Modeling Branes in Warped Extra-Dimension

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Outline*

- Brief review of Randall-Sundrum model (RS2) with single brane in infinite extra-dimension.
- Generalized RS2: a single brane model with different bulk cosmological constants on each side of the brane.
- Smooth/thick brane generalization of singular brane (RS2-like) models.
- Two thick brane model:
  - Solution to hierarchy problem.
  - 4D effective gravity.
- Conclusions.

* Based on JHEP **01**, 177 (2013) and work in progress.

RS model with one brane (RS2)

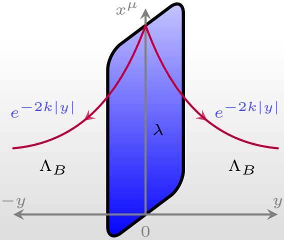
Randall and Sundrum proposed a model with one D3-brane embedded in an infinite extra-dimension. [hep-th/9906064]

- RS metric is

$$ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2, \qquad \text{where} \quad k \approx M.$$

- RS2 action is

$$\mathcal{S}_{RS2} = \int dx^5 \sqrt{-g}\Big\{2M^3 R - \Lambda_B - \lambda\delta(y)\Big\}$$



- RS metric is solution if $\Lambda_B = -24M^3k^2$ and $\lambda = 24M^3k$. $\left[\lambda = \sqrt{-24M^3\Lambda_B}\right]$

- 4D effective gravity is recovered as $M_{Pl}^2 = M^3/k$ is finite.

Generalized RS2

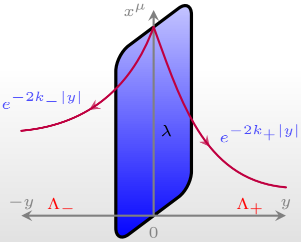
We assume a single D3-brane embedded in an infinite extra-dimension with different cosmological constants on each side.

- The action for generalized RS2 is

$$\mathcal{S} = \int dx^5 \sqrt{-g} \Big\{ 2M^3 R - \Lambda_+ \theta(y) - \Lambda_- \theta(-y) - \lambda \delta(y) \Big\}$$

- Metric ansatz

$$ds^2 = e^{2A(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2$$

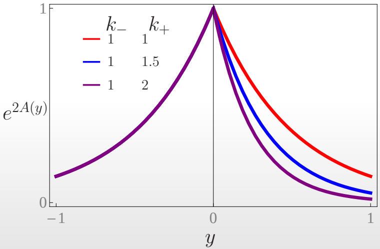


$\Lambda_+ \neq \Lambda_-$

- Exponentially falling behaviour of warped factor on both sides of brane requires $\Lambda_\pm = -24M^3k_\pm^2$ and $\lambda = 12M^3(k_+ + k_-)$. $\left[\lambda = \sqrt{-12M^3(\Lambda_+ + \Lambda_-)}\right]$

- The warp function $A(y)$ is

$$A(y) = \begin{cases} -|y|k_+ & y > 0 \\ -|y|k_- & y < 0 \end{cases}$$



4D Effective gravity

$$M_{Pl}^2 = \frac{M^3}{2k_-} + \frac{M^3}{2k_+}$$

– 4D Plank mass is finite,
– 4D gravity can be reproduced.

Features of RS2

Attractive feature of RS2

- RS2 (generalized RS2) provides an alternative to compactification.

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"Un"-attractive feature of RS2

- Infinitesimally thin brane considered in RS model has *no dynamical mechanism* of generating it in the model.

Goal

Smoothing the generalized RS2 model

We introduce a scalar field $\phi(y)$ with 5D gravity such that it mimics in *brane limit*:

- single brane with positive tension and
- different cosmological constants on each side of the brane.

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- different cosmological constants on each side of the brane.

Two thick branes model

A scalar field $\phi(y)$, in the presence of 5D gravity, mimics in *brane limit*:

- two branes with positive tension and
- which can address the hierarchy problem.

Smoothing the generalized RS2

5D scalar-gravity action

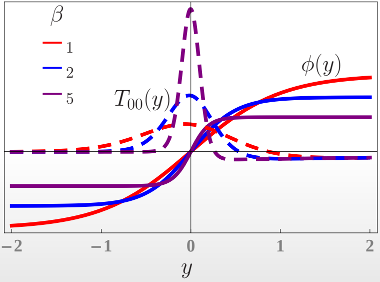
$$\mathcal{S} = \int dx^5 \sqrt{-g} \left\{ 2M^3 R - \frac{1}{2} g^{MN} \nabla_M \phi \nabla_N \phi - V(\phi) \right\}$$

- Metric ansatz

$$ds^2 = e^{2A(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2$$

- Scalar field profile

$$\phi(y) = \frac{\kappa}{\sqrt{\beta}} \tanh(\beta y)$$



Background solutions

- Equations of motion for the above action and metric ansatz are

$$24M^3(A')^2 = \frac{1}{2}(\phi')^2 - V(\phi),$$
$$12M^3A'' + 24M^3(A')^2 = -\frac{1}{2}(\phi')^2 - V(\phi),$$
$$\phi'' + 4A'\phi' - \frac{dV}{d\phi} = 0.$$

- For the given ϕ, the scalar potential $V(\phi)$ can be written as,

$$V(\phi) = \frac{1}{2}\left(\frac{\partial W(\phi)}{\partial\phi}\right)^2 - \frac{1}{6M^3}W(\phi)^2.$$

- Superpotential $W(\phi)$ satisfies

$$\phi' = \frac{\partial W(\phi)}{\partial\phi} \qquad \& \qquad A' = -\frac{1}{12M^3}W(\phi).$$

- The superpotential $W(\phi)$ for $\phi(y) = \frac{\kappa}{\sqrt{\beta}} \tanh(\beta y)$ is

$$W(y) = \kappa^2 \left\{ \tanh(\beta y) - \frac{1}{3} \tanh^3(\beta y) \right\} + W_0$$

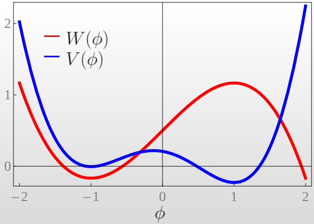
- The warped function $A(y)$

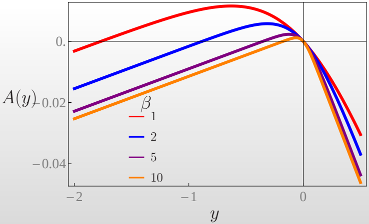
$$A(y) = \frac{-\kappa^2}{72 M^3 \beta} \left(\tanh^2(\beta y) + \ln \cosh^4(\beta y) \right) - \frac{W_0}{12 M^3} y$$

- Brane limit

$$A(y) \approx -k_\pm |y|, \qquad \beta \to \infty.$$

where $k_\pm = \frac{1}{24M^3}\lambda \pm \frac{W_0}{12M^3}$, with $\lambda \equiv \frac{4}{3}\kappa^2$.





Stability of background solutions

- Linearized tensor perturbations are,

$$g_{\mu\nu}(x,y) = e^{2A(y)}\eta_{\mu\nu}(x) + H_{\mu\nu}(x,y).$$

- $H_{\mu\nu}(x,y)$ can be written as $H_{\mu\nu}(x,z) = e^{ip\cdot x}\bar{H}_{\mu\nu}(z)$, where $-p^2 = m^2$ and "z" is defined as $dz = e^{-A(y)}dy$.

- Equation of motion for $H_{\mu\nu}(z)$,

$$\Big(-\partial_z^2 + U(z)\Big)\bar{H}_{\mu\nu}(z) = m^2\bar{H}_{\mu\nu}(z),$$

$$\mathcal{Q}^\dagger\mathcal{Q}\bar{H}_{\mu\nu}(z) = m^2\bar{H}_{\mu\nu}(z),$$

where $U(z) = \frac{9}{4}\dot{A}^2(z) + \frac{3}{2}\ddot{A}(z)$ and $\mathcal{Q} = \Big(\partial_z - \frac{3}{2}\dot{A}\Big)$.

- Quantum Mechanics form for the tensor modes forbids the existence of any tachyonic modes with negative mass^2, $m^2 < 0$.

- Hence our background solution is stable.

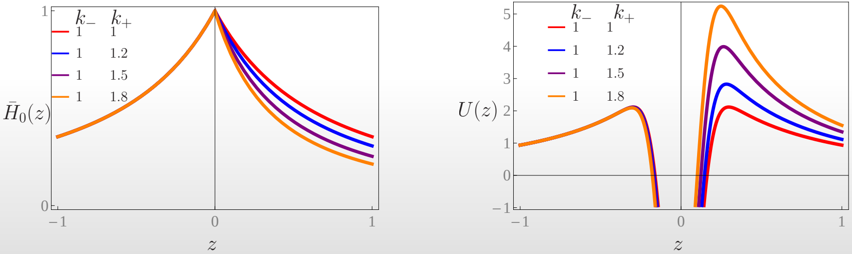
Localization of the 4D graviton

Zero-mode of tensor perturbations

The zero-mode of the tensor fluctuation $\bar{H}_0(z)$, which correspond to the 4D graviton, is localized on the thick brane and is given by

$$\bar{H}_0(z) = e^{\frac{3}{2}A(z)}.$$

- The shape of schrödinger like potential $U(z) = \frac{9}{4}\dot{A}^2(z) + \frac{3}{2}\ddot{A}(z)$ implies that the low mass KK modes will not affect the 4D gravity.



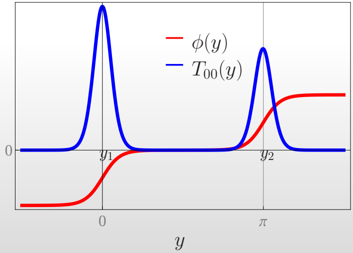
Two thick branes

5D scalar-gravity action

$$\mathcal{S} = \int dx^5 \sqrt{-g} \left\{ 2M^3 R - \frac{1}{2} g^{MN} \nabla_M \phi \nabla_N \phi - V(\phi) \right\},$$

- Metric ansatz: $ds^2 = e^{2A(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2$.
- Scalar field profile

$$\phi(y) = \frac{\kappa_1}{\sqrt{\beta}} \tanh(\beta(y - y_1)) + \frac{\kappa_2}{\sqrt{\beta}} \tanh(\beta(y - y_2)).$$



Background solutions

- The superpotential method described above is also applicable for our two thick brane scenario.
- Superpotential $W(\phi)$ for the double kink-like $\phi(y)$ is

$$W(y) = \kappa_1^2 \left(\tanh[\beta(y-y_1)] - \frac{1}{3}\tanh^3[\beta(y-y_1)] \right) + \kappa_2^2 \left(\tanh[\beta(y-y_2)] - \frac{1}{3}\tanh^3[\beta(y-y_2)] \right) + W_0.$$

- The warped factor $A(y)$

$$A(y) = -\frac{1}{12M^3}\int dy W(y),$$

$$A(y) \approx -k|y|, \qquad \beta \to \infty,$$

where $k = \frac{1}{12M^3}(\lambda_1 + \lambda_2 - W_0)$.

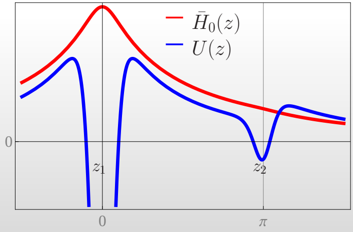
Localization of the 4D graviton

Zero-mode of tensor perturbations

The zero-mode of the tensor fluctuation $\bar{H}_0(z)$, which correspond to the 4D graviton, is

$$\bar{H}_0(z) = e^{\frac{3}{2}A(z)}.$$

- In this scenario the zero-mode of tensor perturbation is localized on the UV-brane.
- The effective 4D gravity on the both branes is well behaved as it is also observed Lykken and Randall for singular branes. [hep-th/9908076]



Hierarchy problem

- Why gravity is so weak as compared to the other fundamental forces?
- Why $m_{EW} \sim 10^3$ GeV $\ll M_{Pl} \sim 10^{19}$ GeV?

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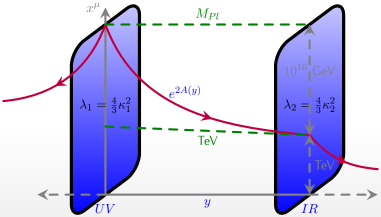
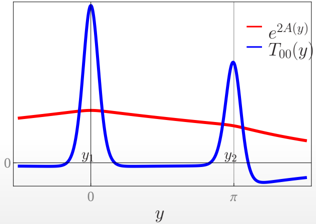
- Why $m_{EW} \sim 10^3$ GeV $\ll M_{Pl} \sim 10^{19}$ GeV?
- **Solution:** If $m_0 \sim M_{Pl} \sim 10^{19}$ GeV then the physical mass m is:

$$m = e^{A(y_2)} m_0 \sim \text{ TeV} \qquad \text{with} \qquad -A(y_2) \approx k y_2 \sim 30.$$





- In the brane limit it mimics Lykken-Randall model with two positive tension branes.

[hep-th/9908076]

Conclusions

- A generalized RS2 model with different bulk cosmological constants on both sides is considered. It is shown that in such a model 4D General Relativity can be recovered.
- The smooth generalizations of RS-like models are obtained with a scalar field.
- Two thick brane model offers an elegant and simple solution to the hierarchy problem assuming the SM fields are localized.
- The background solutions are stable and the effective 4D gravity can be reproduced on the thick brane scenarios.
- We found that RS2-like models can be mimicked by any scalar field $\phi(y)$ if it is monotonic and ϕ'^2 is integrable.

Thank you for your attention

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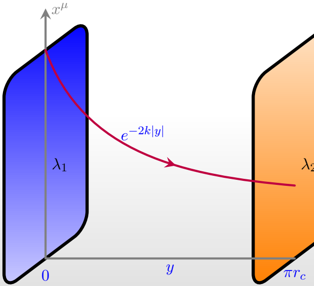
Extra Slides

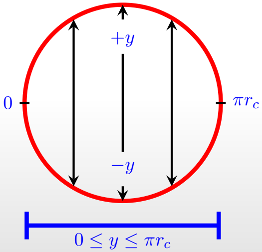
RS model with two branes (RS1)

Randall and Sundrum proposed a model with two D3-branes on the S_1/Z_2 orbifold along the extra-dimension. [hep-ph/9905221]

- The RS metric is

$$ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2, \qquad \text{where} \quad k \approx M.$$

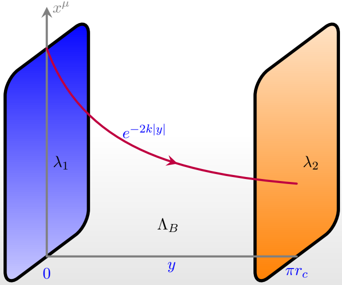




RS1 action

$$\mathcal{S}_{RS1} = \int dx^4 \int_0^{\pi r_c} dy \sqrt{-g} \Big\{ 2M^3 R - \Lambda_B - \lambda_1 \delta(y) - \lambda_2 \delta(y - \pi r_c) \Big\}$$

where $\Lambda_B = -24M^3k^2$ and $\lambda_1 = -\lambda_2 = 24M^3k$.



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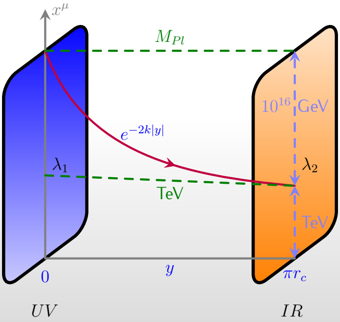
- **Solution:** If $m_0 \sim M_{Pl} \sim 10^{19}$ GeV then physical mass is:

$$m = e^{-\pi k r_c} m_0 \sim TeV \qquad \text{with} \qquad k r_c \approx 10.$$



Obstacles

- Gibbons, Kallosh and Linde pointed out that the non-trivial periodic solutions of a scalar field like RS1 in the absence of singular branes ($\lambda_i = 0$) are impossible in the standard GR. The consistency relation $\oint \phi' \cdot \phi' = 0$ must hold, hence only $\phi = const.$ solution is possible. [hep-th/0011225]

- DeWolfe, Freedman, Gubser and Karch showed that only branes with positive tension could be constructed by a real scalar field configuration. [hep-th/9909134]

Hence, the compactified smooth generalization of the RS1 is *impossible* in the standard Einstein-Hilbert gravity.