XXXVII International Conference of Theoretical Physics

MATTER TO THE DEEPEST

Lepton Mixing and Flavour Symmetries: a status report

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original part based on F. Feruglio, C. Hagedorn and R. Ziegler
"Lepton Mixing Parameters from Discrete and CP Symmetries"
arXiv:1211.5560, see also 1303.7178

global fit before 2012

	Lisi [Neutel2011] [0806.22517update]	Schwetz et al. [1103.0734]
$\sin^2\vartheta_{12}$	$0.307^{+0.018}_{-0.016}$	$0.312^{+0.017}_{-0.015}$
$\sin^2\vartheta_{23}$	$0.42^{+0.09}_{-0.04}$	0.51 ± 0.06 [NO] 0.52 ± 0.06 [IO]
$\sin^2\vartheta_{13}$	$0.014^{+0.009}_{-0.008}$	$0.010^{+0.009}_{-0.006}$ [NO] $0.013^{+0.009}_{-0.007}$ [IO]
$\Delta m^2_{21}\ (eV^2)$	$(7.54^{+0.25}_{-0.22}) \times 10^{-5}$	$(7.59^{+0.20}_{-0.18}) \times 10^{-5}$
$\lvert\Delta m^2_{31}\ (eV^2)\rvert$	$(2.36^{+0.12}_{-0.10}) \times 10^{-3}$	$(2.45 \pm 0.09) \times 10^{-3}$ [NO] $(2.34^{+0.10}_{-0.09}) \times 10^{-3}$ [IO]

TBM
1/3
1/2
0

experimental error on ϑ_{12} [1σ] is 0.02 rad <-> 1 degree
TB prediction for ϑ_{12} agrees within 1.5 σ
same for the other angles

suggests that there is a limit of the underlying theory where lepton mixing angles become simple [e.g. $V_{CKM}=1$ when λ_C is sent to zero]

Tribimaximal Mixing [Harrison, Perkins and Scott 2002]

$$U_{PMNS} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} + \text{(small corrections)}$$

by unitarity

tri-bimaximal [TB] mixing pattern, completely different from the quark mixing pattern: two angles are large

$$\nu_3 = \frac{-\nu_\mu + \nu_\tau}{\sqrt{2}}$$ maximal

$$\nu_2 = \frac{\nu_e + \nu_\mu + \nu_\tau}{\sqrt{3}}$$ trimaximal

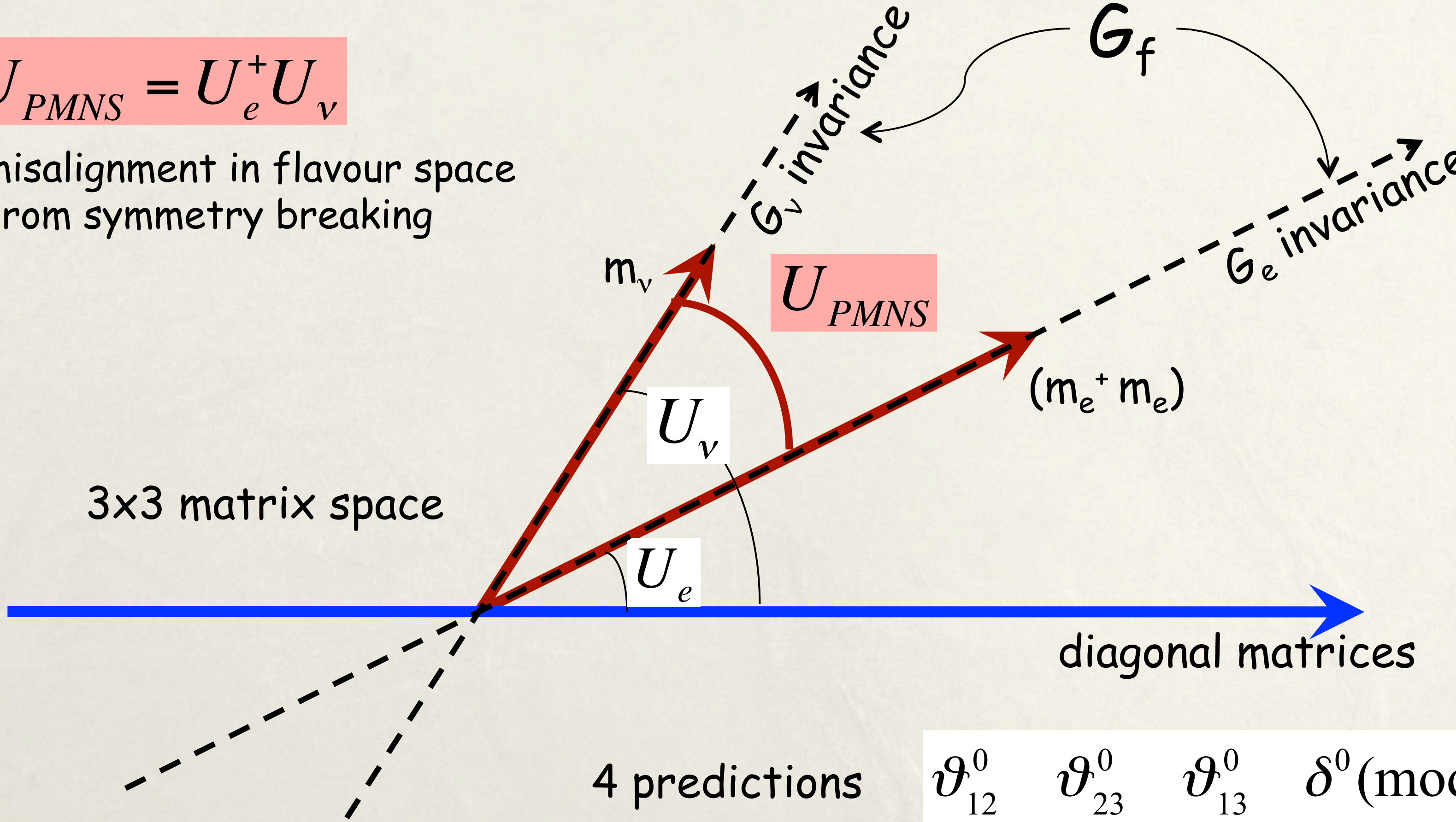
- can be a useful 1st order approximation to data, U^0_{PMNS}, related to some limit of the underlying theory

Mixing patterns U^0_{PMNS} from discrete symmetries

[Ma and Rajasekaran 0106291]

$$U_{PMNS} = U_e^+ U_\nu$$

misalignment in flavour space
from symmetry breaking



4 predictions ϑ^0_{12} ϑ^0_{23} ϑ^0_{13} $\delta^0 (\mathrm{mod}\ \pi)$

- the most general group leaving $\nu^T m_\nu \nu$ invariant, and m_i unconstrained

$$G_\nu = Z_2 \times Z_2$$

Majorana neutrinos imply G_ν discrete!

- G_e can be continuous but the simplest choice is G_e discrete

$$G_e = \begin{cases} Z_2 \times Z_2 \\ Z_n \quad n \geq 3 \end{cases}$$

Some mixing patterns

[Lam 1104.0055]

$$G_\nu = Z_2 \times Z_2$$

G_f	G_e	U_{PMNS}	$\sin^2\vartheta_{23}$	$\sin\vartheta_{13}$	$\sin^2\vartheta_{12}$	
A_4	Z_3	[M]	$1/2$	$1/\sqrt{3}$	$1/2$	
S_4	Z_3	[TB]	$1/2$	0	$1/3$	TBM
	Z_4 $(Z_2 \times Z_2)'$	[BM]	$1/2$	0	$1/2$	
A_5	Z_3	[GR_1]	$1/2$	0	0.127	
	Z_5	[GR_2]	$1/2$	0	0.276	[GR_2<-> Kajiyama, Raidal, Strumia 2007]
	$(Z_2 \times Z_2)'$	[GR_3]	0.276	0.309	0.276	
		[Exp 3σ]	$0.34 \div 0.67$	$0.13 \div 0.17$	$0.27 \div 0.34$	

-- a long way to promote a candidate pattern to a complete model

-- general feature $U_{PMNS} = U^0_{PMNS} + O(u) \quad u \equiv \frac{\langle\varphi\rangle}{\Lambda} < 1$

-- neutrino masses fitted, not predicted.

expectation for $U^0_{PMNS}=U_{TB}$

$$\begin{cases} \vartheta^0_{13} = 0 \\ \vartheta^0_{23} = \dfrac{\pi}{4} \end{cases} \Rightarrow \begin{cases} \vartheta_{13} = \text{O(few degrees)} \\ \vartheta_{23} = \text{close to } \dfrac{\pi}{4} \end{cases}$$

not to spoil the agreement with ϑ_{12}

wrong!

2011/2012 breakthrough

[see Przewłocki and Lavader talks]

-- LBL experiments searching for $\nu_\mu \to \nu_e$ conversion
-- SBL reactor experiments searching for anti-ν_e disappearance

	Lisi [NeuTel 2013]	[1209.3023] [G-Garcia, Maltoni, Salvado, Schwetz]
$\sin^2\vartheta_{13}$	$0.0241^{+0.0025}_{-0.0025}$ (NO) $0.0244^{+0.0023}_{-0.0025}$ (IO)	$0.0227^{+0.0023}_{-0.0024}$
$\sin^2\vartheta_{23}$	$0.386^{+0.024}_{-0.021}$ (NO) $0.392^{+0.039}_{-0.022}$ (IO)	$0.413^{+0.037}_{-0.025} \oplus 0.594^{+0.021}_{-0.022}$

⇒ 10σ away from 0

⇒ hint for non maximal ϑ_{23}

$\vartheta_{13} \approx 0.15$ rad $\approx 9^0$

simplest models based on discrete symmetries
reproducing TBM at LO are ruled out

other possibilities ?

1

Mixing angles from abelian symmetries: $G_f=U(1)_{FN}$

[Froggatt,Nielsen 1979]

lessons from the quark sector: mass ratios and mixing angles are small, hierarchical parameters

$$\frac{m_u}{m_t} << \frac{m_c}{m_t} << 1 \qquad \frac{m_d}{m_b} << \frac{m_s}{m_b} << 1 \qquad |V_{ub}| << |V_{cb}| << |V_{us}| \equiv \lambda < 1$$

easily reproduced by $G_f=U(1)_{FN}$ and $H_f=\{1\}$

mass ratios and mixing angles are powers of a small SB parameter λ

flavon	Q_{FN}
φ	-1

$U(1)_{FN}$ broken by

$$\lambda = \frac{<\varphi>}{\Lambda_f} \approx 0.2$$

$$\frac{\varphi^4}{\Lambda_f^4} v\, d^c d$$

assign decreasing, non-negative, charges to fermions of increasing generations

field	Q_{FN}
q, u^c	$(3,2,0)$
d^c	$(1,0,0)$

$$y_d = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

unbroken $U(1)_{FN}$

entries up to unknown O(1) coefficients

$$y_d = \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda \\ \lambda^3 & \lambda^2 & 1 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

broken $U(1)_{FN}$

order of magnitude of most mass ratios and mixing angles correctly reproduced can be extended to the lepton sector where evidence for hierarchy mainly comes from charged leptons

field	Q_{FN}
e^c	(3,2,0)
l	(1,0,0)

$$y_e \approx y_d^T$$

$$m_\nu \propto \begin{pmatrix} \lambda^2 & \lambda & \lambda \\ \lambda & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix}$$

ϑ_{23}	ϑ_{13}	ϑ_{12}	$\Delta m^2_{12} / \Delta m^2_{23}$
1	λ	λ	1
1	λ	1	λ^2

if det(23)≈λ

Anarchy (neutrino sector is structure-less)
[Hall, Murayama, Weiner 1999]

field	Q_{FN}
l	(0,0,0)

$$m_\nu \propto \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

mixing angles and mass ratios are random O(1) quantities

$$|U_{PMNS}| \approx \begin{pmatrix} 0.8 & 0.5 & 0.2 \\ 0.4 & 0.6 & 0.6 \\ 0.4 & 0.6 & 0.8 \end{pmatrix}$$

predicts ϑ_{13} not tiny and ϑ_{23} not maximal

consistent with data

$G_f=U(1)_{FN}$

- compatible with SU(5) grand unification (comprises quarks and leptons)
- compatible with known solutions to the gauge hierarchy problem (SUSY, RS,...)
- large number of independent O(1) parameters
- no testable predictions beyond order-of-magnitude accuracy

2 add large corrections $O(\vartheta_{13}) \approx 0.2$

- predictability is lost since in general correction terms are many
- new dangerous sources of FC/CPV if NP is at the TeV scale

3 change discrete group G_f

- solutions exist
  special forms of Trimaximal mixing

$$U^0 = U_{TB} \times \begin{pmatrix} \cos\alpha & 0 & e^{i\delta}\sin\alpha \\ 0 & 1 & 0 \\ -e^{-i\delta}\sin\alpha & 0 & \cos\alpha \end{pmatrix}$$

G_f	$\Delta(96)$	$\Delta(384)$	$\Delta(600)$
α	$\pm\pi/12$	$\pm\pi/24$	$\pm\pi/15$
$\sin^2\vartheta_{13}^0$	0.045	0.011	0.029

$\delta^0 = 0, \pi$ (no CP violation) and
α "quantized" by group theory

F.F., C. Hagedorn, R. de A.Toroop
hep-ph/1107.3486 and hep-ph/1112.1340
Lam 1208.5527 and 1301.1736
Holthausen1, Lim and Lindner 1212.2411
Neder, King, Stuart 1305.3200
Hagedorn, Meroni, Vitale 1307.5308]

too big groups?

4 relax symmetry requirements

[Hernandez,Smirnov 1204.0445]

G_e as before

$G_\nu = Z_2$

2 predictions:
2 combinations of $\vartheta_{12}^0 \quad \vartheta_{23}^0 \quad \vartheta_{13}^0$

leads to testable sum rules

$$\sin^2\vartheta_{23} = \frac{1}{2} + \frac{1}{\sqrt{2}}\sin\vartheta_{13}\cos\delta_{CP} + O(\sin^2\vartheta_{13})$$

[He, Zee 2007 and 2011, Grimus, Lavoura 2008, Grimus, Lavoura, Singraber 2009, Albright, Rodejohann 2009, Antusch, King, Luhn, Spinrath 2011, King, Luhn 2011, G. Altarelli, F.F., L. Merlo and E. Stamou hep-ph/1205.4670]

5 include CP in the SB pattern

not a new idea:
residual symmetry of neutrino mass matrix can include a non-trivial action of CP in flavor space [see Grimus talk]

-- μτ reflection symmetry in flavor basis: [Harrison, Scott 2002/2004, Grimus, Lavoura 2003, Babu, Ma, Valle 2002; Ferreira, Grimus, Lavoura, Ludl 1206.7072]

$$\nu_\mu \leftrightarrow \bar{\nu}_\tau \quad \Rightarrow \quad \sin\vartheta_{23} = \cos\vartheta_{23} \qquad \sin\vartheta_{13}\cos\delta = 0 \qquad \sin\alpha = \sin\beta = 0$$

δ maximal

-- combination of S_4 and CP [Mohapatra, Nishi 1208.2875; Krishnan, Harrison, Scott 1211.2000]

generalized CP transformations

[Bernabeu, Branco, Gronau 1986; Ecker, Grimus, Neufeld 1987/1988]

on left-handed lepton doublets l

$$l'(x) = X\, l^*(x_{CP})$$

X is a 3 x 3 unitary matrix in flavor space such that

$$X X^* = 1 \qquad \text{(I)}$$

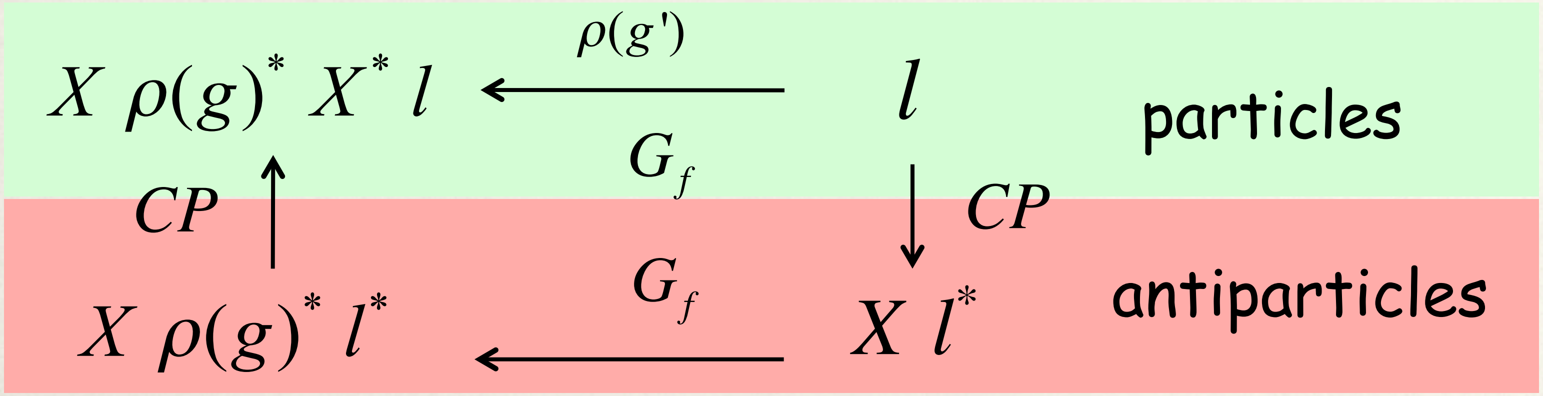
G_f flavor symmetry group

$$l'(x) = \rho(g)\, l(x)$$

ρ realizes a 3-dimensional unitary representation of G_f $\quad g \in G_f \rightarrow \rho(g)$

formalism [F. Feruglio, C. Hagedorn and R. Ziegler arXiv:1211.5560]

consistency condition [see Grimus, Rebelo 1995 for gauge symmetries and Holthausen, Lindner, Schmidt 1211.6953]



$\Rightarrow$ $(X^{-1}\rho(g)\,X)^* = \rho(g')$ **(II)** $g' \neq g$ in general

we assume

$$G_{CP} = G_f \rtimes CP$$

$\Rightarrow$ $(X^{-1}Z\,X)^* = Z$ **(III)**

to consistently define G_ν

G_e generated by Q_i

$G_\nu = Z_2 \times CP$ generated by (Z, X)

given G_f

(I) + (II) + (III)

- represent a set of constraints on the admissible X (i.e. CP transformations)
- in general several physically non-equivalent solutions for X exist

consequences of residual invariance

$$Q_i^+ \, (m_l^+ m_l) \, Q_i = (m_l^+ m_l)$$

$$Z^T m_\nu \, Z = m_\nu$$

$$X \, m_\nu \, X = m_\nu^*$$

⇒

mixing angles and CP phases

$$U^0_{PMNS}(Q_i, Z, X; \vartheta)$$

$$(\vartheta^0_{12}, \vartheta^0_{23}, \vartheta^0_{13}, \delta^0, \alpha^0, \beta^0)$$

predicted in terms of a single real parameter $0 \le \vartheta \le 2\pi$

- the formalism is completely invariant under any change of basis in field space
- the results only depend on G_{CP} and the residual symmetries specified by Q_i, Z and X.

how it works in practice? → $G_f = S_4$

generators of of S_4 in 3' representation

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} \qquad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix} \quad \omega \equiv e^{i\frac{2\pi}{3}} \qquad U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

most general solution to constraints I, II and III found in [F, Hagedorn, Ziegler 1211.5560]

two examples

case	$Q_i\,(G_e)$	Z	X
I	$T\,(Z_3)$	S	X_1
IV	$T\,(Z_3)$	SU	X_1

$$X_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

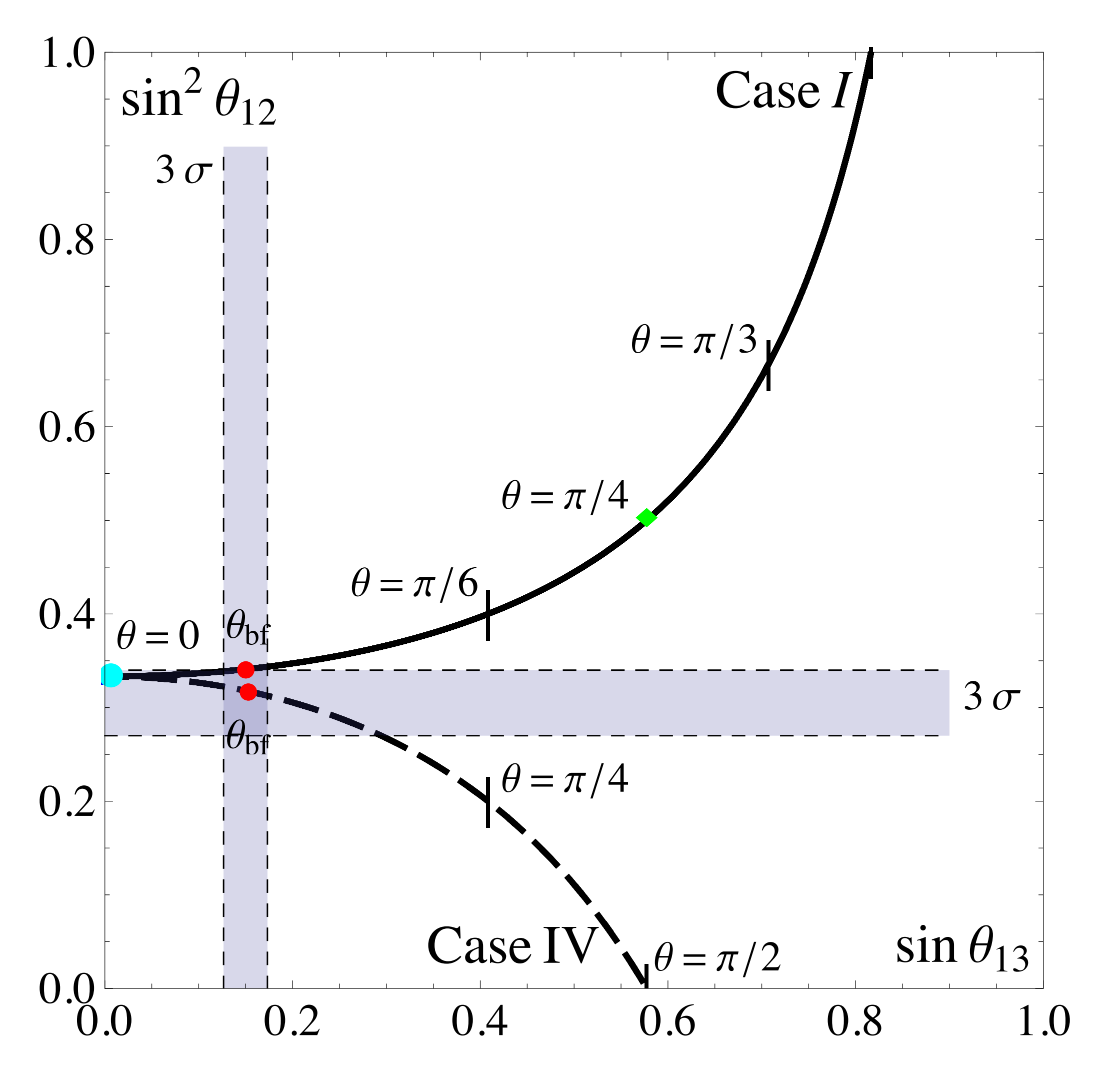
→ 2 specific realizations of μτ reflection symmetry

$$\sin^2 \vartheta_{23}^0 = \frac{1}{2} \qquad |\sin\delta^0| = 1$$

$$\sin\alpha^0 = 0$$

$$\sin\beta^0 = 0$$

predictions



explicit model realizations of case I recently built in SUSY context

[Ding, King, Luhn and Stuart 1303.6180; F. Feruglio, C. Hagedorn and R. Ziegler 1303.7178; Ding, King, and Stuart 1307.4212]

$\chi^2_{MIN} = 18.4(26.7)$
for case I
$\vartheta_{23} < \pi/4 (> \pi/4)$

$\chi^2_{MIN} = 10.2(18.5)$
for case IV
$\vartheta_{23} < \pi/4 (> \pi/4)$

I and IV could provide examples of "geometrical" CP violation since $|\sin\delta^0|=1$ only depends on the SB pattern

conclusion

- big progress on the experimental side:
  -- precisely measured ϑ_{13}: many σ away from zero!
  -- potentially interesting implications on ϑ_{23}
  -- sterile neutrinos waiting for exp. checks

- on the theory side:
  flavour symmetries are a useful tool but
  no compelling and unique picture have emerged so far
  present data can be described within widely different frameworks

- models based on "anarchy" and/or its variants - $U(1)_{FN}$ models - in good shape:
  neutrino mass ratios and mixing angles just random $O(1)$ quantities

- models based on discrete symmetries are less supported by data now
  and modifications of simplest realizations are required
  -- add large corrections $O(\vartheta_{13})\approx 0.2$
  -- move to large discrete symmetry groups G_f such as $\Delta(96)$ $\Delta(384)$ …
  -- relax symmetry requirements
  -- include CP in the SB pattern:
  residual invariance G_e and $G_\nu = Z_2 \times CP$
  determines all mixing angles and phases in terms of a single parameter
  if $G_f=S_4$ several realistic mixing patterns are found

back up slides

2011/2012 breakthrough

-- LBL experiments searching for $\nu_\mu \to \nu_e$ conversion [see Fogli's talk]
-- SBL reactor experiments searching for anti-ν_e disappearance

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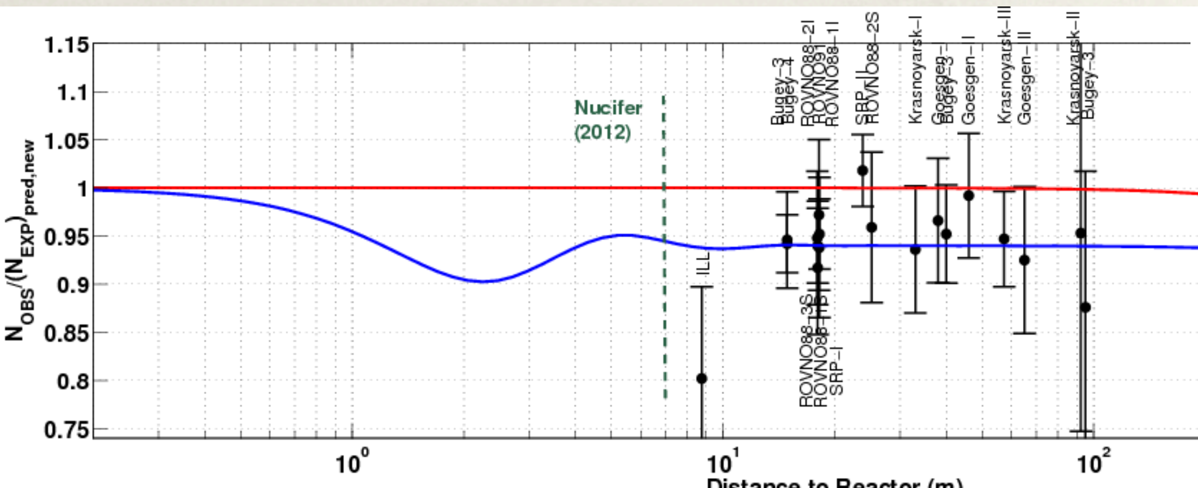
→ 10σ away from 0 — impact on flavor symmetry (part 3)

→ hint for non maximal ϑ_{23}

sterile neutrinos coming back

1 reactor anomaly (anti-ν_e disappearance)

re-evaluation of reactor anti-ν_e flux: new estimate 3.5% higher than old one



$$(\Phi_{exp} - \Phi_{th}) / \Phi_{th} \approx -6\%$$

[th. uncertainty?]

very SBL L ≲ 100 m

$$\vartheta_{es} \approx 0.2$$

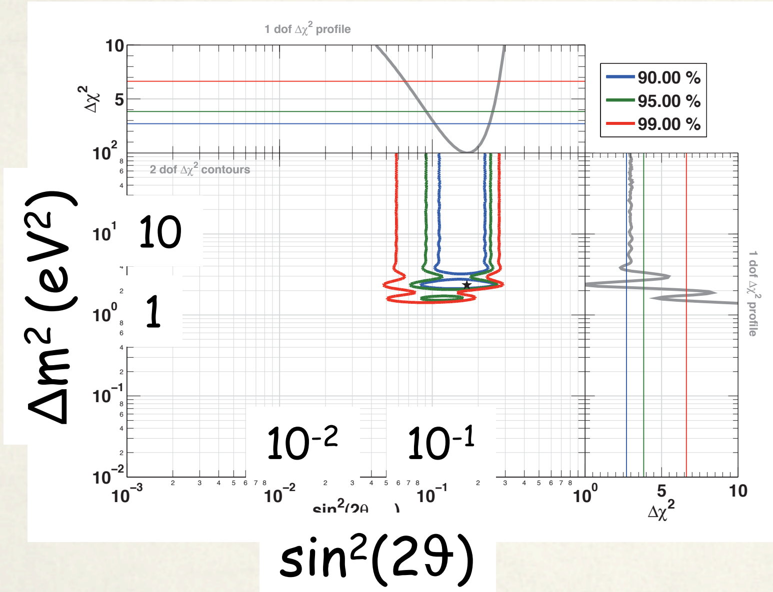
$$\Delta m^2 \approx m_s^2 \geq 1\, eV^2$$

supported by the Gallium anomaly

ν_e flux measured from high intensity radioactive sources in Gallex, Sage exp

$$\nu_e + {}^{71}Ga \rightarrow {}^{71}Ge + e^-$$

[error on σ or on Ge extraction efficiency]



most recent cosmological limits

[depending on assumed cosmological model, data set included,...]

relativistic degrees of freedom at recombination epoch

$$N_{eff} = 3.30 \pm 0.27$$

[Planck, WMAP, BAO, high multiple CMB data]

fully thermalized non relativistic ν

$$N_{eff} < 3.80 \quad (95\%\, CL)$$

$$m_s < 0.42\, eV \quad (95\%\, CL)$$

2 long-standing claim

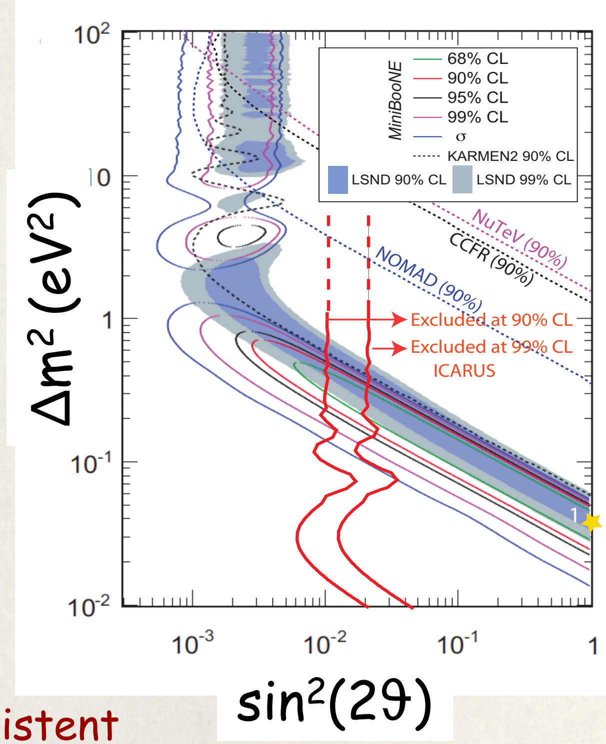
evidence for ν_μ -> ν_e appearance in accelerator experiments

exp		$E(MeV)$	$L(m)$	
LSND	$\bar{\nu}_\mu \rightarrow \bar{\nu}_e$	$10 \div 50$	30	3.8σ
MiniBoone	$\nu_\mu \rightarrow \nu_e$ $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$	$300 \div 3000$	541	3.8σ [signal from low-energy region]

parameter space limited by negative results from Karmen and ICARUS

$$\vartheta_{e\mu} \approx 0.035$$

$$\Delta m^2 \approx 0.5\, eV^2$$

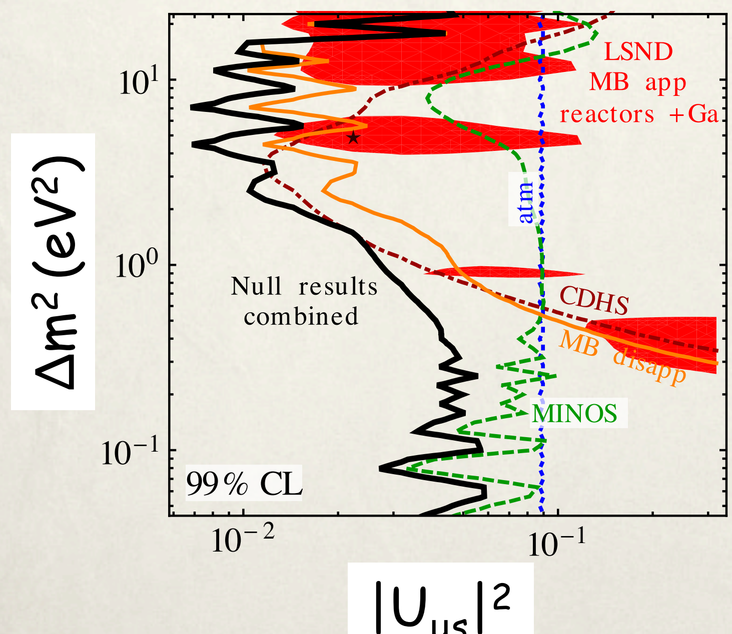


3 interpretation in 3+1 scheme: inconsistent (more than 1s disfavored by cosmology)

$$\underbrace{\vartheta_{e\mu}}_{0.035} \approx \underbrace{\vartheta_{es}}_{0.2} \times \vartheta_{\mu s} \Rightarrow \vartheta_{\mu s} \approx 0.2$$

predicted suppression in ν_μ disappearance experiments: undetected

by ignoring LSND/Miniboone data the reactor anomaly can be accommodated by $m_s \geq 1$ eV and $\vartheta_{es} \approx 0.2$
[not suitable for WDM, more on this later]



how it works? example $G_f = S_4$

generators in 3' representation

$$S = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \qquad T = \frac{1}{2}\begin{pmatrix} 1 & \sqrt{2} & 1 \\ \sqrt{2} & 0 & -\sqrt{2} \\ -1 & \sqrt{2} & -1 \end{pmatrix} \qquad U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

all possibilities are exhausted by choosing

G_e	Q_i
Z_3	T
Z_4	STU
$Z_2 \times Z_2$	$TST^2S,\ UT^2$

G_ν	Z	X
$Z_2 \times CP$	S	$X_i \quad (i = 1,...,6)$
$Z_2 \times CP$	SU	$X_i \quad (i = 1,...,4)$
$Z_2 \times CP$	U	$X_i \quad (i = 1,...,4)$

solutions to the constraints (I,II,III):

$$X_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad X_2 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad X_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad X_4 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$X_5 = \begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{pmatrix} \quad X_6 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

sample of results

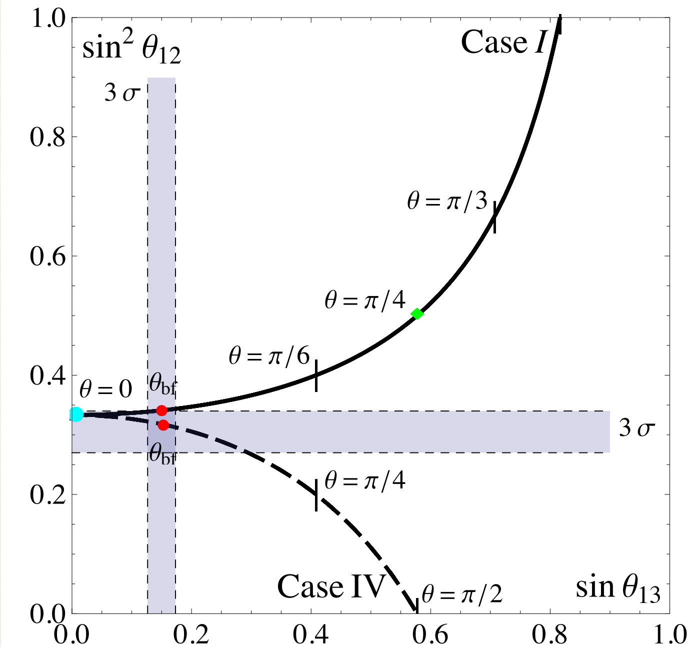
case	$Q_i\ (G_e)$	Z	X
I	$T\ (Z_3)$	S	X_1
IV	$T\ (Z_3)$	SU	X_1

I and IV could provide examples of "geometrical" CP violation since $|\sin\delta^0|=1$ only depends on the SB pattern

2 specific realizations of μτ reflection symmetry

$$\sin^2\vartheta_{23}^0 = \frac{1}{2} \qquad |\sin\delta^0| = 1$$

$$\sin\alpha^0 = 0$$
$$\sin\beta^0 = 0$$



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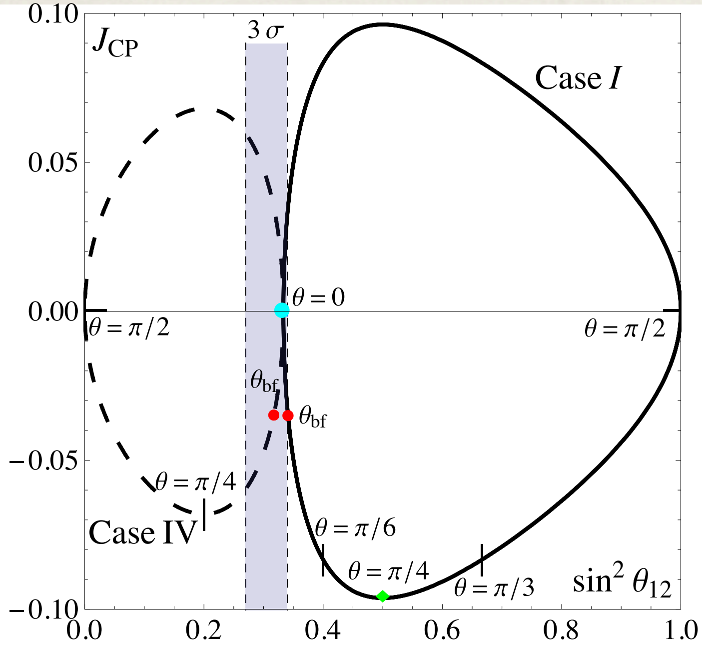
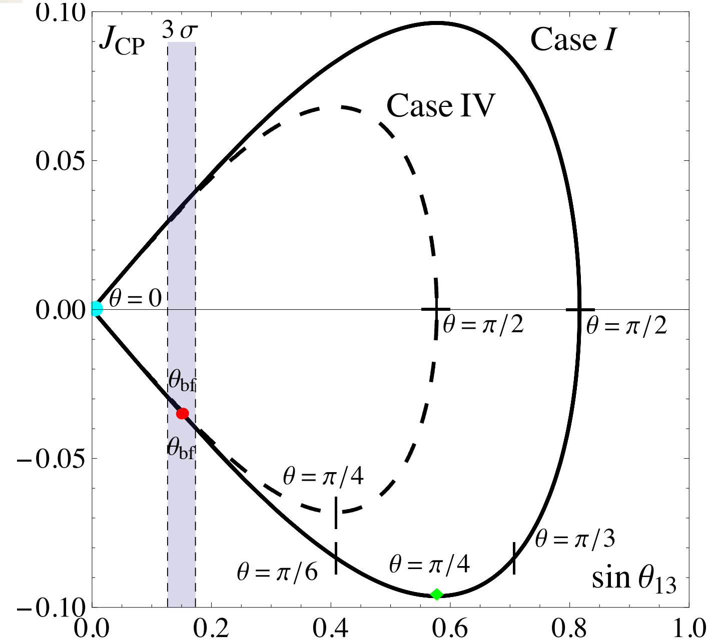




case	$Q_i\ (G_e)$	Z	X
II	$T\ (Z_3)$	S	X_3
V	$T\ (Z_3)$	SU	X_2

$\sin\delta^0 = 0$

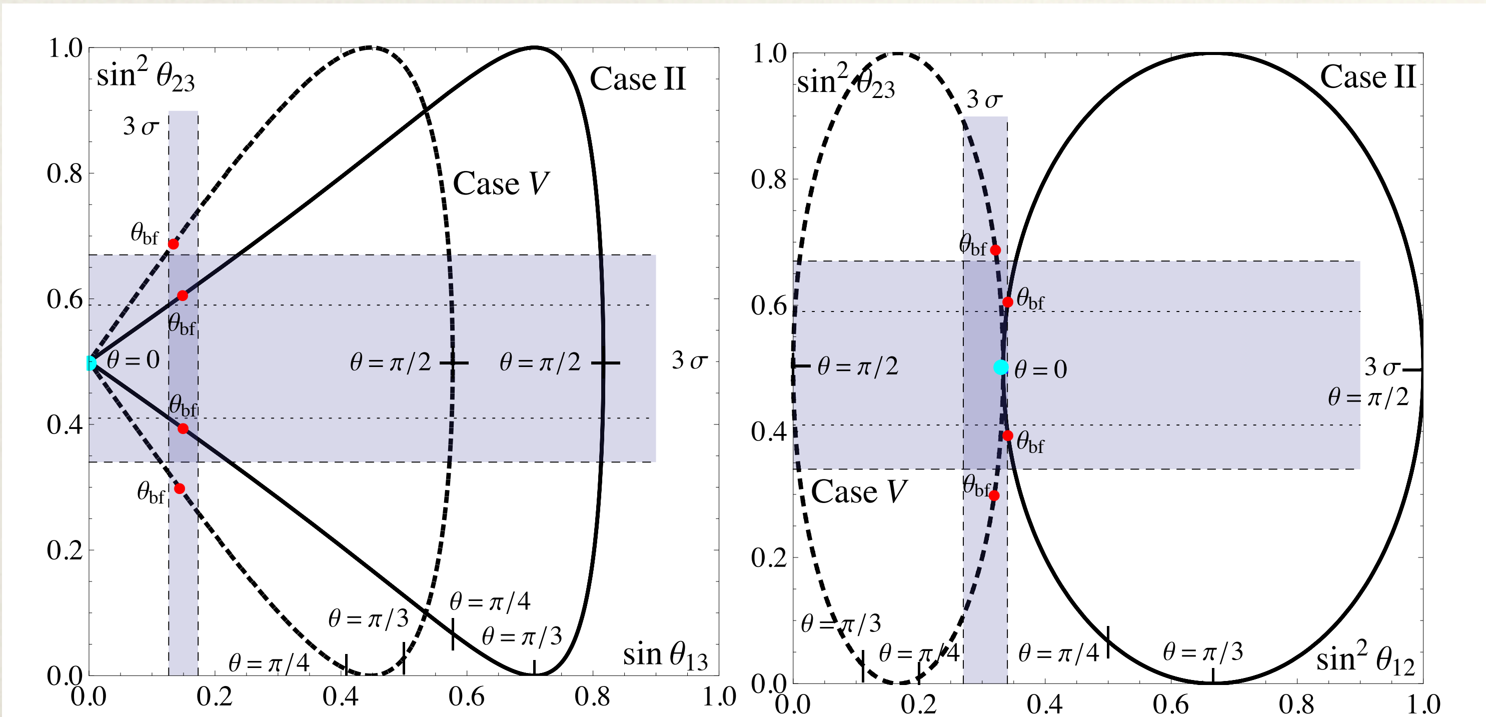
$\sin\alpha^0 = 0$

$\sin\beta^0 = 0$

CP is conserved because X_3 is an accidental symmetry of the whole lepton sector in both cases II and V

$\chi^2_{MIN} = 10.3(10.5)$
for case II
$\vartheta_{23} < \pi/4 (> \pi/4)$

$\chi^2_{MIN} = 16.1(27.2)$
for case V
$\vartheta_{23} < \pi/4 (> \pi/4)$



($\sin^2\vartheta^0_{12}$, $\sin\vartheta^0_{13}$) correlation is the same as for cases I and IV

a non-realistic case with full dependence on ϑ

	III	
$\sin^2\theta_{13}$	$\frac{1}{3}\left(1-\frac{\sqrt{3}}{2}\sin 2\theta\right)$	
$\sin^2\theta_{12}$	$\frac{2}{4+\sqrt{3}\sin 2\theta}$	
$\sin^2\theta_{23}$	$\frac{2}{4+\sqrt{3}\sin 2\theta}$	$1-\frac{2}{4+\sqrt{3}\sin 2\theta}$
$\lvert J_{CP}\rvert$	$\frac{\lvert\cos 2\theta\rvert}{6\sqrt{3}}$	
$\lvert\sin\delta\rvert$	$\left\lvert\frac{(4+\sqrt{3}\sin 2\theta)\cos 2\theta\sqrt{4-2\sqrt{3}\sin 2\theta}}{5+3\cos 4\theta}\right\rvert$	
$\lvert\sin\alpha\rvert$	$\left\lvert\frac{\sqrt{3}+2\sin 2\theta}{2+\sqrt{3}\sin 2\theta}\right\rvert$	
$\lvert\sin\beta\rvert$	$\left\lvert\frac{4\sqrt{3}\cos 2\theta}{5+3\cos 4\theta}\right\rvert$	
$\theta_{\rm bf}$	$0.785\ \ \theta_{23}<\pi/4$	$0.785\ \ \theta_{23}>\pi/4$
$\chi^2_{\rm min}$	106.7	110.5
$\sin^2\theta_{13}(\theta_{\rm bf})$	0.045	
$\sin^2\theta_{12}(\theta_{\rm bf})$	0.349	
$\sin^2\theta_{23}(\theta_{\rm bf})$	0.349	0.651
$\lvert J_{CP}\rvert(\theta_{\rm bf})$	0	
$\lvert\sin\delta\rvert(\theta_{\rm bf})$	0	
$\lvert\sin\alpha\rvert(\theta_{\rm bf})$	1	
$\lvert\sin\beta\rvert(\theta_{\rm bf})$	0	

case	$Q_i\ (G_e)$	Z	X
III	$T\ (Z_3)$	S	X_5

best fit value of ϑ

$\chi^2_{min} > 100$ since $\sin^2\vartheta^0_{13} \geq 0.045$

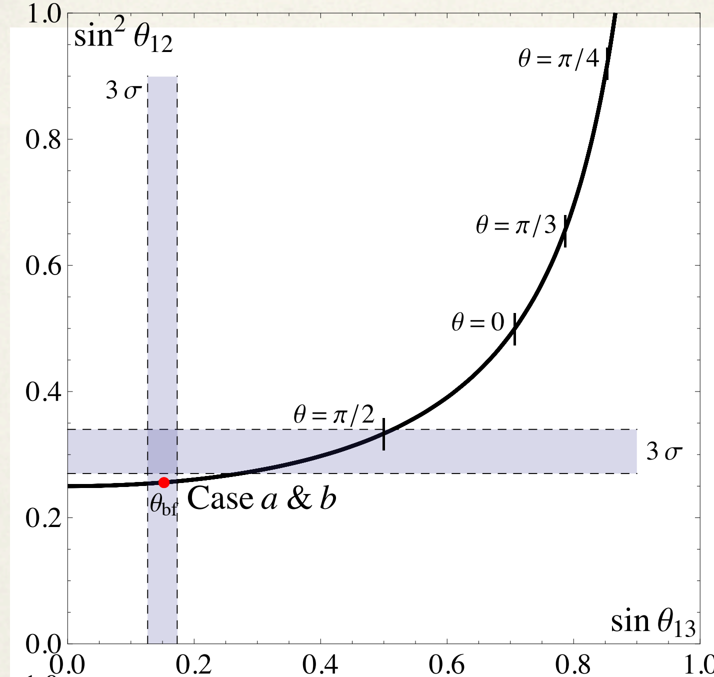
case	$Q_i\ (G_e)$	Z	X
VI	$STU\ \ (Z_4)$	U	X_2
VII	$TST^2S, UT^2\ \ (Z_2 \times Z_2)$	U	X_1

$\sin\delta^0 = 0$

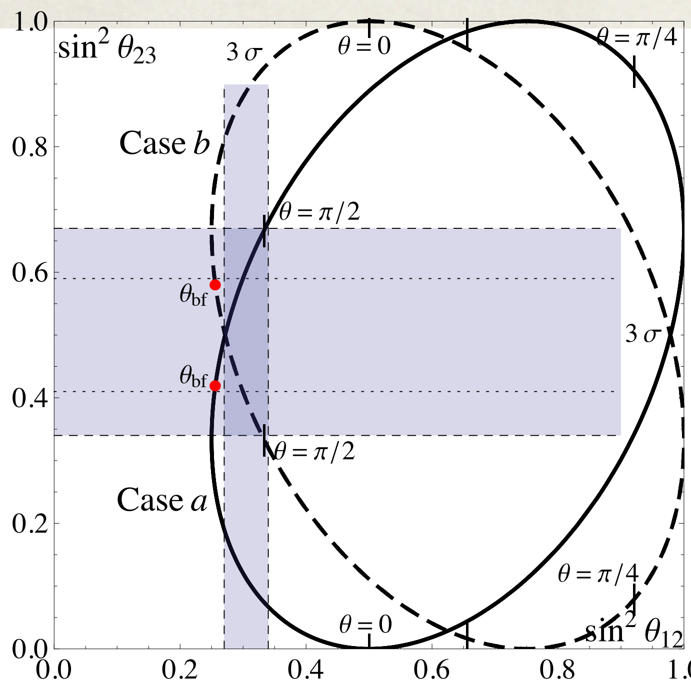
$\sin\alpha^0 = 0$

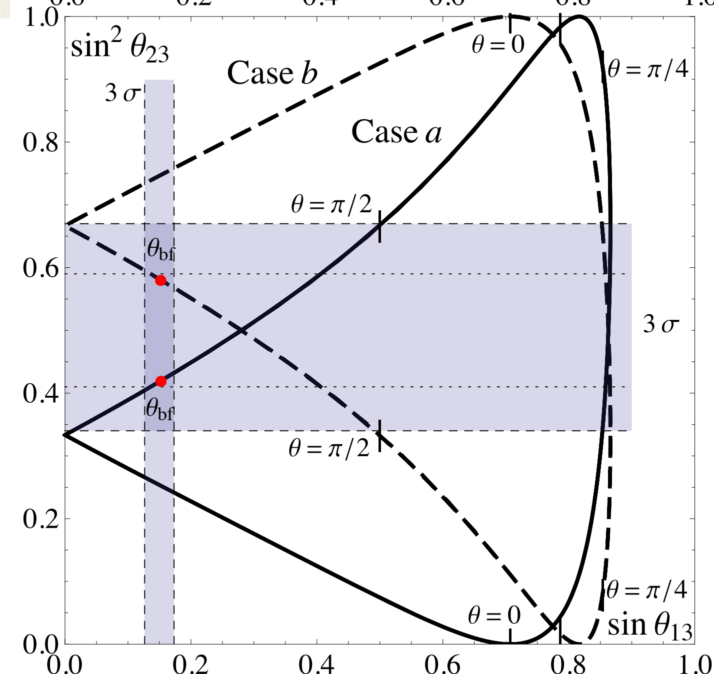
$\sin\beta^0 = 0$

CP is conserved because
X is an accidental symmetry
of the whole lepton sector
in both cases VI and VII



$\chi^2_{MIN} = 11.6(11.7)$
for case a(b)





$\vartheta_{13} > 0$ from any discrete symmetry, at the LO ?

[de Adelhart Toorop, F, Hagedorn 1107.3486]

how to "deform" A_4 and/or S_4? no continuous parameter

abstract definition
in terms of generators
and relations

$$S^2 = (ST)^3 = T^n = 1$$

$n = 3 \quad A_4$

$n = 4 \quad S_4$

both subgroups of the (infinite) modular group Γ $\quad S^2 = (ST)^3 = 1$

we looked for other subgroups of Γ, the so-called finite modular groups
[there are only six of them admitting three dimensional irreducible
representations]

$$\Delta(96): \qquad S^2 = (ST)^3 = T^8 = 1 \qquad \left(ST^{-1}ST\right)^3 = 1$$

$$\Delta(384): \qquad S^2 = (ST)^3 = T^{16} = 1 \qquad \left(ST^{-1}ST\right)^3 = 1$$

$\Delta(384)$

$$S = \frac{1}{2}\begin{pmatrix} 0 & \sqrt{2} & \sqrt{2} \\ \sqrt{2} & -1 & 1 \\ \sqrt{2} & 1 & -1 \end{pmatrix}$$

$$T = \begin{pmatrix} \omega_{16}^{14} & 0 & 0 \\ 0 & \omega_{16}^{5} & 0 \\ 0 & 0 & \omega_{16}^{13} \end{pmatrix} \qquad \omega_{16} = e^{i\frac{\pi}{8}}$$

$$G_\nu = Z_2 \times Z_2$$

generated by (S, ST^8ST^8)

$$G_e = Z_3$$

generated by ST

$$|U_{PMNS}| = \frac{1}{\sqrt{3}}\begin{pmatrix} \sqrt{4+\sqrt{2}+\sqrt{6}}/2 & 1 & \sqrt{4-\sqrt{2}-\sqrt{6}}/2 \\ \sqrt{4+\sqrt{2}-\sqrt{6}}/2 & 1 & \sqrt{4-\sqrt{2}+\sqrt{6}}/2 \\ \sqrt{1-1/\sqrt{2}} & 1 & \sqrt{1+1/\sqrt{2}} \end{pmatrix}$$

[by exchanging 2nd and 3rd rows in U_{PMNS}]

[M3]

$$\sin^2\vartheta_{13} = (4-\sqrt{2}-\sqrt{6})/12 \approx 0.011$$

$$\sin^2\vartheta_{23} = \frac{(4-\sqrt{2}+\sqrt{6})}{(8+\sqrt{2}+\sqrt{6})} \approx 0.424$$

$$\sin^2\vartheta_{12} = \frac{4}{(8+\sqrt{2}+\sqrt{6})} \approx 0.337$$

$$\delta_{CP} = 0$$

[M4]

$$\sin^2\vartheta_{13} = (4-\sqrt{2}-\sqrt{6})/12 \approx 0.011$$

$$\sin^2\vartheta_{23} = \frac{(4+2\sqrt{2})}{(8+\sqrt{2}+\sqrt{6})} \approx 0.576$$

$$\sin^2\vartheta_{12} = \frac{4}{(8+\sqrt{2}+\sqrt{6})} \approx 0.337$$

$$\delta_{CP} = \pi$$

$\Delta(96)$

$$S = \frac{1}{2}\begin{pmatrix} 0 & \sqrt{2} & \sqrt{2} \\ \sqrt{2} & -1 & 1 \\ \sqrt{2} & 1 & -1 \end{pmatrix}$$

$$T = \begin{pmatrix} \omega_8^6 & 0 & 0 \\ 0 & \omega_8^7 & 0 \\ 0 & 0 & \omega_8^3 \end{pmatrix} \qquad \omega_8 = e^{i\frac{\pi}{4}}$$

$$G_\nu = Z_2 \times Z_2$$

generated by (S, ST^4ST^4)

$$G_e = Z_3$$

generated by ST

$$|U_{PMNS}| = \frac{1}{\sqrt{3}}\begin{pmatrix} (\sqrt{3}+1)/2 & 1 & (\sqrt{3}-1)/2 \\ (\sqrt{3}-1)/2 & 1 & (\sqrt{3}+1)/2 \\ 1 & 1 & 1 \end{pmatrix}$$

[M1]

$$\sin^2\vartheta_{13} = (2-\sqrt{3})/6 \approx 0.045$$

$$\sin^2\vartheta_{23} = (5+2\sqrt{3})/13 \approx 0.651$$

$$\sin^2\vartheta_{12} = (8-2\sqrt{3})/13 \approx 0.349$$

$$\delta_{CP} = \pi$$

[M2] [by exchanging 2nd and 3rd rows in U_{PMNS}]

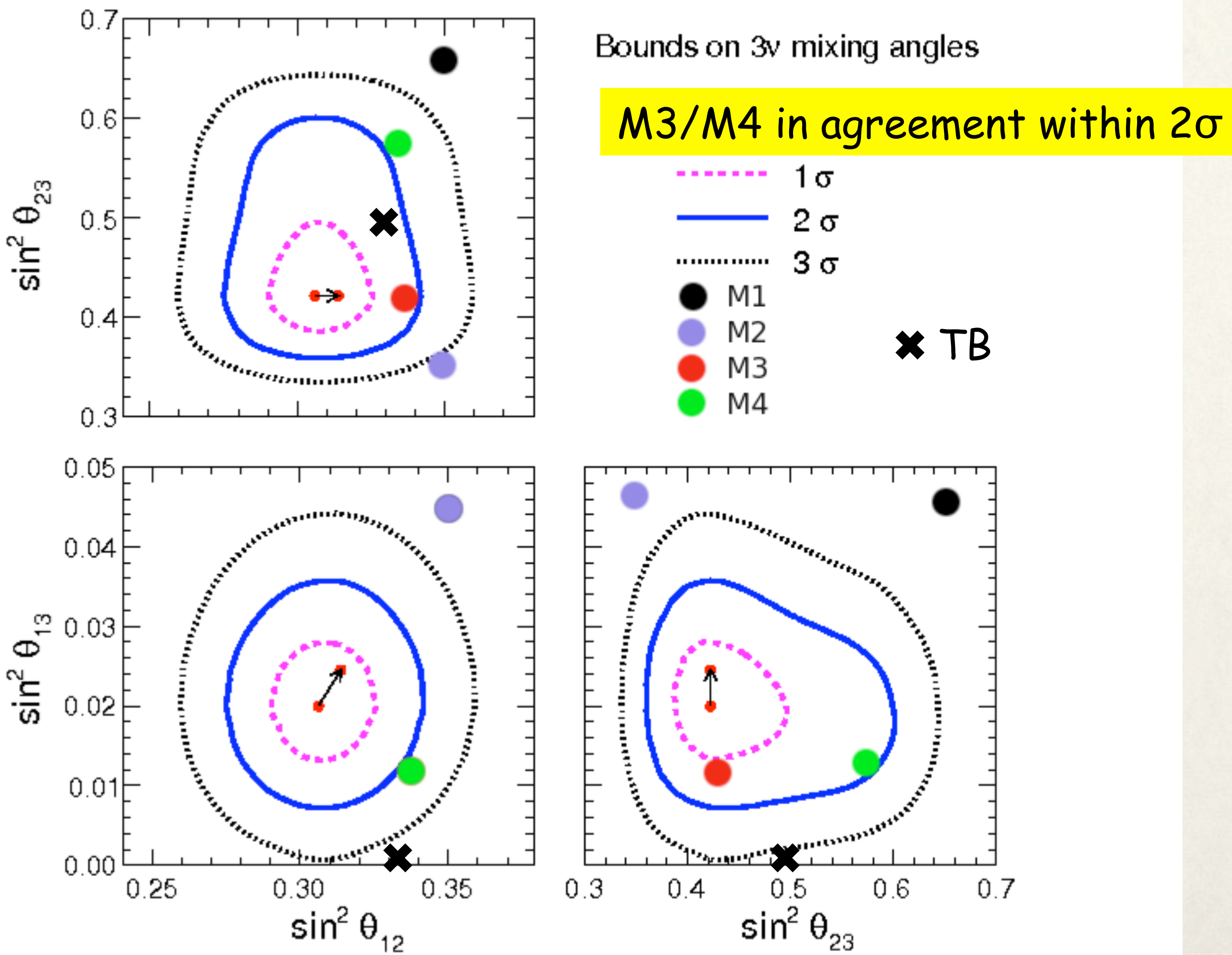
$$\sin^2\vartheta_{13} = (2-\sqrt{3})/6 \approx 0.045$$

$$\sin^2\vartheta_{23} = (8-2\sqrt{3})/13 \approx 0.349$$

$$\sin^2\vartheta_{12} = (8-2\sqrt{3})/13 \approx 0.349$$

$$\delta_{CP} = 0$$

[de Adelhart Toorop, F, Hagedorn 1107.3486]



[contours from Fogli, Lisi, Marrone, Palazzo, Rotunno 1106.6028]

relation to TB mixing

$$U_{PMNS} = U_{TB} U_{13}(\alpha)$$

$$U_{13}(\alpha) = \begin{pmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{pmatrix}$$

choosing $|\alpha|=\pi/24, \pi/12$ we reproduce the mixing pattern M3, M4, M1, M2
the angle α is not a free parameter: it is fixed by group theory

for α generic [Trimaximal mixing]
[He, Zee 2007 and 2011, Grimus, Lavoura 2008, Grimus, Lavoura, Singraber 2009, Albright, Rodejohann 2009, Antusch, King, Luhn, Spinrath 2011, King, Luhn 2011]

$$\sin^2\vartheta_{12} = \frac{1}{2+\cos 2\alpha} \approx \frac{1}{3} + \frac{2\alpha^2}{9} + \ldots$$

$$\sin^2\vartheta_{23} = \frac{1}{2} - \frac{\sqrt{3}\sin 2\alpha}{4+2\cos 2\alpha} \approx \frac{1}{2} - \frac{\alpha}{\sqrt{3}} + \ldots$$

$$\sin^2\vartheta_{13} = \frac{2}{3}\sin^2\alpha \approx \frac{2}{3}\alpha^2 + \ldots$$

deviation from TB is linear in α for $\sin^2\theta_{23}$, whereas is quadratic for $\sin^2\theta_{12}$, the best measured angle

LFV - signatures of discrete symmetries

discrete symmetries are weaker than continuous ones such as MFV, SO(3)...
and allow for G_f-invariant and LFV operators

in all models: l~3 of G_f

	A_4	S_4	A_5
$\frac{1}{\Lambda_{NP}^2}(\overline{\tau}\overline{\mu}ee+...)$	Yes	Yes	Yes
$\frac{1}{\Lambda_{NP}^2}(\overline{\tau}\overline{e}\mu\mu+...)$	Yes	No	No
$\frac{1}{\Lambda_{NP}^2}(\overline{\mu}\overline{e}\tau\tau+...)$	Yes	No	No

selection rule $\Delta L_e \Delta L_\mu \Delta L_\tau = 0, \pm 2$

$\tau^- \to \mu^+ e^- e^-$ in A_4, S_4, A_5

$\tau^- \to e^+ \mu^- \mu^-$ in A_4

$BR(\tau^- \to \mu^+ e^- e^-) < 2.0 \times 10^{-8}$

$BR(\tau^- \to e^+ \mu^- \mu^-) < 2.3 \times 10^{-8}$

$\Lambda_{NP} > 10$ TeV

$m_{NP} > 500$ GeV $\quad (m_{NP} = g\Lambda_{NP}/4\pi)$

- in simplest realizations of the above groups these operators are not generated at the LO

$$\frac{BR(\tau^- \to \mu^+ e^- e^-)}{BR(\tau^- \to \mu^+ \mu^- \mu^-)} = O(u^4) \qquad \frac{BR(\tau^- \to e^+ \mu^- \mu^-)}{BR(\tau^- \to \mu^+ \mu^- \mu^-)} = O(u^2 \frac{m_\mu}{m_\tau})$$

LFV - radiative decays $l_i \to l_j\gamma$

$G_f = A_4 \times SUSY\ldots$

from loops of SUSY particles

1. allowing for the most general slepton mass matrix compatible with pattern of flavour symmetry breaking. For instance [in super-"CKM" basis]

$$\hat{m}^2_{LL} = \begin{pmatrix} n & n_{12}u^2 & n_{13}u^2 \\ n_{12}u^2 & n & n_{23}u^2 \\ n_{13}u^2 & n_{23}u^2 & n \end{pmatrix} m^2_{SUSY} + \ldots$$

m^2_{XY} (X,Y=L,R) are almost diagonal

off-diagonal terms $(\delta_{ij})_{XY}$ local in $u=\langle\varphi\rangle/\Lambda$

$$R_{ij} = \frac{BR(l_i \to l_j\gamma)}{BR(l_i \to l_j\nu_i\bar{\nu}_j)} = \begin{cases} \dfrac{6m_W^4\alpha_{em}}{\pi m^4_{SUSY}}\left|w_{ij}u\right|^2 & \text{[generic]} \\ \dfrac{6m_W^4\alpha_{em}}{\pi m^4_{SUSY}}\left[\left|w^{(1)}_{ij}u^2\right|^2 + \dfrac{m_j^2}{m_i^2}\left|w^{(2)}_{ij}u\right|^2\right] & \text{[restricted class of models]} \end{cases}$$

[$w^{(1,2)}_{ij}$ are known O(1) functions of SUSY parameters]

$$BR(\mu \to e\gamma) \approx BR(\tau \to \mu\gamma) \approx BR(\tau \to e\gamma)$$

independently from $u \approx \vartheta_{13}$

$\tau \to \mu\gamma$ and $\tau \to e\gamma$ below future experimental sensitivity

relatively light sparticle spectrum still allowed

2. further contributions to slepton mass matrices if ν masses come from type I see-saw [ss], through RGE running

if $G_f = A_4, S_4, A_5$

$$\left(\delta^{ss}_{\mu e}\right)_{LL} = -\frac{\left(3+a_0^2\right)y^2}{8\pi^2} U_{\mu 2} U^*_{e2} \log\frac{m_2}{m_1} + O(u)$$

$$\left(\delta^{ss}_{\tau e}\right)_{LL} = -\frac{\left(3+a_0^2\right)y^2}{8\pi^2} U_{\tau 2} U^*_{e2} \log\frac{m_2}{m_1} + O(u)$$

$$\left(\delta^{ss}_{\tau\mu}\right)_{LL} = -\frac{\left(3+a_0^2\right)y^2}{8\pi^2} \left[U_{\tau 2} U^*_{\mu 2} \log\frac{m_2}{m_1} + U_{\tau 3} U^*_{\mu 3} \log\frac{m_3}{m_1}\right] + O(u)$$

at LO they only depend on the smallest m_i at variance with the general case [18 ss parameters]

$$\left[Y_\nu^+ \log\left(\frac{M_X^2}{MM^+}\right) Y_\nu\right]_{ij}$$

Example: A$_4$ x SUSY+ see-saw [Hagedorn, Molinaro, Petcov 0911.3605]

Normal Ordering

$$BR(\mu \to e\gamma) \approx BR(\tau \to e\gamma) \approx O(10^{-1}) BR(\tau \to \mu\gamma)$$

$$\left(\delta^{ss}_{\mu e}\right)_{LL} \approx 10^{-2}$$

-- tanβ small
-- relatively heavy sparticles
-- μ-> eγ close to the present bound

Inverted Ordering

$$BR(\mu \to e\gamma) \approx BR(\tau \to e\gamma) << BR(\tau \to \mu\gamma)$$

yet $R_{\tau\mu}$ above 10^{-9} practically excluded

observation of τ->μγ [$R_{\tau\mu}>10^{-9}$] rules out the A$_4$ x SUSY model

A_4 models with special corrections

group theoretical origin of TB mixing suggests how to modify $\vartheta_{13} \approx 0.1$ while keeping ϑ_{12} almost unchanged

assume $G_e = Z_3$ (generated by T) and $G_\nu = Z_2$ (generated by S) i.e. remove S' generator

-- natural in the context of A_4 that contains S and T, but not S'
-- explicit constructions proposed before T2K,… [Lin 2009]
-- starting from the full $G_\nu = Z_2 \times Z_2$, the parity S' can be broken at a high scale

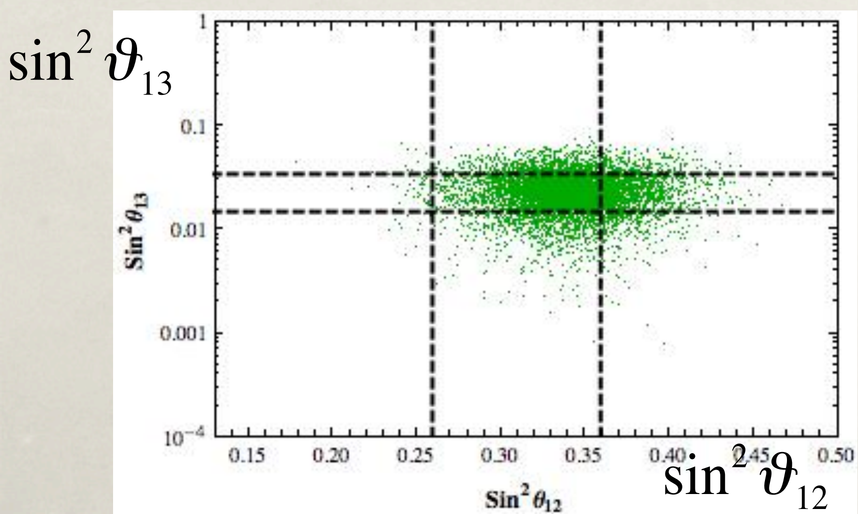
$$U^0 = U_{TB} \times \begin{pmatrix} \cos\alpha & 0 & e^{i\delta}\sin\alpha \\ 0 & 1 & 0 \\ -e^{-i\delta}\sin\alpha & 0 & \cos\alpha \end{pmatrix}$$

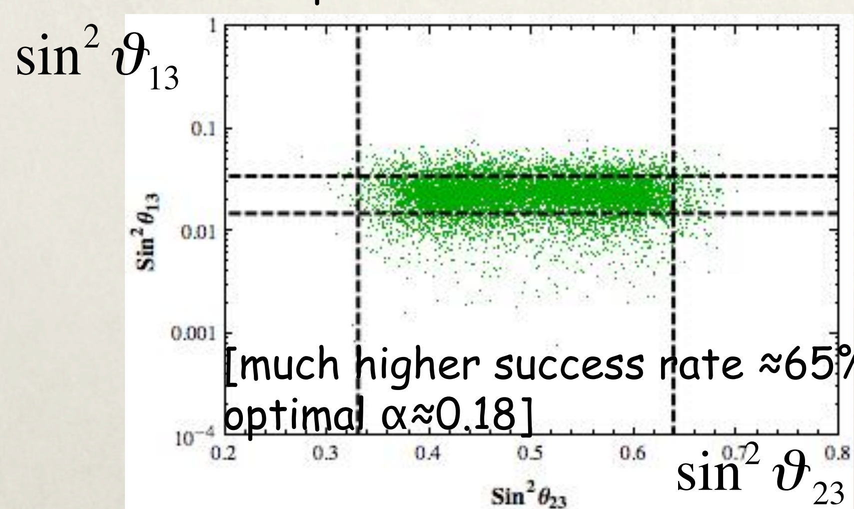
$$0 \le \alpha \le \pi/2$$
$$0 < \delta \le 2\pi$$

$$\sin\vartheta_{13} = \sqrt{2/3}\,\alpha + \ldots$$
$$\sin^2\vartheta_{12} = 1/3 + 2/9\,\alpha^2 + \ldots$$
$$\sin^2\vartheta_{23} = 1/2 + \alpha/\sqrt{3}\,\cos\delta + \ldots$$
$$\delta_{CP} = \delta$$

Trimaximal mixing
gives back TB when α=0

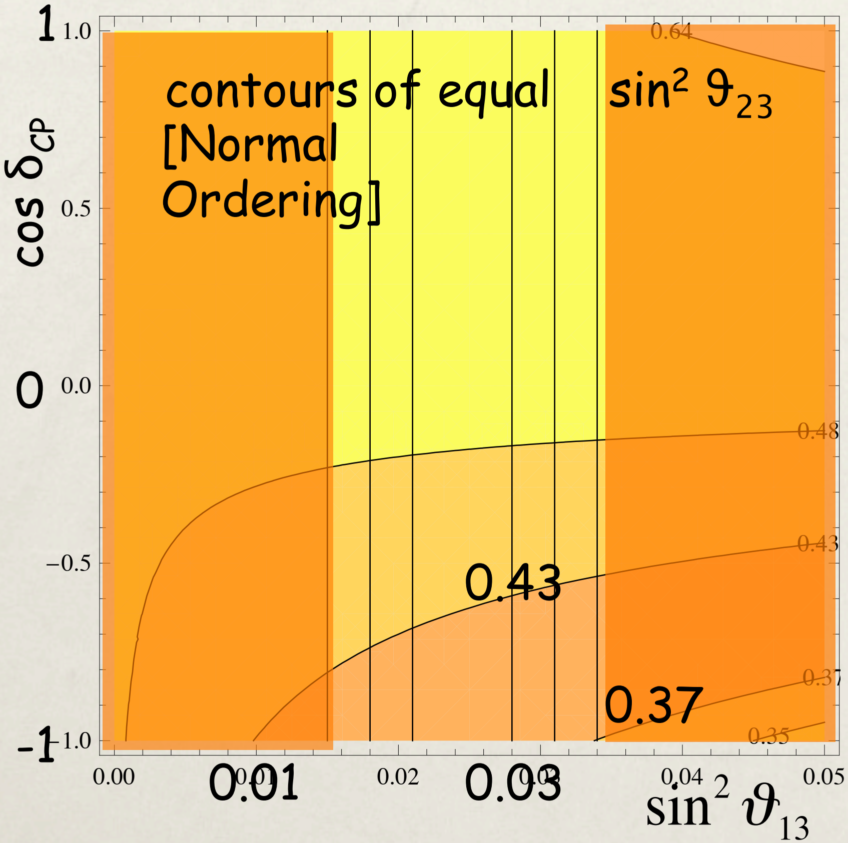
[assuming α=0.1 and expanding in powers of α]





from the previous relations

$$\sin^2\vartheta_{23} = \frac{1}{2} + \frac{1}{\sqrt{2}}\sin\vartheta_{13}\cos\delta_{CP} + O(\sin^2\vartheta_{13})$$



indication for $\sin^2\vartheta_{23} \approx 0.4$ would favor $-1 < \cos\delta_{CP} < -0.5$

can be tested by measuring δ_{CP} and improving on $\sin^2\vartheta_{23}$

Trimaximal ansatz proposed with different motivations by many authors

[He, Zee 2007 and 2011, Grimus, Lavoura 2008, Grimus, Lavoura, Singraber 2009, Albright, Rodejohann 2009, Antusch, King, Luhn, Spinrath 2011, King, Luhn 2011]

[similar tests can be realized in S_4 (TM) and A_5 (GR)
more possibilities by enforcing $G_\nu = Z_2$ generated by SxS']

corrections to $U^0_{PMNS}=U_{BM}$ realized in S_4

in this case removing S' would not help since it would maintain ϑ_{12} very close to π/4, i.e. the LO BM prediction

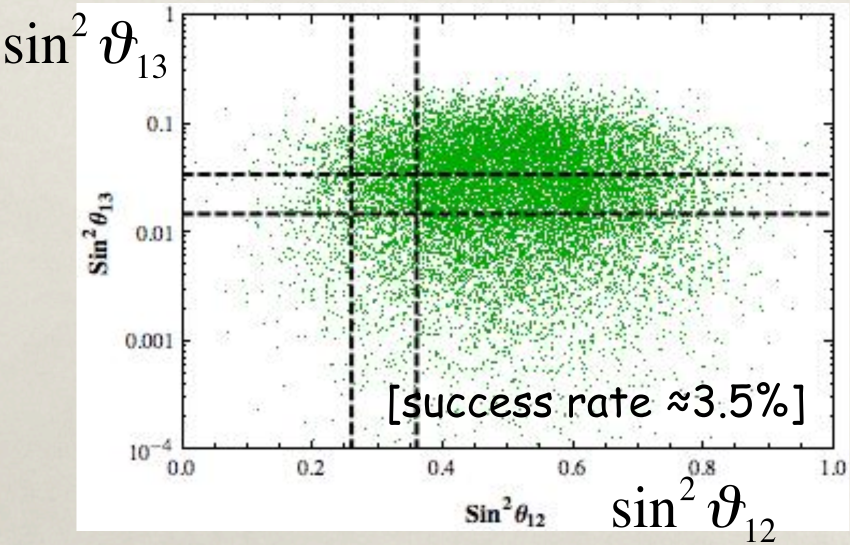
as observed long ago, the most efficient correction is of the following type

$$U_{BM} \rightarrow U_e^+ U_{BM}$$

a correction from the charged lepton sector, mainly through rotations in the 12 and 13 sectors, to preserve $\vartheta_{23}=\pi/4$

several existing models incorporate this idea, in particular in the context of $G_f=S_4$

S_4 model with $U^0_{PMNS}=U_{BM}$ and typical O(0.1) corrections from U_e
[size of the corrections – 0.17 - optimized to maximize the success rate]



-- a tuning of the parameters in U_e is needed to reproduce both ϑ_{13} and ϑ_{12} otherwise $\sin^2\vartheta_{12}$ ranges from 0.2 up to 0.8

-- required tuning is worse than in A_4 model with typical O(0.1) corrections

S_4 models with special corrections

BM mixing can also arise from S_4 when $G_e=Z_2\times Z_2$ (generated by T,T') and $G_\nu=Z_2\times Z_2$ (generated by S,S') [FHT2]

$$T=\begin{pmatrix}+1 & 0 & 0\\ 0 & +1 & 0\\ 0 & 0 & -1\end{pmatrix} \qquad T'=\begin{pmatrix}+1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & +1\end{pmatrix}$$

assume $G_e=Z_2$ (generated by T) and $G_\nu=Z_2\times Z_2$ (generated by S,S')
i.e. remove T' generator

-- starting from the full $G_e=Z_2\times Z_2$, the parity T' can be broken at a high scale

$$U^0=\begin{pmatrix}\cos\alpha & -e^{i\delta}\sin\alpha & 0\\ e^{-i\delta}\sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{pmatrix}\times U_{BM}$$

$$0\le\alpha\le\pi/2$$
$$0<\delta\le 2\pi$$

$$\sin\vartheta_{13}=\alpha/\sqrt{2}+\ldots$$
$$\sin^2\vartheta_{12}=1/2+\alpha\cos\delta/\sqrt{2}+\ldots$$
$$\sin^2\vartheta_{23}=1/2-\alpha^2/4\ldots$$
$$\delta_{CP}=-\delta$$

reasonable correction if charged leptons are similar to quarks, i.e. dominant mixing is in 12 sector

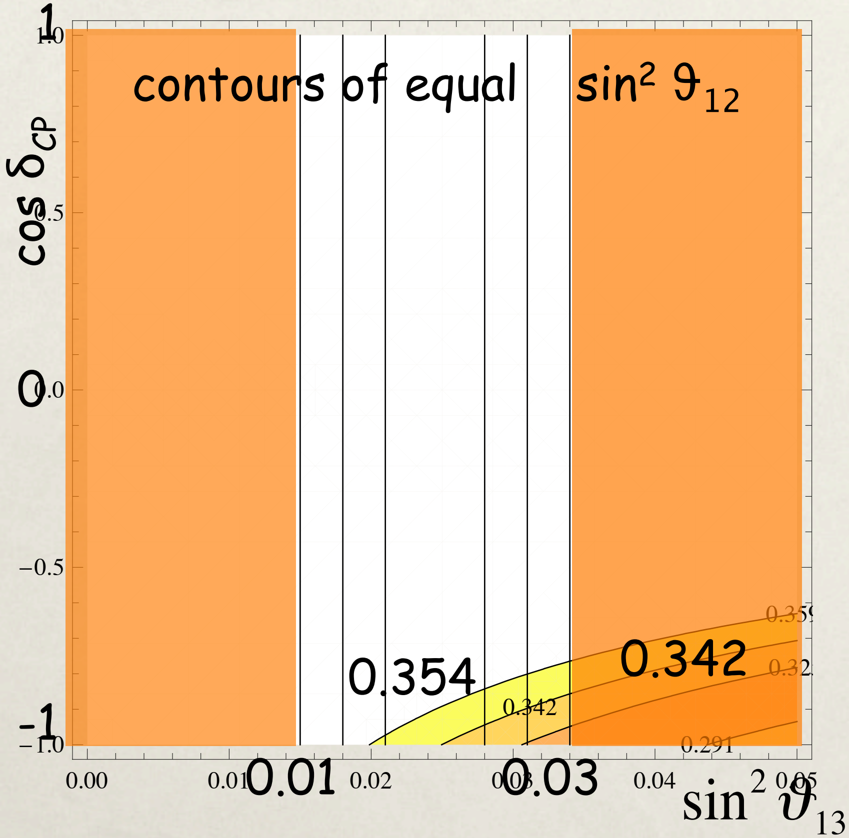
[assuming α=0.1 and expanding in powers of α]

from the previous relations

$$\sin^2\vartheta_{12} = \frac{1}{2} + \sin\vartheta_{13}\cos\delta_{CP} + O(\sin^2\vartheta_{13})$$

since $\vartheta_{13}=O(\lambda_C)$ this realizes a form of QLC

[Raidal 0404046
Minakata, Smirnov 0405088]



reduced parameter space still allowed

strong preference for $\delta_{CP}=\pi$
[no CP violation in lepton sector]
and for the higher side of $\sin^2\vartheta_{12}$

testable by measuring δ_{CP}

[Frampton, Petcov, Rodejohann 0401206
Altarelli, F, Masina 0402155
Romanino 0402508,
Marzocca, Petcov, Romanino, Spinrath 1108.0614]

So far $U_{PMNS} = U^0_{PMNS}$ + corrections