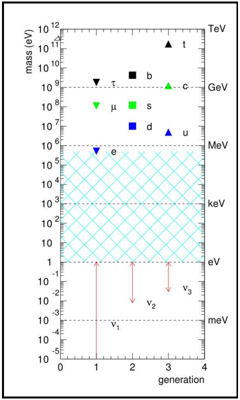
Attempts to explain neutrino masses and mixings using finite horizontal symmetry groups

B. Dziewit, S. Zajac, M. Zralek

University of Silesia

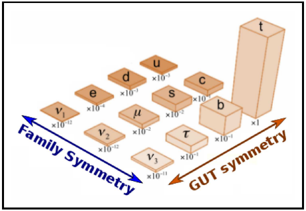
Matter to the deepest
06.09.2013 Ustron

Introduction



Hierarchy of mass

Hierarchy of mass is an open problem in elementary particle physics



Symmetry

One can try to relate this problems with symmetry.

Introduction

Up to now we don't know any fundamental rule(s) giving relations between Yukawa couplings and Higgs expectation values.

... short wish list ...

From such a symmetry we expect that it will give:

- **relations** between Yukawa couplings
- in the case of New Physics: **relations** between vacuum Higgs expectation values . . .
- ... and as a consequence **relations** among lepton masses, quark masses, mixing angles, CP phases ...

How to find horizontal symmetry?

Indications for strong symmetry breaking

- big differences between quark masses in the same generation
- ... and between different generations
- different charged lepton masses

In quark sector mixing is small (perturbation?)

Why neutrino mixing?

Neutrino mixing is relatively big so it may be probably the best place to search non-perturbation, non-breaking, fundamental symmetry.

How to find horizontal symmetry?

There are many ways one can introduce family symmetry into a model.

Continuous	Discrete
U(1), SU(3)	S_3, A_4, T', S_4, A_5, $\Delta(24)$

In general, we can classify two types of methods:

- „**bottom-up**":
  experimental data $\rightarrow U_{PMNS} \rightarrow u_i \rightarrow G_i \rightarrow G_F$
- „**top-down**":
  $G_F \rightarrow G_i \rightarrow u_i \rightarrow U_{PMNS}$

For νSM

Let there exist flavour symmetry G_F, we have :

- For each $\Psi = \{L_L, \nu_R\}$ there exist 3 dimensional representation of G_F: $(\mathbf{A}^{\Psi})$
- For each Higgs multiplet Φ there exist $(N_d \times N_d)$ dimensional representation of G_F: $(\mathbf{A}^{\Phi})$

Applying such a symmetry to Yukawa term:

$$L_Y = - \sum_{\alpha,\beta=e,\mu,\tau}^{3} h^{\nu}_{\alpha\beta} \left[\overline{L}_{\alpha L} \Phi \nu_{\beta R}\right] \Rightarrow - \sum_{i=1}^{N_d} \overline{L}_{\chi L} (\tilde{h}^{\nu}_{k})_{\chi,\delta} \Phi_k \nu_{\delta R} = L'_Y$$

$$\boxed{L'_{\alpha L} = (A^L)_{\alpha,\chi} L_{\chi L} \quad \nu'_{\beta R} = (A^{\nu})_{\beta,\delta} \nu_{\delta R} \quad \phi'_i = (A^{\phi})_{i,k} \phi_k}$$

For νSM

For symmetry we have: $L_Y = L'_Y \Rightarrow \tilde{h}^\nu_k = h^\nu_k$ so:

$$\sum_{i=1}^{N_d} \left(A^{L\dagger} h_i^\nu \left(A^\phi\right)_{i,k} A^\nu \right)_{\chi,\delta} = \left(h_k^\nu\right)_{\chi,\delta}$$

$$M^\nu_{\alpha,\beta} = \frac{1}{\sqrt{2}} \sum_{i=1}^{N_d} v_i \left(h_i^\nu\right)_{\alpha,\beta} \quad \text{for one Higgs boson:} \;\; M^\nu = \frac{1}{\sqrt{2}} v h^\nu$$

$$M^{\nu\prime} = A^{L\dagger} \left(\frac{1}{\sqrt{2}} \sum_{i,k=1}^{N_d} v_i h_k^\nu \left(A^\phi\right)_{k,i} \right) A^\nu = M^\nu$$

For one Higgs boson

Neutrino mass matrix

$$M^{\nu\prime} = A^{L\dagger} M^{\nu} A^{\nu} = M^{\nu} \tag{1}$$

For only one 3-dim representation of G_F:

$$A^L = A^{\nu} = A \Leftrightarrow A^{\dagger} M^{\nu} A = M^{\nu} \Leftrightarrow [M^{\nu}, A] = 0$$

$$A = G_1^a G_2^b G_3^c$$

where G_i are generators of G_F group and we have:

$$\boxed{[\mathbf{M}^{\nu}, \mathbf{G_i}] = \mathbf{0}}$$

For one Higgs boson

In the base:

$$M^l = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} \Rightarrow U^l = I \Rightarrow U_{PMNS} = U^{l\dagger} U^\nu = U^\nu$$

$$U_{PMNS} = (u_1, u_2, u_3)$$

G_F generators can be expressed by:

$$G_1 = u_1 u_1^\dagger - u_2 u_2^\dagger - u_3 u_3^\dagger$$

$$G_2 = -u_1 u_1^\dagger + u_2 u_2^\dagger - u_3 u_3^\dagger$$

$$G_3 = -u_1 u_1^\dagger + u_2 u_2^\dagger + u_3 u_3^\dagger$$

Tribimaximal

In the days when the reactor angle of neutrino mixing was thought to be zero and the atmospheric angle maximal, mixings could be taken to be tribimaximal, and explained by A_4

$$U_{TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Old data:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} 0.816 & 0.577 & 0.0 \\ -0.408 & 0.577 & 0.707 \\ 0.408 & -0.577 & 0.707 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

CORRESPONDS TO:

$$\theta_{13} = 0, \theta_{23} = (\pi/4), \theta_{12} = arcsin(1/\sqrt{3})$$

A_4 group generators:

$$G_1 = \begin{pmatrix} 1 & -2 & 2 \\ -2 & -2 & 1 \\ 2 & -1 & -2 \end{pmatrix}$$

$$G_2 = \begin{pmatrix} -1 & 2 & -2 \\ 2 & -1 & -2 \\ -2 & -2 & -1 \end{pmatrix}$$

$$G_3 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Current status

Recent global fits and direct measurements show the reactor angle to be non-negligible and the atmospheric angle possibly non-maximal

Recent data: $$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} 0.819 & 0.551 & 0.158 \\ -0.512 & 0.581 & 0.632 \\ 0.256 & -0.599 & 0.758 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Many suggestions have been advanced to explain the new data.

There is no longer a group that can produce all the mixing data

...

Open questions

- ....
- neutrino mass scheme — normal or inverted hierarchy
- Dirac or Majorana nature $\Rightarrow U_{PMNS}$ parametrization
- precise experimental data, mixing angles, masses
- sterile neutrino (?), (3+1, or 3+2 scheme)
- ...

Instead of summary: prospects

- Current tests for new experimental data . . .
- Consider models with more Higgs bosons (doublet, double doublet, triplets, . . .)
- Different symmetry in lepton and neutrino sector . . .
- Models with sterile neutrinos . . .