# SEMILEPTONIC B DECAYS, QUARK MASSES, AND CKM

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#### S.L. DECAYS DETERMINE $|V_{ub}|$ and $|V_{cb}|$





 $B \rightarrow \tau \nu$  is not yet competitive

Since several years, exclusive decays prefer smaller  $|V_{ub}|$  and  $|V_{cb}|$ 

### THE UNITARITY TRIANGLE

#### $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$



sin2 $\beta$  is measured directly in CP asymmetries in B $\rightarrow$ J/ $\Psi$ Ks

#### inclusives vs exclusives



Marcella Bona QMUI

### INCLUSIVE VS EXCLUSIVE B DECAYS



#### **INCLUSIVE DECAYS: BASICS**



- Simple idea: inclusive decays do not depend on final state, long distance dynamics of the B meson factorizes. An OPE allows to express it in terms of B meson matrix elements of local operators
- The Wilson coefficients are perturbative, matrix elements of local ops parameterize non-pert physics: *double series in α<sub>s</sub>, Λ/m<sub>b</sub>*
- Lowest order: decay of a free *b*, linear  $\Lambda/m_b$  absent. Depends on  $m_{b,c}$ , 2 parameters at O(1/m<sub>b</sub><sup>2</sup>), 2 more at O(1/m<sub>b</sub><sup>3</sup>)...

$$\mu_{\pi}^{2}(\mu) = \frac{1}{2M_{B}} \left\langle B \left| \overline{b} (i \overline{D})^{2} b \right| B \right\rangle_{\mu} \qquad \mu_{G}^{2}(\mu) = \frac{1}{2M_{B}} \left\langle B \left| \overline{b} \frac{i}{2} \sigma_{\mu\nu} G^{\mu\nu} b \right| B \right\rangle_{\mu} \right\rangle$$

### THE TOTAL WIDTH IN THE OPE

$$\Gamma[B \to X_c l\bar{\nu}] = \Gamma_0 \ g(r) \left[ 1 + \frac{\alpha_s}{\pi} c_1(r) + \frac{\alpha_s^2}{\pi^2} c_2(r) - \frac{\mu_\pi^2}{2m_b^2} + c_G(r) \frac{\mu_G^2}{m_b^2} + c_D(r) \frac{\rho_D^3}{m_b^3} + c_{LS}(r) \frac{\rho_{LS}^3}{m_b^3} + O\left(\alpha_s \frac{\mu_{\pi,G}^2}{m_b^2}\right) + O\left(\frac{1}{m_b^4}\right) \right]$$

$$\Gamma_0 = \frac{G_F^2 |V_{cb}|^2 m_b^5}{192\pi^3} \qquad r = \frac{m_c^2}{m_b^2}$$

OPE valid for inclusive enough measurements, away from perturbative singularities moments

Present implementations include all terms through  $O(\alpha_s^2, 1/m_b^3)$ :  $m_{b,c,} \mu^2_{\pi,G,} \rho^3_{D,LS}$  6 parameters

#### **EXTRACTION OF THE OPE PARAMETERS**



Global shape parameters (first moments of the distributions) tell us about B structure,  $m_b$  and  $m_c$ , total rate about  $|V_{cb}|$ 

OPE parameters describe universal properties of the B meson and of the quarks  $\rightarrow$  useful in many applications (rare decays,  $V_{ub},...$ )

## NEW SEMILEPTONIC FITS

Schwanda, PG, 1307.4551

- first fits to include all O(αs<sup>2</sup>) corrections and Czarnecki, Pak, Melnikov, Biswas 2008-10
- reassessment of theoretical errors, study of their correlations
- new external constraints: precise heavy quark mass determinations
- kinetic scheme calculation based on PG, 1107.3100; Uraltsev & PG,hep-ph/0401063

Previous fits: Buchmuller, Flaecher hep-ph/0507253, Bauer et al, hep-ph/0408002 (1S scheme)

#### **THEORETICAL ERRORS DOMINATE**



#### **THEORETICAL CORRELATIONS**



Correlations between theory errors of moments with different cuts difficult to estimate



- I. 100% correlations (unrealistic but used so far)
- 2. corr. computed from low-order expressions
- 3. constant factor 0<ξ<1 for 100MeV step
- 4. same as 3. but larger for larger cuts

always assume different central moments uncorrelated

#### **THEORETICAL CORRELATIONS**



#### CHARM MASS DETERMINATIONS



Remarkable improvement in recent years.  $m_c$  can be used as precise input to fix  $m_b$ 

### **RESULTS: BOTTOM MASS**



The fits give  $m_b^{kin}(1\text{GeV})=4.541(23)\text{GeV}$ , independent of th corr. scheme translation error  $m_b^{kin}(1\text{GeV})=m_b(m_b)+0.37(3)\text{GeV}$ 

### FIT RESULTS

th. corr. scenario	$m_b^{kin}$	$m_c^{(3G)}$	$eV)\mu_{\pi}^{2}$	$ ho_D^3$	$\mu_G^2$	$ ho_{LS}^3$	$\mathrm{BR}_{c\ell\nu}(\%)$	$10^3  V_{cb} $
4.	4.541	0.987	0.414	0.154	0.340	-0.147	10.65	42.42
uncertainty	0.023	0.013	0.078	0.045	0.066	0.098	0.16	0.86

Without mass constraints  $m_b^{kin}(1 \text{ GeV}) - 0.85 \overline{m}_c(3 \text{ GeV}) = 3.701 \pm 0.019 \text{ GeV}$ 

- results depend little on assumption for correlations and choice of inputs, 2% determination of V<sub>cb</sub>
- 20-30% determination of the OPE parameters



#### **HIGHER ORDER EFFECTS**

- Reliability of the method depends on our ability to control higher order effect and quark-hadron duality violations.
- Purely perturbative corrections complete  $O(\alpha_s^2)$ included, small residual error Melnikov, Czarnecki, Pak, PG
- **Power corrections**  $O(1/m_Q^{4,5})$  known but involve many new parameters, numerical relevance under study. In vacuum saturation approx small effect on  $V_{cb}$ Mannel, Turczyk, Uraltsev
- **Mixed** perturbative corrections to power suppressed coefficients at  $O(\alpha_s/m_b^2)$  almost finished, already known for  $b \rightarrow s\gamma$ Becher, Boos, Lunghi, Alberti, Ewerth, Nandi, PG

## $O(\alpha_s/m_b^2)$ EFFECTS

 $W_3^{(\pi,n)} = \frac{5}{3}\hat{q}_0 \,\frac{dW_3^{(n)}}{d\hat{q}_0}$ 

Boos,Becher,Lunghi 2007 Alberti,Ewerth,Nandi,PG 2012

They can be in part computed using reparameterization invariance which relates different orders in the HQET

$$W_{i} = W_{i}^{(0)} + \frac{\mu_{\pi}^{2}}{2m_{b}^{2}}W_{i}^{(\pi,0)} + \frac{\mu_{G}^{2}}{2m_{b}^{2}}W_{i}^{(G,0)} + \frac{C_{F}\alpha_{s}}{\pi} \left[W_{i}^{(1)} + \frac{\mu_{\pi}^{2}}{2m_{b}^{2}}W_{i}^{(\pi,1)} + \frac{\mu_{G}^{2}}{2m_{b}^{2}}W_{i}^{(G,1)}\right]$$

For i=3 RPI at all orders

$$- rac{\hat{q}^2 - \hat{q}_0^2}{3} rac{d^2 W_3^{(n)}}{d\hat{q}_0^2}$$

Manohar 2010

good testing ground for the calculation. Proliferation of power divergences, up to  $1/u^3$ , and complex kinematics  $(q^2, q_0, m_c, m_b)$ 

 $W_i^{(G,1)}$  are now ready! new results soon



### **EXCLUSIVE DECAY** $B \rightarrow D^* \ell \nu$

At zero recoil, where rate vanishes, the ff is

$$\mathcal{F}(1) = \eta_A \left[ 1 + O\left(\frac{1}{m_c^2}\right) + \dots \right]$$

Recent progress in measurement of slopes and shape parameters, exp error only ~2%

The ff F(1) cannot be experimentally determined. Lattice QCD is the best hope to compute it. Only one unquenched Lattice calculation:

Laiho et al 2010

$$|V_{cb}| = 39.05(0.7)(0.6)10^{-3}$$

2.1% error (adding in quadrature) ~2.7 $\sigma$  or ~8% from inclusive determination B→Dlv has larger errors: new  $|V_{cb}|=40.2(2.0)\times10^{-3}$ <u>at non-zero recoil!</u> Qiu et al, Lattice 2013

### ZERO RECOIL SUM RULE



Heavy quark sum rules put bounds on the zero recoil form factor F(1) for  $B \rightarrow D^*$  Shifman, Vainshtein, Uraltsev 1996

- $\mathcal{F}(1) = \sqrt{I_0(\varepsilon_M) I_{inel}(\varepsilon_M)} \qquad \qquad \mathcal{F}(1) \le \sqrt{I_0(\varepsilon_M)}$ Unitarity bound  $\mathcal{F}(1) < 0.935$
- Starting point OPE for axial vector current at zero recoil: expansion of *I*<sub>0</sub> in 1/*m*<sub>c</sub> and 1/*m*<sub>b</sub> and α<sub>s</sub>
- Recent calculation incorporates higher order effects and estimates inelastic contributions Mannel, Uraltsey, PG 2012
- Estimate of inelastic (non-resonant) contribution is hard

### THE INELASTIC CONTRIBUTION

$$I_1(\varepsilon_M) = -\frac{1}{2\pi i} \oint_{|\varepsilon| = \varepsilon_M} T(\varepsilon) \varepsilon d\varepsilon \qquad I_{inel}(\varepsilon_M) = \frac{I_1(\varepsilon_M)}{\bar{\varepsilon}}$$

 $\overline{\epsilon}$  represents the average excitation energy mainly controlled by the lowest radial (1/2<sup>+</sup>) and D-wave (3/2<sup>+</sup>) excitations, therefore about 700MeV

**OPE:** 
$$I_1 = \frac{-(\rho_{\pi G}^3 + \rho_A^3)}{3m_c^2} + \frac{-2\rho_{\pi \pi}^3 - \rho_{\pi G}^3}{3m_c m_b} + \frac{\rho_{\pi \pi}^3 + \rho_{\pi G}^3 + \rho_A^3 + \rho_A^3}{4} \left(\frac{1}{m_c^2} + \frac{2}{3m_c m_b} + \frac{1}{m_b^2}\right) + \mathcal{O}\left(\frac{1}{m_Q^3}\right)$$

in terms of little known non-local correlators of the form

$$\frac{i}{2M_B} \int d^4x \langle B|T\{O_i(x), O_j(x)\}|B\rangle' \qquad O \sim \bar{b} \,\pi_k \pi_l \, b$$

 $\rho_{\pi\pi}^{3} + \rho_{\pi G}^{3} + \rho_{S}^{3} + \rho_{A}^{3} \ge 0$ 

each of them is integral of spectral function with specific spin structure e.g.  $\rho_{\pi\pi}^3 = \int_{\omega>0} d\omega \frac{\rho_p^{(\frac{1}{2}^+)}}{\omega}$ 

#### ESTIMATING THE NON-LOCAL GUYS

Hyperfine splitting

$$\Delta M_Q^2 = M_{Q^*}^2 - M_Q^2 = \frac{4}{3} c_G(m_Q) \mu_G^2 + \frac{2}{3} \frac{\rho_{\pi G}^3 + \rho_A^3 - \rho_{LS}^3 + 2\bar{\Lambda}\mu_G^2}{m_Q} + O\left(\frac{1}{m_Q^2}\right)$$

**Experimentally**  $\Delta M_B^2 \simeq \Delta M_D^2$ 

within a ~25% uncertainty

 $ho_{\pi G}^3 + 
ho_A^3 \approx -0.45 {
m GeV}^3$ 

From  $\overline{M}_B - \overline{M}_D$  and moments fits  $\rho_{\pi G}^3 + \rho_A^3 \lesssim -0.33 \text{GeV}^3$ 

with somewhat larger uncertainty

These are strong indications that non-local guys are larger than expected. Based on a BPS expansion we get a minimum  $I_{inel}(\varepsilon_M \sim 0.75 \text{GeV}) \gtrsim 0.14 \pm 0.03$ 

using the <u>lowest</u> value of *I*<sub>inel</sub> and interpreting the total uncertainty as gaussian which leads to

$$\mathcal{F}(1) = 0.86 \pm 0.02$$

which leads to  $V_{cb}$ =40.9(1.1)10<sup>-3</sup> in good agreement with inclusive  $V_{cb}$ 

## V<sub>cb</sub> summary



## Vub DETERMINATIONS

#### **Inclusive: 5-6% total error**

HFAG 2012	Average $ V_{ub} x10^3$
DGE	$4.45(15)_{\rm ex}^{+15}$ -16
BLNP	$4.40(15)_{\rm ex}^{+19}_{-21}$
GGOU	$4.39(15)_{\rm ex}^{+12}$ -14

Exclusive: 10-15% total error

$$|V_{ub}| = (3.25 \pm 0.31) \times 10^{-3}$$
MILC
$$|V_{ub}| = \left(3.50^{+0.38}_{-0.33}\Big|_{th.} \pm 0.11\Big|_{exp.}\right) \times 10^{-3}$$

LCSR, Khodjamirian et al, see also Bharucha

#### $B \rightarrow \pi lv$ data <u>poorly consistent!</u>

2.7-3 $\sigma$  from B $\rightarrow\pi$ lv (MILC-FNAL) 2 $\sigma$  from B $\rightarrow\pi$ lv (LCSR, Siegen) 2.5-3 $\sigma$  from UTFit 2011

UT fit (without direct  $V_{ub}$ ):  $V_{ub}$ =3.64(13) 10<sup>-3</sup>

The discrepancy here is around 25% !!

## Vub IN THE GGOU APPROACH

PG,Giordano,Ossola,Uraltsev

![](_page_23_Figure_2.jpeg)

## **NEW PHYSICS?**

LR models can explain a difference between inclusive and exclusive V<sub>ub</sub> determinations (Chen,Nam)

Also in MSSM (Crivellin)

BUT the RH currents affect predominantly the exclusive  $V_{ub}$ , making the conflict between  $V_{ub}$ and  $\sin 2\beta$  ( $\psi K_s$ ) stronger...

![](_page_24_Figure_4.jpeg)

Buras, Gemmler, Isidori 1007.1993

### SUMMARY

- Theoretical efforts to improve the OPE approach to semileptonic decays continue, more results soon. No sign of inconsistency in this approach so far.
- New fit results: interesting m<sub>b</sub> determination based on precise m<sub>c</sub>
- HQSR calculation of zero recoil  $B \rightarrow D^*$  form factor agrees with inclusive determination of  $V_{cb}$ , unlike FNAL lattice one
- Exclusive/incl. tension in  $V_{ub}$  remains misterious (2-3 $\sigma$ ). It could be explained by right-handed current... Belle-II will increase significantly the statistics for  $b \rightarrow ul\nu$  decays. Measurement of spectra will enable direct constraints on shape function(s).

#### **BACK-UP SLIDES**

### **PERTURBATIVE EFFECTS**

- $O(\alpha_s)$  implemented by all groups De Fazio, Neubert
- Running coupling  $O(\alpha_s^2\beta_0)$  (PG,Gardi,Ridolfi) in GGOU, DGE lead to -5% & +2%, resp. in  $|V_{ub}|$
- Complete  $O(\alpha_s^2)$  in the SF region Asatrian, Greub, Pecjak-Bonciani, Ferroglia-Beneke, Huber, Li G. Bell 2008
- In BLNP leads to up 8% increase in  $V_{ub}$  related to resummation, not yet included by HFAG. It is an **artefact** of this approach.

• $P_+ <$	0.66	GeV:
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	$\Gamma_u^{(0)}$	$\mu_h$	$\mu_i$
NLO	60.37	$^{+3.52}_{-3.37}$	$^{+3.81}_{-6.67}$
NNLO	52.92	$^{+1.46}_{-1.72}$	$^{+0.09}_{-2.79}$

#### Greub, Neubert, Pecjak arXiv:0909.1609

•  $P_+ < 0.66$  GeV:

Fixed-Order	$\Gamma_u^{(0)}$	$\mu$
NLO	49.11	$+5.43 \\ -9.41$
NNLO	49.53	$^{+0.13}_{-4.01}$

**NEW**: full phase space O(α<sub>s</sub><sup>2</sup>) calculation Brucherseifer, Caola, Melnikov, arXiv:1302.0444

Confirms non-BLM/BLM approx 20% over relevant phase space

#### **EXCLUSIVE** $V_{ub}$ **FROM** $B \rightarrow \pi l v$

Here there is no preferred point in phase space. Lattice and light-cone sum rules estimate form factor.

Recent lattice based: MILC collaboration

#### Recent sum-rules based:

Khodjamirian, Mannel,Offen,Wang 2011 see also Bharucha

Precision is improved by fitting lattice/LCSR together with data

Experimental data are not well consistent

 $|V_{ub}| = (3.25 \pm 0.31) \times 10^{-3}$ 

$$|V_{ub}| = \left(3.50^{+0.38}_{-0.33}\Big|_{th.} \pm 0.11\Big|_{exp.}\right) \times 10^{-3}$$

![](_page_28_Figure_9.jpeg)

#### THE TOTAL $B \rightarrow X_{U} \ell \nu$ WIDTH

$$\begin{split} \Gamma[\bar{B} \to X_{u}e\bar{\nu}] &= \frac{G_{H}^{2}m_{b}^{5}}{192\pi^{3}}|V_{ub}|^{2} \left[1 + \frac{\alpha_{s}}{\pi}p_{u}^{(1)}(\mu) + \frac{\alpha_{s}^{2}}{\pi^{2}}p_{u}^{(2)}(r,\mu) - \frac{\mu_{\pi}^{2}}{2m_{b}^{2}} - \frac{3\mu_{G}^{2}}{2m_{b}^{2}} \\ &+ \left(\frac{77}{6} + 8\ln\frac{\mu_{WA}^{2}}{m_{b}^{2}}\right)\frac{\rho_{D}^{3}}{m_{b}^{3}} + \frac{3\rho_{LS}^{3}}{2m_{b}^{3}} + \frac{32\pi^{2}}{m_{b}^{3}}B_{WA}(\mu_{WA})\right] \\ &+ O(\alpha_{s}\frac{\mu_{\pi,G}^{2}}{m_{b}^{2}}) + O(\frac{1}{m_{b}^{4}})^{\bullet} \end{split}$$
Using the results of the fit, V<sub>ub</sub> build be extracted if we had the total width...

CC

Weak Annihilation, severely constrained from D decays, see Kamenik, PG, arXiv:1004.0114

#### ZERO RECOIL SUM RULE

 $T(\varepsilon) = \frac{i}{6M_B} \int d^4x e^{-ix_0(M_B - M_D^* - \varepsilon)} \langle B|TJ_A^k(x)J_{Ak}(0)|B\rangle$ 

$$\varepsilon = M_X - M_{D^*}$$

$$I_0(\varepsilon_M) = -\frac{1}{2\pi i} \oint_{|\varepsilon| = \varepsilon_M} T(\varepsilon) d\varepsilon = \mathcal{F}^2(1) + I_{inel}(\varepsilon_M)$$

$$Inelastic non-resonant piece I_{inel}(\varepsilon_M) = \frac{1}{2\pi i} \int_{0+}^{\varepsilon_M} \operatorname{disc} T(\varepsilon) d\varepsilon$$

$$\varepsilon$$

$$\mathcal{F}(1) = \sqrt{I_0(\varepsilon_M) - I_{inel}(\varepsilon_M)}$$

$$\mathcal{F}(1) \leq \sqrt{I_0(\varepsilon_M)}$$

Unitarity bound