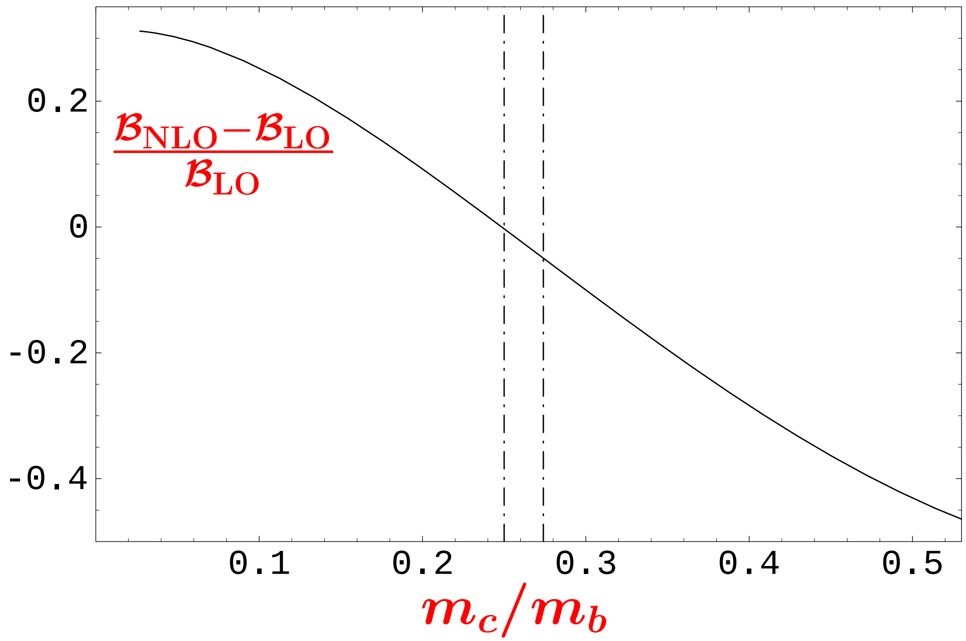
$$K_{17}^{(2)}(r,E_0) = -\frac{1}{6}K_{27}^{(2)}(r,E_0) + A_1 + F_1(r,E_0) + \left(\frac{94}{81} - \frac{3}{2}K_{27}^{(1)} - \frac{3}{4}K_{78}^{(1)}\right) L_b - \frac{34}{27}L_b^2,$$

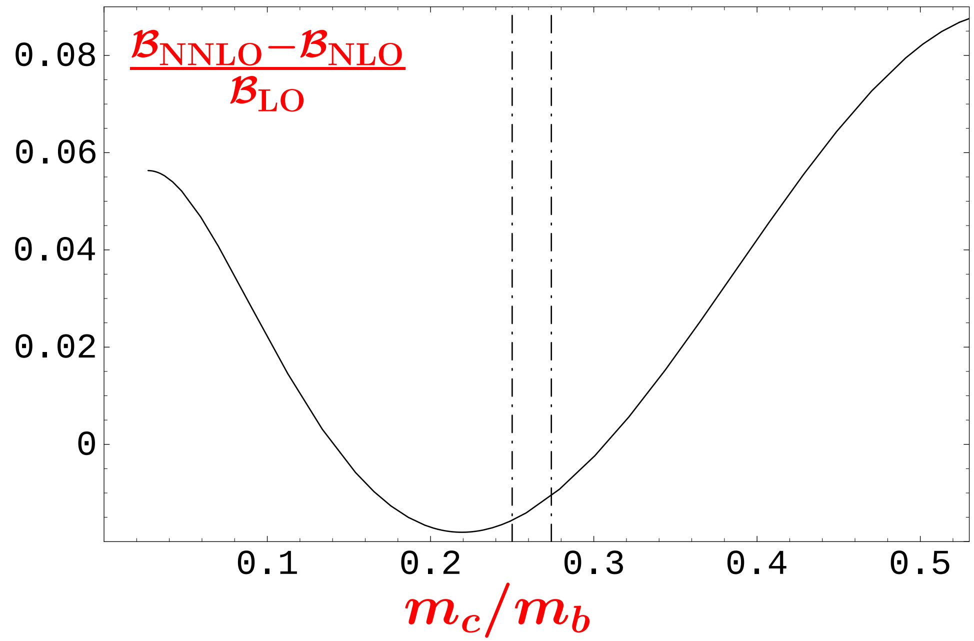
where $F_i(0,0) \equiv 0$, $A_1 \simeq 22.605$, $A_2 \simeq$ ~~-37.314~~ -81.179 (?) from the present calculation.

Correction due to $\mathcal{O}(\epsilon)$ term in one of the master integrals. Currently being cross-checked.

Next, we interpolate in m_c by assuming that $F_i(r,0)$ are linear combinations of $f_q(r,0)$, $f_{NLO}(r,0)$, $r\frac{d}{dr}f_{NLO}(r,0)$ and a constant term. The known large-r behaviour of F_i [hep-ph/0609241] and the condition $F_i(0,0) \equiv 0$ fix these linear combinations in a unique manner.

Relative NLO and NNLO QCD corrections to $\mathcal{B}(\bar{B} \to X_s\gamma)$ and their dependence on m_c/m_b



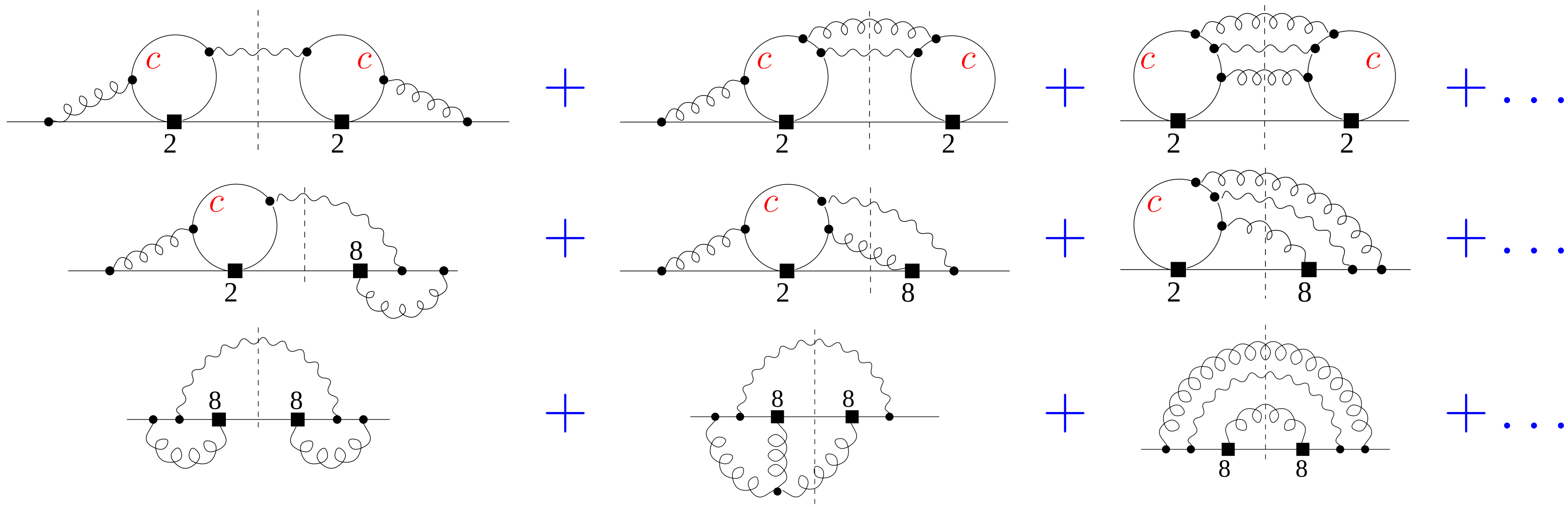


Interferences not involving the photonic dipole operator are treated as follows:

K_{22}**:** (and analogous K_{11} & K_{12})

K_{28}**:** (and analogous K_{18})

K_{88}**:**



Two-particle cuts are known (just $|\mathrm{NLO}|^2$).

Three- and four-particle cuts are known in the BLM approximation only. The NLO+(NNLO BLM) corrections are not big $(+3.8\%)$.

Incorporating other perturbative contributions evaluated after the previous phenomenological analysis in hep-ph/0609232:

1. Four-loop mixing (current-current) $\to$ (gluonic dipole)

   M. Czakon, U. Haisch, MM, JHEP 0703 (2007) 008 [hep-ph/0612329]

2. Diagrams with massive quark loops on the gluon lines

   R. Boughezal, M. Czakon and T. Schutzmeier, JHEP 0709 (2007) 072 [arXiv:0707.3090]
   H. M. Asatrian, T. Ewerth, H. Gabrielyan and C. Greub, Phys. Lett. B 647 (2007) 173 [hep-ph/0611123]
   T. Ewerth, Phys. Lett. B 669 (2008) 167 [arXiv:0805.3911]

3. Complete interference (photonic dipole)–(gluonic dipole)

   H. M. Asatrian, T. Ewerth, A. Ferroglia, C. Greub and G. Ossola, Phys. Rev. D 82 (2010) 074006 [arXiv:1005.5587]

4. New BLM corrections to contributions from 3-body and 4-body final states for interferences not involving the photonic dipole:

   A. Ferroglia and U. Haisch, Phys. Rev. D 82 (2010) 094012 [arXiv:1009.2144]
   MM and M. Poradziński, Phys. Rev. D 83 (2011) 014024 [arXiv:1009.5685]

5. LO contributions from $b \to s\gamma q\bar{q}$, $(q = u, d, s)$ from the four quark operators ("penguin" ones or CKM-suppressed ones).

   M. Kamiński, MM and M. Poradziński, Phys. Rev. D 86 (2012) 094004 [arXiv:1209.0965]

Taking into account new non-perturbative analyses:

M. Benzke, S. J. Lee, M. Neubert and G. Paz, JHEP 1008 (2010) 099 [arXiv:1003.5012]
T. Ewerth, P. Gambino and S. Nandi, Nucl. Phys. B 830 (2010) 278 [arXiv:0911.2175]

Updating the parameters (Parametric uncertertainties go down to 2.4%)

P. Gambino, C. Schwanda, arXiv:1307.4551

Summary

- Combining the recently calculated NNLO QCD and NLO EW corrections to $\mathcal{B}(\bar{B}_s \to \mu^+\mu^-)$ will allow for a significant reduction of the residual perturbative uncertainties.

- The dominant NNLO corrections to $\mathcal{B}(\bar{B} \to X_s\gamma)$ will soon be known not only in the large m_c limit, but also at $m_c = 0$. If the current result survives, no reduction of uncertainties with respect to the 2006 estimate is expected, except for the parametric one.